Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 8(10): 24132–24152.
DOI: 10.3934/math.20231230
Received: 17 April 2023
Revised: 19 June 2023
Accepted: 23 June 2023
Published: 09 August 2023

Research article

Some algebraic properties on rough neutrosophic matrix and its application to multi-criteria decision-making

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Abstract: Rough set theory is a method of information processing for database systems. The neutrosophic matrix is a generalization of the fuzzy matrix, especially in handling indeterminacy situations. The concept of matrix theory and its energy in the neutrosophic environment help to determine the value of the uncertain matrix. In this paper, we correlate the rough set theory with the neutrosophic matrix theory to introduce the rough neutrosophic matrix (RNM). In this structure, lower and upper approximation neutrosophic matrices are used to deal with uncertain situations. We demonstrate that the given matrix plays a different role in decision-making situations and defined the proposed matrix's determinant, adjoint, algebraic properties and operations. Finally, derived the ranking function for a rough neutrosophic matrix's energy. The new multi-criteria decision-making (MCDM) approach was presented with the ranking formula, which was utilized to rank the alternatives, and numerical examples were provided to show how the proposed matrix and its energy could be applied to an MCDM problem.

Keywords: rough neutrosophic matrix; algebraic properties; determinant of RNM; matrix energy; multi-criteria decision-making
Mathematics Subject Classification: 60L70, 90B50

1. Introduction

Zadeh [1] initially introduced fuzzy sets, fuzzy membership functions, and fuzzy logic in 1965. Pawlak [2, 3] first proposed the concept of a rough set in 1982. Its idea is the upper and lower approximations of a set, a pair of sets that approximate sets. In this case, those approximations are based on the equivalence relation. He connects the fuzzy set concept with the rough set concept in 1985. The concepts of fuzzy rough sets and rough set approaches for imperfect information systems were introduced in 1990 and 1998, respectively [4, 5]. Smarandache [6, 7] introduced the neutrosophic set in 1998. He developed the ideas of neutrosophic sets and logic to address the problem of indeterminacy specifically. Additionally, he developed single, interval, and multi-valued neutrosophic sets, as well as hesitant and dual hesitant neutrosophic sets.

Neutrosophic set and rough set theory will both be effective strategies for dealing with partial, ambiguous, uncertain, and inaccurate data. In 2014, the rough neutrosophic set was first proposed by Broumi et al. [8]. In this study, they described the basic neutrosophic sets and how they function. The interval valued neutrosophic rough set was subsequently proposed in 2015 [9]. The same year, Mondal and Pramanik [10] demonstrated a crude grey relational analysis-based approach for neutrosophic MADM. This study defines the rough neutrosophic decision matrix and applies it to the resolution of an MCDM problem. A rough neutrosophic field was the subject of several different proposals in 2017 [11, 12]. They investigated MCDM in rough single-valued neutrosophic sets, rough neutrosophic multisets, and rough neutrosophic sets with coefficient correlation. The innovative multi-granulation neutrosophic rough set over single-valued and its applications were discussed by Bo et al. [13] in 2018. The same year, a rough neutrosophic set was employed in medical diagnostics by Samuel and Narmadhagnanam [14] The use of medical diagnostics to determine the patient's care is covered in this paper. Zhang et al. [15] released a paper with work on medical conditions focusing on single-valued neutrosophic indeterminate rough multisets in two universes. Pi-distance of rough neutrosophic sets for medical diagnosis was published in 2019 by Samuel et al. [16]. The goal of the research is to use a rough neutrosophic set to assess the patient's condition and establish a casual link between the sickness and their symptoms. The concepts of neutrosophic single-valued rough sets, neutrosophic soft sets with rough set theory and incorporating topology [17, 18] will be developed further in 2021. As a result, the rough set is significant in each area of the neutrosophic field. In 2022, Subha et al. [19] introduce the idea of rough neutrosophic sets in rings. Additionally, they demonstrate that the upper and lower approximations of the neutrosophic subring and its examples.

Michael G. Thomason introduced the concept of fuzzy matrice theory in 1977. It can be used in a variety of situations. The matrix approach is well known to provide an extra benefit in resolving the issue. In 2002, Pal et al. [20] created specifically intuitionistic fuzzy matrices. Furthermore, it is challenging to assess the worth of membership and non-membership at some stage. Fuzzy and neutrosophic relational maps were introduced in 2004 by Kandasamy and Smarandache [21]. They also incorporated square neutrosophic matrices into this. They developed the neutrosophic matrix and related algebraic operations in 2014 [22]. The neutrosophic square matrix algebraic creativity was published by Abobala et al. [23] in 2021. This paper presents the algebraic functions of neutrosophic matrices, and this work presents the requirements of the inverse of a square neutrosophic matrix using its determinant. To address many uncertainty-related issues, the complex neutrosophic matrix, a novel idea, is introduced by Poonia and Bajaj [24] in 2021. They have offered several algebraic operations based on the suggested matrix, including addition, union, subtraction, and many others that will be useful for developing essential ideas. The multi-valued neutrosophic matrix was presented by Martina and Deepa [25] in 2021. In this, the operations and properties of the proposed matrix were discussed, and they created the linguistic variable for that matrix, which was then applied in the neutrosophic simplified TOPSIS method.

In 2009, Christi DiStefano et al. proposed the concept of matrix energy. They came up with the formula for the energy of the matrix. A generalization of the energy of a graph is the energy of a matrix. Bravoa et al. [26] presented a study titled "Energy of Matrixes" in 2017 and defined several theorems regarding matrix energy in addition to upper and lower bounds. In 2013, Vijayabalaji and

Balaji [27] introduced the concept of rough matrix theory in the decision-making field. Neutrosophic soft metric matrices are introduced, and various operations on such matrices are defined by Khan et al. [28] in 2021. Also, they create an approach using neutrosophic soft metric matrices and use it to solve a problem with decision-making. A novel fuzzy neutrosophic soft complement matrix, the trace, and some essential functions of the fuzzy neutrosophic soft matrix are presented by SheebaMaybell and Shanmugapriya [29] in 2022. They examine a few concepts and attributes of fuzzy neutrosophic soft matrices. The energy concept was then applied to the rough neutrosophic matrix [30] in 2022. This paper defines the energy of a rough neutrosophic matrix and applies it to a multi-criteria decision-making problem.

The rough matrix in a fuzzy environment applies to solving decision-making problems. In this case, the MCDM problem of ranking the alternatives underperformed in indeterminate situations. When we use a neutrosophic matrix in a rough structure, it will handle the uncertain area of ranking the alternatives. As a result, matrix theory is a frequently used notion in the neutrosophic field, and the concept of energy is also widely developed in matrix theory. In the neutrosophic environment, there is no study involving the matrix energy. So, we concentrate on the neutrosophic matrix and its energy, especially in the rough matrix theory where we apply the neutrosophic context. The main contribution of this paper is to create a rough neutrosophic matrix and build a ranking formula for a rough neutrosophic set, which was applied to the new MCDM method to evaluate the alternatives and provide a good result. At this stage, the results will be more valuable than the earlier study.

We provide the rough neutrosophic matrix's determinant, adjoint, operations and algebraic properties in Sections 3 and 4. The ranking functions of the rough neutrosophic set were derived in Section 5. The novel multi-criteria decision-making method for a rough neutrosophic matrix was given in Section 6. It was numerically illustrated in Section 7 by a multi-criteria problem. The comparative results were presented in Section 8. Finally, the results and conclusion were given.

2. Basic definitions

Definition 1. *Rough set [2].*

Let U be the universal set and R be an equivalence relation on U. The collection of all equivalence classes of U under R is called an approximation space and it is defined as A= U/R.

Let X be a subset of U. Let $\underline{A}(X)$ *and* $\overline{A}(X)$ *be lower and upper approximation of X in A, which are denoted as follows:*

$$\underline{A}(X) = \{a \in U : [a]_R \subseteq X\},$$
$$\overline{A}(X) = \{a \in U : [a]_R \cap X \neq \emptyset\},$$

where $[a]_R$ *denotes the equivalence class of R containing an element a.*

The pair $A(X) = (\underline{A}(X), \overline{A}(X))$ *is called the rough set of X in A.*

Definition 2. *Neutrosophic set [21].*

Let U be the universal set and a be an element in U. The degree of truth, indeterminacy and falsity membership functions are denoted by T_S*,* I_S *and* F_S*. Then the neutrosophic set S can be defined as*

$$S = \{\langle a, T_S(a), I_S(a), F_S(a) : a \in U\rangle\},$$

where

$$0 \leq T_S(a) + I_S(a) + F_S(a) \leq 3$$

and T_S is the truth membership function, I_S is the indeterminacy membership function, F_S is the false membership function, every function lies between [0,1] in U.

Definition 3. *Rough neutrosophic set [8].*

Let U be the universal set and a be an element in U. Let R be an equivalence relation on U and S be the neutrosophic set in U with truth membership function T_S, indeterminacy function I_S and false membership function F_S. The lower and upper approximations of S in U/R is denoted by $\underline{N}(S)$ and $\overline{N}(S)$ and they are defined as follows:

$$\underline{N}(S) = \left\{\left\langle a, T_{\underline{N}(S)}(a), I_{\underline{N}(S)}(a), F_{\underline{N}(S)}(a)\right\rangle \ : b \in [a]_R, a \in U\right\},$$
$$\overline{N}(S) = \left\{\left\langle a, T_{\overline{N}(S)}(a), I_{\overline{N}(S)}(a), F_{\overline{N}(S)}(a)\right\rangle \ : b \in [a]_R, a \in U\right\},$$

where

$$\begin{array}{c|c} T_{\underline{N}(S)}(a) = \bigwedge\limits_{b\in[a]_R} T_S(b) & T_{\overline{N}(S)}(a) = \bigvee\limits_{b\in[a]_R} T_S(b) \\ I_{\underline{N}(S)}(a) = \bigvee\limits_{b\in[a]_R} I_S(b) & I_{\overline{N}(S)}(a) = \bigwedge\limits_{b\in[a]_R} I_S(b) \\ F_{\underline{N}(S)}(a) = \bigvee\limits_{b\in[a]_R} F_S(b) & F_{\overline{N}(S)}(a) = \bigwedge\limits_{b\in[a]_R} F_S(b) \end{array},$$

where

$$0 \leq T_{\underline{N}(S)}(a) + I_{\underline{N}(S)}(a) + F_{\underline{N}(S)}(a) \leq 3$$

and

$$0 \leq T_{\overline{N}(S)}(a) + I_{\overline{N}(S)}(a) + F_{\overline{N}(S)}(a) \leq 3,$$

where $\bigwedge$ means "min" and $\bigvee$ means "max" and $T_S(a)$, $I_S(a)$ and $F_S(a)$ are truth, indeterminacy, false membership function of a on neutrosophic set S. Therefore, $\underline{N}(S)$ and $\overline{N}(S)$ are two neutrosophic sets in U. The pair $(\underline{N}(S), \overline{N}(S))$ is called the rough neutrosophic set in U/R.

If $\underline{N}(S) = \overline{N}(S)$ for any $a \in U$, then S is called definable neutrosophic set.

Definition 4. *Rough matrix [27].*

We can define a rough matrix $R_M = [r_{ij}]$ of order $m \times n$ as follows

$$R_M = [r_{ij}] = \begin{pmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \dots & r_{mn} \end{pmatrix},$$

where, each $r_{ij} \in \mu_x^R$. μ_x^R is rough membership function.

Definition 5. *Neutrosophic matrix [22].*

A neutrosophic matrix P of order $m \times n$ is defined as

$$P = [\left\langle T_{ijp}, I_{ijp}, F_{ijp}\right\rangle]_{m\times n},$$

where, T_{ijp}, I_{ijp} and F_{ijp} are called truth, indeterminacy and false membership of ij^{th} in P, which satisfying the condition

$$0 \leq T_{ijp} + I_{ijp} + F_{ijp} \leq 3.$$

For simplicity, we write $[P_{ij}]_{m\times n}$, where,

$$P_{ij} = \left\langle T_{ijp}, I_{ijp}, F_{ijp} \right\rangle.$$

Definition 6. *Energy of neutrosophic matrix [30].*

The neutrosophic matrix can be expressed by three matrices, the first matrix includes the entries a_{ij} as truth membership values, the second matrix includes the entries b_{ij} as indeterminacy membership values, and the third matrix includes the entries c_{ij} as false membership values. It is denoted as

$$P(N) = \left\langle P(T_{ij}), P(I_{ij}), P(F_{ij}) \right\rangle$$

and $a_{ij} \in P(T_{ij})$, $b_{ij} \in P(I_{ij})$ and $c_{ij} \in P(F_{ij})$.

Then the neutrosophic matrix's energy is defined as

$$\begin{aligned} E[P(N)] &= \left\langle E[P(T_{ij})], E[P(I_{ij})], E[P(F_{ij})] \right\rangle \\ &= \left\langle \sum_{i=1}^{n} \left|\lambda_i - \mu_\lambda\right|, \sum_{i=1}^{n} \left|\zeta_i - \mu_\zeta\right|, \sum_{i=1}^{n} \left|\eta_i - \mu_\eta\right| \right\rangle, \end{aligned}$$

where λ_i is the truth matrix eigenvalues, ζ_i is the indeterminacy matrix eigenvalues, and η_i is the false matrix eigenvalues, where $(i = 1, 2, \cdots, n)$, μ_λ, μ_ζ and μ_η are the mean values of λ_i, ζ_i and η_i respectively.

3. Rough neutrosophic matrix and its operations

In this section, we present rough neutrosophic matrix, its determinant, adjoint and its operations.

Definition 7. *Rough neutrosophic matrix.*

A rough neutrosophic matrix is defined as $D = \left\langle \underline{D}_{ij}(S), \overline{D}_{ij}(S) \right\rangle$ with order of $m \times n$. where $\underline{D}_{ij}$ is a lower approximation and $\overline{D}_{ij}$ is a upper approximation of the neutrosophic set S. It can be expressed as

$$D = \left\langle \underline{D}_{ij}(S), \overline{D}_{ij}(S) \right\rangle = \left\langle \left(\underline{T}_{ij}(S), \underline{I}_{ij}(S), \underline{F}_{ij}(S) \right), \left(\overline{T}_{ij}(S), \overline{I}_{ij}(S), \overline{F}_{ij}(S) \right) \right\rangle_{m\times n},$$

where, $\underline{T}_{ij}(S), \underline{I}_{ij}(S)$ and $\underline{F}_{ij}(S)$ are ij^{th} values of truth, indeterminacy and false membership matrices of lower approximation and $\overline{T}_{ij}(S)$, $\overline{I}_{ij}(S)$ and $\overline{F}_{ij}(S)$ are ij^{th} values of truth, indeterminacy and false membership matrices of upper approximation of rough neutrosophic matrix D, which statisfy the conditions

$$\begin{aligned} 0 &\leq \underline{T}_{ij}(S) + \underline{I}_{ij}(S) + \underline{F}_{ij}(S) \leq 3, \\ 0 &\leq \overline{T}_{ij}(S) + \overline{I}_{ij}(S) + \overline{F}_{ij}(S) \leq 3. \end{aligned}$$

Example 1. *Let we define the rough neutrosophic matrix D with order of* 4×2.

$$D = \begin{bmatrix} \langle(0.2, 0.5, 0.2), (0.3, 0.4, 0.8)\rangle & \langle(0.1, 0.3, 0.4), (0.5, 0.6, 0.7)\rangle \\ \langle(0.3, 0.4, 0.7), (0.5, 0.3, 0.7)\rangle & \langle(0.1, 0.2, 0.5), (0.3, 0.2, 0.7)\rangle \\ \langle(0.2, 0.6, 0.9), (0.7, 0.3, 0.1)\rangle & \langle(0.3, 0.5, 0.6), (0.9, 0.3, 0.5)\rangle \\ \langle(0.5, 0.7, 0.9), (0.1, 0.2, 0.4)\rangle & \langle(0.4, 0.5, 0.1), (0.2, 0.3, 0.4)\rangle \end{bmatrix}.$$

Definition 8. *Determinant of rough neutrosophic matrix.*

The determinant of RNM D of order $n \times n$ *denoted by det(D) or* $|D|$ *and it is defined as*

$$\begin{aligned} |D| &= \sum_{\sigma \in S_n} \sigma\{(\underline{\alpha}_{1\sigma(1)}(x), \underline{\beta}_{1\sigma(1)}(x), \underline{\gamma}_{1\sigma(1)}(x)), \cdots, (\underline{\alpha}_{n\sigma(n)}(x), \underline{\beta}_{n\sigma(n)}(x), \underline{\gamma}_{n\sigma(n)}(x))\} \\ &= \sum_{\sigma \in S_n} \sigma\{(\overline{\alpha}_{1\sigma(1)}(x), \overline{\beta}_{1\sigma(1)}(x), \overline{\gamma}_{1\sigma(1)}(x)), \cdots, (\overline{\alpha}_{n\sigma(n)}(x), \overline{\beta}_{n\sigma(n)}(x), \overline{\gamma}_{n\sigma(n)}(x))\} \\ &= \sum_{\sigma \in S_n} \sigma \prod_{i=1}^{n} \left\langle \left(\underline{d}_{i\sigma(i)}, \overline{d}_{i\sigma(i)}\right) \right\rangle, \end{aligned}$$

where, $i = 1, 2, \cdots, n$,

$$\underline{d}_{i\sigma(i)} = \underline{\alpha}_{i\sigma(i)}(x), \underline{\beta}_{i\sigma(i)}(x), \underline{\gamma}_{i\sigma(i)}(x)$$

and

$$\overline{d}_{i\sigma(i)} = \overline{\alpha}_{i\sigma(i)}(x), \overline{\beta}_{i\sigma(i)}(x), \overline{\gamma}_{i\sigma(i)}(x).$$

Where every element of x belongs to D and S_n *denotes the symmetric group of all permutations of* $\{1, 2, \cdots, n\}$.

Definition 9. *Adjoint of RNM.*

The adjoint of RNM D of order $n \times n$ *is denoted by adj D. It is denoted by* $A_{ij} = \left|D_{ji}\right|$, *where* D_{ji} *is transpose of D. It can be written by*

$$A_{ij} = \sum_{\sigma \in S_{n_i n_j}} \prod_{t \in n_j} \left\langle \left(\underline{d}^T_{t\sigma(t)}, \underline{d}^I_{t\sigma(t)}, \underline{d}^F_{t\sigma(t)}\right), \left(\overline{d}^T_{t\sigma(t)}, \overline{d}^I_{t\sigma(t)}, \overline{d}^F_{t\sigma(t)}\right) \right\rangle,$$

where $S_{n_i n_j}$ *is the set of all permutations of* n_j *over* n_i *and* $n_j = \{1, 2, \cdots, n\}$.

Example 2. *Let define the rough neutrosophic matrix D with order of* 2×2.

$$D = \begin{bmatrix} \langle(0.2, 0.5, 0.2), (0.3, 0.4, 0.8)\rangle & \langle(0.1, 0.3, 0.4), (0.5, 0.6, 0.7)\rangle \\ \langle(0.3, 0.4, 0.7), (0.5, 0.3, 0.7)\rangle & \langle(0.1, 0.2, 0.5), (0.3, 0.2, 0.7)\rangle \end{bmatrix},$$

$$\begin{aligned} |D| &= [(0.2, 0.5, 0.2), (0.3, 0.4, 0.8)].[(0.1, 0.2, 0.5), (0.3, 0.2, 0.7)] \\ &+ [(0.3, 0.4, 0.7), (0.5, 0.3, 0.7)].[(0.1, 0.3, 0.4), (0.5, 0.6, 0.7)] \\ &= (min\{0.2, 0.1\}, max\{0.5, 0.2\}, max\{0.2, 0.5\}), (min\{0.3, 0.3\}, max\{0.4, 0.2\}, max\{0.8, 0.7\}) \\ &+ (min\{0.3, 0.1\}, max\{0.4, 0.3\}, max\{0.7, 0.4\}), (min\{0.5, 0.5\}, max\{0.3, 0.6\}, max\{0.7, 0.7\}) \\ &= [(0.1, 0.5, 0.5), (0.3, 0.4, 0.8)] + [(0.1, 0.4, 0.7), (0.5, 0.6, 0.7)] \\ &= (max\{0.1, 0.1\}, min\{0.5, 0.4\}, min\{0.5, 0.7\}), (max\{0.3, 0.5\}, min\{0.4, 0.6\}, min\{0.8, 0.7\}) \\ &= [(0.1, 0.4, 0.5), (0.5, 0.4, 0.7)], \\ adjD &= \left|D_{ji}\right| = [(0.1, 0.4, 0.5), (0.5, 0.4, 0.7)]. \end{aligned}$$

3.1. Algebraic operations of rough neutrosophic matrix

Let D and C be two rough neutrosophic matrices that are denoted by

$$D = [(\underline{d}_{ij}^T, \underline{d}_{ij}^I, \underline{d}_{ij}^F), (\overline{d}_{ij}^T, \overline{d}_{ij}^I, \overline{d}_{ij}^F)]_{m\times n}$$

and

$$C = [(\underline{c}_{ij}^T, \underline{c}_{ij}^I, \underline{c}_{ij}^F), (\overline{c}_{ij}^T, \overline{c}_{ij}^I, \overline{c}_{ij}^F)]_{m\times n},$$

then,

(i) Addition

$$D + C = \left(max\{\underline{d}_{ij}^T, \underline{c}_{ij}^T\}, min\{\underline{d}_{ij}^I, \underline{c}_{ij}^I\}, min\{\underline{d}_{ij}^F, \underline{c}_{ij}^F\}\right), \left(max\{\overline{d}_{ij}^T, \overline{c}_{ij}^T\}, min\{\overline{d}_{ij}^I, \overline{c}_{ij}^I\}, min\{\overline{d}_{ij}^F, \overline{c}_{ij}^F\}\right).$$

(ii) Multiplication

$$D.C = \left(min\{\underline{d}_{ij}^T, \underline{c}_{ij}^T\}, max\{\underline{d}_{ij}^I, \underline{c}_{ij}^I\}, max\{\underline{d}_{ij}^F, \underline{c}_{ij}^F\}\right), \left(min\{\overline{d}_{ij}^T, \overline{c}_{ij}^T\}, max\{\overline{d}_{ij}^I, \overline{c}_{ij}^I\}, max\{\overline{d}_{ij}^F, \overline{c}_{ij}^F\}\right).$$

(iii) Subraction

$$D - C = \left(\{\underline{d}_{ij}^T - \underline{c}_{ij}^T\}, \{\underline{d}_{ij}^I - \underline{c}_{ij}^I\}, \{\underline{d}_{ij}^F - \underline{c}_{ij}^F\}\right), \left(\{\overline{d}_{ij}^T - \overline{c}_{ij}^T\}, \{\overline{d}_{ij}^I - \overline{c}_{ij}^I\}, \{\overline{d}_{ij}^F - \overline{c}_{ij}^F\}\right),$$

$$\underline{d}_{ij}^T - \underline{c}_{ij}^T = \begin{cases} \underline{d}_{ij}^T, & \text{if } \underline{d}_{ij}^T \geq \underline{c}_{ij}^T, \\ 0, & \text{otherwise,} \end{cases} \qquad \overline{d}_{ij}^T - \overline{c}_{ij}^T = \begin{cases} \overline{d}_{ij}^T, & \text{if } \overline{d}_{ij}^T \geq \overline{c}_{ij}^T, \\ 0, & \text{otherwise,} \end{cases}$$

$$\underline{d}_{ij}^I - \underline{c}_{ij}^I = \begin{cases} \underline{d}_{ij}^I, & \text{if } \underline{d}_{ij}^I \geq \underline{c}_{ij}^I, \\ 0, & \text{otherwise,} \end{cases} \qquad \overline{d}_{ij}^I - \overline{c}_{ij}^I = \begin{cases} \overline{d}_{ij}^I, & \text{if } \overline{d}_{ij}^I \geq \overline{c}_{ij}^I, \\ 0, & \text{otherwise,} \end{cases}$$

$$\underline{d}_{ij}^F - \underline{c}_{ij}^F = \begin{cases} \underline{d}_{ij}^F, & \text{if } \underline{d}_{ij}^F < \underline{c}_{ij}^F, \\ 0, & \text{otherwise,} \end{cases} \qquad \overline{d}_{ij}^F - \overline{c}_{ij}^F = \begin{cases} \overline{d}_{ij}^F, & \text{if } \overline{d}_{ij}^F < \overline{c}_{ij}^F, \\ 0, & \text{otherwise.} \end{cases}$$

(iv) Element wise addition

$$D \oplus C = \left(\{\underline{d}_{ij}^T + \underline{c}_{ij}^T - \underline{d}_{ij}^T.\underline{c}_{ij}^T\}, \{\underline{d}_{ij}^I.\underline{c}_{ij}^I\}, \{\underline{d}_{ij}^F.\underline{c}_{ij}^F\}\right), \left(\{\overline{d}_{ij}^T + \overline{c}_{ij}^T - \overline{d}_{ij}^T.\overline{c}_{ij}^T\}, \{\overline{d}_{ij}^I.\overline{c}_{ij}^I\}, \{\overline{d}_{ij}^F.\overline{c}_{ij}^F\}\right).$$

(v) Element wise multiplication

$$D \odot C = \left(\{\underline{d}_{ij}^T.\underline{c}_{ij}^T\}, \{\underline{d}_{ij}^I + \underline{c}_{ij}^I - \underline{d}_{ij}^I.\underline{c}_{ij}^I\}, \{\underline{d}_{ij}^F + \underline{c}_{ij}^F - \underline{d}_{ij}^F.\underline{c}_{ij}^F\}\right), \left(\{\overline{d}_{ij}^T.\overline{c}_{ij}^T\}, \{\overline{d}_{ij}^I + \overline{c}_{ij}^I - \overline{d}_{ij}^I.\overline{c}_{ij}^I\}, \{\overline{d}_{ij}^F + \overline{c}_{ij}^F - \overline{d}_{ij}^F.\overline{c}_{ij}^F\}\right).$$

(vi) Scalar multiplication

$$\lambda D = \left\langle \left(\{1 - (1 - \underline{d}_{ij}^T)\}^\lambda, \{\underline{d}_{ij}^I\}^\lambda, \{\underline{d}_{ij}^F\}^\lambda\right), \left(\{1 - (1 - \overline{d}_{ij}^T)\}^\lambda, \{\overline{d}_{ij}^I\}^\lambda, \{\overline{d}_{ij}^F\}^\lambda\right) \right\rangle.$$

(vii) Power of matrix

$$D^\lambda = \left(\{\underline{d}_{ij}^T\}^\lambda, \{1 - (1 - \underline{d}_{ij}^I)\}^\lambda, \{1 - (1 - \underline{d}_{ij}^F)\}^\lambda\right), \left(\{\overline{d}_{ij}^T\}^\lambda, \{1 - (1 - \overline{d}_{ij}^I)\}^\lambda, \{1 - (1 - \overline{d}_{ij}^F)\}^\lambda\right).$$

(viii) Average

$$D@C = \left\langle \left(\frac{\underline{d}_{ij}^T + \underline{c}_{ij}^T}{2}, \frac{\underline{d}_{ij}^I + \underline{c}_{ij}^I}{2}, \frac{\underline{d}_{ij}^F + \underline{c}_{ij}^F}{2} \right), \left(\frac{\overline{d}_{ij}^T + \overline{c}_{ij}^T}{2}, \frac{\overline{d}_{ij}^I + \overline{c}_{ij}^I}{2}, \frac{\overline{d}_{ij}^F + \overline{c}_{ij}^F}{2} \right) \right\rangle.$$

(ix) Complement of D

$$D^c = \left\langle \left(\underline{d}_{ij}^T, 1 - \underline{d}_{ij}^I, \underline{d}_{ij}^F \right), \left(\overline{d}_{ij}^T, 1 - \overline{d}_{ij}^I, \overline{d}_{ij}^F \right) \right\rangle.$$

(x) Transpose of D

$$tr(D) = \left\langle \left(\underline{d}_{ji}^T, \underline{d}_{ji}^I, \underline{d}_{ji}^F \right), \left(\overline{d}_{ji}^T, \overline{d}_{ji}^I, \overline{d}_{ji}^F \right) \right\rangle.$$

4. Algebaric properties of rough neutrosophic matrix

In this section, we present the algebaric properties of rough neutrosophic matrix, which are commutative and associative property, identity property, distributive property, properties of transpose and determinant of rough neutrosophic matrix.

Proposition 1. *If B, C and D are three rough neutrosophic matrices of same order, then it satisfies the following properties*

Commutativity	*Associativity,*
(i) $D+C = C+D$,	*(i)* $(D+C)+B=D+(C+B)$,
(ii) $D.C=C.D$,	*(ii)* $(D.C).B=D.(C.B)$,
(iii) $D \oplus C = C \oplus D$,	*(iii)* $(D \oplus C) \oplus B = D \oplus (C \oplus B)$,
(iv) $D \odot C = C \odot D$,	*(iv)* $(D \odot C) \odot B = D \odot (C \odot B)$.

Proof. Let

$$D = [(\underline{d}_{ij}^T, \underline{d}_{ij}^I, \underline{d}_{ij}^F), (\overline{d}_{ij}^T, \overline{d}_{ij}^I, \overline{d}_{ij}^F)]_{m\times n}$$

and

$$C = [(\underline{c}_{ij}^T, \underline{c}_{ij}^I, \underline{c}_{ij}^F), (\overline{c}_{ij}^T, \overline{c}_{ij}^I, \overline{c}_{ij}^F)]_{m\times n}.$$

Commutativity:

$$(i)\ D + C = \left(max\{\underline{d}_{ij}^T, \underline{c}_{ij}^T\}, min\{\underline{d}_{ij}^I, \underline{c}_{ij}^I\}, min\{\underline{d}_{ij}^F, \underline{c}_{ij}^F\} \right), \left(max\{\overline{d}_{ij}^T, \overline{c}_{ij}^T\}, min\{\overline{d}_{ij}^I, \overline{c}_{ij}^I\}, min\{\overline{d}_{ij}^F, \overline{c}_{ij}^F\} \right),$$
$$C + D = \left(max\{\underline{c}_{ij}^T, \underline{d}_{ij}^T\}, min\{\underline{c}_{ij}^I, \underline{d}_{ij}^I\}, min\{\underline{c}_{ij}^F, \underline{d}_{ij}^F\} \right), \left(max\{\overline{c}_{ij}^T, \overline{d}_{ij}^T\}, min\{\overline{c}_{ij}^I, \overline{d}_{ij}^I\}, min\{\overline{c}_{ij}^F, \overline{d}_{ij}^F\} \right),$$

so D+C = C+D. Similarly we can prove the other commutative properties (ii)–(iv).
Associativity:

$$\begin{aligned}(i)\ (D + C) + B &= \left[\left(max\{\underline{d}_{ij}^T, \underline{c}_{ij}^T\}, min\{\underline{d}_{ij}^I, \underline{c}_{ij}^I\}, min\{\underline{d}_{ij}^F, \underline{c}_{ij}^F\} \right), \left(max\{\overline{d}_{ij}^T, \overline{c}_{ij}^T\}, min\{\overline{d}_{ij}^I, \overline{c}_{ij}^I\}, min\{\overline{d}_{ij}^F, \overline{c}_{ij}^F\} \right) \right] \\ &+ (\underline{b}_{ij}^T, \underline{b}_{ij}^I, \underline{b}_{ij}^F), (\overline{b}_{ij}^T, \overline{b}_{ij}^I, \overline{b}_{ij}^F) \\ &= \left(max\{\underline{d}_{ij}^T, \underline{c}_{ij}^T, \underline{b}_{ij}^T\}, min\{\underline{d}_{ij}^I, \underline{c}_{ij}^I, \underline{b}_{ij}^I\}, min\{\underline{d}_{ij}^F, \underline{c}_{ij}^F, \underline{b}_{ij}^F\} \right), \left(max\{\overline{d}_{ij}^T, \overline{c}_{ij}^T, \overline{b}_{ij}^T\}, min\{\overline{d}_{ij}^I, \overline{c}_{ij}^I, \overline{b}_{ij}^I\}, \overline{c}_{ij}^F, \overline{b}_{ij}^F\} \right),\end{aligned}$$

$$D+(C+B)=(\underline{d}^T_{ij},\underline{d}^I_{ij},\underline{d}^F_{ij}),(\overline{d}^T_{ij},\overline{d}^I_{ij},\overline{d}^F_{ij})+\Big[\Big(max\{\underline{c}^T_{ij},\underline{b}^T_{ij}\},min\{\underline{c}^I_{ij},\underline{b}^I_{ij}\},min\{\underline{c}^F_{ij},\underline{b}^F_{ij}\}\Big),$$
$$\Big(max\{\overline{c}^T_{ij},\overline{b}^T_{ij}\},min\{\overline{c}^I_{ij},\overline{b}^I_{ij}\},min\{\overline{c}^F_{ij},\overline{b}^F_{ij}\}\Big)\Big]$$
$$=\Big(max\{\underline{d}^T_{ij},\underline{c}^T_{ij},\underline{b}^T_{ij}\},min\{\underline{d}^I_{ij},\underline{c}^I_{ij},\underline{b}^I_{ij}\},min\{\underline{d}^F_{ij},\underline{c}^F_{ij},\underline{b}^F_{ij}\}\Big),\Big(max\{\overline{d}^T_{ij},\overline{c}^T_{ij},\overline{b}^T_{ij}\},min\{\overline{d}^I_{ij},\overline{c}^I_{ij},\overline{b}^I_{ij}\},\overline{c}^F_{ij},\overline{b}^F_{ij}\}\Big),$$

so (D+C)+B=D+(C+B). Similarly we can prove the other associative properties (ii)–(iv). □

Proposition 2. *Let D be a rough neutrosophic matrix. Then it satisfies the following identity property.*

$$(i)\ D+I_a=I_a+D=D,$$
$$(ii)\ D.I_m=I_m.D=D.$$

Proof. (i) Let I_a be the additive identity which is the zero rough neutrosophic matrix. It is denoted by $I_a=\langle(0,1,1),(0,1,1)\rangle$,

$$D+I_a=\Big(max\{\underline{d}^T_{ij},0\},min\{\underline{d}^I_{ij},1\},min\{\underline{d}^F_{ij},1\}\Big),\Big(max\{\overline{d}^T_{ij},0\},min\{\overline{d}^I_{ij},1\},min\{\overline{d}^F_{ij},1\}\Big),$$
$$I_a+D=\Big(max\{0,\underline{d}^T_{ij}\},min\{1,\underline{d}^I_{ij}\},min\{1,\underline{d}^F_{ij}\}\Big),\Big(max\{0,\overline{d}^T_{ij}\},min\{1,\overline{d}^I_{ij}\},min\{1,\overline{d}^F_{ij}\}\Big),$$

so $D+I_a=I_a+D=D$.

(ii) Let I_m be the multiplicative identity which is the unit rough neutrosophic matrix. It is denoted by $I_m=\langle(1,0,0),(1,0,0)\rangle$,

$$D.I_m=\Big(min\{\underline{d}^T_{ij},0\},max\{\underline{d}^I_{ij},1\},max\{\underline{d}^F_{ij},1\}\Big),\Big(min\{\overline{d}^T_{ij},0\},max\{\overline{d}^I_{ij},1\},max\{\overline{d}^F_{ij},1\}\Big),$$
$$I_m.D=\Big(min\{0,\underline{d}^T_{ij}\},max\{1,\underline{d}^I_{ij}\},max\{1,\underline{d}^F_{ij}\}\Big),\Big(min\{0,\overline{d}^T_{ij}\},max\{1,\overline{d}^I_{ij}\},max\{1,\overline{d}^F_{ij}\}\Big),$$

so $D.I_m=I_m.D=D$. Hence it satisfies the identity property. □

Proposition 3. *Let B, C and D be three rough neutrosophic matrices, then it satisfies the following distributive properties:*

$$(i)\ D.(C+B)=D.C+D.B,$$
$$(D+C).B=D.B+C.B.$$
$$(ii)\ D\odot(C\oplus B)=D\odot C\oplus D\odot B,$$
$$(D\oplus C)\odot B=D\odot B\oplus C\odot B.$$

Proof. Let us assume the elements of D, C and B matrices

$$\underline{d}^T_{ij}>\underline{c}^T_{ij}>\underline{b}^T_{ij},\ \ \underline{d}^I_{ij}>\underline{c}^I_{ij}>\underline{b}^I_{ij},\ \ \underline{d}^F_{ij}>\underline{c}^I_{ij}>\underline{b}^F_{ij},$$
$$\overline{d}^T_{ij}>\overline{c}^T_{ij}>\overline{b}^T_{ij},\ \ \overline{d}^I_{ij}>\overline{c}^I_{ij}>\overline{b}^I_{ij},\ \ \overline{d}^F_{ij}>\overline{c}^I_{ij}>\overline{b}^F_{ij},$$
$$C+B=[(\underline{c}^T_{ij},\underline{c}^I_{ij},\underline{c}^F_{ij}),(\overline{c}^T_{ij},\overline{c}^I_{ij},\overline{c}^F_{ij})]+[(\underline{b}^T_{ij},\underline{b}^I_{ij},\underline{b}^F_{ij}),(\overline{b}^T_{ij},\overline{b}^I_{ij},\overline{b}^F_{ij})],$$
$$C+B=\Big[\Big(max\{\underline{c}^T_{ij},\underline{b}^T_{ij}\},min\{\underline{c}^I_{ij},\underline{b}^I_{ij}\},min\{\underline{c}^F_{ij},\underline{b}^F_{ij}\}\Big),\Big(max\{\overline{c}^T_{ij},\overline{b}^T_{ij}\},min\{\overline{c}^I_{ij},\overline{b}^I_{ij}\},min\{\overline{c}^F_{ij},\overline{b}^F_{ij}\}\Big)\Big]$$
$$=\Big[(\underline{c}^T_{ij},\underline{b}^I_{ij},\underline{b}^F_{ij}),(\overline{c}^T_{ij},\overline{b}^I_{ij},\overline{b}^F_{ij})\Big],$$

$$
\begin{aligned}
D.(C+B) &= \left[(\underline{d}^T_{ij}, \underline{d}^I_{ij}, \underline{d}^F_{ij}), (\overline{d}^T_{ij}, \overline{d}^I_{ij}, \overline{d}^F_{ij})\right].\left[(\underline{c}^T_{ij}, \underline{b}^I_{ij}, \underline{b}^F_{ij}), (\overline{c}^T_{ij}, \overline{b}^I_{ij}, \overline{b}^F_{ij})\right],\\
D.(C+B) &= \left[\left(min\{\underline{d}^T_{ij}, \underline{c}^T_{ij}\}, max\{\underline{d}^I_{ij}, \underline{b}^I_{ij}\}, max\{\underline{d}^F_{ij}, \underline{b}^F_{ij}\}\right), \left(min\{\overline{d}^T_{ij}, \overline{c}^T_{ij}\}, max\{\overline{d}^I_{ij}, \overline{b}^I_{ij}\}, max\{\overline{d}^F_{ij}, \overline{b}^F_{ij}\}\right)\right]\\
&= \left[(\underline{c}^T_{ij}, \underline{d}^I_{ij}, \underline{d}^F_{ij}), (\overline{c}^T_{ij}, \overline{d}^I_{ij}, \overline{d}^F_{ij})\right],\\
D.C &= \left[\left(min\{\underline{d}^T_{ij}, \underline{c}^T_{ij}\}, max\{\underline{d}^I_{ij}, \underline{c}^I_{ij}\}, max\{\underline{d}^F_{ij}, \underline{c}^F_{ij}\}\right), \left(min\{\overline{d}^T_{ij}, \overline{c}^T_{ij}\}, max\{\overline{d}^I_{ij}, \overline{c}^I_{ij}\}, max\{\overline{d}^F_{ij}, \overline{c}^F_{ij}\}\right)\right]\\
&= \left[(\underline{c}^T_{ij}, \underline{d}^I_{ij}, \underline{d}^F_{ij}), (\overline{c}^T_{ij}, \overline{d}^I_{ij}, \overline{d}^F_{ij})\right],\\
D.B &= \left[\left(min\{\underline{d}^T_{ij}, \underline{b}^T_{ij}\}, max\{\underline{d}^I_{ij}, \underline{b}^I_{ij}\}, max\{\underline{d}^F_{ij}, \underline{b}^F_{ij}\}\right), \left(min\{\overline{d}^T_{ij}, \overline{b}^T_{ij}\}, max\{\overline{d}^I_{ij}, \overline{b}^I_{ij}\}, max\{\overline{d}^F_{ij}, \overline{b}^F_{ij}\}\right)\right]\\
&= \left[(\underline{b}^T_{ij}, \underline{d}^I_{ij}, \underline{d}^F_{ij}), (\overline{b}^T_{ij}, \overline{d}^I_{ij}, \overline{d}^F_{ij})\right],\\
D.C + D.B &= \left[\left(max\{\underline{c}^T_{ij}, \underline{b}^T_{ij}\}, min\{\underline{d}^I_{ij}, \underline{d}^I_{ij}\}, min\{\underline{d}^F_{ij}, \underline{d}^F_{ij}\}\right), \left(max\{\overline{c}^T_{ij}, \overline{b}^T_{ij}\}, min\{\overline{d}^I_{ij}, \overline{d}^I_{ij}\}, min\{\overline{d}^F_{ij}, \overline{d}^F_{ij}\}\right)\right]\\
&= \left[(\underline{c}^T_{ij}, \underline{d}^I_{ij}, \underline{d}^F_{ij}), (\overline{c}^T_{ij}, \overline{d}^I_{ij}, \overline{d}^F_{ij})\right],
\end{aligned}
$$

so D.(C+B) = D.C+D.B, and also it satifies (D+C).B = D.B+C.B.

Similarly we can prove the (ii) $D \odot (C \oplus B) = D \odot C \oplus D \odot B$, $(D \oplus C) \odot B = D \odot B \oplus C \odot B$. □

Proposition 4. *Let D and C be two rough neutrosophic matrices, tr(D) and tr(C) be the transpose of RNM D and C respectively and det(D) and det(C) be the determinant of RNM D and C respectively. Then it satisfies the following:*

$$
\begin{aligned}
(i)\ & D + D = D,\\
(ii)\ & tr(D) = tr(tr(D)),\\
(iii)\ & tr(D + C) = tr(D) + tr(C),\\
& tr(D.C) = tr(C).tr(D),\\
(vi)\ & det(D + C) = det(D) + det(C),\\
& det(D.C) = det(D).det(C).
\end{aligned}
$$

Proof.

$$
\begin{aligned}
(i)\ D &= \left[(\underline{d}^T_{ij}, \underline{d}^I_{ij}, \underline{d}^F_{ij}), (\overline{d}^T_{ij}, \overline{d}^I_{ij}, \overline{d}^F_{ij})\right],\\
D + D &= \left(max\{\underline{d}^T_{ij}, \underline{d}^T_{ij}\}, min\{\underline{d}^I_{ij}, \underline{d}^I_{ij}\}, min\{\underline{d}^F_{ij}, \underline{d}^F_{ij}\}\right), \left(max\{\overline{d}^T_{ij}, \overline{d}^T_{ij}\}, min\{\overline{d}^I_{ij}, \overline{d}^I_{ij}\}, min\{\overline{d}^F_{ij}, \overline{d}^F_{ij}\}\right)\\
&= \left[(\underline{d}^T_{ij}, \underline{d}^I_{ij}, \underline{d}^F_{ij}), (\overline{d}^T_{ij}, \overline{d}^I_{ij}, \overline{d}^F_{ij})\right]\\
&= D.\\
(ii)\ D &= \left[\left(\underline{d}^T_{ij}, \underline{d}^I_{ij}, \underline{d}^F_{ij}\right), \left(\overline{d}^T_{ij}, \overline{d}^I_{ij}, \overline{d}^F_{ij}\right)\right],\\
tr(D) &= \left[\left(\underline{d}^T_{ji}, \underline{d}^I_{ji}, \underline{d}^F_{ji}\right), \left(\overline{d}^T_{ji}, \overline{d}^I_{ji}, \overline{d}^F_{ji}\right)\right],\\
tr(tr(D)) &= \left[\left(\underline{d}^T_{ij}, \underline{d}^I_{ij}, \underline{d}^F_{ij}\right), \left(\overline{d}^T_{ij}, \overline{d}^I_{ij}, \overline{d}^F_{ij}\right)\right],\\
tr(tr(D)) &= D.
\end{aligned}
$$

(iii) Let we take the elements of D and C are

$$\underline{d}_{ij}^T > \underline{c}_{ij}^T,\ \underline{d}_{ij}^I > \underline{c}_{ij}^I,\ \underline{d}_{ij}^F > \underline{c}_{ij}^I$$

and

$$\overline{d}_{ij}^T > \overline{c}_{ij}^T,\ \overline{d}_{ij}^I > \overline{c}_{ij}^I,\ \overline{d}_{ij}^F > \overline{c}_{ij}^I.$$

$$\begin{aligned}
D + C &= \left[\left(max\{\underline{d}_{ij}^T, \underline{c}_{ij}^T\}, min\{\underline{d}_{ij}^I, \underline{c}_{ij}^I\}, min\{\underline{d}_{ij}^F, \underline{c}_{ij}^F\} \right), \left(max\{\overline{d}_{ij}^T, \overline{c}_{ij}^T\}, min\{\overline{d}_{ij}^I, \overline{c}_{ij}^I\}, min\{\overline{d}_{ij}^F, \overline{c}_{ij}^F\} \right) \right], \\
D + C &= \left[(\underline{d}_{ij}^T, \underline{c}_{ij}^I, \underline{c}_{ij}^F), (\overline{d}_{ij}^T, \overline{c}_{ij}^I, \overline{c}_{ij}^F) \right], \\
tr(D + C) &= \left[(\underline{d}_{ji}^T, \underline{c}_{ji}^I, \underline{c}_{ji}^F), (\overline{d}_{ji}^T, \overline{c}_{ji}^I, \overline{c}_{ji}^F) \right], \\
tr(D) &= \left[(\underline{d}_{ji}^T, \underline{d}_{ji}^I, \underline{d}_{ji}^F), (\overline{d}_{ji}^T, \overline{d}_{ji}^I, \overline{d}_{ji}^F) \right], \\
tr(C) &= \left[(\underline{c}_{ji}^T, \underline{c}_{ji}^I, \underline{c}_{ji}^F), (\overline{c}_{ji}^T, \overline{c}_{ji}^I, \overline{c}_{ji}^F) \right], \\
tr(D) + tr(C) &= \left[\left(max\{\underline{d}_{ji}^T, \underline{c}_{ji}^T\}, min\{\underline{d}_{ji}^I, \underline{c}_{ji}^I\}, min\{\underline{d}_{ji}^F, \underline{c}_{ji}^F\} \right), \left(max\{\overline{d}_{ji}^T, \overline{c}_{ji}^T\}, min\{\overline{d}_{ji}^I, \overline{c}_{ji}^I\}, min\{\overline{d}_{ji}^F, \overline{c}_{ji}^F\} \right) \right] \\
&= \left[(\underline{d}_{ji}^T, \underline{c}_{ji}^I, \underline{c}_{ji}^F), (\overline{d}_{ji}^T, \overline{c}_{ji}^I, \overline{c}_{ji}^F) \right],
\end{aligned}$$

so tr(D+C) = tr(D)+tr(C) and also tr(D.C) = tr(C).tr(D).

Similarly we can prove the (iv). □

Hence the rough neutrosophic matrices satisfies all the properties.

5. Ranking methods for rough neutrosophic matrix's energy

In this section, we provide some ranking measures for the energy of rough neutrosophic matrix. The presented formula are helps to rank the matrix.

Definition 10. *Energy of rough neutrosophic matrix [30].*

Let

$$D(N) = \left\langle D(\underline{N}_{ij}(S)),\ D(\overline{N}_{ij}(S)) \right\rangle$$

be the square rough neutrosophic matrix with the order $n \times n$, where, $D(\underline{N}_{ij}(S))$ and $D(\overline{N}_{ij}(S))$ are a lower and upper approximation of the neutrosophic set S.

This matrix can be separated by six matrices, the first three matrices belong to a lower approximation that includes the elements $\underline{a}_{ij}$, $\underline{b}_{ij}$, $\underline{c}_{ij}$ another three matrices belong to the upper approximation that includes the elements $\overline{a}_{ij}$, $\overline{b}_{ij}$, $\overline{c}_{ij}$. where $\underline{a}_{ij}$, $\overline{a}_{ij}$ are truth membership values, $\underline{b}_{ij}$, $\overline{b}_{ij}$ are indeterminacy membership values and $\underline{c}_{ij}$, $\overline{c}_{ij}$ are false membership values. The matrix can be written as

$$\begin{aligned}
D(N) &= \left\langle D(\underline{N}_{ij}(S)), D(\overline{N}_{ij}(S)) \right\rangle \\
&= \left\langle (D(\underline{T}_{ij}(S)), D(\underline{I}_{ij}(S)), D(\underline{F}_{ij}(S))), (D(\overline{T}_{ij}(S)), D(\overline{I}_{ij}(S)), D(\overline{F}_{ij}(S))) \right\rangle,
\end{aligned}$$

where the elements, $\underline{a}_{ij} \in D(\underline{T}_{ij}(S))$, $\underline{b}_{ij} \in D(\underline{I}_{ij}(S))$, $\underline{c}_{ij} \in D(\underline{F}_{ij}(S))$, $\overline{a}_{ij} \in D(\overline{T}_{ij}(S))$, $\overline{b}_{ij} \in D(\overline{I}_{ij}(S))$, and $\overline{c}_{ij} \in D(\overline{F}_{ij}(S))$.

Then the rough neutrosophic matrix's energy defined as

$$E[D(N)] = \left(E[D(\underline{T}_{ij}(S))], E[D(\underline{I}_{ij}(S))], E[D(\underline{F}_{ij}(S))]\right), \left(E[D(\overline{T}_{ij}(S))], E[D(\overline{I}_{ij}(S))], E[D(\overline{F}_{ij}(S))]\right),$$

$$E[D(N)] = \left(\sum_{i=1}^{n}\left|\underline{\lambda}_i - \mu_{\underline{\lambda}}\right|, \sum_{i=1}^{n}\left|\underline{\zeta}_i - \mu_{\underline{\zeta}}\right|, \sum_{i=1}^{n}\left|\underline{\eta}_i - \mu_{\underline{\eta}}\right|\right), \left(\sum_{i=1}^{n}\left|\overline{\lambda}_i - \mu_{\overline{\lambda}}\right|, \sum_{i=1}^{n}\left|\overline{\zeta}_i - \mu_{\overline{\zeta}}\right|, \sum_{i=1}^{n}\left|\overline{\eta}_i - \mu_{\overline{\eta}}\right|\right),$$

where, $\underline{\lambda}_i$, $\underline{\zeta}_i$ *and* $\underline{\eta}_i$ *are the truth, indeterminacy, and false eigenvalues of lower approximation matrices and* $\overline{\lambda}_i$, $\overline{\zeta}_i$, $\overline{\eta}_i$ *are the truth, indeterminacy, and false eigenvalues of upper approximation matrices.* $\mu_{\underline{\lambda}}, \mu_{\underline{\zeta}}, \mu_{\underline{\eta}}, \mu_{\overline{\lambda}}, \mu_{\overline{\zeta}}$ *and* $\mu_{\overline{\eta}}$ *are mean values of the eigen values* $\underline{\lambda}_i$, $\underline{\zeta}_i$, $\underline{\eta}_i$, $\overline{\lambda}_i$, $\overline{\zeta}_i$ *and* $\overline{\eta}_i$ *respectively.*

Example 3. *Let D be a square rough neutrosophic matrix with order* 2×2.

$$D = \begin{bmatrix} \langle(0.2, 0.5, 0.2), (0.3, 0.4, 0.8)\rangle & \langle(0.1, 0.3, 0.4), (0.5, 0.6, 0.7)\rangle \\ \langle(0.3, 0.4, 0.7), (0.5, 0.3, 0.7)\rangle & \langle(0.1, 0.2, 0.5), (0.3, 0.2, 0.7)\rangle \end{bmatrix},$$

$$D(\underline{T}_{ij}) = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.1 \end{bmatrix}, \quad D(\underline{I}_{ij}) = \begin{bmatrix} 0.5 & 0.3 \\ 0.4 & 0.2 \end{bmatrix}, \quad D(\underline{F}_{ij}) = \begin{bmatrix} 0.2 & 0.4 \\ 0.7 & 0.5 \end{bmatrix},$$

$$D(\overline{T}_{ij}) = \begin{bmatrix} 0.3 & 0.5 \\ 0.5 & 0.3 \end{bmatrix}, \quad D(\overline{I}_{ij}) = \begin{bmatrix} 0.4 & 0.6 \\ 0.3 & 0.2 \end{bmatrix}, \quad D(\overline{F}_{ij}) = \begin{bmatrix} 0.8 & 0.7 \\ 0.7 & 0.7 \end{bmatrix}.$$

The eigen values of truth lower matrix $\underline{\lambda_i} = 0.3303, -0.0303$ *and mean* $\mu_{\underline{\lambda_i}} = 0.15$. *The energy of* $D(\underline{T}_{ij})$ *= 0.3606. Similarly find other matrices.*

So E[D] = [(0.3606,0.7550,1.1),(1,0.8718,1.4036)].

Definition 11. *Score function and accuracy function.*

Let $S = \langle T_S(a), I_S(a), F_S(a)\rangle$ *be a neutrosophic set in universal set U and every element* $a \in U$. *Then the score function [31] of neutrosophic set is defined by*

$$\sigma_S(a) = \frac{1 + T_S(a) - 2I_S(a) - F_S(a)}{2}.$$

The accuracy function [32] of neutrosophic set is defined by

$$\tau_S(a) = T_S(a) - 2I_S(a) - F_S(a).$$

From this existing formula we defined the score function and accuracy function for rough neutrosophic set.

Let

$$N(S) = \left\langle \left(\underline{T}_{N(S)}(a), \underline{I}_{N(S)}(a), \underline{F}_{N(S)}(a)\right), \left(\overline{T}_{N(S)}(a), \overline{I}_{N(S)}(a), \overline{F}_{N(S)}(a)\right)\right\rangle.$$

Then the score function of rough neutrosophic set is defined by

$$\sigma_{N(S)}(a) = \left(\frac{1}{2}\right) \frac{1 + \underline{T}_{N(S)}(a) - 2\underline{I}_{N(S)}(a) - \underline{F}_{N(S)}(a) + 1 + \overline{T}_{N(S)}(a) - 2\overline{I}_{N(S)}(a) - \overline{F}_{N(S)}(a)}{2}$$

$$= \frac{2 + \underline{T}_{N(S)}(a) + \overline{T}_{N(S)}(a) - 2[\underline{I}_{N(S)}(a) + \overline{I}_{N(S)}(a)] - [\underline{F}_{N(S)}(a) + \overline{F}_{N(S)}(a)]}{4}. \quad (5.1)$$

The accuracy function of rough neutrosophic set is defined by

$$\tau_{N(S)}(a) = \left(\frac{1}{2}\right)\left(\underline{T}_{N(S)}(a) - 2\underline{I}_{N(S)}(a) - \underline{F}_{N(S)}(a) + \overline{T}_{N(S)}(a) - 2\overline{I}_{N(S)}(a) - \overline{F}_{N(S)}(a)\right)$$
$$= \frac{\underline{T}_{N(S)}(a) + \overline{T}_{N(S)}(a) - 2[\underline{I}_{N(S)}(a) + \overline{I}_{N(S)}(a)] - [\underline{F}_{N(S)}(a) + \overline{F}_{N(S)}(a)]}{2}. \quad (5.2)$$

6. The multi-criteria decision-making method using the ranking of rough neutrosophic matrix's energy

This section addresses a method for choosing the best option among several alternatives using the energy of rough neutrosophic matrix. Over x criteria, consider the set of r alternatives. There are a group of y experts who evaluated the alternatives. So we set $F = \{F_1, F_2, \cdots F_r\}$, $C = \{C_1, C_2, \cdots C_x\}$ and experts (EX) $= \{EX_1, EX_2, \cdots EX_y\}$.

Step 1: Each expert provided the weighted values of m criteria as well as the rating values for each alternative on each criterion. We use a matrix to represent each alternate rating and weight value.

Assume the expert ratings for x criteria as a matrix with order $x \times y$ for the weight of criteria W.

$$\begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_x \end{array} \begin{array}{c} \begin{array}{cccc} EX_1 & EX_2 & \dots & EX_y \end{array} \\ \begin{pmatrix} \langle \upsilon_{11}, \nu_{11}, \omega_{11} \rangle & \langle \upsilon_{12}, \nu_{12}, \omega_{12} \rangle & \dots & \langle \upsilon_{1y}, \nu_{1y}, \omega_{1y} \rangle \\ \langle \upsilon_{21}, \nu_{21}, \omega_{21} \rangle & \langle \upsilon_{22}, \nu_{22}, \omega_{22} \rangle & \dots & \langle \upsilon_{2y}, \nu_{2y}, \omega_{2y} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \upsilon_{x1}, \nu_{x1}, \omega_{x1} \rangle & \langle \upsilon_{x2}, \nu_{x2}, \omega_{x2} \rangle & \dots & \langle \upsilon_{xy}, \nu_{xy}, \omega_{xy} \rangle \end{pmatrix} \end{array}.$$

Assume the expert ratings for alternative A_1 as a matrix with order $y \times x$ for the alternative A_1.

$$\begin{array}{c} \\ EX_1 \\ EX_2 \\ \vdots \\ EX_y \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_x \end{array} \\ \begin{pmatrix} \langle u_{11}, v_{11}, w_{11} \rangle & \langle u_{12}, v_{12}, w_{12} \rangle & \dots & \langle u_{1x}, v_{1x}, w_{1x} \rangle \\ \langle u_{21}, v_{21}, w_{21} \rangle & \langle u_{22}, v_{22}, w_{22} \rangle & \dots & \langle u_{2x}, v_{2x}, w_{2x} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle u_{y1}, v_{y1}, w_{y1} \rangle & \langle u_{y2}, v_{y2}, w_{y2} \rangle & \dots & \langle u_{yx}, v_{yx}, w_{yx} \rangle \end{pmatrix} \end{array}.$$

Step 2: Determine the weights of experts. Let $EX_1, EX_2, \cdots, EX_n$ be the experts, they have individuals weights. Take

$$EX_1 = \langle a_1, b_1, c_1 \rangle, \ EX_2 = \langle a_2, b_2, c_2 \rangle, \cdots, \ EX_y = \langle a_y, b_y, c_y \rangle.$$

Step 3: Define the matrix with a rough neutrosophic set for criteria and alternatives. A rough neutrosophic matrix for criteria is created to express the connection between each criterion and the weight of experts.

$$W(C_1 EX_1) = \left[(min(a_1, \upsilon_{11}), max(b_1, \nu_{11}), max(c_1, \omega_{11})), (max(a_1, \upsilon_{11}), min(b_1, \nu_{11}), min(c_1, \omega_{11}))\right]$$
$$= \left\langle (\underline{\upsilon}_{11}, \underline{\nu}_{11}, \underline{\omega}_{11}), (\overline{\upsilon}_{11}, \overline{\nu}_{11}, \overline{\omega}_{11}) \right\rangle,$$

$$W = \begin{matrix} & EX_1 & \cdots & EX_y \\ C_1 & \left\langle(\underline{\upsilon}_{11}, \underline{\nu}_{11}, \underline{\omega}_{11}), (\overline{\upsilon}_{11}, \overline{\nu}_{11}, \overline{\omega}_{11})\right\rangle & \cdots & \left\langle(\underline{\upsilon}_{1y}, \underline{\nu}_{1y}, \underline{\omega}_{1y}), (\overline{\upsilon}_{1y}, \overline{\nu}_{1y}, \overline{\omega}_{1y})\right\rangle \\ C_2 & \left\langle(\underline{\upsilon}_{21}, \underline{\nu}_{21}, \underline{\omega}_{21}), (\overline{\upsilon}_{21}, \overline{\nu}_{21}, \overline{\omega}_{21})\right\rangle & \cdots & \left\langle(\underline{\upsilon}_{2y}, \underline{\nu}_{2y}, \underline{\omega}_{2y}), (\overline{\upsilon}_{2y}, \overline{\nu}_{2y}, \overline{\omega}_{2y})\right\rangle \\ \vdots & \vdots & \ddots & \vdots \\ C_x & \left\langle(\underline{\upsilon}_{x1}, \underline{\nu}_{x1}, \underline{\omega}_{x1}), (\overline{\upsilon}_{x1}, \overline{\nu}_{x1}, \overline{\omega}_{x1})\right\rangle & \cdots & \left\langle(\underline{\upsilon}_{xy}, \underline{\nu}_{xy}, \underline{\omega}_{xy}), (\overline{\upsilon}_{xy}, \overline{\nu}_{xy}, \overline{\omega}_{xy})\right\rangle \end{matrix}.$$

A rough neutrosophic matrix for alternatives is created to express the connection between each criterion and alternatives.

$$F_1(EX_1C_1) = (min(\upsilon_{11}, u_{11}), max(\nu_{11}, v_{11}), max(\omega_{11}, w_{11})), (max(\upsilon_{11}, u_{11}), min(\nu_{11}, v_{11}), min(\omega_{11}, w_{11}))$$
$$= \left\langle(\underline{u}_{11}, \underline{v}_{11}, \underline{w}_{11}), (\overline{u}_{11}, \overline{v}_{11}, \overline{w}_{11})\right\rangle,$$

$$F_1 = \begin{matrix} & C_1 & \cdots & C_x \\ EX_1 & \left\langle(\underline{u}_{11}, \underline{v}_{11}, \underline{w}_{11}), (\overline{u}_{11}, \overline{v}_{11}, \overline{w}_{11})\right\rangle & \cdots & \left\langle(\underline{u}_{1x}, \underline{v}_{1x}, \underline{w}_{1x}), (\overline{u}_{1x}, \overline{v}_{1x}, \overline{w}_{1x})\right\rangle \\ EX_2 & \left\langle(\underline{u}_{21}, \underline{v}_{21}, \underline{w}_{21}), (\overline{u}_{21}, \overline{v}_{21}, \overline{w}_{21})\right\rangle & \cdots & \left\langle(\underline{u}_{2x}, \underline{v}_{2x}, \underline{w}_{2x}), (\overline{u}_{2x}, \overline{v}_{2x}, \overline{w}_{2x})\right\rangle \\ \vdots & \vdots & \ddots & \vdots \\ EX_y & \left\langle(\underline{u}_{y1}, \underline{v}_{y1}, \underline{w}_{y1}), (\overline{u}_{y1}, \overline{v}_{y1}, \overline{w}_{y1})\right\rangle & \cdots & \left\langle(\underline{u}_{yx}, \underline{v}_{yx}, \underline{w}_{yx}), (\overline{u}_{yx}, \overline{v}_{yx}, \overline{w}_{yx})\right\rangle \end{matrix}.$$

Step 4: Change the non-square matrix toward a square matrix in this stage. The above matrix W is separated by six matrices that are a lower three and upper three approximations of truth, indeterminacy and false matrix are denoted by ($W(\underline{T}_{ij})$, $W(\underline{I}_{ij})$, $W(\underline{F}_{ij})$) and ($W(\overline{T}_{ij})$, $W(\overline{I}_{ij})$, $W(\overline{F}_{ij})$). Similarly, F_1 matrix expressed as ($F_1(\underline{T}_{ij})$, $F_1(\underline{I}_{ij})$, $F_1(\underline{F}_{ij})$) and ($F_1(\overline{T}_{ij})$, $F_1(\overline{I}_{ij})$, $F_1(\overline{F}_{ij})$).

$$F_1(\underline{T}_{ij})_{y\times x} * W(\underline{T}_{ij})_{x\times y} = F_1(\underline{T}) = \begin{pmatrix} \underline{u\upsilon}_{11} & \underline{u\upsilon}_{12} & \cdots & \underline{u\upsilon}_{1n} \\ \underline{u\upsilon}_{21} & \underline{u\upsilon}_{22} & \cdots & \underline{u\upsilon}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{u\upsilon}_{y1} & \underline{u\upsilon}_{y2} & \cdots & \underline{u\upsilon}_{yy} \end{pmatrix}_{y\times y}.$$

Step 5: Calculate the matrix's energy using the notion of energy of a rough neutrosophic matrix. For one alternative, we obtained six energies for the lower three and upper three approximations of truth, indeterminacy, and false matrix.

$$E(F_1) = \left\langle\left(E[F_1(\underline{T})], E[F_1(\underline{I})], E[F_1(\underline{F})]\right), \left(E[F_1(\overline{T})], E[F_1(\overline{I})], E[F_1(\overline{F})]\right)\right\rangle.$$

Step 6: Repeat this procedure for k possible alternatives. Approximate neutrosophic matrix energies of $E[F_1], E[F_2], \cdots, E[F_r]$ were obtained for each alternative.

Step 7: Determine the values of the score function and accuracy function for ranking the rough neutrosophic energy values by Eqs (5.1) and (5.2).

Finally, we rank the alternatives in descending order by these values.

7. Numerical examples

For example, we take the problem of selecting a good factory in a particular area that is good in all criteria. We take 8 factories as alternatives ($F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8$), the criteria are:

C_1 - Safety and security of workers,
C_1 - Environment,
C_3 - Use of products.

The problem is evaluated by a group of 4 decision makers (EX_1, EX_2, EX_3, EX_4) who are experts in factory management in that area. Table 1 shows the linguistic variables for neutrosophic numbers which help experts to evaluate the alternatives.

Table 1. Linguistic variables for neutrosophic numbers.

S. No	Linguistic variable (Code)	Neutrosophic numbers
1	Very Bad- VB	$\langle 0.1, 0.9, 1\rangle$
2	Bad - B	$\langle 0.3, 0.7, 0.75\rangle$
3	Medium - M	$\langle 0.5, 0.5, 0.5\rangle$
4	Good - G	$\langle 0.8, 0.3, 0.25\rangle$
5	Very Good - VG	$\langle 1, 0.1, 0\rangle$

Step 1: The ratings of weights of criteria and alternatives are given by the experts. It is shown in Tables 2 and 3.

Table 2. Weights of criteria.

Criteria	EX_1	EX_2	EX_3	EX_4
C_1	G	VG	G	M
C_2	G	VG	M	G
C_3	M	G	VG	B

Table 3. Ratings for each alternative.

Factories	Experts	C_1	C_2	C_3	Factories	Experts	C_1	C_2	C_3
	EX_1	G	M	G		EX_1	M	VG	G
F_1	EX_2	VG	G	M	F_5	EX_2	G	B	G
	EX_3	G	M	B		EX_3	G	VG	G
	EX_4	M	M	G		EX_4	B	M	M
	EX_1	G	G	G		EX_1	G	G	VG
F_2	EX_2	M	G	M	F_6	EX_2	M	B	G
	EX_3	B	G	M		EX_3	VB	B	M
	EX_4	M	VB	G		EX_4	G	M	VG
	EX_1	M	G	G		EX_1	B	M	VB
F_3	EX_2	VB	B	M	F_7	EX_2	M	G	M
	EX_3	G	VG	G		EX_3	VG	B	G
	EX_4	VG	G	B		EX_4	G	M	M
	EX_1	G	G	B		EX_1	G	B	M
F_4	EX_2	VG	B	M	F_8	EX_2	VG	G	G
	EX_3	G	M	M		EX_3	M	G	M
	EX_4	G	VG	B		EX_4	B	VG	G

Step 2: The weights of experts:

$$EX_1 = VG = \langle 1, 0.1, 0 \rangle, \qquad EX_2 = G = \langle 0.8, 0.3, 0.25 \rangle,$$

$$EX_3 = G = \langle 0.8, 0.3, 0.25 \rangle, \quad EX_4 = M = \langle 0.5, 0.5, 0.5 \rangle.$$

Step 3: By the above values determine the rough neutrosophic matrix for criteria and alternatives. A rough neutrosophic matrix for criteria is created to express the connection between each criterion and the weight of experts. It is shown in Table 4. A rough neutrosophic matrix for alternatives is created to express the connection between each criterion and alternatives. It is shown in Table 5.

Table 4. Rough neutrosophic matrix for criteria.

Criteria	EX_1	EX_2
C_1	$\langle(0.8, 0.3, 0.25), (1, 0.1, 0)\rangle$	$\langle(0.8, 0.3, 0.25), (1, 0.1, 0)\rangle$
C_2	$\langle(0.8, 0.3, 0.25), (1, 0.1, 0)\rangle$	$\langle(0.8, 0.3, 0.25), (1, 0.1, 0)\rangle$
C_3	$\langle(0.5, 0.5, 0.5), (1, 0.1, 0)\rangle$	$\langle(0.8, 0.3, 0.25), (0.8, 0.3, 0.25)\rangle$
Criteria	EX_3	EX_4
C_1	$\langle(0.8, 0.3, 0.25), (0.8, 0.3, 0.25)\rangle$	$\langle(0.5, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle$
C_2	$\langle(0.5, 0.5, 0.5), (0.8, 0.3, 0.25)\rangle$	$\langle(0.5, 0.5, 0.5), (0.8, 0.3, 0.25)\rangle$
C_3	$\langle(0.8, 0.3, 0.25), (1, 0.1, 0)\rangle$	$\langle(0.3, 0.7, 0.75), (0.5, 0.5, 0.5)\rangle$

Table 5. Rough neutrosophic matrix for alternative.

F_1	C_1	C_2
EX_1	$\langle(0.8, 0.3, 0.25), (0.8, 0.3, 0.25)\rangle$	$\langle(0.5, 0.5, 0.5), (0.8, 0.3, 0.25)\rangle$
EX_2	$\langle(1, 0.1, 0), (1, 0.1, 0)\rangle$	$\langle(0.8, 0.3, 0.25), (1, 0.1, 0)\rangle$
EX_3	$\langle(0.8, 0.3, 0.25), (0.8, 0.3, 0.25)\rangle$	$\langle(0.5, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle$
EX_4	$\langle(0.5, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle$	$\langle(0.5, 0.5, 0.5), (0.8, 0.3, 0.25)\rangle$
F_1	C_3	
EX_1	$\langle(0.5, 0.5, 0.5), (0.8, 0.3, 0.25)\rangle$	
EX_2	$\langle(0.5, 0.5, 0.5), (0.8, 0.3, 0.25)\rangle$	
EX_3	$\langle(0.3, 0.7, 0.75), (1, 0.1, 0)\rangle$	
EX_4	$\langle(0.3, 0.7, 0.75), (0.8, 0.3, 0.25)\rangle$	

Step 4: Multiplying the truth lower matrices of alternative and weights of criteria

$$F_1(\underline{T}) = F_1(\underline{T}_{ij})_{y\times x} * W(\underline{T}_{ij})_{x\times y} = \begin{pmatrix} 0.8 & 0.5 & 0.5 \\ 1.0 & 0.8 & 0.5 \\ 0.8 & 0.5 & 0.3 \\ 0.5 & 0.5 & 0.3 \end{pmatrix} * \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.5 \\ 0.8 & 0.8 & 0.5 & 0.5 \\ 0.5 & 0.8 & 0.8 & 0.3 \end{pmatrix},$$

$$F_1(\underline{T}) = \begin{pmatrix} 1.29 & 1.44 & 1.29 & 0.80 \\ 1.69 & 1.84 & 1.60 & 1.05 \\ 1.19 & 1.28 & 1.13 & 0.74 \\ 0.95 & 1.04 & 0.89 & 0.59 \end{pmatrix},$$

$\underline{\lambda}_i$ = 4.8712 , -0.0324, 0.0113 and 0. Energy of truth lower approximation matrix = 7.3173.

Step 5: We determine the energy of lower and upper approximation of truth, indeterminacy and false matrices.

Energy of

$$F_1 = [(7.3173, 3.4893, 3.3375), (12.3286, 1.2554, 0.8043)].$$

Similaly we find the energy for every alternatives.

Step 6: Energy values for each alternative.

F_2 = [(6.3293 , 3.9318 , 3.8498), (12.7576, 1.2032, 0.7652)],
F_3 = [(6.0865, 3.5045 , 3.2391), (12.8080, 1.2251, 0.7675)],
F_4 = [(7.1963, 3.3538, 3.1291), (11.8958, 1.2829, 0.7936)],
F_5 = [(7.0609, 3.4929, 3.3041), (13.0764, 1.2774, 0.9186)],
F_6 = [(5.8110, 3.8487, 3.6749), (13.0672, 0.9653, 0.5098)],
F_7 = [(6.0898, 3.8056, 3.6584), (11.9034, 1.2723, 0.7651)],
F_8 = [(7.3234, 3.4354, 3.2567), (12.5012, 1.1168, 0.6461)].

Step 7: Determine the values of the score function and accuracy function for rough neutrosophic matrix energy values. The score function and accuracy function of F_1 is calculated by using Eqs (5.1) and (5.2). In the same way, other alternatives are calculated.

$$\begin{aligned}
Score\ function\ of\ F_1 &= \frac{2 + 7.3173 + 12.3286 - 2(3.4893 + 1.2554) - (3.3375 + 0.8043)}{4}, \\
\sigma(F_1) &= 2.0037, \\
Accuracy\ function\ of\ F_1 &= \frac{7.3173 + 12.3286 - 2(3.4893 + 1.2554) - (3.3375 + 0.8043)}{2}, \\
\tau(F_1) &= 3.0074.
\end{aligned}$$

The ranking order is as follows $F_8 > F_5 > F_1 > F_4 > F_3 > F_6 > F_2 > F_7$. In Table 6, it was displayed. Alternative 8 is best to compare to others. So Factory 8 is selected as a good factory in that area. Figure 1 shows the bar chart of both the score function and accuracy function of each alternative which represents by MATLAB.

Table 6. Ranking of alternatives.

Factories	Score funtion	Accuracy function	Ranking
F_1	2.0037	3.0074	3
F_2	1.5504	2.1009	7
F_3	1.8572	2.7143	5
F_4	1.9740	2.9480	4
F_5	2.0935	3.1869	2
F_6	1.7664	2.5328	6
F_7	1.3535	1.7069	8
F_8	2.2043	3.4087	1

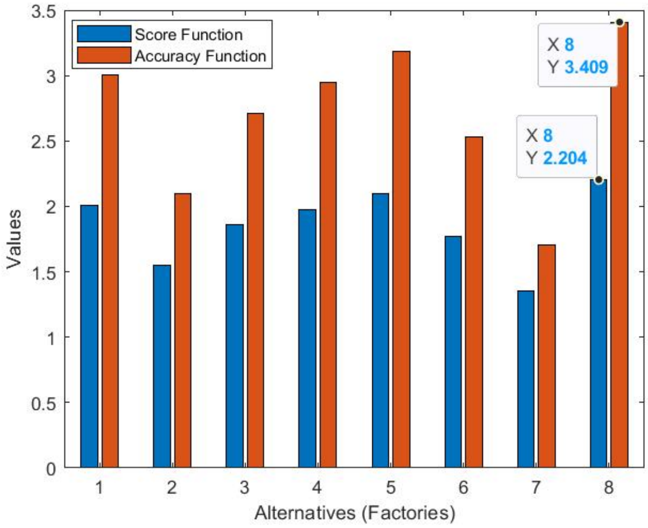

Figure 1. Barchat of ranking.

8. Comparative results

In a existing MCDM methods TOPSIS stay an good role with taken decision. So we solve our taken problem with neutrosophic numbers by neutrosophic simplified-TOPSIS method [25]. We got the same ranking order when the problem is solve by TOPSIS method. The final ranking formula for TOPSIS method given below

$$R = \frac{N^-}{N^+ + N^-},$$

where, N^+ is max ideal solution and N^- is min ideal solution.

The final ranking values and order and the proposed method ranking values and order are displayed in Table 7. This comparison of results proves that the proposed method is accurate and that the ranking will remain the same. The ranking order of both results: $F_8 > F_5 > F_1 > F_4 > F_3 > F_6 > F_2 > F_7$.

Table 7. Comparative results.

	TOPSIS result				Proposed method result		
Factories	N^+	N^-	R	Order	$\sigma(F)$	$\tau(F)$	Order
F_1	0.072	0.383	0.8418	3	2.0037	3.0074	3
F_2	0.378	0.077	0.1692	7	1.5504	2.1009	7
F_3	0.31	0.145	0.3187	5	1.8572	2.7143	5
F_4	0.204	0.251	0.5516	4	1.9740	2.9480	4
F_5	0.049	0.406	0.8923	2	2.0935	3.1869	2
F_6	0.368	0.087	0.1912	6	1.7664	2.5328	6
F_7	0.446	0.009	0.0198	8	1.3535	1.7069	8
F_8	0.009	0.446	0.9802	1	2.2043	3.4087	1

9. Conclusions

The combination of the rough matrix and the neutrosophic matrix is used to deal with uncertain situations in multi-criteria decision-making problems. This paper achieves good results with the proposed method by using the rough neutrosophic matrix and ranking formula in the MCDM environment. The properties and operations of rough neutrosophic matrices were proven. The problem is solved using the energy of a rough neutrosophic matrix; the problem is about selecting the best factory in a particular area that satisfies some criteria set by the experts. As a result, factory 8 is chosen as a good factory. The outcome of the problem is represented by the MATLAB figure. The outcome was compared with the TOPSIS approach. The ranking will remain the same. Furthermore, we will extend the concept of a rough neutrosophic matrix and its energy to other types of rough matrices, such as interval-valued and multi-valued rough neutrosophic matrices.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This research was funded by the Vellore Institute of Technology, Vellore, India. We would like to thank the Vellore Institute of Technology for funding support and encouragement to carry out this research work. We wish to thank the editor and reviewers for their valuable comments and suggestions.

Conflict of interest

The authors declare no conflicts of interest.

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