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Research article

Research on digital transformation strategy and subsidy mechanism of manufacturing supply chain based on differential game

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Abstract: In the current research on the impact of the government on the digital transformation of enterprises, scholars have neglected the synergies of the supply chain and the mechanisms for regulating and assessing the subsequent behaviour of enterprises. Therefore, this paper uses differential games to study the optimal strategy of cost sharing in manufacturing supply chains during the digital transformation of upstream and downstream firms with government subsidies. The equilibrium game strategy is used with the government and parts of the supply chain as players under different models of cooperation, independent operation and government awards and punishments. For each model, the equilibrium results are solved, compared and simulated. The results show that the government reward and punishment mechanism can effectively suppress the "double marginal effect" and improve digitalization and benefits of enterprises. Meanwhile, the benefit level of enterprises under the government reward and punishment mechanism is related to the target digitalization level and the strength of the reward and punishment. When the target digitization level is relatively low at the early stage of digital transformation, the government reward and punishment mechanism is more likely to promote the level of benefits of enterprises. When the target digitalization level is relatively high in the middle and late stages of digital transformation, the government needs to reasonably set the strength of the reward and punishment in order to effectively promote digitalization and economic development.

Keywords: digital transformation; supply chain; differential game; government subsidy; profit sharing

Mathematics Subject Classification: 91A23

1. Introduction

In the era of digital economy, data is the current core production factor and digital transformation has become an important development method for the production of the global digital economy. The use of cloud computing, big data, artificial intelligence and other digital technologies to widely empower various industries and fields has become an important force to promote quality change, efficiency change and power change in the national economic development [1]. Internationally, various major economies have introduced digital strategies in the hope of using digital transformation to enhance the competitiveness of traditional industrial policies. The United States enacted the National Strategy for Critical and Emerging Technologies in 2020 to become a world leader in critical and emerging technologies and to build technology alliances to achieve technology risk management. In the same year, France also published "Making France a Breakthrough Technology-Led Economy". During the 14th five-year plan, the digital transformation of manufacturing industry has become an important goal for the development of China's digital economy, and it is also an important support of the industrial supply chain and a new development pattern as the economy advances [2]. The 14th fiveyear plan clearly identifies smart manufacturing development as a national strategy to promote the digital transformation of production methods. In the years following this announcement, policies have been promulgated one after another to support digital transformation as a driver of improving the manufacturing industry. The digital transformation of the manufacturing industry has become a strategic response to the changes of the times and the urgent demand of national economic development. With the continuous innovation and improvement of many new digital technologies, and with the impact of the COVID-19 epidemic on the traditional market, China's manufacturing industry faces a great pressure to develop and respond to competition. Digital transformation has become the inevitable path to the survival and development of the manufacturing industry in the era of the digital economy. Currently, the overall level of digitalization in China's manufacturing industry is still low; moreover, many SMBs lack sufficient funds for digital transformation. Additionally, promoting digital transformation needs to play a good role in government and effectively improve the development environment. Based on the abovementioned views, we need to investigate the digital transformation decisions of manufacturing enterprises to achieve digital transformation and to upgrade manufacturing enterprises to improve digitalization and promote the development of the digital economy.

In recent years, an increasing number of enterprises have taken digital transformation as a development goal to shape market competitiveness. To study the relationship between manufacturing and digitalization, Nayal et al. [3] used an empirical analysis of the automotive manufacturing industry to confirm that supply chain collaboration can promote the digital transformation of manufacturing enterprises, thus effectively improve enterprise performance. Second, with regard to the transformation strategy, Kong Cunyu [4] analyzed the "triple dilemma" faced by the manufacturing industry during transformation and proposed accelerating digital transformation by promoting the open trade of manufacturing subjects. Additionally, for the path aspect, Wu Changqi et al. [5] argued that the path of digital transformation in traditional manufacturing enterprises must go through the digitalization of the industrial ecosystem, which constitutes the value loop of the production chain, to realize the integration of the whole. At the same time, it can be seen that with the increase in uncertainty in today's global environment, competition among enterprises is gradually evolving into supply chain competition [6]. Li Xiaojing et al. [7] indicated that with the deep development of the global industrial division of labor,

enterprises must consider the development of their own supply chain to achieve a competitive advantage, especially for large manufacturing enterprises. For the current state of China's manufacturing enterprises, through efforts to digitize due to long supply chains with many links in the production process, data silos easily manifest between upstream and downstream enterprises [8], resulting in insufficient data application and suppressed data value, etc. The information along the industrial supply chains cannot be effectively integrated and utilized to maximize benefits. Feng Weiyi [9] indicated that the core competitiveness of manufacturing enterprises should be improved by combining each link in the development of the manufacturing industry with digital elements, rather than limiting the digital transformation of the manufacturing industry to individual enterprises; moreover, for manufacturing enterprises, supply chain management affects all aspects of business operations. China's current manufacturing industry still has obvious shortcomings in the supply chain system. At present, many scholars have put forward the viewpoint of a digital supply chain, and believe that a digital industrial supply chain is the optimal realization path to reunderstand digital transformation of manufacturing enterprises. Existing studies have explored theoretical aspects of supply chain digital transformation. Zhai Weifeng et al. [10] explored the interaction between the supply chain and digitalization based on the principle of innovation diffusion and proposed that building a digital supply chain is a key initiative for manufacturing enterprises to be self-sufficient in the digital economy. After the COVID-19 epidemic, the necessity and urgency of the digital development of the supply chain was even more demonstrated, and the digital supply chain has now become the core of competitiveness in the manufacturing industry [11].

Meanwhile, in recent years, due to the development needs of the national industrial economy and the impact of the COVID-19 epidemic, a number of digital transformation subsidy policies have been introduced across the country to support the relief and development of manufacturing enterprises. Accordingly, some scholars have started to consider the impact of government subsidies on the digital transformation of enterprises. The percentage of domestic enterprises that have a good foundation in information technology and the effective implementation of digital transformation is relatively low, and most enterprises do not have realistic conditions for digital transformation. For most enterprises, digital transformation requires significant capital investment and the introduction of new digital technologies and talent [12]. Through empirical analysis, Wu Fei et al. [13] indicated that the government can effectively help the digital transformation of enterprises by means of financial science and technology expenditures. Chen et al. [14] further analyzed the significant incentive of government subsidies on the digitization of manufacturing firms through linked financing constraints that promote R&D investment and innovation output from the perspective of direct resource supplementation and indirect signaling. Government subsidy incentive policies have become an important driving force for the digital transformation of manufacturing enterprises and have a guiding and supporting role in their initial transformation. Hao Zeng [15] explored the incentive policy for the digital transformation of adapted enterprises using the prospective effective tax rate and proposed differentiated incentive policies that give full play to the combinatorial effect of policies. Utilizing automobile manufacturing enterprises, Fan Decheng et al. [16] discovered that the trapping effect of unreasonable subsidy policies would lead to reverse guidance, "subsidy-seeking" investment and overinvestment, thus leading government subsidies to inadvertently crowd out the digital transformation of enterprises. Most of the relevant studies have researched individual enterprises and ignored the synergistic effect between upstream and downstream enterprises in the industrial supply chain. Through empirical tests, Yu et al. [17] demonstrated that the positive effect of government subsidies on the digital transformation

of upstream enterprises can significantly promote the digital transformation of downstream enterprises through industrial chain synergy. Therefore, consideration of the government subsidy effect needs to include the synergy among supply chain members; however, the aforementioned studies only start with an empirical analysis, and there are fewer studies that use differential games to explore the impact of subsidies on the digital transformations of enterprises in the manufacturing supply chain. It does not consider the impact of government subsidies on supply chain member firms when the government is an endogenous variable, and the analysis of the sensitivity of the supply chain member firms to government subsidies under different decision-making modes. Additionally, there is a lack of research on the regulation and evaluation of corporate transformation subsidies, which does not take into account the evaluation and assessment of the efficiency of government subsidies, and the government lacks effective regulation of the behaviour of companies after they have received subsidies.

In summary, many theoretical studies related to the digital transformation of the manufacturing industry and government subsidies have been proposed in the literature; however, there are still some problems that need further discussion. At present, most scholars mainly study the implementation and strategy of the digital transformation of the manufacturing industry based on a macroscopic perspective using a literature review and empirical analysis [18], ignoring microlevel issues such as conflicts of interest and values among enterprises in digital transformation. In the literature, government subsidies are mostly targeted at individual enterprises, and there is a lack of research on the impact of the overall synergy in the supply chain on government subsidies. Moreover, subsidies are mostly treated as either fixed values or exogenous variables, and the government is not brought into the game scenario as a game party to consider the impact of subsidy policy changes on the transformation dynamics and decision-making of supply chain members. Starting from these observations, this paper uses a dynamic perspective, considering that the digitalization level of enterprises will change over time under the influence of multiple parties. The differential game model is a dynamic way to analyze the cooperative decisions of game parties in a continuous time [19]. Differential game models are widely used when multiple parties are competing in a game problem in continuous time; therefore, we use the differential game model in this article. From the perspective of supply chain management with upstream and downstream enterprises as an example, and considering the current problems of data silos, insufficient core technology, lack of digital facilities and lack of motivation for the digital transformation of the manufacturing industry, the level of digital transformation is used as a static variable to measure data integration and digital technology in the manufacturing supply chain. The government and upstream and downstream enterprises in the manufacturing supply chain are constructed as the main players [20]. This paper also introduces a government reward and punishment mechanism to assess the performance of enterprises' behaviour after receiving subsidies. It also considers the role of government rewards and penalties in reducing the cost and improving the level of digital transformation, and constructs a multi-stage government subsidy model. The game equilibrium of upstream and downstream enterprises in the manufacturing supply chain is studied under four situations: collaborative decision-making, independent decision-making, the dynamic study better describes the ongoing game and balancing strategies of supply chain members government reward and punishment decision-making. Most current research has been conducted from a static perspective, but digital transformation behaviour has certain external economies, and dynamic research is better able to describe the ongoing game and balancing strategies of supply chain members. Additionally, paper provides a comparative analysis of the decision equilibrium solutions for the three models when the system either reaches or does not reach a steady state, making the study more practical. These scenarios

provide a theoretical reference for the government to reasonably formulate a cost-subsidy policy to promote digital transformation in manufacturing and for enterprises to choose digital transformation to reconstruct the manufacturing supply chain with the participation of multiple parties. Compared to existing studies, the innovations in this paper are the introduction of government incentives and penalties, and the consideration of the impact of digitisation levels and prices on market demand. The paper also discusses the dynamic analysis of equilibrium game outcomes for supply chain member firms under government subsidies and regulatory mechanisms, and considers the strength of the reward and punishment and the impact of the time factor on the level of digitisation and the level of efficiency. The results of the simulation analysis are made more practical by classifying and discussing the steady state and non-steady state situations of the supply chain.

2. Model construction and solution

Hypothesis 1. In this paper, we consider the upstream and downstream enterprises in the manufacturing supply chain as two groups of suppliers and manufacturers. Suppliers are upstream enterprises providing either raw materials or semifinished products for downstream manufacturers, who then assemble and produce finished industrial products for sale to the market. $E_S(t)$, $E_M(t)$, denote the degree of digital transformation efforts of suppliers and manufacturers at moment t, respectively, indicating the degree of their willingness to pay for digital upgrades in manpower, equipment and data sharing and integration efforts. Second, the digital transformation efforts of suppliers and manufacturers will affect the digital level of the whole supply chain, which changes with time. At the same time, the aging and associated depreciation of digital equipment and the loss of digital technicians will lead to a certain decay in the digital level of enterprises, and the differential equation for the change in their digital level is as follows:

$$L(t) = \alpha_s E_s(t) + \alpha_M E_M(t) - \delta L(t), \qquad (1)$$

where L(t) indicates the digitalization level of the whole supply chain at time *t*. In other words, the digitalization of the supply chain includes the levels of digital technology application and data sharing and integration both upstream and downstream from the enterprise. The initial digitization level is $L(0) = L_0 \ge 0$. α_s and α_M denote the impact coefficients of the digital transformation activities of suppliers and manufacturers on the digitalization level, respectively. $\delta > 0$ denotes the degree of natural decay of the digital level.

Hypothesis 2. The cost of digital transformation for suppliers and manufacturers marginally increases with the degree of effort at digital transformation, such that they may be represented as follows:

$$C_{s} = \frac{1}{2}\mu_{s}E_{s}^{2}; C_{M} = \frac{1}{2}\mu_{M}E_{M}^{2}, \qquad (2)$$

where $\mu_s > 0$ and $\mu_M > 0$ denote the digital transformation cost for suppliers and manufacturers, respectively.

Hypothesis 3. Digital transformation can effectively improve production efficiency, reduce manufacturing costs and improve the quality of products. These cumulative changes will enhance market competitiveness and eventually increase market demand and allow enterprises to occupy a greater share of the market. According to EI Ouardighi & Kogan [21], market demand is influenced

by price and nonprice factors, and the two factors can be separated and multiplied; therefore, this paper assumes that the market demand function is as follows:

$$D(t) = \beta L(t)(A - bP(t)), \qquad (3)$$

where D(t) denotes the market demand of the product at moment *t*, and P(t) denotes the selling price of the product at moment *t*. $\beta > 0$ indicates the market competitiveness resulting from the digital transformation and upgrading of enterprises. b > 0 denotes the coefficient of influence of product price on market demand. A > 0 indicates the maximum size of the market allowed, and $A - bP(t) \ge 0$.

Hypothesis 4. Suppliers and manufacturers in the supply chain are rational decision-makers based on complete information, and suppliers and manufacturers have the same discount factor in an infinite time horizon, where $\rho > 0$. Both ends of the supply chain seek an optimal decision to maximize their own interests in an infinite time horizon. The government provides transformation subsidies in the ratio of $\theta_s(t)$ and $\theta_M(t)$ for suppliers and manufacturers, respectively, to promote digital transformation in the manufacturing industry, thereby creating a digital supply chain and achieving high-quality development of our economy.

2.1. Collaborative decision-making (strategy C)

Collaborative decision making is an idealized model of decision making in a game model. Under the influence of government requirements and support incentives, suppliers and manufacturers reach synergistic cooperation, sign contracts, and share data upstream and downstream, all with the goal of maximizing the overall benefits of the supply chain for their own digital transformation. The game order has the government first subsidizing the optimal transformation costs of upstream and downstream enterprises (suppliers and manufacturers), and then the suppliers and manufacturers maximize the overall profit of the supply chain according to the subsidy policy provided by the government. The decision function of the supply chain as a whole is as follows:

$$J_{C}^{C} = \max_{E_{S},E_{M}} \int_{0}^{\infty} e^{-\rho t} \left[P(A-bP)\beta L - \frac{1}{2}(1-\theta_{S}^{C})\mu_{S}E_{S}^{2} - \frac{1}{2}(1-\theta_{M}^{C})\mu_{M}E_{M}^{2} \right] dt \,.$$
(4)

Government subsidies are more conducive to the digital transformation of enterprises and the improvement of their market competitiveness. Consequently, the essence of the government's objective function is to make supply chain enterprises maximize their inputs and outputs through subsidies [22]. Therefore, this paper assumes that the objective function of the government is the sum of the benefits of the supply chain member enterprises minus the government subsidies:

$$J_{G}^{C} = \max_{E_{S}, E_{M}} \int_{0}^{\infty} e^{-\rho t} [P(A-bP)\beta L - \frac{1}{2}\mu_{S}E_{S}^{2} - \frac{1}{2}\mu_{M}E_{M}^{2}]dt.$$
(5)

Proposition 1. Equilibrium strategies for suppliers, manufacturers, and the government under collaborative decision-making.

(1) The optimal equilibrium strategies for digital transformation inputs, government subsidies, and product prices for suppliers and manufacturers under collaborative decision-making are as follows:

$$\begin{cases} E_{S}^{C^{**}} = \frac{A^{2}\beta\alpha_{S}}{4b\mu_{S}(\delta+\rho)(1-\theta_{S}^{C^{*}})}, & E_{M}^{C^{**}} = \frac{A^{2}\beta\alpha_{M}}{4b\mu_{M}(\delta+\rho)(1-\theta_{M}^{C^{*}})}, \\ \theta_{S}^{C^{*}} = 0, & \theta_{M}^{C^{*}} = 0, \\ P^{C^{*}} = \frac{A}{2b}. \end{cases}$$
(6)

(2) The optimal trajectory of the digitalization level of the supply chain is as follows:

$$L^{C^{**}} = L^{C^{*}}_{RSS} + (L_0 - L^{C^{*}}_{RSS})e^{-\delta t} .$$
⁽⁷⁾

Among them, $L_{RSS}^{C*} = \frac{A^2 \beta \alpha_s^2}{4b \delta \mu_s (\delta + \rho)(1 - \theta_s^C)} + \frac{A^2 \beta \alpha_M^2}{4b \delta \mu_M (\delta + \rho)(1 - \theta_M^C)}$.

(3) The overall optimal profit function of the supply chain is as follows:

$$J_{C}^{C^{**}} = \frac{A^{2}\beta}{4b(\delta+\rho)}L + \frac{A^{4}\beta^{2}\alpha_{s}^{2}}{32b^{2}\mu_{s}(\delta+\rho)^{2}(1-\theta_{s}^{C})} + \frac{A^{4}\beta^{2}\alpha_{M}^{2}}{32b^{2}\mu_{M}(\delta+\rho)^{2}(1-\theta_{M}^{C})}.$$
(8)

Property 1. From Proposition 1, it is clear that the optimal degree of digital transformation inputs and the overall optimal profit of suppliers and manufacturers are positively related to market size A, the coefficient of impact of digital transformation activities on the level of digitalization (α_S or α_M) and the coefficient of impact of supply chain digitalization on market demand β ; conversely, it is negatively related to the price sensitivity coefficient b, the degree of natural decay of digitalization δ , the discount factor ρ and the coefficient of own cost (E_S or E_M). Therefore, an increase in the level of digitalization can make companies more profitable, and thus promote their investment in digital transformation, while companies also limit and control the degree of investment in digital transformation activities in consideration of their transformation costs.

2.2. Independent decision-making (strategy N)

To ensure the reference ability and completeness of the model, the independent decision model is introduced as a benchmark for comparison with other decisions. Under the independent decision, the supplier and the manufacturer, as two independent individuals, both upstream and downstream enterprises make decisions with the goal of maximizing their own profits. The supplier provides the either semifinished products or raw materials to the manufacturer for further assembly and processing at wholesale price ω ; finally, the manufacturer sells the product to the market at market price p [23]. The decision function of the supplier, manufacturer and government is as follows:

$$J_{S}^{N} = \max_{E_{S}} \int_{0}^{\infty} e^{-\rho t} [\omega(A - bP)\beta L - \frac{1}{2}(1 - \theta_{S}^{N})\mu_{S}E_{S}^{2}]dt, \qquad (9)$$

$$J_{M}^{N} = \max_{E_{M}} \int_{0}^{\infty} e^{-\rho t} [(P - \omega)(A - bP)\beta L - \frac{1}{2}(1 - \theta_{M}^{N})\mu_{M}E_{M}^{2}]dt, \qquad (10)$$

$$J_G^N = \max_{E_S, E_M} \int_0^\infty e^{-\rho t} [P(A - bP)\beta L - \frac{1}{2}\mu_S E_S^2 - \frac{1}{2}\mu_M E_M^2] dt .$$
(11)

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Proposition 2. Equilibrium strategies for upstream and downstream firms in the supply chain and the government under independent decision-making.

(1) The optimal equilibrium strategies for digital transformation inputs, government transformation subsidy rates, wholesale prices, and product sales prices for upstream and downstream firms are as follows:

$$\begin{cases} E_{S}^{N^{**}} = \frac{A^{2}\beta\alpha_{S}}{8b\mu_{S}(\delta+\rho)(1-\theta_{S}^{N^{*}})}, \quad E_{M}^{N^{**}} = \frac{A^{2}\beta\alpha_{M}}{16b\mu_{M}(\delta+\rho)(1-\theta_{M}^{N^{*}})}, \\ \theta_{S}^{N^{*}} = \frac{1}{3}, \quad \theta_{M}^{N^{*}} = \frac{2}{3}, \\ P^{N^{*}} = \frac{3A}{4b}, \quad \omega^{N^{*}} = \frac{A}{2b}. \end{cases}$$
(12)

(2) The optimal trajectory of the digitalization level of the supply chain is as follows:

$$L^{N^*} = L^N_{RSS} + (L_0 - L^N_{RSS})e^{-\delta t}.$$
 (13)

Among them, $L_{RSS}^{N} = \frac{A^{2}\beta\alpha_{s}^{2}}{8b\delta\mu_{s}(\delta+\rho)(1-\theta_{s})} + \frac{A^{2}\beta\alpha_{M}^{2}}{16b\delta\mu_{M}(\delta+\rho)(1-\theta_{M})}.$

(3) The supplier, manufacturer, and overall supply chain profit optimality function is as follows:

$$\begin{cases} J_{s}^{N^{**}} = \frac{A^{2}\beta}{8b(\delta+\rho)}L + \frac{A^{4}\beta^{2}\alpha_{s}^{2}}{128b^{2}\mu_{s}(\delta+\rho)^{2}(1-\theta_{s}^{N^{*}})} + \frac{A^{4}\beta^{2}\alpha_{M}^{2}}{128b^{2}\mu_{M}(\delta+\rho)^{2}(1-\theta_{M}^{N^{*}})}, \\ J_{M}^{N^{**}} = \frac{A^{2}\beta}{16b(\delta+\rho)}L + \frac{A^{4}\beta^{2}\alpha_{s}^{2}}{128b^{2}\mu_{s}(\delta+\rho)^{2}(1-\theta_{s}^{N^{*}})} + \frac{A^{4}\beta^{2}\alpha_{M}^{2}}{512b^{2}\mu_{M}(\delta+\rho)^{2}(1-\theta_{M}^{N^{*}})}, \\ J_{C}^{N^{**}} = \frac{3A^{2}\beta}{16b(\delta+\rho)}L + \frac{A^{4}\beta^{2}\alpha_{s}^{2}}{64b^{2}\mu_{s}(\delta+\rho)^{2}(1-\theta_{s}^{N^{*}})} + \frac{5A^{4}\beta^{2}\alpha_{M}^{2}}{512b^{2}\mu_{M}(\delta+\rho)^{2}(1-\theta_{M}^{N^{*}})}. \end{cases}$$
(14)

2.3. Government reward and punishment mechanism (strategy M)

Traditional manufacturing enterprises need to digitally upgrade hardware, software and other equipment before and in the middle of digital transformation, and introduce and train digital-related talents, which requires a lot of resources. Therefore, government subsidies can, to a certain extent, reduce the cost of enterprise transformation and improve the willingness of enterprises to digital transformation.

However, problems such as information asymmetry between government and enterprises can also cause reverse guidance and have a crowding-out effect on enterprise transformation. Therefore, for the problems of subsidy-seeking behavior, strategic transformation and excessive investment by enterprises is required. Therefore, for the problems of subsidy-seeking behavior, strategic transformation and excessive investment by enterprises, in this paper, we set up a government acceptance mechanism, in which the government provides subsidies to enterprises for transformation in the early stage, and after a certain development stage, the government will accept the digital transformation results of subsidized enterprises and implement rewards and punishments according to their digital level. After reaching a certain development stage, the government will accept the digital transformation results of the subsidized enterprises. According to its digitalization level, the government will implement rewards and punishments, where $v(L-L_G)$, in order to achieve the purpose of project supervision of enterprises and reduce the adverse impact of government subsidies. At the same time, multi-level subsidies and incentives are used to improve the transformation momentum of manufacturing enterprises.

Suppliers and manufacturers share responsibility for digital transformation to develop in the same frequency and to realize the digitalization of the supply chain. Therefore, the supplier will bear the government rewards and penalties in the acceptance stage as $\lambda v(L-L_G)$, the manufacturer will bear the rewards and penalties as $(1-\lambda)v(L-L_G)$, and the supplier and manufacturer objective benefit functions are as follows:

$$J_{S}^{M} = \max_{E_{S}} \int_{0}^{\infty} e^{-\rho t} [\omega(A - bP)\beta L - \frac{1}{2}\mu_{S}(1 - \theta_{S}^{M})E_{S}^{2} + \lambda v(L - L_{G})]dt, \qquad (15)$$

$$J_{M}^{M} = \max_{E_{M}} \int_{0}^{\infty} e^{-\rho t} [(P - \omega)(A - bP)\beta L - \frac{1}{2}\mu_{M}(1 - \theta_{M}^{M})E_{M}^{2} + (1 - \lambda)v(L - L_{G})]dt, \qquad (16)$$

$$J_{G}^{M} = \max_{E_{M}} \int_{0}^{\infty} e^{-\rho t} [P(A-bP)\beta L - \frac{1}{2}\mu_{S}E_{S}^{2} - \frac{1}{2}\mu_{M}E_{M}^{2} + v(L-L_{G})]dt.$$
(17)

Proposition 2. Equilibrium strategies for upstream and downstream firms in the supply chain and the government under government and punishment mechanism.

(1) The optimal equilibrium strategies for digital transformation inputs, government transformation subsidy rates, wholesale prices, and product sales prices for upstream and downstream firms are as follows:

$$\begin{cases} E_{S}^{M^{**}} = \frac{(A^{2}\beta + 8b\lambda v)\alpha_{S}}{8b\mu_{S}(\delta + \rho)(1 - \theta_{S}^{M})}, & E_{M}^{M^{**}} = \frac{[A^{2}\beta + 16(1 - \lambda)bv]\alpha_{M}}{16b\mu_{M}(\delta + \rho)(1 - \theta_{M}^{M})}, \\ P^{M^{*}} = \frac{3A}{4b}, & \omega^{M^{*}} = \frac{A}{2b}, \\ \theta_{S}^{M} = \frac{A^{2}\beta + (1 - \lambda)16bv}{3A^{2}\beta + 16bv}, \\ \theta_{M}^{M} = \frac{2A^{2}\beta + 16\lambda bv}{3A^{2}\beta + 16bv}. \end{cases}$$
(18)

(2) The optimal trajectory of the digitalization level of the supply chain is as follows:

$$L^{M^*} = L^M_{RSS} + (L_0 - L^M_{RSS})e^{-\hat{\alpha}}.$$
 (19)

Among them, $L_{RSS}^{M} = \frac{(A^2\beta + 8b\lambda v)\alpha_s^2}{8b\delta\mu_s(\delta + \rho)(1 - \theta_s^M)} + \frac{[A^2\beta + 16b(1 - \lambda)v]\alpha_M^2}{16b\delta\mu_M(\delta + \rho)(1 - \theta_M^M)}).$

(3) The supplier, manufacturer, and overall supply chain profit optimality function is as follows:

$$\begin{cases} J_{S}^{M^{**}} = \frac{A^{2}\beta + 8b\lambda v}{8b(\delta + \rho)}L + \frac{(A^{2}\beta + 8b\lambda v)^{2}\alpha_{S}^{2}}{128b^{2}\mu_{S}(1 - \theta_{S}^{M})(\delta + \rho)^{2}} + \frac{(A^{2}\beta + 8b\lambda v)[A^{2}\beta + 16(1 - \lambda)bv]\alpha_{M}^{2}}{128b^{2}\mu_{M}(\delta + \rho)^{2}(1 - \theta_{M}^{M})} - \lambda vL_{G}, \\ J_{M}^{M^{**}} = \frac{A^{2}\beta + 16b(1 - \lambda)v}{16b(\delta + \rho)}L + \frac{(A^{2}\beta + 8b\lambda v)[A^{2}\beta + 16(1 - \lambda)bv]\alpha_{S}^{2}}{128b^{2}\mu_{S}(\delta + \rho)^{2}(1 - \theta_{S}^{M})} + \frac{[A^{2}\beta + 16(1 - \lambda)bv]^{2}\alpha_{M}^{2}}{512b^{2}\mu_{M}(\delta + \rho)^{2}(1 - \theta_{M}^{M})} - (1 - \lambda)vL_{G}, \\ J_{C}^{M^{**}} = \frac{3A^{2}\beta + 16bv}{16b(\delta + \rho)}L + \frac{(A^{2}\beta + 8b\lambda v)[A^{2}\beta + 4(2 - \lambda)bv]\alpha_{S}^{2}}{64b^{2}\mu_{S}(\delta + \rho)^{2}(1 - \theta_{S}^{M})} + \frac{[A^{2}\beta + 16(1 - \lambda)v][5A^{2}\beta + 16(1 + \lambda)bv]\alpha_{M}^{2}}{512b^{2}\mu_{M}(\delta + \rho)^{2}(1 - \theta_{M}^{M})} - vL_{G}. \end{cases}$$

$$(20)$$

From Proposition 3, it is clear that the government can further enhance the digitalization of the supply chain after implementing the reward and punishment mechanism, while bringing higher benefits to supply chain member firms.

The enhanced efficiency is related to the strength of government rewards and punishments v and the proportion of rewards and punishments allocated λ .

2.4. Comparative analysis of model results

Corollary 1. When the rate of government subsidies to suppliers and manufacturers for digital transformation is zero, comparing of the level of digital transformation inputs under models C, N and M are as follows:

When
$$0 < \lambda < \frac{2}{5}$$
, $\frac{3}{16(1-\lambda)} < \frac{1}{8\lambda}$.
When $0 < v < \frac{3A^2\beta}{16b(1-\lambda)}$, $E_S^{C^*} > E_S^{M^*} > E_S^{N^*}$, $E_M^{C^*} > E_M^{M^*} > E_M^{N^*}$.
When $\frac{3A^2\beta}{16b(1-\lambda)} \le v < \frac{A^2\beta}{8b\lambda}$, $E_S^{M^*} \ge E_S^{C^*} > E_S^{N^*}$, $E_M^{C^*} > E_M^{M^*} > E_M^{N^*}$.
When $v \ge \frac{A^2\beta}{8b\lambda}$, $E_S^{M^*} \ge E_S^{C^*} > E_S^{N^*}$, $E_M^{M^*} \ge E_M^{C^*} > E_M^{N^*}$. Similarly, we can get the case of $\frac{3}{16(1-\lambda)} > \frac{1}{8\lambda}$.

when $\frac{2}{5} \le \lambda < 1$, which is not described due to the space problem.

Corollary 2. When the rate of government subsidies to suppliers and manufacturers for digital transformation is zero, the results of comparing of steady-state digitization levels under modes C, N and M are as follows:

When $0 < \lambda < \frac{2}{5}$, $\frac{3}{16(1-\lambda)} < \frac{1}{8\lambda}$. When $0 < v < \frac{A^2\beta}{8\lambda b}$, $L^{C^*} > L^{M^*} > L^{N^*}$.

When $v \ge \frac{A^2 \beta}{8\lambda b}$, $L^{M^*} \ge L^{C^*} > L^{N^*}$. Similarly, we can get the case of $\frac{3}{16(1-\lambda)} > \frac{1}{8\lambda}$, when $\frac{2}{5} \le \lambda < 1$, which is not described due to the space problem.

Corollary 2 shows the lowest level of supply chain digitization under mode *N*. When $v = \max[\frac{A^2\beta}{8b\lambda}, \frac{3A^2\beta}{16b(1-\lambda)}]$, the level of supply chain digitization under mode *M* reaches the level of mode *C*. The level of supply chain digitization under mode *M* increases with the increase of *v*.

Corollary 3. When the rate of government subsidies to suppliers and manufacturers for digital

transformation is zero, the results of comparing of the optimal benefit steady state values of the supply chain under models C, N and M are as follows:

When
$$L_{G} > L_{G1}$$
, $v_{2} > 0$.
Therefore, when $0 < v < v_{2}$, $J_{C}^{C^{*}} > J_{C}^{N^{*}} > J_{C}^{M^{*}}$.
When $v_{2} \le v < v_{1}$, $J_{C}^{C^{*}} > J_{C}^{N^{*}} > J_{C}^{N^{*}}$,
When $v \ge v_{1}$, $J_{C}^{M^{*}} \ge J_{C}^{C^{*}} > J_{C}^{N^{*}}$.
When $0 < L_{G} \le L_{G1}$, $v_{2} \le 0$.
Therefore, when $0 < v < v_{1}$, $J_{C}^{C^{*}} > J_{C}^{M^{*}} > J_{C}^{N^{*}}$. When $v \ge v_{1}$, $J_{C}^{M^{*}} \ge J_{C}^{C^{*}} > J_{C}^{N^{*}}$.
Among them, $0 < v_{2} < v_{1}$,
 $v_{1} = \frac{(\rho + \delta)}{2[f\lambda + g(1 - \lambda)]} [l_{1} + \sqrt{l_{1}^{2} + 4[f\lambda + g(1 - \lambda)](\frac{5fn^{2}}{128} + \frac{13gn^{2}}{256})}]$, $v_{2} = \frac{(\delta + \rho)l_{2}}{f\lambda + g(1 - \lambda)}$,
 $n = \frac{A^{2}\beta}{b(\delta + \rho)}$, $f = \frac{\alpha_{s}^{2}}{\mu_{s}\delta}$, $g = \frac{\alpha_{M}^{2}}{\mu_{M}\delta}$, $l_{1} = L_{G}(\rho + \delta) - \frac{1}{16}nf(3\lambda + 2) - \frac{1}{16}ng(4 - 3\lambda)$,
 $l_{v} = (\delta + \rho)L_{v} - \frac{1}{2}(3\lambda + 2)nf - \frac{1}{2}(4 - 3\lambda)ng$,

$$L_{G1} = \frac{(3\lambda + 2)nf + (4 - 3\lambda)ng}{16(\delta + \rho)} + [L_0 - \frac{(3\lambda + 2)nf + (4 - 3\lambda)ng}{16(\delta + \rho)}]e^{-\rho t}.$$

Corollary 3 shows that when the target digitization level set by the government $L_G > L_{G1}$ conditions, the actual digitization level of the supply chain is lower than the target digitization level when $0 < v < v_2$, and all supply chain member firms are penalized. When the overall supply chain benefit of mode M is greater than that of mode N in the case of $v_2 \le v$, the government subsidy mechanism can improve the overall benefit of the enterprise. When $v = v_1$, the overall supply chain benefit of mode M reaches the level of mode C. Under the condition that the target digitization level $L_G \le L_{G1}$, the overall supply chain benefit of mode M is greater than that of mode M is greater than that of mode M is greater than that penalty or reward.

Corollary 4. The comparative results of the comparison of government subsidy rates under models C, N and M after obtaining government subsidies are

$$\theta_C < \theta_M < \theta_N$$

Corollary 4 shows that the rate of government subsidies under model M is higher than under model N. Model M is effective in reducing the rate of government subsidies and in reducing government expenditure. At the same time, the rate of government subsidies received by firms under model M is inversely proportional to the proportion of rewards and penalties borne by the firms themselves. The higher the up-front government subsidiy rate, the lower the proportion of rewards and penalties that are instead assessed.

Corollary 5. The comparative results of the degree of digital input and digital level of enterprises under models *C*, *N* and *M* after obtaining government subsidies are as follows:

When
$$0 < v < \frac{A^2 \beta}{16b}$$
, $E_S^{C^*} > E_S^{M^*} > E_S^{N^*}$, $E_M^{C^*} > E_M^{M^*} > E_M^{N^*}$; $L^{C^*} > L^{M^*} > L^{N^*}$.
When $v \ge \frac{A^2 \beta}{16b}$, $E_S^{M^*} \ge E_S^{C^*} > E_S^{N^*}$, $E_M^{M^*} \ge E_M^{C^*} > E_M^{N^*}$; $L^{M^*} \ge L^{C^*} > L^{N^*}$.

Corollary 5 shows that the degree of digital input and the level of digitalization of firms under model N is the lowest after the government participates in the game. Combined with Corollary 1, when $v = \frac{A^2\beta}{16b}$, the degree of digital input and the level of digitization of enterprises under mode M reaches

the level of mode C. Additionally, $\frac{A^2\beta}{16b} < \min[\frac{A^2\beta}{16\lambda b}, \frac{3A^2\beta}{16(1-\lambda)b}]$, the threshold of government rewards and penalties is reduced compared to when government is an exogenous variable.

Corollary 6. The results of the comparison of steady-state values of optimal r benefits for supplier and manufacture under models *C*, *N* and *M* after receiving government subsidies are as follows:

Under the condition of $L_G > L_{G2}$, $v_5 > 0$. Therefore, when $0 < v < v_5$, $J_S^{C^*} > J_S^{N^*} > J_S^{M^*}$. When $v_5 \le v < v_3$, $J_S^{C^*} > J_S^{N^*} \ge J_S^{N^*}$. When $v \ge v_3$, $J_S^{M^*} \ge J_S^{C^*} > J_S^{N^*}$.

Under the condition of $0 < L_G \le L_{G2}$, $v_5 \le 0$. Therefore, when $0 < v < v_3$, $J_S^{C^*} > J_S^{M^*} > J_S^{N^*}$. When $v > v_3$, $J_S^{M^*} \ge J_S^{C^*} > J_S^{N^*}$.

Under the condition of $L_G > L_{G3}$, $v_6 > 0$. Therefore, when $0 < v < v_6$, $J_M^{C^*} > J_M^{N^*} > J_M^{M^*}$. When $v_6 \le v < v_4$, $J_M^{C^*} > J_M^{M^*} \ge J_M^{N^*}$. When $v \ge v_4$, $J_M^{M^*} \ge J_M^{C^*} > J_M^{N^*}$.

Under the condition of $0 < L_G \le L_{G2}$, $v_6 \le 0$. Therefore, when $0 < v < v_4$, $J_M^{C^*} > J_M^{M^*} > J_M^{N^*}$. When $v > v_4$, $J_M^{M^*} \ge J_M^{C^*} > J_M^{N^*}$.

Among them, $v_5 < v_3$, $v_6 < v_4$,

$$v_{3} = \frac{(\rho + \delta)}{2\lambda(f + g)} [l_{3} + \sqrt{l_{3}^{2} + 4(f + g)(\frac{fn^{2}}{128} + \frac{gn^{2}}{128})}], \quad l_{3} = \lambda(\delta + \rho)L_{G} - \frac{(2 + 3\lambda)}{16}n(f + g),$$
$$= \frac{(\rho + \delta)}{(1 + 1)^{2}} [l_{4} + \sqrt{l_{3}^{2} + 4(f + g)(\frac{5fn^{2}}{128} + \frac{5gn^{2}}{128})}], \quad l_{4} = (1 - \lambda)(\delta + \rho)L_{G} - \frac{(4 - 3\lambda)}{16}n(f + g),$$

$$v_4 = \frac{(\rho + \delta)}{2(f+g)(1-\lambda)} [l_4 + \sqrt{l_4^2 + 4(f+g)(\frac{5fn^2}{256} + \frac{5gn^2}{256})}], \quad l_4 = (1-\lambda)(\delta + \rho)L_G - \frac{(4-3\lambda)}{16}n(f+g),$$

$$\begin{split} v_5 &= (\delta + \rho) \frac{\lambda(\delta + \rho)L_G - \frac{(2+3\lambda)}{16}nf - \frac{(2+3\lambda)}{16}ng}{(f+g)\lambda}, \\ L_{G2} &= \frac{(2+3\lambda)(nf+ng)}{16\lambda(\delta + \rho)} + [L_0 - \frac{(2+3\lambda)(nf+ng)}{16\lambda(\delta + \rho)}]e^{-\rho t}, \end{split}$$

$$v_{6} = (\delta + \rho) \frac{(1 - \lambda)(\delta + \rho)L_{G} - \frac{(4 - 3\lambda)}{16}nf - \frac{(4 - 3\lambda)}{16}ng}{(1 - \lambda)(f + g)}$$
$$L_{G3} = \frac{(4 - 3\lambda)(nf + ng)}{16(1 - \lambda)(\delta + \rho)} + [L_{0} - \frac{(4 - 3\lambda)(nf + ng)}{16(1 - \lambda)(\delta + \rho)}]e^{-\rho t}.$$

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Corollary 6 shows that under the condition of $L_G > L_{G2}$, $v_5 > 0$ and $0 < v < v_5$, the actual digitization level under mode M is smaller than the target digitization level, and the lowest benefit is obtained by the supplier under mode M. When $v_5 \le v < v_3$, the supplier under mode N receives the lowest benefit, and when $v = v_3$, the supplier under mode M receives the benefit up to the level under mode C; similarly, under the condition of $L_G \le L_{G3}$, $v_5 \le 0$ and $0 < v < v_3$, the lowest benefit is obtained by the supplier under mode N, when the benefit obtained by the supplier under mode M receives the benefit obtained by the supplier under mode N, when the benefit obtained by the supplier under mode M.

3. Example analysis

To better prove the above proposition, this paper uses MATLAB software to conduct numerical simulations of the above decision equilibrium results. By assigning values to variables, the comparison of enterprise decisions under different game conditions can be further visualized, and the role of different cooperation contracts in promoting the digital transformation and upgrading of the supply chain can be further analyzed. This section analyses the equilibrium strategies of the supply chain in steady state and in non-steady state by means of arithmetical examples.

3.1. Steady-state analysis

With reference to the current data of manufacturing enterprises, the simulation parameters are assumed to be $\alpha_S=0.5$, $\alpha_M=0.6$, $\delta=1.5$, $\mu_S=2$, $\mu_M=3$, $\beta=0.4$, A=50, b=5, $\rho=0.9$, $L_0=10$, $\varphi=0.4$, $\lambda=0.6$. To ensure the significance of the steady-state strategy values, this section calculates the strategies for each of the three models when the strength of government rewards and penalties b = 0, 10, 20 and 30, respectively, and the results are presented in Table 1.

			0 0		
Mode	θ	v	L	J_S	J_M
С	0	-	122.5	3162.5	3121.5
N	0	-	46.25	1232.81	665.62
М	0	0	46.25	1232.81	665.62
	0	10	58.55	1631.75	814.87
	0	20	70.85	2110.22	1054.11
	0	30	83.15	2758.51	1302.42

 Table 1. Sensitivity analysis of the strength of government rewards and sanctions.

Table 1 represents a comparison of the three game models when the government's upfront subsidy rate is zero, and the results of their analyses are as follows:

(1) The lowest level of steady-state digitisation is achieved under mode N. This mode is not conducive to greater benefits for suppliers and manufacturers. With increased government rewards and penalties, mode M can effectively improve digitisation. However, there is still a gap between the level of digitisation and the ideal mode C. The impact of increased government rewards and penalties on the level of digitisation is not high. This suggests that in the absence of sufficient up-front government subsidies, reliance on government reward and punishment mechanisms alone has a more limited impact on the level of digitisation.

(2) With the change in government incentives and penalties, the level of benefits for both suppliers

and manufacturers increased. However, the increase in the level of benefits for suppliers was greater than the increase in the level of benefits for manufacturers. The government's implementation of a reward and punishment mechanism for upstream enterprises can more effectively increase the level of corporate benefit.

Figures 1–5 analyze when the time tends to infinity and the supply chain system reaches a steady state. With the government as the game party and participating in the game model, the effect of government reward and punishment b on the equilibrium game outcome of enterprises in three different decision models is shown.

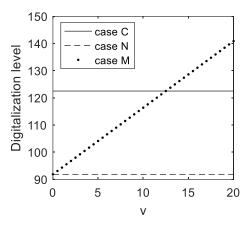


Figure 1. The level of digitization varies with *v*.

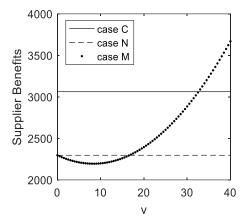


Figure 2. Optimal benefit of supplier varies with v.

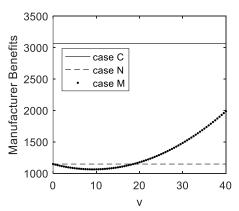


Figure 3. Optimal benefit of manufacturer varies with v.

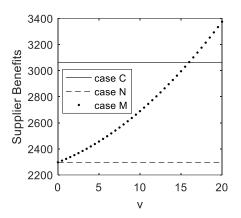


Figure 4. Optimal benefit of supplier varies with v.

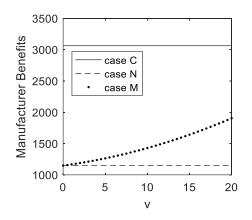


Figure 5. Optimal benefit of manufacturer varies with v.

Figure 1 shows that the digitization level under mode M gradually increases with the increase of government incentives and penalties b. Additionally, the digitization level under mode M is always greater than that under mode N. Therefore, the reward and punishment mechanism can effectively improve the digitization level of the supply chain.

Figures 2–5 show that (1) when the target digitization level $L_G > \max[L_{G2}, L_{G3}]$ set by the government when the target digital level set by the government. As v increases, the optimal benefit level of suppliers under model M exhibits a decreasing and then increasing trend, further proving Corollary 5. However, the overall trend still shows an increasing trend. Moreover, comparing Figures 2 and 3, the supplier, as an upstream firm in the supply chain, has a significantly higher growth trend in his optimal benefit than the manufacturer. (2) The optimal benefit for the firm under model M is related to the level of government rewards and penalties, the level of digitization of the target. When the target digitization level is $L_G \leq \max[L_{G2}, L_{G3}]$ set by the government, the actual digitization level gradually increases as v increases.

Regardless of the government penalties and incentives for firms, the optimal benefit for firms under model M is higher than model N. It can be seen that when the target digitization level $L_G \leq \max[L_{G2}, L_{G3}]$ set by the government, the government subsidy mechanism can effectively motivate enterprises to increase the degree of digitalization investment and improve the level of digitalization, thus increasing the benefits of enterprises. (3) Comparing Figures 2–5, the higher the goal set by the government, the greater the need for greater rewards and penalties to achieve the same level of benefits.

3.2. Non-steady state analysis analysis

To reflect the time characteristics, v=12, $L_G=100$ are given in this paper. Additionally, we plot the level of supply chain digitisation and the level of optimal firm utility against the temporal characteristics for each of the three decision models. The results of the analysis are as follows.

Figure 6 shows that the digitisation levels of the supply chain converge to their respective steadystate values over time. The lowest digitisation level is found under mode N. Under mode M, the digitisation level is higher than under mode N, whether or not steady state is reached. In the early stages of the non-steady state, the digitisation level under mode M can reach the same level as under mode C. Combining with Figure 1, as reward and punishment b increase, mode M will reach the digitisation level of mode C.

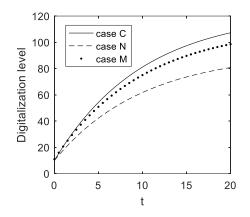


Figure 6. Comparison of digitization levels.

Figures 7–9 show the optimal benefits for when government incentives and penalties are certain. (1) In both steady and non-steady states, the optimal utility of both suppliers and producers are higher in mode M than in mode N, and the utility of firms is effectively enhanced.

(2) Government participation can effectively improve the interests of firms, while the improvement for suppliers of upstream firms is greater under mode M. Combined with the steady state analysis in the previous section, it can be seen that the level of interests of suppliers of upstream firms is always higher than that of manufacturers of downstream firms, regardless of whether the supply chain reaches steady state or not.

(3) In the early stage of the non-steady state, the level of benefits of suppliers under mode M may reach the same level as that under mode C. As the supply chain tends to steady state, the gap between the level of benefits of suppliers under mode M and mode C gradually widens. This shows that in the early stages of a company's digital transformation. The subsidy efficiency of mode M is highest and mode M is able to maximise the level of benefit to suppliers.

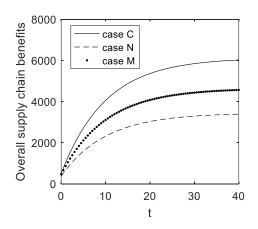


Figure 7. Comparison of total supply chain benefits.

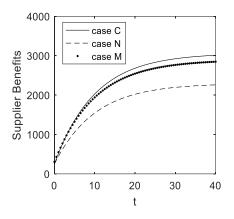


Figure 8. Comparison of supplier benefits.

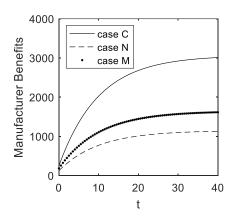


Figure 9. Comparison of manufacturer benefits.

4. Conclusions and recommendations

This paper introduces the government as a player into the transformation game, and considers the influence of government subsidies on the cost of digital transformation among manufacturing supply chain member enterprises. Moreover, this paper introduces the mechanism of government incentives

and penalties It constructs a three-party game model with the government, suppliers and manufacturers as players, studies the coordination strategies for digital transformation between upstream and downstream enterprises alongside the utility of government subsidies; it ultimately provides a cooperative transformation strategy for upstream and downstream enterprises considering the formulation of government subsidy policy. The paper provides a theoretical basis for the formulation of cooperative transformation strategies and the role of government subsidy policies among upstream and downstream enterprises in the supply chain. Taking the digital transformation level of enterprises as the state variable and marginal profit as the dynamic variable, three different decision models are constructed. The influence of the level of digital transformation and product sales prices on market demand are both considered, and through comparative analysis of the differential game models, the following conclusions are obtained.

(1) Previous literature on government subsidies has rarely considered the evaluation and monitoring mechanisms of government subsidies. This paper introduces a government reward and punishment mechanism to evaluate the digital transformation effectiveness of subsidised firms, in response to the government's increase in upfront subsidies. The analysis proves that it is effective in guiding members of the supply chain. The level of digital investment, the level of digitalisation, and the level of benefits of enterprises all increase with the increase in government rewards and penalties. The upfront subsidies provided by the government can increase the level of benefits for suppliers and manufacturers, as well as the supply chain as a whole. The innovative introduction of government incentives and penalties in this paper further increases their level of digitisation and benefits. Comparing to previous studies, this paper takes a more comprehensive view of the situation of government subsidies to firms and discusses the lack of effective assessment of the efficiency of government subsidies to firms and the reverse direction of government to firms.

(2) Under the government reward and punishment model, suppliers and manufacturers share the responsibility for digital transformation, which to some extent weakens the strength of government rewards. The degree of digital investment and the level of benefits obtained by supply chain member firms are positively related to the proportion of their own reward and punishment allocation.

(3) When the target level of digitisation set by the government is high, there will be a threshold for the strength of government rewards and punishments in the government reward and punishment model, and when the strength of rewards and punishments is low, the level of benefits to the firm will instead be reduced to a small extent. On the other hand, when the government sets a relatively low target level of digitisation, the level of benefits to the enterprise may increase regardless of whether the government rewards and punishments are punitive or rewarding to suppliers and manufacturers. Therefore, under the conditions of a relatively low target level of digitisation set by the government and at the beginning of digital transformation, the supervisory effect of the reward and punishment mechanism received by the firm can increase the benefits by increasing the level of digitisation, regardless of whether the firm is rewarded or punished.

(4) In the discrete model, suppliers always receive a higher level of benefits than manufacturers, and the rate of increase in the level of benefits for suppliers is higher than for manufacturers. By comparing the existing literature, it can be concluded that the original may be that upstream enterprises have a price advantage and their prices have a greater impact on market demand, resulting in upstream enterprises often gain more benefits more easily with the same level of digital input. Therefore, for the synergistic development of supply chain members, it may be possible to reach some kind of trade agreement between upstream and downstream enterprises in the supply chain, which will increase the

overall benefits of the supply chain, reduce internal transaction costs, and facilitate the development of digital transformation of downstream enterprises.

(5) According to the steady-state and non-steady-state analyses, the level of enterprise benefits under the reward and punishment model is related to both the target digitalisation level and the strength of government rewards and punishments. If the target level of digitisation is too low or too high, the level of benefits to enterprises will be affected, given the same level of rewards and punishments. Over time, no matter how much enterprises invest or how much government subsidy is provided, the level of commitment to digital transformation in the same model will not change significantly. Therefore, enterprises need to allocate resources rationally, and the government needs to set targets, rewards and punishments, and allocate subsidies rationally according to different circumstances. It is important to avoid non-economic effects and waste of resources caused by unreasonable levels of investment in digital transformation, and government incentives and penalties.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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