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Research article

Asymptotic synchronization analysis of fractional-order octonion-valued neural networks with impulsive effects

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Abstract: This paper deals with a class of fractional-order octonion-valued neural networks (FOOVNNs) with impulsive effects. Firstly, although the multiplication of octonion numbers does not satisfy the commutativity and associativity, we don't need to separate an octonion-valued system into eight real-valued systems. Secondly, by applying the appropriate Lyapunov function, and inequality techniques, we obtain the global asymptotical synchronization of FOOVNNs. Finally, we give two illustrative examples to illustrate the feasibility of the proposed method.

Keywords: octonion algebra; synchronization; fractional-order; neural networks; impulsive effects
Mathematics Subject Classification: 34A08, 34A37, 34K24, 34K25

1. Introduction

As well known, the neural network model has been widely studied over the past few decades. Neural networks are extensively applied in many fields, such as pattern recognition, signal processing, image recognition, and so on (see [1–8]). Compared with integer order calculus, fractional calculus was incorporated into neural networks, it is called fractional-order neural networks (FONNs). Recently, under the research of many scholars, many achievements have been made in the dynamic behaviors of FONNs, especially stability. For example, many authors have explored stability and finite-time stability (see [9–17]). In the real world, many evolutionary processes, which are usually subject to short-time perturbations, exhibit impulsive effects. In the exploration of the neural network, the phenomenon of impulse is often considered, and many scholars have got many good results of FONNs with impulsive effects (see [18–20]).

Octonion-valued neural network, as a generalization of real-valued, complex-valued, and quaternion-valued neural network, was first introduced by Popa (see [21]). Octonion-valued neural network is not a special case of Clifford-value neural network, because octonion algebra is non-commutative, but also nonassociative. In the application of octonion-valued neural networks, some interesting results were presented (see [22–27]). Some authors have discussed fractional-order octonion-valued neural networks (FOOVNNs) (see [28]).

In the dynamical behaviors of FONNs, synchronization, as a hot and interesting topic in FONNs, plays a key role and has attracted the attention of many scholars. So far, many researchers have explored delayed FORNNs and produced interesting results on synchronization. For instance, many good results on synchronization for FONNs have been reported (see [29–38]).

With inspiration from the previous researches, to fill the gap in the research field of synchronization of FOOVHNNs with impulsive effects, the work of this article comes from three main motivations. (1) Recently, in [28], some scholars have explored FOOVNNs via the decomposition method. However, there are few results of FOOVNNs. (2) In [32–34], some authors have discussed the synchronization of fractional-order quaternion-valued neural networks via the decomposition method. But there has been no paper on the synchronization of FOOVNNs with impulsive effects. (3) Synchronization is a significant dynamical property for differential equations, thus it is worth exploring the synchronization. Therefore, it is worthwhile to investigate the synchronization of FOOVNNs with impulsive effects via the non-decomposition method and the Lyapunov function method.

Compared with the previous literatures, the main contributions of this article are listed as follows. (1) Firstly, this is the first time to explore the synchronization of FOOVNNs with impulsive effects. (2) Secondly, the multiplication of octonion numbers does not satisfy the commutativity and associativity, we don't need to separate the octonion-valued system into four complex-valued systems or eight real-valued systems. (3) Thirdly, unlike [28, 32–34], in this paper, we explore the synchronization of FOOVNNs via the non-decomposition method. (4) Fourthly, our method in this paper can be used to explore the stability and synchronization of other types of FOOVNNs.

This paper is organized as follows: In Section 2, we introduce some definitions and Lemmas. In Section 3, we establish some sufficient conditions for global asymptotical synchronization for System (2.1) and System (2.2). In Section 4, one numerical example is provided to verify the effectiveness of the theoretical results. Finally, we draw a conclusion in Section 5.

Notations: $\mathbb{R}$ denotes the set of real numbers, $\mathbb{C}$ denotes the set of complex numbers, $\mathbb{O}$ denotes the set of octonion numbers, $\mathbb{O}^n$ denotes the n dimensional octonion numbers. For $x = \sum_{p=0}^{7} [x]_p \in \mathbb{O}$, its norm as $\|x\|_{\mathbb{O}} = |x|$. For $x = (x_1, x_2, \cdots, x_n) \in \mathbb{O}^n$, its norm as $\|x\|_{\mathbb{O}^n} = \sum_{i=1}^{n} \|x_i\|_{\mathbb{O}}$.

2. Preliminaries

In this section, we shall first recall some fundamental definitions and lemmas.

The algebra of octonion is defined as

$$\mathbb{O} = \left\{ x = \sum_{p=0}^{7} [x]_p e_p \middle| [x]_0, [x]_1, \cdots, [x]_7 \in \mathbb{R} \right\},$$

where e_p are the octonion units, $0 \le p \le 7$, and when $p = 0$, we have $e_0 = 1$. The octonion units obey the octonion multiplication rules: $e_p e_q = -e_q e_p \neq e_q e_p, \forall 0 < p \neq q \le 7$, from which we deduce that

$\mathbb{O}$ is not commutative, and that $(e_pe_q)e_k = -e_p(e_qe_k) \neq e_p(e_qe_k)$, for k, p, q distinct, $0 < k, p, q \leq 7$, or $e_pe_q \neq \pm e_k$, thus $\mathbb{O}$ is also not associative.

Octonion addition is defined by $x + y = \sum_{p=0}^{7}([x]_p + [y]_p)e_p$, scalar multiplication is given by $\alpha x = \sum_{p=0}^{7}(\alpha[x]_p)e_p$, and octonion multiplication is given by the multiplication of the octonion units:

$\times$	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_0	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_1	e_1	$-e_0$	e_3	$-e_2$	e_5	$-e_4$	$-e_7$	e_6
e_2	e_2	$-e_3$	$-e_0$	e_1	e_6	e_7	$-e_4$	$-e_5$
e_3	e_3	e_2	$-e_1$	$-e_0$	e_7	$-e_6$	e_5	$-e_4$
e_4	e_4	$-e_5$	$-e_6$	$-e_7$	$-e_0$	e_1	e_2	e_3
e_5	e_5	e_4	$-e_7$	e_6	$-e_1$	$-e_0$	$-e_3$	e_2
e_6	e_6	e_7	e_4	$-e_5$	$-e_2$	e_3	$-e_0$	$-e_1$
e_7	e_7	$-e_6$	e_5	e_4	$-e_3$	$-e_2$	e_1	$-e_0$

The conjugate of an octonion x is defined as $\bar{x} = [x]_0e_0 - \sum_{p=1}^{7}[x]_pe_p$, its norm as $|x| = \sqrt{x\bar{x}} = \sqrt{\sum_{p=0}^{7}[x]_p^2}$, and its inverse as $x^{-1} = \frac{\bar{x}}{|x|^2}$. We can now see that $\mathbb{O}$ is a normed division algebra, and it can be proved that the only three division algebras that can be defined over the reals are the complex, quaternion, and octonion algebras.

Definition 2.1. *[39] The Caputo fractional derivative of order α for a function $f(t)$ is given as follows:*

$$D^{\alpha}f(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-s)^{n-\alpha-1}f^{(n)}(s)ds,$$

where $t \geq 0$, n is a positive integer, $n-1 < \alpha < n$.

Particularly, when $0 < \alpha < 1$,

$$D^{\alpha}f(t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t (t-s)^{-\alpha}f'(s)ds.$$

In this paper, we will consider the FOOVNNs as following model:

$$\begin{cases} D^{\alpha}x_i(t) & = -c_ix_i(t) + \sum_{j=1}^{n} a_{ij}f_j(x_j(t)) + U_i, \quad t \geq 0, \quad t \neq t_k, \\ \Delta x_i(t_k) & = I_{ik}(t_k, x_i(t_k)), \end{cases} \tag{2.1}$$

where $0 < \alpha < 1$, $i = 1, 2, \cdots, n$, $x_i(t) \in \mathbb{O}$ is the state vector of the ith unit at time t, $c_i > 0$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs, respectively. $a_{ij} \in \mathbb{O}$ denote the strength of connectivity, respectively. The activation functions $f_j \in \mathbb{O}$ show how the jth neuron reacts to input, $U_i \in \mathbb{O}$ denotes the ith component of an external input source introduced from outside the network to the unit i at time t. The fixed moments of the impulse times $\{t_k : k \in \mathbb{N}\}$ satisfy $0 \le t_0 < t_1 < \cdots < t_k < \cdots$, with $t_k \to \infty$ as $k \to \infty$. At the points of discontinuity t_k of the solution $t \mapsto x_i(t)$, we assume that $x_i(t) \equiv x_i(t_k^-)$. It is clear that, in general, the derivatives $D^\alpha x_i(t_k)$ do not exist. On the other hand, the right limits and the left limits of $x_i(t_k)$ exist, but they are unequal, and the difference $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$.

Next, consider System (2.1) as drive system, then the response system is given by

$$\begin{cases} D^\alpha y_i(t) &= -c_i y_i(t) + \sum\limits_{j=1}^{n} a_{ij} f_j(y_j(t)) + U_i + \epsilon_i(t), \ \ t \ge 0, \ \ t \ne t_k, \\ \Delta y_i(t_k) &= I_{ik}(t_k, y_i(t_k)), \end{cases} \tag{2.2}$$

where $i = 1, 2, \cdots, n$, $\epsilon_i \in \mathbb{O}$ is a state-feedback controller, other notations are the same as those in System (2.1).

For $i = 1, 2, \ldots, n$, let $z_i = y_i - x_i$, then from (2.1) and (2.2), the error system is given by

$$\begin{cases} D^\alpha z_i(t) &= -c_i z_i(t) + \sum\limits_{j=1}^{n} a_{ij}\Big(f_j(y_j(t)) - f_j(x_j(t))\Big) + \epsilon_i(t), \ \ t \ge 0, t \ne t_k, \\ \Delta z_i(t_k) &= I_{ik}(t_k, y_i(t_k)) - I_{ik}(t_k, x_i(t_k)). \end{cases} \tag{2.3}$$

The controller ϵ_i is designed as

$$\epsilon_i(t) \ \ = \ \ -d_i z_i(t) + \sum_{j=1}^{n} b_{ij} g_j(z_j(t)), \tag{2.4}$$

where $i = 1, 2, \cdots, n$, $d_i > 0$, $b_{ij}, g_j \in \mathbb{O}$.

In this paper, we need to introduce the following assumptions:

(H_1) For $i = 1, 2, \cdots, n$, $k \in \mathbb{N}$, the impulsive operators

$$I_{ik}(t_k, x_i(t_k)) = -\gamma_{ik} x_i(t_k), \ \ 0 \le \gamma_{ik} \le 2;$$

(H_2) For $j = 1, 2, \cdots, n$, there exist positive constants L_f, L_g such that

$$|f_j(u) - f_j(v)| \le L_f |u - v|,$$

$$|g_j(u) - g_j(v)| \le L_g |u - v|;$$

(H_3) There exists positive constant μ such that

$$\mu := 2c_i^- + 2d_i^- - \sum_{j=1}^{n} |a_{ij}|^2 - nL_f^2 - \sum_{j=1}^{n} |b_{ij}|^2 - nL_g^2 > 0,$$

where

$$c_i^- = \min_{1 \le i \le n}\{c_i\}, \ \ d_i^- = \min_{1 \le i \le n}\{d_i\}.$$

Lemma 2.1. *Set* $x(t) \in \mathbb{O}$ *be a differentiable function. Then, we have:*

$$D^\alpha\Big(x(t)\overline{x(t)}\Big) \le x(t)(D^\alpha \overline{x(t)}) + \overline{x(t)}(D^\alpha x(t)), \ \ t \ge 0$$

where $0 < \alpha < 1$.

Proof. Let $x(t) = \sum_{p=0}^{7}[x(t)]_p e_p$, where $[x(t)]_0, [x(t)]_1, \cdots, [x(t)]_7 \in \mathbb{R}$, the conjugate of an octonion $x(t)$ is defined as $\overline{x(t)} = [x(t)]_0 e_0 - \sum_{p=1}^{7}[x(t)]_p e_p$, then we have

$$\begin{aligned} D^{\alpha}\Big(x(t)\overline{x(t)}\Big) &= D^{\alpha}\Big(\sum_{p=0}^{7}[x(t)]_p^2\Big) \\ &= \sum_{p=0}^{7} D^{\alpha}[x(t)]_p^2 \\ &\le \sum_{p=0}^{7} 2[x(t)]_p D^{\alpha}[x(t)]_p, \end{aligned}$$

$$\begin{aligned} & x(t)(D^{\alpha}\overline{x(t)}) + \overline{x(t)}(D^{\alpha}x(t)) \\ = & \Big([x(t)]_0 e_0 + \sum_{p=1}^{7}[x(t)]_p e_p\Big)\Big(D^{\alpha}[x(t)]_0 e_0 - \sum_{p=1}^{7} D^{\alpha}[x(t)]_p e_p\Big) \\ & + \Big([x(t)]_0 e_0 - \sum_{p=1}^{7}[x(t)]_p e_p\Big)\Big(D^{\alpha}[x(t)]_0 e_0 + \sum_{p=1}^{7} D^{\alpha}[x(t)]_p e_p\Big) \\ = & \sum_{p=0}^{7} 2[x(t)]_p D^{\alpha}[x(t)]_p. \end{aligned}$$

Hence, we have

$$D^{\alpha}\Big(x(t)\overline{x(t)}\Big) \le x(t)(D^{\alpha}\overline{x(t)}) + \overline{x(t)}(D^{\alpha}x(t)), \quad t \ge 0.$$

The proof is completed. □

Lemma 2.2. *For all $u, v \in \mathbb{O}$, and any real constant $\eta > 0$, the following inequality holds:*

$$u\bar{v} + \bar{u}v \le \eta u\bar{u} + \eta^{-1}v\bar{v}.$$

Proof. Set $\theta \in \mathbb{R}$, and $\theta \ne 0$, we consider

$$(\theta u - \theta^{-1}v)\overline{(\theta u - \theta^{-1}v)} = \theta^2 u\bar{u} - u\bar{v} - v\bar{u} + \theta^{-2}v\bar{v} \ge 0,$$

that is, we have that

$$u\bar{v} + v\bar{u} \le \theta^2 u\bar{u} + \theta^{-2}v\bar{v}.$$

Letting $\eta = \theta^2$,

$$u\bar{v} + \bar{u}v \le \eta u\bar{u} + \eta^{-1}v\bar{v}.$$

The proof is completed. □

Lemma 2.3. *[40] Let $V(t) \in \mathbb{R}$ be a continuously differentiable and nonnegative function, satisfying*

$$D^{\alpha}V(t) \le -\mu V(t),$$

where $0 < \alpha < 1$. If $\mu > 0$, then $\lim_{t\to+\infty} V(t) = 0$, $t \ge 0$.

3. Main results

In this section, we will study the synchronization for System (2.1) and System (2.2), based on Lyapunov function method.

Theorem 3.1. *Assume that assumption* (H_2) *holds. If the following condition is satisfied:*

$$0 < \sum_{j=1}^{n} \frac{|a_{ij}|L_f}{c_i^-} < 1.$$

Then System (2.1) *exists a unique equilibrium point.*

Proof. To prove that there exists a unique equilibrium point of System (2.1), we consider the a mapping $\Phi : \mathbb{O}^n \to \mathbb{O}^n$, namely,

$$\Phi(u) = (\Phi_1(u), \Phi_2(u), \cdots, \Phi_n(u))^T,$$

where $\Phi_1(u) \in \mathbb{O}$, $u = (u_1, u_2, \cdots, u_n)^T \in \mathbb{O}^n$, and

$$\Phi_1(u) = \sum_{j=1}^{n} a_{ij} f_j\Big(\frac{u_j}{c_j}\Big) + U_i, \;\; i = 1, 2, \cdots, n.$$

Set $u = (u_1, u_2, \cdots, u_n)^T \in \mathbb{O}^n$ and $v = (v_1, v_2, \cdots, v_n)^T \in \mathbb{O}^n$, we have

$$\begin{aligned} \|\Phi(u) - \Phi(v)\|_{\mathbb{O}^n} &= \sum_{i=1}^{n} \|\Phi_i(u) - \Phi_i(v)\|_{\mathbb{O}} \\ &= \sum_{i=1}^{n} \Big\| \sum_{j=1}^{n} a_{ij} f_j\Big(\frac{u_j}{c_j}\Big) - \sum_{j=1}^{n} a_{ij} f_j\Big(\frac{v_j}{c_j}\Big) \Big\|_{\mathbb{O}} \\ &\le \sum_{i=1}^{n} \sum_{j=1}^{n} |a_{ij}| \Big\| f_j\Big(\frac{u_j}{c_j}\Big) - f_j\Big(\frac{v_j}{c_j}\Big) \Big\|_{\mathbb{O}} \\ &\le \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{|a_{ij}|L_f}{c_i^-} \|u_j - v_j\|_{\mathbb{O}} \\ &\le \sum_{j=1}^{n} \frac{|a_{ij}|L_f}{c_i^-} \|u - v\|_{\mathbb{O}^n}. \end{aligned}$$

Hence, Φ is a contraction mapping on $\mathbb{O}^n$, which means that Φ exists a unique fixed point u^*, such that $\Phi(u^*) = u^*$, that is

$$\sum_{j=1}^{n} a_{ij} f_j\Big(\frac{u_j^*}{c_j}\Big) + U_i = u_i^*, \;\; i = 1, 2, \cdots, n.$$

Consider $c_i x_i^* = u_i^*$, we have

$$-c_i x_i^* + \sum_{j=1}^{n} a_{ij} f_j(x_j^*) + U_i = 0, \;\; i = 1, 2, \cdots, n.$$

Therefore, System (2.1) exists a unique equilibrium point x_i^*. □

Theorem 3.2. *Assume that assumptions* (H_1)–(H_3) *hold. If the following condition is satisfied:*

$$g_j(0) = 0, \quad j = 1, 2, \cdots, n.$$

Then System (2.1) *and System* (2.2) *are asymptotically synchronized.*

Proof. We construct a Lyapunov function as follow:

$$V(t) = \sum_{i=1}^{n} z_i(t)\overline{z_i(t)}. \tag{3.1}$$

For $i = 1, 2, \cdots, n$, from assumption (H_1), we have

$$\begin{aligned} z_i(t_k^+) &= \Delta z_i(t_k) + z_i(t_k^-) \\ &= I_{ik}\big(t_k, y_i(t_k)\big) - I_{ik}\big(t_k, x_i(t_k)\big) + y_i(t_k^-) - x_i(t_k^-) \\ &= -\gamma_{ik} y_i(t_k) + y_i(t_k) - \Big(- \gamma_{ik} x_i(t_k) + x_i(t_k)\Big) \\ &= (1 - \gamma_{ik}) z_i(t_k). \end{aligned} \tag{3.2}$$

Hence,

$$\begin{aligned} V(t_k^+) &= \sum_{i=1}^{n} z_i(t_k^+)\overline{z_i(t_k^+)} \\ &= \sum_{i=1}^{n} |z_i(t_k^+)|^2 \\ &= \sum_{i=1}^{n} |(1 - \gamma_{ik})|^2 |z_i(t_k)|^2 \\ &\le \sum_{i=1}^{n} |z_i(t_k)|^2 \\ &= \sum_{i=1}^{n} z_i(t_k)\overline{z_i(t_k)} \\ &= V(t_k). \end{aligned}$$

According to Lemma 2.1, Lemma 2.2, assumption (H_2), and assumption (H_3), for $i = 1, 2, \cdots, n$, when $t \ge 0$, we have

$$\begin{aligned} D^{\alpha} V(t) &= \sum_{i=1}^{n} D^{\alpha}\Big(z_i(t)\overline{z_i(t)}\Big) \\ &\le \sum_{i=1}^{n} \Big\{ z_i(t) D^{\alpha}\Big(\overline{z_i(t)}\Big) + \overline{z_i(t)} D^{\alpha}\Big(z_i(t)\Big) \Big\} \\ &= \sum_{i=1}^{n} \Big\{ z_i(t) \Big[- c_i \overline{z_i(t)} + \sum_{j=1}^{n} \overline{a_{ij}\Big(f_j(y_j(t)) - f_j(x_j(t))\Big)} \end{aligned}$$

$$
\begin{aligned}
& -d_i\overline{z_i(t)} + \sum_{j=1}^{n} \overline{b_{ij}g_j(z_j(t))}\Big] + \overline{z_i(t)}\Big[- c_i z_i(t) \\
& + \sum_{j=1}^{n} a_{ij}\Big(f_j(y_j(t)) - f_j(x_j(t))\Big) - d_i z_i(t) \\
& + \sum_{j=1}^{n} b_{ij}g_j(z_j(t))\Big]\Big\} \\
= \ & \sum_{i=1}^{n}\Big\{ - c_i\Big(z_i(t)\overline{z_i(t)} + \overline{z_i(t)}z_i(t)\Big) - d_i\Big(z_i(t)\overline{z_i(t)} \\
& + \overline{z_i(t)}z_i(t)\Big) + \sum_{j=1}^{n}\Big[z_i(t)\overline{a_{ij}}\overline{\Big(f_j(y_j(t)) - f_j(x_j(t))\Big)} \\
& + \overline{z_i(t)}a_{ij}\Big(f_j(y_j(t)) - f_j(x_j(t))\Big)\Big] \\
& + \sum_{j=1}^{n}\Big[z_i(t)\overline{b_{ij}g_j(z_j(t))} + \overline{z_i(t)}b_{ij}g_j(z_j(t))\Big]\Big\} \\
\leq \ & \sum_{i=1}^{n}\Big\{ - 2c_i^{-} z_i(t)\overline{z_i(t)} - 2d_i^{-} z_i(t)\overline{z_i(t)} + \sum_{j=1}^{n}\Big(z_i(t)\overline{a_{ij}}\Big) \\
& \times\Big(\overline{z_i(t)}a_{ij}\Big) + \sum_{j=1}^{n}\Big(f_j(y_j(t)) - f_j(x_j(t))\Big) \\
& \times\overline{\Big(f_j(y_j(t)) - f_j(x_j(t))\Big)} + \sum_{j=1}^{n}\Big(z_i(t)\overline{b_{ij}}\Big) \\
& \times\Big(\overline{z_i(t)}b_{ij}\Big) + \sum_{j=1}^{n} g_j(z_j(t))\overline{g_j(z_j(t))}\Big\} \\
\leq \ & \sum_{i=1}^{n}\Big\{ - 2c_i^{-} z_i(t)\overline{z_i(t)} - 2d_i^{-} z_i(t)\overline{z_i(t)} + \sum_{j=1}^{n} |a_{ij}|^2 \\
& \times z_i(t)\overline{z_i(t)} + \sum_{j=1}^{n} L_f^2 z_j(t)\overline{z_j(t)} + \sum_{j=1}^{n} |b_{ij}|^2 \\
& \times z_i(t)\overline{z_i(t)} + \sum_{j=1}^{n} L_g^2 z_j(t)\overline{z_j(t)}\Big\} \\
\leq \ & \sum_{i=1}^{n}\Big\{ - 2c_i^{-} - 2d_i^{-} + \sum_{j=1}^{n} |a_{ij}|^2 + nL_f^2 \\
& + \sum_{j=1}^{n} |b_{ij}|^2 + nL_g^2\Big\} \cdot z_i(t)\overline{z_i(t)} \\
\leq \ & -\mu V(t),
\end{aligned}
\tag{3.3}
$$

where

$$\mu := 2c_i^- + 2d_i^- - \sum_{j=1}^{n} |a_{ij}|^2 - nL_f^2 - \sum_{j=1}^{n} |b_{ij}|^2 - nL_g^2 > 0.$$

Therefore, according to Lemma 2.3, we can get $\lim_{t\to+\infty} V(t) = 0$. This implies that the drive System (2.1) and the response System (2.2) can achieve asymptotic synchronization. The proof is completed. □

4. Illustrative examples

In this section, we give two examples to illustrate the feasibility and effectiveness of main results.

Example 4.1. *Consider the following FOOVNNs with two neurons as the drive system:*

$$\begin{cases} D^\alpha x_i(t) &= -c_i x_i(t) + \sum\limits_{j=1}^{2} a_{ij} f_j(x_j(t)) + U_i, \ \ t \geq 0, \ \ t \neq t_k, \\ x_1(t_k^+) &= 0.45 x_1(t_k), \\ x_2(t_k^+) &= 0.55 x_2(t_k), \end{cases} \tag{4.1}$$

the response system is given by

$$\begin{cases} D^\alpha y_i(t) &= -c_i y_i(t) + \sum\limits_{j=1}^{2} a_{ij} f_j(y_j(t)) + U_i + \epsilon_i(t), \ \ t \geq 0, \ \ t \neq t_k, \\ y_1(t_k^+) &= 0.45 y_1(t_k), \\ y_2(t_k^+) &= 0.55 y_2(t_k), \end{cases} \tag{4.2}$$

and the controller ϵ_p is designed as

$$\epsilon_i(t) \ \ = \ \ -d_i z_i(t) + \sum_{j=1}^{2} b_{ij} g_j(z_j(t)), \tag{4.3}$$

where $i = 1, 2$, $\alpha = 0.85$, $c_1 = 23$, $c_2 = 25$, $d_1 = 13$, $d_2 = 20$, and

$$a_{11} = (1, 2, 2, 1, 1, -1, 2, 2)^T, \ \ a_{12} = (2, 2, 1, 1, -1, 1, -2, 2)^T,$$

$$a_{21} = (1, 1, -1, -1, 2, 2, 1, 1)^T, \ \ a_{22} = (2, 1, 1, 2, 2, -2, 1, -1)^T,$$

$$b_{11} = (2, 2, -1, -1, 2, 2, 1, 1)^T, \ \ b_{12} = (-1, 2, 2, -2, 1, 2, 1, 1)^T,$$

$$b_{21} = (1, 2, 2, -2, -1, 1, 2, 2)^T, \ \ b_{22} = (2, 2, -2, -2, 1, 1, 2, 2)^T,$$

$$U_1 = (2, 3, -4, -5, 6, 8, 9, 10)^T, \ \ U_1 = (4, 2, 3, 5, 9, -10, 8, 7)^T,$$

$$f_j = \sum_{p=0}^{7} \frac{1}{2} \sin([x_j]_p) e_p, \ \ g_j = \sum_{p=0}^{7} \frac{1}{3} \cos([z_j]_p) e_p$$

and the impulsive moments constrained by $0 < t_1 < t_2 < t_3 < \cdots$, $t_{k+1} - t_k = 0.75$, $\lim_{k\to+\infty} t_k = +\infty$. *Let* $t_1 = \frac{1}{2} + \frac{\pi}{5}$, *and by calculating, we have*

$$L_f = \frac{1}{2}, \ L_g = \frac{1}{3},$$

and

$$\begin{aligned} \mu \ &:= \ 2c_i^- + 2d_i^- - \sum_{j=1}^{n} |a_{ij}|^2 - nL_f^2 \\ &\quad - \sum_{j=1}^{n} |b_{ij}|^2 - nL_g^2 > 4.7222 > 0. \end{aligned}$$

It is not difficult to verify that all conditions (H_1)–(H_3) *are satisfied. Therefore, by Theorem* 3.2, *the Systems* (4.1) *and* (4.2) *are globally asymptotically synchronized, which is shown in Figures 1–6.*

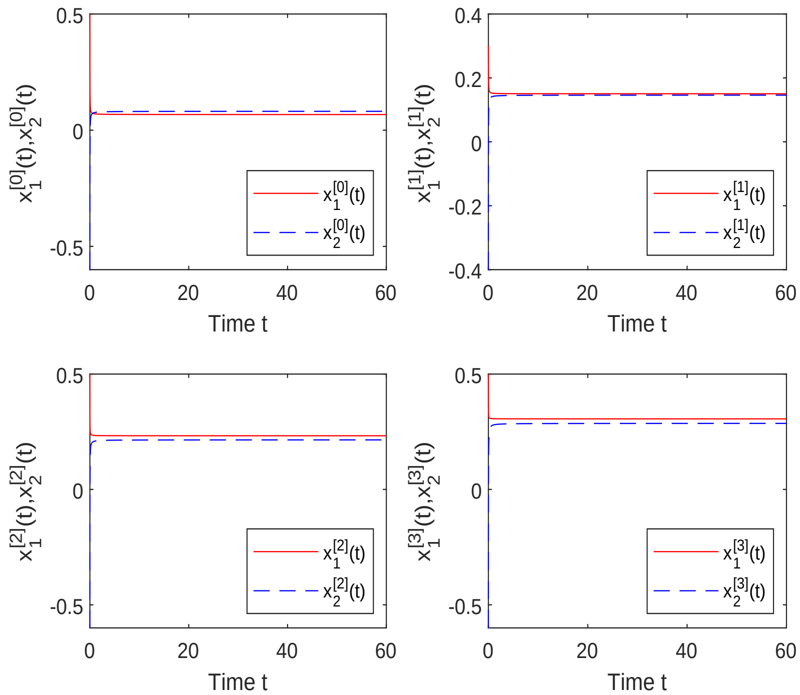

Figure 1. The states of $x_i^{[p]}(t)$, $i = 1, 2$, p = 0,1,2,3.

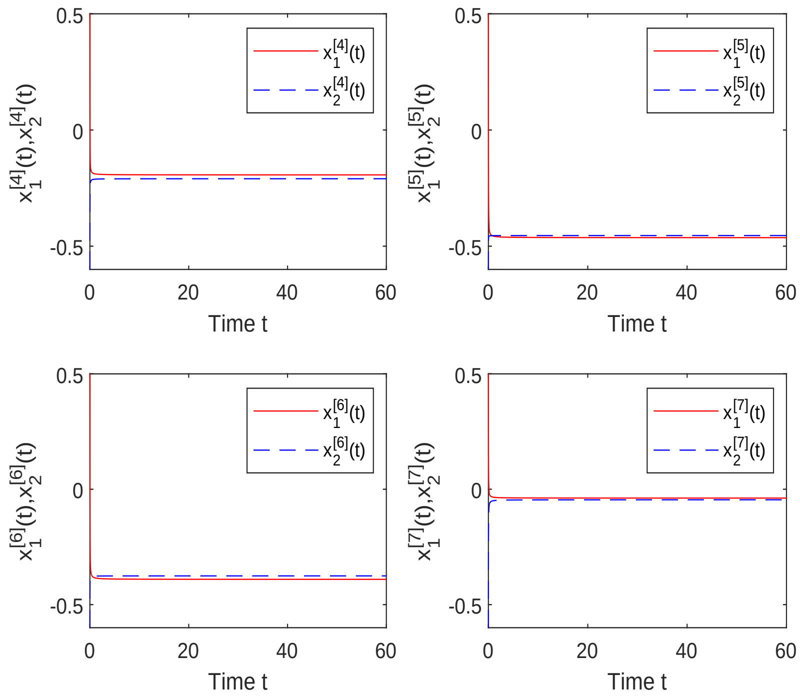

Figure 2. The states of $x_i^{[p]}(t)$, $i = 1, 2$, p = 4,5,6,7.

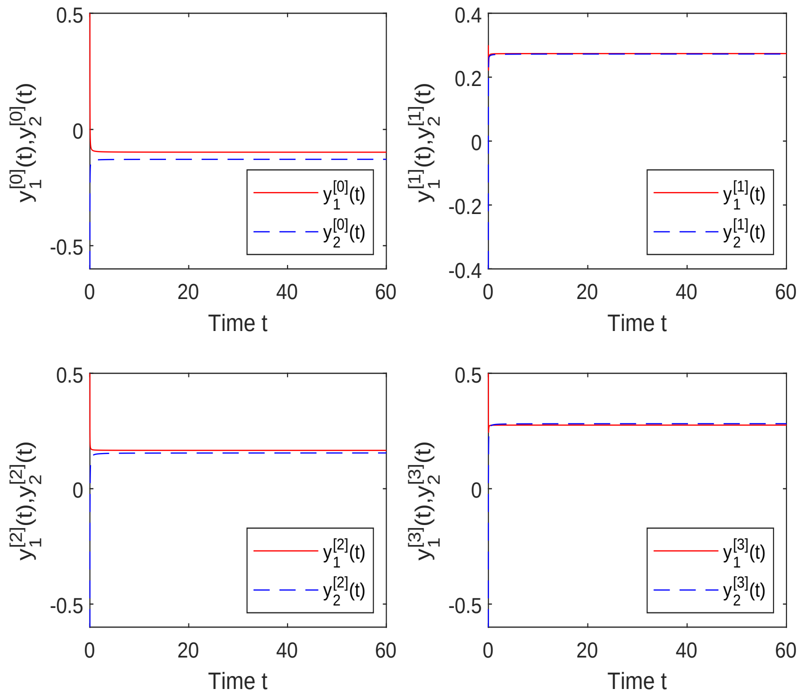

Figure 3. The states of $y_i^{[p]}(t)$, $i = 1, 2$, p = 0,1,2,3.

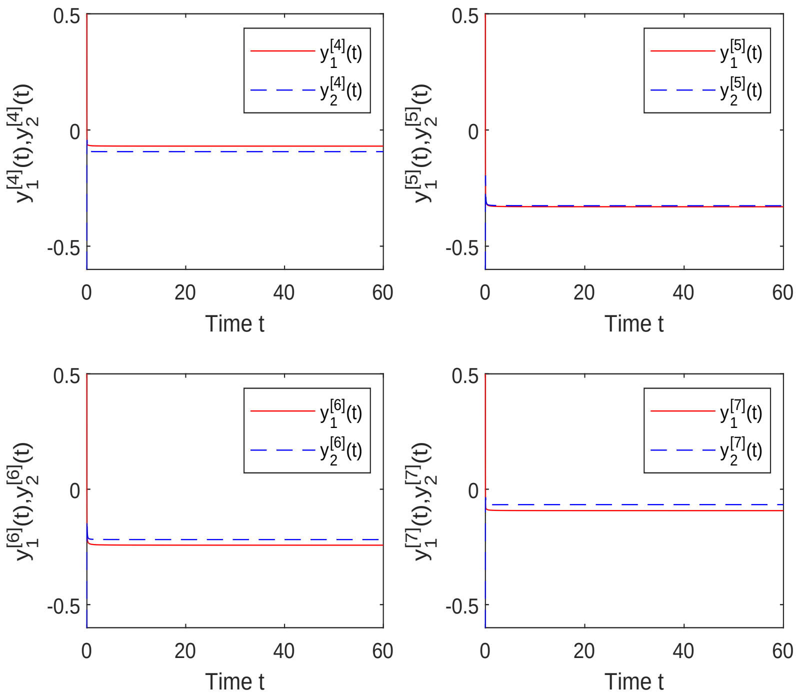

Figure 4. The states of $y_i^{[p]}(t)$, $i = 1, 2$, p = 4,5,6,7.

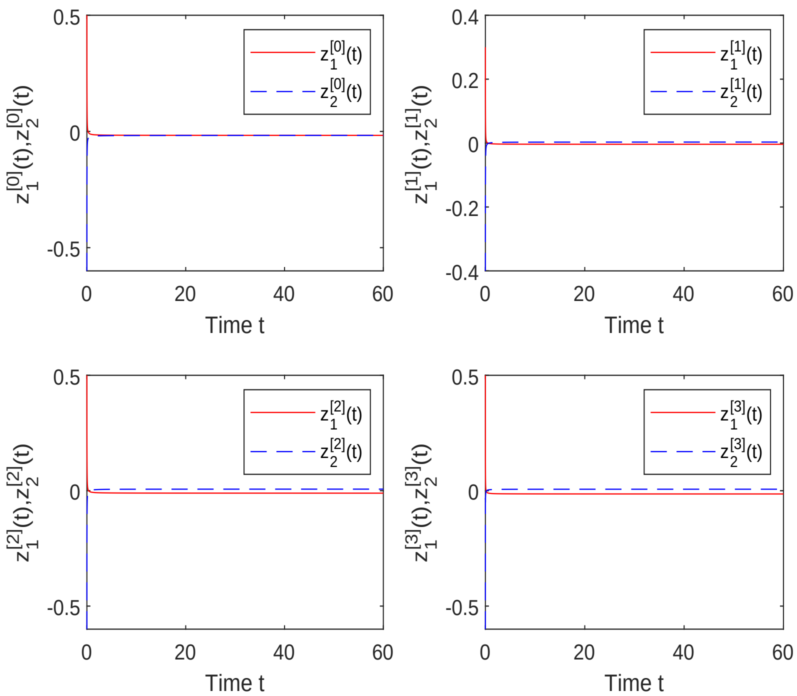

Figure 5. Synchronization errors $z_i^{[p]}(t)$, $i = 1, 2$, p = 0,1,2,3.

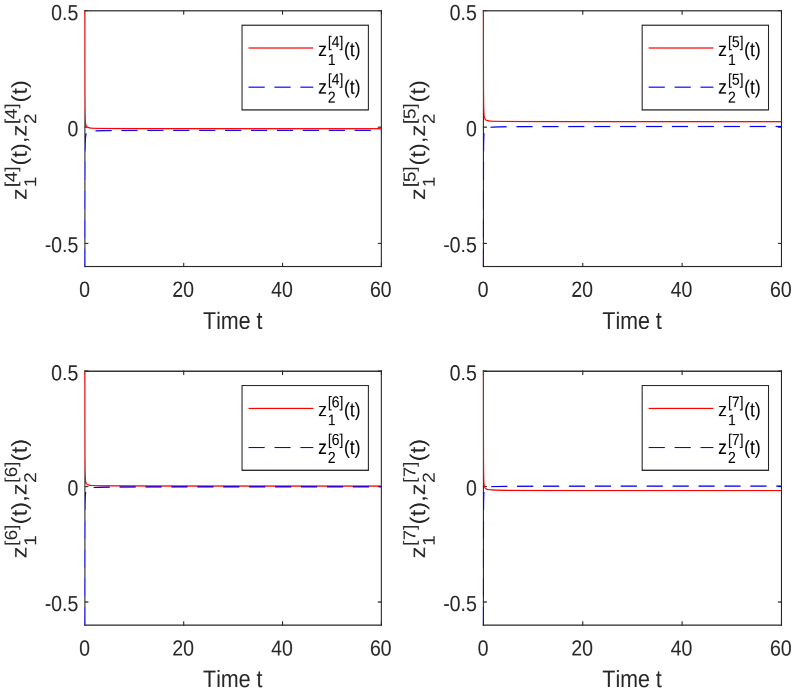

Figure 6. Synchronization errors $z_i^{[p]}(t)$, $i = 1, 2$, p = 4,5,6,7.

Example 4.2. *Consider the same FOOVNNs with two neurons in* (4.1)–(4.3)*:*

$$\alpha = 0.85,\ c_1 = 1.4,\ c_2 = 1.7,\ d_1 = 1.1,\ d_2 = 1.5,$$

and

$$\begin{aligned}
a_{11} &= (-0.3, 0.2, 0.25, 0.15, 0.4, -0.3, 0.17, 0.4)^T,\\
a_{12} &= (0.22, 0.3, 0.6, 0.18, -0.6, 0.19, -0.2, 0.4)^T,\\
a_{21} &= (0.4, 0.4, -0.6, -0.3, 0.25, 0.2, 0.18, 0.5)^T,\\
a_{22} &= (0.2, 0.5, 0.4, 0.2, 0.2, -0.5, 0.17, -0.5)^T,\\
b_{11} &= (0.22, 0.2, -0.15, -0.17, 0.3, 0.5, 0.5, 0.4)^T,\\
b_{12} &= (-0.18, 0.2, 0.22, -0.3, 0.1, 0.2, 0.3, 0.4)^T,\\
b_{21} &= (0.3, 0.2, 0.4, -0.5, -0.6, 0.7, 0.4, 0.4)^T,\\
b_{22} &= (0.2, 0.9, -0.8, -0.2, 0.4, 0.5, 0.6, 0.7)^T,\\
U_1 &= (0.2, 0.3, -0.4, -0.5, 0.6, 0.8, 0.9, 0.1)^T,\\
U_1 &= (0.4, 0.2, 0.3, 0.5, 0.9, -0.1, 0.8, 0.7)^T,\\
f_j &= \sum_{p=0}^{7} \frac{1}{2}\cos([x_j]_p)e_p,\quad g_j = \sum_{p=0}^{7} \frac{1}{3}\sin([z_j]_p)e_p
\end{aligned}$$

and the impulsive moments constrained by $0 < t_1 < t_2 < t_3 < \cdots$, $t_{k+1} - t_k = 0.75$, $\lim_{k\to+\infty} t_k = +\infty$. *Let* $t_1 = \frac{1}{2} + \frac{\pi}{5}$, *and by calculating, we have*

$$L_f = \frac{1}{2},\ L_g = \frac{1}{3},$$

and

$$\mu \ := \ 2c_i^- + 2d_i^- - \sum_{j=1}^{n} |a_{ij}|^2 - nL_f^2 - \sum_{j=1}^{n} |b_{ij}|^2 - nL_g^2 > 0.3056 > 0.$$

It is not difficult to verify that all conditions (H_1)*–*(H_3) *are satisfied. Therefore, by Theorem* 3.2*, the Systems* (4.1) *and* (4.2) *are globally asymptotically synchronized, which is shown in Figures 7–12.*

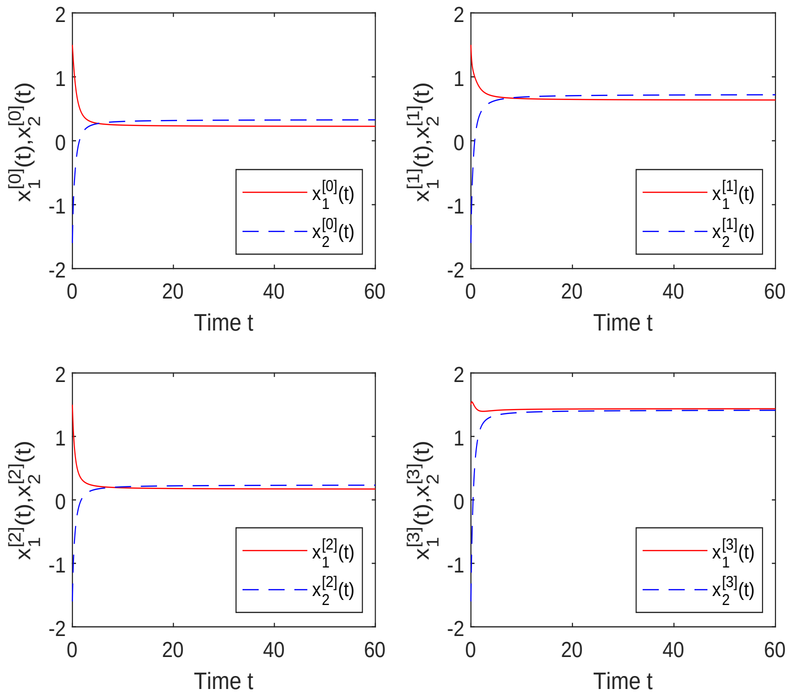

Figure 7. The states of $x_i^{[p]}(t)$, $i = 1, 2$, p = 0,1,2,3.

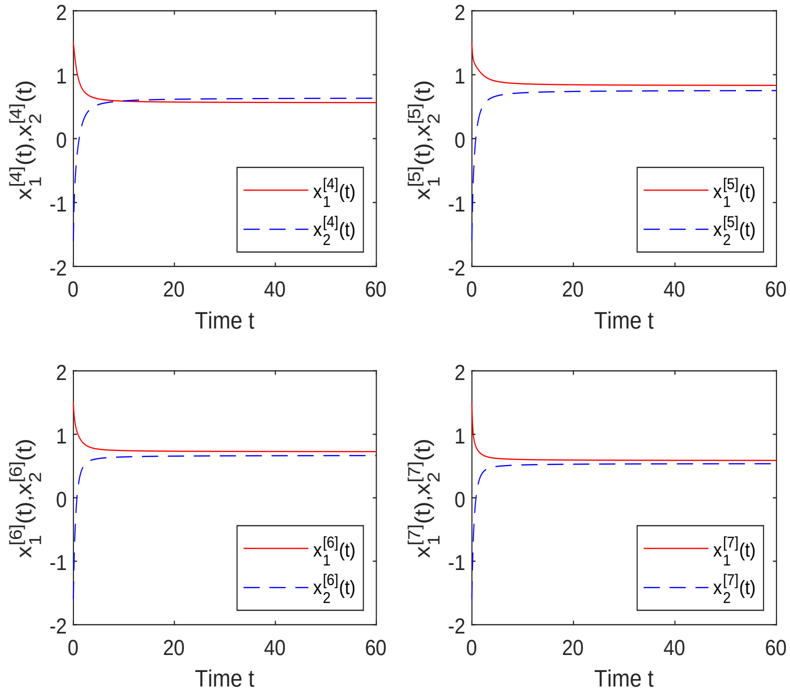

Figure 8. The states of $x_i^{[p]}(t)$, $i = 1, 2$, p = 4,5,6,7.

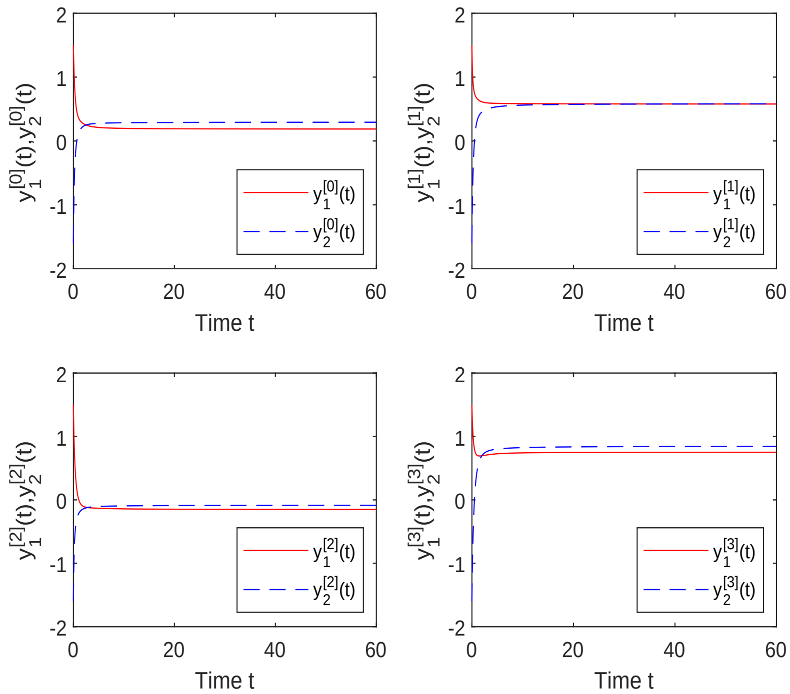

Figure 9. The states of $y_i^{[p]}(t)$, $i = 1, 2$, p = 0,1,2,3.

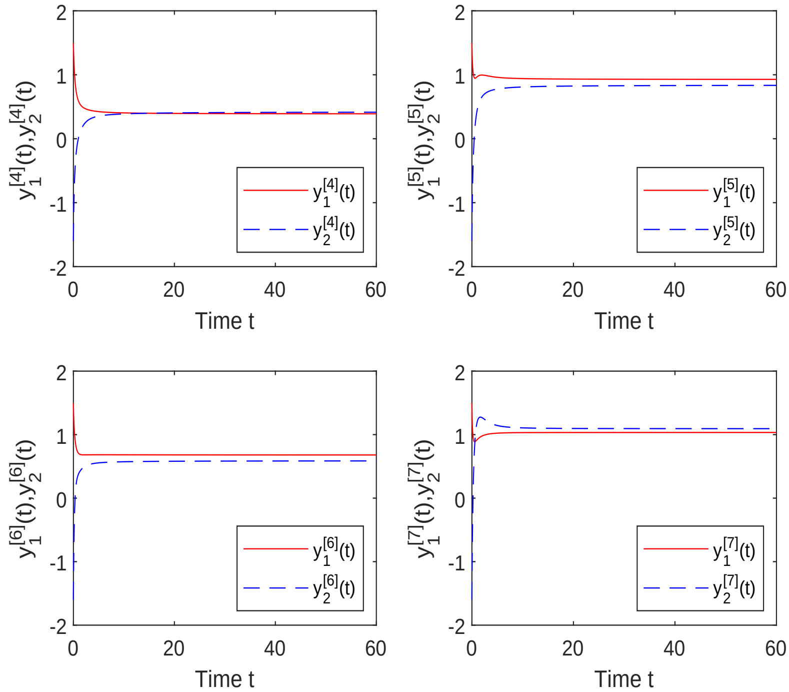

Figure 10. The states of $y_i^{[p]}(t)$, $i = 1, 2$, p = 4,5,6,7.

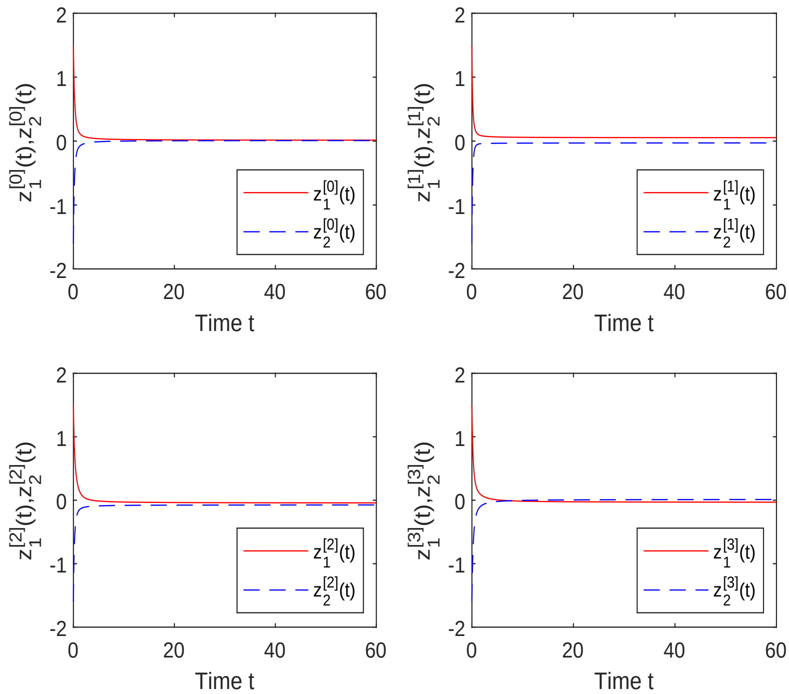

Figure 11. Synchronization errors $z_i^{[p]}(t)$, $i = 1, 2$, p = 0,1,2,3.

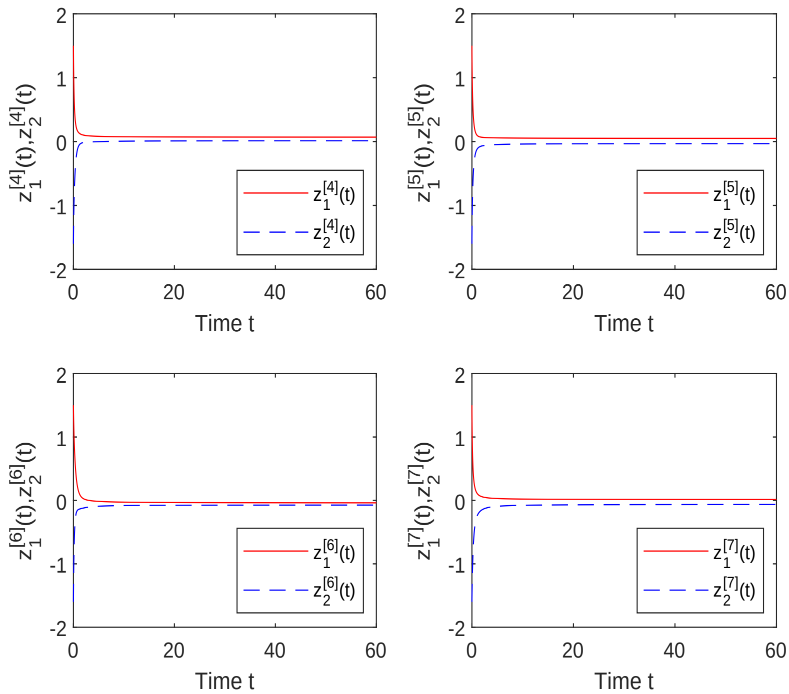

Figure 12. Synchronization errors $z_i^{[p]}(t)$, $i = 1, 2$, p = 4,5,6,7.

5. Conclusions

In this paper, we deal with a class of FOOVNNs with impulsive effects. Unlike complex and quaternion numbers, the multiplication of octonion numbers is non-commutative, but also nonassociative. Therefore, it is very difficult to study octonion-valued neural networks. Considering this fact, without separating the octonion-valued system into four complex-valued or eight real-valued systems. We can obtain the global asymptotical synchronization of FOOVNNs by constructing the appropriate Lyapunov function and using the non-decomposition method. We give two illustrative examples to illustrate the feasibility of the proposed method, and our method can be extended to explore other types of FOOVNNs.

Acknowledgments

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Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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