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Research article

New single traveling wave solution of the Fokas system via complete discrimination system for polynomial method

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Abstract: In this paper, the traveling wave solution of the Fokas system which represents the irregular pulse propagation in monomode optical fibers is studied by using the complete discriminant system method of polynomials. Firstly, the Fokas system is simplified into nonlinear ordinary differential equations by using the traveling wave transformation. Secondly, the Jacobian function solutions, the trigonometric function solutions, the hyperbolic function solutions and the rational function solution of Fokas system are obtained by using the complete discriminant system method of polynomials. Finally, in order to show the propagation of Fokas system in monomode optical fibers, three-dimensional diagram, two-dimensional diagram, contour plot and density plot of some solutions are drawn by using Maple software.

Keywords: Fokas system; traveling wave solution; complete discrimination system
Mathematics Subject Classification: 35C05, 35C07, 35R11

1. Introduction

Nonlinear partial differential equation is a very important branch of the nonlinear science, which has been called the foreword and hot topic of current scientific development. In theoretical science and practical application, nonlinear partial differential is used to describe the problems in the fields of optics, mechanics, communication, control science and biology [1–9]. At present, the main problems in the study of nonlinear partial differential equations are the existence of solutions, the stability of solutions, numerical solutions and exact solutions. With the development of research, especially the study of exact solutions of nonlinear partial differential equations has important theoretical value and application value. In the last half century, many important methods for constructing exact solutions of nonlinear partial differential equations have been proposed, such as the planar dynamic system method [10], the Jacobi elliptic function method [11], the bilinear transformation method [12], the complete discriminant system method for polynomials [13], the unified Riccati equation method [14], the generalized Kudryashov method [15], and so on [16–24].

There is no unified method to obtain the exact solution of nonlinear partial differential equations. Although predecessors have obtained some analytical solutions with different methods, no scholar has studied the system with complete discrimination system for polynomial method.

The Fokas system is a very important class of nonlinear partial differential equations. In this article, we focus on the Fokas system, which is given as follows [25–37]

$$\begin{cases} ip_t + r_1 p_{xx} + r_2 pq = 0, \\ r_3 q_y - r_4(|p|^2)_x = 0, \end{cases} \tag{1.1}$$

where $p = p(x, y, t)$ and $q = q(x, y, t)$ are the complex functions which stand for the nonlinear pulse propagation in monomode optical fibers. The parameters r_1, r_2, r_3 and r_4 are arbitrary non-zero constants, which are coefficients of nonlinear terms in Eq (1.1) and reflect different states of optical solitons.

This paper is arranged as follows. In Section 2, we describe the method of the complete discrimination system for polynomial method. In Section 3, we substitute traveling wave transformation into nonlinear ordinary differential equations and obtain the different new single traveling wave solutions for the Fokas system by complete discrimination system for polynomial method. At the same time, we draw some images of solutions. In Section 4, the main results are summarized.

2. Description of the method

First, we consider the following partial differential equations:

$$\begin{cases} F(u, v, u_x, u_t, v_x, v_t, u_{xx}, u_{xt}, u_{tt}, \cdots) = 0, \\ G(u, v, u_x, u_t, v_x, v_t, u_{xx}, u_{xt}, u_{tt}, \cdots) = 0, \end{cases} \tag{2.1}$$

where F and G is polynomial function which is about the partial derivatives of each order of $u(x,t)$ and $v(x,t)$ with respect to x and t.

Step 1: Taking the traveling wave transformation $u(x,t) = u(\xi), v(x,t) = v(\xi), \xi = kx + ct$ into Eq (2.1), then the partial differential equation is converted to an ordinary differential equation

$$\begin{cases} F(u, v, u', v', u'', v'', \cdots) = 0, \\ G(u, v, u', v', u'', v'', \cdots) = 0. \end{cases} \tag{2.2}$$

Step 2: The above nonlinear ordinary differential equations (2.2) are reduced to the following ordinary differential form after a series of transformations**:**

$$(u')^2 = u^3 + d_2u^2 + d_1u + d_0. \tag{2.3}$$

The Eq (2.3) can also be written in integral form as:

$$\pm(\xi - \xi_0) = \int \frac{du}{\sqrt{u^3 + d_2u^2 + d_1u + d_0}}. \tag{2.4}$$

Step 3: Let $\phi(u) = u^3 + d_2u^2 + d_1u + d_0$. According to the complete discriminant system method of third-order polynomial

$$\begin{cases} \Delta = -27(\frac{2d_2^3}{27} + d_0 - \frac{d_1d_2}{3})^2 - 4(d_1 - \frac{d_2^2}{3})^3, \\ D_1 = d_1 - \frac{d_2^2}{3}, \end{cases} \tag{2.5}$$

the classification of the solution of the equation can be obtained, and the classification of traveling wave solution of the Fokas system will be given in the following section.

3. Traveling wave solution of Eq (1.1)

In the current part, we obtain all exact solutions to Eq (1.1) by complete discrimination system for polynomial method. According to the wave transformation

$$p(x, y, t) = \varphi(\eta)e^{i(\lambda_1 x + \lambda_2 y + \lambda_3 t + \lambda_4)}, q(x, y, t) = \phi(\eta), \eta = x + y - vt, \tag{3.1}$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and v are real parameters, and v represents the wave frame speed.

Substituting the above transformation Eq (3.1) into Eq (1.1), we get

$$\begin{cases} (-v + 2r_1\lambda_1)i\varphi' - \lambda_3\varphi + r_1\varphi'' - r_1\lambda_1^2\varphi + r_2\varphi\phi = 0, \\ r_3\phi' - 2r_4\varphi\varphi' = 0. \end{cases} \tag{3.2}$$

Integrating the second equation in (3.2) and ignoring the integral constant, we get

$$\phi(\eta) = \frac{r_4\varphi^2(\eta)}{r_3}. \tag{3.3}$$

Substituting Eq (3.3) into the first equation in (3.2) and setting $v = 2r_1\lambda_1$, we get the following:

$$r_1\varphi'' - (\lambda_3 + r_1\lambda_1^2)\varphi + \frac{r_2r_4\varphi^3}{r_3} = 0. \tag{3.4}$$

Multiplying φ' both sides of the Eq (3.4), then integrating once to get

$$(\varphi')^2 = a_4\varphi^4 + a_2\varphi^2 + a_0, \tag{3.5}$$

where $a_4=-\frac{r_2r_4}{2r_1r_3}, a_2=\frac{\lambda_3+r_1\lambda_1^2}{r_1}$, a_0 is the arbitrary constant.

Let $\varphi=\pm\sqrt{(4a_4)^{-\frac{1}{3}}\omega},\ b_1=4a_2(4a_4)^{-\frac{2}{3}},\ b_0=4a_0(4a_4)^{-\frac{1}{3}}, \eta_1=(4a_4)^{\frac{1}{3}}\eta.$ (3.6)

Equation (3.5) can be expressed as the following:

$$(\omega'_{\eta_1})^2=\omega^3+b_1\omega^2+b_0\omega. \tag{3.7}$$

Then we can get the integral expression of Eq (3.7)

$$\pm(\eta_1-\eta_0)=\int\frac{d\omega}{\sqrt{\omega(\omega^2+b_1\omega+b_0)}}, \tag{3.8}$$

where η_0 is the constant of integration.

Here, we get the $F(\omega)=\omega^2+b_1\omega+b_0$ and $\Delta=b_1^2-4b_0$. In order to solve Eq (3.7), we discuss the third order polynomial discrimination system in four cases.

Case 1: $\Delta=0$ and $\omega>0$.

When $b_1<0$, the solution of Eq (3.7) is

$$\omega_1=-\frac{b_1}{2}\tanh^2\left(\frac{1}{2}\sqrt{-\frac{b_1}{2}}(\eta_1-\eta_0)\right). \tag{3.9}$$

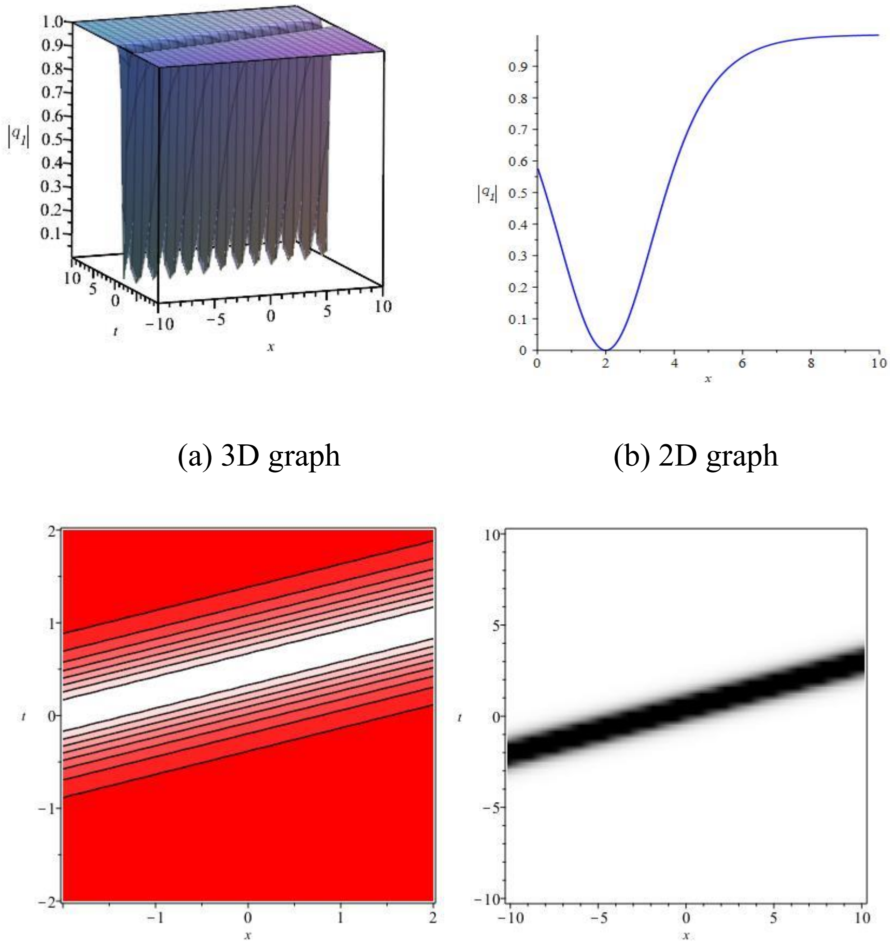
$$\omega_2=-\frac{b_1}{2}\coth^2\left(\frac{1}{2}\sqrt{-\frac{b_1}{2}}(\eta_1-\eta_0)\right). \tag{3.10}$$

Thus, the classification of all solutions of Eq (3.7) is obtained by the third order polynomial discrimination system. The exact traveling wave solutions of the Eq (1.1) are obtained by combining the above solutions and the conditions (3.6) with Eq (3.1), can be expressed as below:

$$p_1(x,y,t)=\pm\sqrt{\frac{r_3(\lambda_3+r_1\lambda_1^2)}{r_2r_4}}\tanh\left(\frac{1}{2}\sqrt{-\frac{2(\lambda_3+r_1\lambda_1^2)}{r_1}\cdot\left(\frac{2r_2r_4}{r_1r_3}\right)^{-\frac{2}{3}}}\cdot\left(\left(\frac{2r_2r_4}{r_1r_3}\right)^{\frac{1}{3}}\eta+\eta_0\right)\right)\cdot e^{i(\lambda_1x+\lambda_2y+\lambda_3t+\lambda_4)}. \tag{3.11}$$

In Eq (3.11), $p_1(x,y,t)$ is a dark soliton solution, it expresses the energy depression on a certain intensity background. Figure 1 depict two-dimensional graph, three-dimensional graph, contour plot and density plot of the solution.

$$q_1(x,y,t)=\frac{\lambda_3+r_1\lambda_1^2}{r_2}\tanh^2\left(\frac{1}{2}\sqrt{-\frac{2(\lambda_3+r_1\lambda_1^2)}{r_1}\cdot\left(\frac{2r_2r_4}{r_1r_3}\right)^{-\frac{2}{3}}}\cdot\left(\left(\frac{2r_2r_4}{r_1r_3}\right)^{\frac{1}{3}}\eta+\eta_0\right)\right) \tag{3.12}$$



(a) 3D graph (b) 2D graph

(c) Contour plot (d) Density plot

Figure 1. Module length graphs of Eq (3.12) when $r_1=-2, r_2=1, r_3=-1, r_4=1, \lambda_1=-1, \lambda_3=3,\ \eta_0=0$.

$$p_2(x,y,t)=\pm\sqrt{\frac{r_3(\lambda_3+r_1\lambda_1^2)}{r_2r_4}}\coth(\frac{1}{2}\sqrt{-\frac{2(\lambda_3+r_1\lambda_1^2)}{r_1}\cdot(\frac{2r_2r_4}{r_1r_3})^{-\frac{2}{3}}}\cdot((\frac{2r_2r_4}{r_1r_3})^{\frac{1}{3}}\eta+\eta_0))\cdot e^{i(\lambda_1x+\lambda_2y+\lambda_3t+\lambda_4)}, \quad (3.13)$$

where $p_1(x,y,t), q_1(x,y,t), p_2(x,y,t), q_2(x,y,t)$ are hyperbolic function solutions. Specially, $p_2(x,y,t)$ is a bright soliton solution.

$$q_2(x,y,t)=\frac{\lambda_3+r_1\lambda_1^2}{r_2}\coth^2(\frac{1}{2}\sqrt{-\frac{2(\lambda_3+r_1\lambda_1^2)}{r_1}\cdot(\frac{2r_2r_4}{r_1r_3})^{-\frac{2}{3}}}\cdot((\frac{2r_2r_4}{r_1r_3})^{\frac{1}{3}}\eta+\eta_0)). \quad (3.14)$$

When $b_1>0,$ the solution of Eq (3.7) is

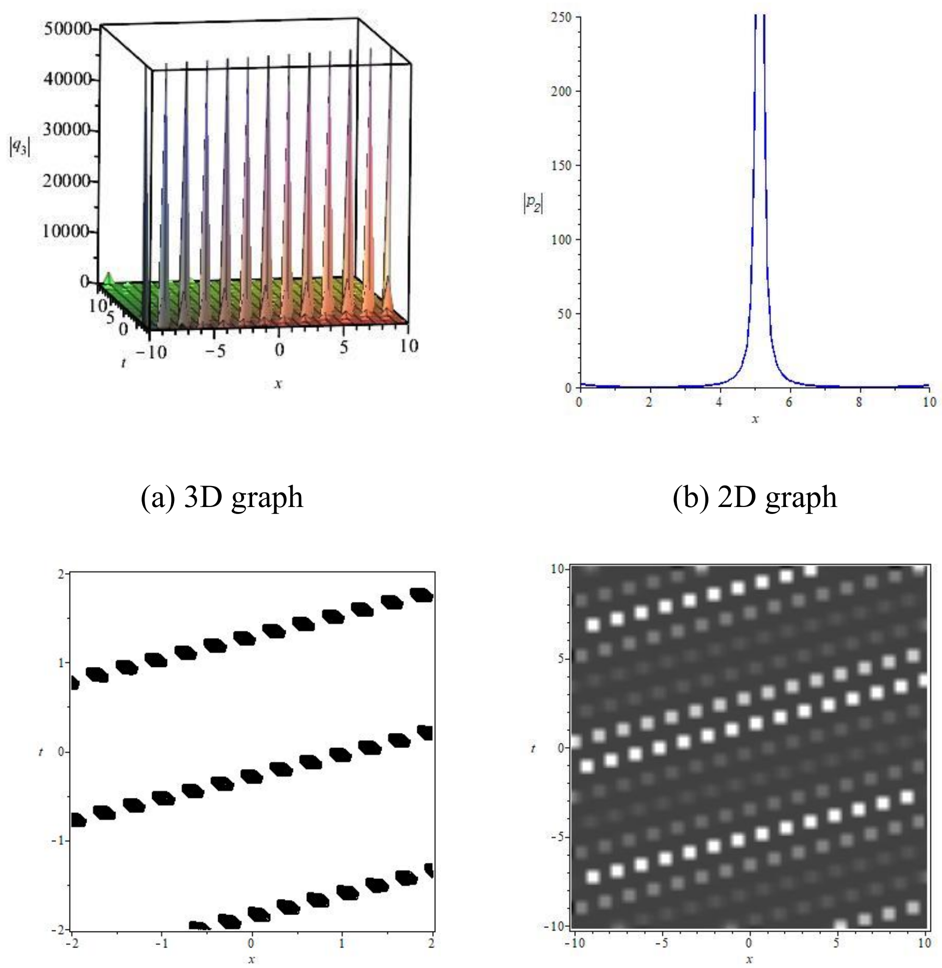
$$\omega_3=\frac{b_1}{2}\tan^2(\frac{1}{2}\sqrt{\frac{b_1}{2}}(\eta_1-\eta_0)). \quad (3.15)$$

The exact traveling wave solutions of the Eq (1.1) can be expressed as below:

$$p_3(x,y,t)=\pm\sqrt{-\frac{r_3(\lambda_3+r_1\lambda_1^2)}{r_2r_4}}\tan(\frac{1}{2}\sqrt{\frac{2(\lambda_3+r_1\lambda_1^2)}{r_1}\cdot(\frac{2r_2r_4}{r_1r_3})^{-\frac{2}{3}}}\cdot((\frac{2r_2r_4}{r_1r_3})^{\frac{1}{3}}\eta+\eta_0))\cdot e^{i(\lambda_1x+\lambda_2y+\lambda_3t+\lambda_4)}. \quad (3.16)$$

$$q_3(x,y,t)=-\frac{\lambda_3+r_1\lambda_1^2}{r_2}\tan^2(\frac{1}{2}\sqrt{\frac{2(\lambda_3+r_1\lambda_1^2)}{r_1}\cdot(\frac{2r_2r_4}{r_1r_3})^{-\frac{2}{3}}}\cdot((\frac{2r_2r_4}{r_1r_3})^{\frac{1}{3}}\eta+\eta_0)). \quad (3.17)$$

In Eq (3.16) and Eq (3.17), $p_3(x,y,t)$ and $q_3(x,y,t)$ are trigonometric function solutions. $q_3(x,y,t)$ is a periodic wave solution, and it Shows the periodicity of $q_3(x,y,t)$ in Figure 2(a), (b).



(a) 3D graph (b) 2D graph

(c) Contour plot (d) Density plot

Figure 2. Module length graphs of Eq (3.17) when $r_1=2, r_2=1, r_3=1, r_4=1, \lambda_1=1, \lambda_3=-1,\ \eta_0=0$.

When $b_1=0$, the solution of Eq (3.7) is

$$\omega_4=\frac{4}{(\eta_1-\eta_0)^2}. \quad (3.18)$$

The exact traveling wave solutions of the Eq (1.1) can be expressed as below:

$$p_4(x,y,t)=\pm\sqrt{-(\frac{2r_2r_4}{r_1r_3})^{-\frac{1}{3}}}\frac{2}{(\frac{2r_2r_4}{r_1r_3})^{\frac{1}{3}}\eta+\eta_0}e^{i(\lambda_1x+\lambda_2y+\lambda_3t+\lambda_4)},\tag{3.19}$$

$$q_4(x,y,t)=-\frac{r_4}{r_3}(\frac{2r_2r_4}{r_1r_3})^{-\frac{1}{3}}\frac{4}{((\frac{2r_2r_4}{r_1r_3})^{\frac{1}{3}}\eta+\eta_0)^2},\tag{3.20}$$

where $p_4(x,y,t)$ is exponential function solution, and $q_4(x,y,t)$ is rational function solution.

Case 2: $\Delta=0$ and $b_0=0$.

When $\omega>-b_1$ and $b_1<0$, the solution of Eq (3.7) is

$$\omega_5=\frac{b_1}{2}\tanh^2(\frac{1}{2}\sqrt{\frac{b_1}{2}}(\eta_1-\eta_0))-b_1.\tag{3.21}$$

$$\omega_6=\frac{b_1}{2}\coth^2(\frac{1}{2}\sqrt{\frac{b_1}{2}}(\eta_1-\eta_0))-b_1.\tag{3.22}$$

The exact traveling wave solutions of the Eq (1.1) can be expressed as below:

$$p_5(x,y,t)=\pm\sqrt{-\frac{r_3(\lambda_3+r_1\lambda_1^2)}{r_2r_4}(\tanh^2(\frac{1}{2}\sqrt{\frac{2(\lambda_3+r_1\lambda_1^2)}{r_1}\cdot(\frac{2r_2r_4}{r_1r_3})^{-\frac{2}{3}}}\cdot((\frac{2r_2r_4}{r_1r_3})^{\frac{1}{3}}\eta+\eta_0))-2)}\cdot e^{i(\lambda_1x+\lambda_2y+\lambda_3t+\lambda_4)},\tag{3.23}$$

$$q_5(x,y,t)=-\frac{\lambda_3+r_1\lambda_1^2}{r_2}\tanh^2(\frac{1}{2}\sqrt{\frac{2(\lambda_3+r_1\lambda_1^2)}{r_1}\cdot(\frac{2r_2r_4}{r_1r_3})^{-\frac{2}{3}}}\cdot((\frac{2r_2r_4}{r_1r_3})^{\frac{1}{3}}\eta+\eta_0))+\frac{2(\lambda_3+r_1\lambda_1^2)}{r_2},\tag{3.24}$$

$$p_6(x,y,t)=\pm\sqrt{-\frac{r_3(\lambda_3+r_1\lambda_1^2)}{r_2r_4}(\coth^2(\frac{1}{2}\sqrt{\frac{2(\lambda_3+r_1\lambda_1^2)}{r_1}\cdot(\frac{2r_2r_4}{r_1r_3})^{-\frac{2}{3}}}\cdot((\frac{2r_2r_4}{r_1r_3})^{\frac{1}{3}}\eta+\eta_0))-2)}\cdot e^{i(\lambda_1x+\lambda_2y+\lambda_3t+\lambda_4)},\tag{3.25}$$

$$q_6(x,y,t)=-\frac{\lambda_3+r_1\lambda_1^2}{r_2}\coth^2(\frac{1}{2}\sqrt{\frac{2(\lambda_3+r_1\lambda_1^2)}{r_1}\cdot(\frac{2r_2r_4}{r_1r_3})^{-\frac{2}{3}}}\cdot((\frac{2r_2r_4}{r_1r_3})^{\frac{1}{3}}\eta+\eta_0))+\frac{2(\lambda_3+r_1\lambda_1^2)}{r_2},\tag{3.26}$$

where $p_5(x,y,t), q_5(x,y,t), p_6(x,y,t)$ and $q_6(x,y,t)$ are hyperbolic function solutions.

When $\omega>-b_1$ and $b_1>0$, the solution of Eq (3.7) is

$$\omega_7 = -\frac{b_1}{2}\tan^2\left(\frac{1}{2}\sqrt{-\frac{b_1}{2}}(\eta_1-\eta_0)\right)-b_1. \tag{3.27}$$

The exact traveling wave solutions of the Eq (1.1) can be expressed as below:

$$p_7(x,y,t)=\pm\sqrt{\frac{r_3(\lambda_3+r_1\lambda_1^2)}{r_2r_4}\left(\tan^2\left(\frac{1}{2}\sqrt{-\frac{2(\lambda_3+r_1\lambda_1^2)}{r_1}\cdot\left(\frac{2r_2r_4}{r_1r_3}\right)^{-\frac{2}{3}}}\cdot\left(\left(\frac{2r_2r_4}{r_1r_3}\right)^{\frac{1}{3}}\eta+\eta_0\right)\right)+2\right)}\cdot e^{i(\lambda_1x+\lambda_2y+\lambda_3t+\lambda_4)}, \tag{3.28}$$

$$q_7(x,y,t)=\frac{\lambda_3+r_1\lambda_1^2}{r_2}\tan^2\left(\frac{1}{2}\sqrt{-\frac{2(\lambda_3+r_1\lambda_1^2)}{r_1}\cdot\left(\frac{2r_2r_4}{r_1r_3}\right)^{-\frac{2}{3}}}\cdot\left(\left(\frac{2r_2r_4}{r_1r_3}\right)^{\frac{1}{3}}\eta+\eta_0\right)\right)+\frac{2(\lambda_3+r_1\lambda_1^2)}{r_2}, \tag{3.29}$$

where $p_7(x,y,t)$ and $q_7(x,y,t)$ are trigonometric function solutions.

Case 3: $\Delta>0$ and $b_0\neq 0.$ Let $u<v<s,$ there u,v and s are constants satisfying one of them is zero and two others are the root of $F(\omega)=0.$

When $u<\omega<v,$ we can get the solution of Eq (3.7) is

$$\omega_8 = u+(v-u)sn^2\left(\frac{\sqrt{s-u}}{2}(\eta_1-\eta_0),c\right), \tag{3.30}$$

where $c^2=\dfrac{v-u}{s-u}.$

The exact traveling wave solutions of the Eq (1.1) can be expressed as below:

$$p_8(x,y,t)=\pm\sqrt{-\left(\frac{2r_2r_4}{r_1r_3}\right)^{-\frac{1}{3}}\left[u+(v-u)\cdot sn^2\left(\frac{\sqrt{s-u}}{2}\left(\left(\frac{2r_2r_4}{r_1r_3}\right)^{\frac{1}{3}}\eta+\eta_0\right),c\right)\right]}\cdot e^{i(\lambda_1x+\lambda_2y+\lambda_3t+\lambda_4)}. \tag{3.31}$$

$$q_8(x,y,t)=-\frac{r_4}{r_3}\left(\frac{2r_2r_4}{r_1r_3}\right)^{-\frac{1}{3}}\left[u+(v-u)\cdot sn^2\left(\frac{\sqrt{s-u}}{2}\left(\left(\frac{2r_2r_4}{r_1r_3}\right)^{\frac{1}{3}}\eta+\eta_0\right),c\right)\right]. \tag{3.32}$$

When $\omega>s$, the solution of Eq (3.7) is

$$\omega_9=\frac{-v\cdot sn^2(\sqrt{s-u}(\eta_1-\eta_0)/2,c)+s}{cn^2(\sqrt{s-u}(\eta_1-\eta_0)/2,c)}. \tag{3.33}$$

The exact traveling wave solutions of the Eq (1.1) can be expressed as below:

$$p_9(x,y,t)=\pm\sqrt{-\left(\frac{2r_2r_4}{r_1r_3}\right)^{-\frac{1}{3}}\frac{-v\cdot sn^2\left(\frac{\sqrt{s-u}}{2}\left(\left(\frac{2r_2r_4}{r_1r_3}\right)^{\frac{1}{3}}\eta+\eta_0\right),c)\right]+s}{cn^2\left(\frac{\sqrt{s-u}}{2}\left(\left(\frac{2r_2r_4}{r_1r_3}\right)^{\frac{1}{3}}\eta+\eta_0\right),c\right)}}e^{i(\lambda_1x+\lambda_2y+\lambda_3t+\lambda_4)}. \tag{3.34}$$

$$q_9(x,y,t)=-\frac{r_4}{r_3}\left(\frac{2r_2r_4}{r_1r_3}\right)^{-\frac{1}{3}}\frac{-v\cdot sn^2\left(\frac{\sqrt{s-u}}{2}\left(\left(\frac{2r_2r_4}{r_1r_3}\right)^{\frac{1}{3}}\eta+\eta_0\right),c)\right]+s}{cn^2\left(\frac{\sqrt{s-u}}{2}\left(\left(\frac{2r_2r_4}{r_1r_3}\right)^{\frac{1}{3}}\eta+\eta_0\right),c\right)}. \tag{3.35}$$

Case 4: $\Delta < 0$.

When $\omega > 0$, similarly we get

$$\omega_{10} = \frac{2\sqrt{b_0}}{1 + cn(b_0^{\frac{1}{4}}(\eta_1 - \eta_0), c)} - \sqrt{b_0}, \tag{3.36}$$

where $c^2 = (1 - \frac{b_1\sqrt{b_0}}{2})/2$.

The exact traveling wave solutions of the Eq (1.1) can be expressed as below:

$$p_{10}(x,y,t) = \pm\sqrt{2\sqrt{a_0}(\frac{2r_2r_4}{r_1r_3})^{-\frac{1}{2}}[\frac{-2}{1+cn((4a_0(-\frac{2r_2r_4}{r_1r_3})^{-\frac{1}{3}})^{\frac{1}{4}}((\frac{2r_2r_4}{r_1r_3})^{\frac{1}{3}}\eta+\eta_0),c)}+1]} \cdot e^{i(\lambda_1 x+\lambda_2 y+\lambda_3 t+\lambda_4)}, \tag{3.37}$$

$$q_{10}(x,y,t) = -\frac{r_4}{r_3}2\sqrt{a_0}(\frac{2r_2r_4}{r_1r_3})^{-\frac{1}{2}}[\frac{-2}{1+cn((4a_0(-\frac{2r_2r_4}{r_1r_3})^{-\frac{1}{3}})^{\frac{1}{4}}((\frac{2r_2r_4}{r_1r_3})^{\frac{1}{3}}\eta+\eta_0),c)}+1], \tag{3.38}$$

where $p_8(x,y,t), q_8(x,y,t), p_9(x,y,t), q_9(x,y,t), p_{10}(x,y,t)$ and $q_{10}(x,y,t)$ are Jacobian elliptic function solutions.

4. Conclusions

In this paper, the complete discrimination system of polynomial method has been applied to construct the single traveling wave solutions of the Fokas system. The Jacobian elliptic function solutions, the trigonometric function solutions, the hyperbolic function solutions and the rational function solutions are obtained. The obtained solutions are very rich, which can help physicists understand the propagation of traveling wave in monomode optical fibers. Furthermore, we have also depicted two-dimensional graphs, three-dimensional graphs, contour plots and density plots of the solutions of Fokas system, which explains the state of solitons from different angles.

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Conflict of interest

The authors declare no conflict of interest.

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