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Research article

A novel approach towards Heronian mean operators in multiple attribute decision making under the environment of bipolar complex fuzzy information

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Abstract: One of the most effective and impressive approaches to tackle uncertainty is the theory of bipolar complex fuzzy set (BCFS). The theory of BCFS modified the theory of fuzzy set (FS), bipolar FS (BFS), and complex FS. Further, the Heronian mean (HM) and generalized HM (GHM) give the aggregation operators (AOs), which have the benefits of taking into account the interrelatedness among the parameters. Up till now, in the prevailing literature, these operators are not introduced in the setting of BCFS. Thus, in this article, our goal is to introduce HM and GHM operators under a bipolar complex fuzzy setting. Firstly, we initiate the bipolar complex fuzzy generalized Heronian mean (BCFGHM) operator. Then, a few of its particular cases by changing the values of the parameter to show its supremacy. We also initiate the bipolar complex fuzzy weighted generalized Heronian mean (BCFWGHM) operator. Secondly, we interpret a method called the "multiple attribute decision making" (MADM) procedure by employing the initiated operators. Next, we provide a descriptive example (selection of the finest renewable energy generation project) to portray the applicability and usefulness of the initiated MADM procedure. Finally, to demonstrate the usefulness of the propounded operators and MADM procedure we compare our initiated work with several present operators and MADM techniques.

Keywords: bipolar complex fuzzy sets; generalized Heronian mean operators; decision-making procedure
Mathematics Subject Classification: 03E72, 90B50, 90C31

1. Introduction

The MADM is a procedure that can provide ordering outcomes for the limited alternatives as per the values of their attributes and it is a significant part of decision sciences. As of late, the advancement of social decision-making (DM) and enterprises in all perspectives are connected with the problem of MADM, so it is broadly employed in all sorts of disciplines. In the genuine decision method, a significant issue is a way to describe the attribute values competently and precisely. In reality, because of the intricacy of DM issues and the fuzziness of DM settings, it isn't sufficient to describe the attribute values of each alternative by precise values. To adapt to such kinds of problems, Zadeh [1] explored the conception of the fuzzy set (FS). FS includes the membership grade (MG) with an instruction that MG must be restricted to the unit interval. Mardani et al. [2] explored the DM techniques based on fuzzy aggregation operators (AOs). Merigo and Casanovas [3] presented fuzzy generalized hybrid AOs. The fuzzy AOs in DM with Dempster-Shafer belief were described by Casanovas, Merigo and Ngan et al. [4,5] initiated DM by relying on fuzzy AOs for medical diagnosis.

Since FS merely contains MG, it sometimes becomes problematic to express certain complicated circumstances. For instance, for an issue of judgment, if the outcomes established that an expert presented his opinion in two different aspects i.e., positive and negative aspects. So, it is clear that the FS can't describe the outcome of this issue. To compound this, bipolar FS (BFS) was developed by Zhang [6]. BFS includes the positive MG (PMG) with an instruction that PMG must be restricted to the unit interval $[0, 1]$ and negative MG (NMG) with an instruction that NMG must be restricted to the closed interval $[-1, 0]$. Jana et al. [7] presented bipolar fuzzy (BF) Dombi AOs in MADM. Wei et al. [8] explored BF Hamacher AOs.

The BF Dombi prioritized AOs were invented by Jana et al. [9]. Akram [10] proposed the BF graphs. Samanta and Pal [11] initiated irregular BF graphs. The BF finite state machine was described by Jun and Kavikumar [12]. Alghamdi et al. [13] propounded multi-criteria DM (MCDM) procedures in a BF setting. Also Lame and Alshehri [14] initiated the extension of the VIKOR method for MCHM based on BFS. The BF relations were expressed by Lee and Hur [15]. A Yin Yang BF cognitive TOPSIS method for bipolar disorder diagnosis was explored by Han et al. [16]. Kang and Kang [17] established the BFS notion employed to sub-semigroups. Riaz and Tehrim [18] described multi-attribute group DM (MAGDM) on cubic BF data. The novel approach of the bipolar soft set (SS) (BSS) was introduced by Mahmood [19]. Abdullah et al. [20] initiated BF SS (BFSS).

As FS merely contains an MG in a single dimension (i.e., only amplitude term) and can't handle the data which includes both amplitude term and phase term. To fill this gap, Ramot et al. [21] modified the notion of FS and presented a novel notion of complex FS (CFS) by adding the phase term to the MG of FS with an instruction that MG must be restricted to the unit circle of a complex plane. After that, Tamir et al. [22] explored the cartesian shape of CFS. Bi et al. [23] established complex fuzzy (CF) arithmetic AOs. Akram and Bashir [24] propounded CF ordered weighted quadratic averaging operators. The CF geometric AOs were initiated by Bi et al. [25]. Li et al. [26] explored CF AOs with complex weights.

Alkouri and Salleh [27] presented linguistic variables and hedges on CFSs. Moses et al. [28] introduced linguistic coordinate transformations for CFSs. Tamir and Kandel [29] explored the axiomatic theory of CF logic and CF classes. Various authors presented the conception of graph in the setting of CFS such as Luqman et al. [30], Akram et al. [31], Hameed et al. [32]. Mahmood et al. [33] invented complex hesitant FSs (CHFSs). Akram and Naz [34] propounded complex Pythagorean FS. Their theories mentioned above can't deal with the structure where PMG and NMG are involved in two dimensions at a time because the structure of FS merely contains the MG in a single dimension. The structure of BFS merely contains PMG and NMG in a single dimension, the structure of CFS merely contains MG in two dimensions. To fill this gap, Mahmood and Ur Rehman [35] introduced the idea of bipolar CFS (BCFS). BCFS includes the PMG with both real and unreal parts with an instruction that these parts must be restricted to the unit interval $[0, 1]$ and NMG with both real and unreal parts with an instruction that these parts must be restricted to the closed interval $[-1, 0]$. Later, Mahmood and Ur Rehman [36] introduced a MADM procedure on Dombi AOs under bipolar complex fuzzy (BCF) data. The BCF Hamacher AOs were developed by Mahmood et al. [37]. The AOs for BCFS were established by Mahmood et al. [38]. Mahmood et al. [39] investigated BCF Bonferroni mean (BM) operator.

The HM is a mean sort of aggregation method, which is created to manage precise numerical values [40]. The beneficial quality of the HM is that HM can catch the interconnection of the input values, which settles on it extremely valuable in DM. The HM operators can manage the cooperation between the attribute values. Later Lui [41] modified the HM operators to GHM operators. In the past several years, the HM and GHM have received a ton of consideration from numerous scholars and they employed it in many present notions such as Yu and Wu [42] presented interval-valued intuitionistic fuzzy (IF) (IVIF) HM operators, Lui et al. [43] invented some intuitionistic uncertain linguistic HM operators and their application to DM, Wei et al. [44] explored picture fuzzy (PF) HM operators in MADM. Yu [45] described IF geometric HM AOs. There is a difference among HM operator, Choquet integral (CI), and power average (PA). The main focus of the HM operator is on the aggregated inputs and CI and PA focus on varying the weight vector of the AOs. For a collection of attributes $(\mathcal{A}_j, j = 1, 2, \ldots, n)$ the BM operator may consider the relationship among any pair of attributes $\mathcal{A}_j$ and $\mathcal{A}_k$ $(j \neq k)$. But the BM neglects the relationship of the attribute with itself. Moreover, the relationship among the attributes $\mathcal{A}_j$ and $\mathcal{A}_k$ $(j \neq k)$ is equal to the relationship among the attributes $\mathcal{A}_k$ and $\mathcal{A}_j$ $(j \neq k)$. However, the BM operator treats it independently and the outcomes are consequently superfluity. While the HM operator has the same kind of framework as the BM operator. The HM can handle the above-mentioned two issues of the BM operators. However, the appropriate inputs which must be aggregated by the prevailing HM operators can take the structure of fuzzy number (FN), intuitionistic FN, picture FN, etc. which confine the advantages and usefulness of the HM operators to various other fields. One of the best ways to fill this gap is to expand the HM operator in the environment of BCFN which is the main goal of this study.

In this study, we introduce:

1) GHM and HM operators in the BCFS setting which have the benefits of taking into account the interrelatedness among the parameters and BCFN.
2) BCFGHM operator and its particular cases show its supremacy. Also, introduce the BCFWGHM operator.
3) A MADM procedure based on the interpreted operators for solving DM issues.
4) A descriptive example (selection of the finest renewable energy generation project) to portray the applicability and usefulness of the initiated MADM procedure.

The rest of the manuscript is constructed similar to Section 2, we reviewed the BCFS and its related elementary laws. In Section 3, firstly, the conceptions of HM and GHM are revised, secondly, we expanded the GHM into the environment of BCFSs to introduce BCFGHM and BCFWGHM operators for aggregating BCFNs. In Section 4, We propounded a MADM procedure based on the established BCFGHM, and BCFWGHM operators to cope with BCFN information. Section 5, included a descriptive example exhibiting the benefits and competencies of the interpreted MADM procedure. In Section 6, we demonstrated the usefulness of the propounded operators and interpreted the MADM procedure by competing them with some current work. The concluding remarks are exhibited in Section 7.

2. Preliminaries

Here, we review the BCFS and its related elementary laws. In the following study $\dot{\check{\mathfrak{D}}}$ will denote a universal set.

Definition 1: [35] A BCFS $\dot{\check{T}}$ is of the structure

$$\dot{\check{T}} = \left\{\dot{\mathfrak{y}}, \left(\dot{\Gamma}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}), \dot{\Gamma}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}})\right) \mid \dot{\mathfrak{y}} \in \dot{\check{\mathfrak{D}}}\right\}$$

where $\dot{\Gamma}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}): \dot{\check{\mathfrak{D}}} \to [0,1] + \mathfrak{i}\,[0,1]$ and $\dot{\Gamma}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}): \dot{\check{\mathfrak{D}}} \to [-1,0] + \mathfrak{i}\,[-1,0]$, specified the PMG and NMG in this structure i.e. $\dot{\Gamma}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) = \dot{\delta}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) + \mathfrak{i}\,\dot{\eta}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}})$ and $\dot{\Gamma}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) = \dot{\delta}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) + \mathfrak{i}\,\dot{\eta}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}})$, with $\dot{\delta}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}), \dot{\eta}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) \in [0,1]$ and $\dot{\delta}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}}), \dot{\eta}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) \in [-1,0]$. A BCF number (BCFN) is of the shape

$$\dot{\check{T}} = \left(\dot{\mathfrak{y}}, \left(\dot{\Gamma}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}), \dot{\Gamma}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}})\right)\right) = \left(\dot{\mathfrak{y}}, \dot{\delta}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) + \mathfrak{i}\,\dot{\eta}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}), \dot{\delta}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) + \mathfrak{i}\dot{\eta}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}})\right).$$

Definition 2: [36] In the existence of BCFN

$$\dot{\check{T}} = \left(\dot{\mathfrak{y}}, \left(\dot{\Gamma}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}), \dot{\Gamma}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}})\right)\right) = \left(\dot{\mathfrak{y}}, \dot{\delta}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) + \mathfrak{i}\,\dot{\eta}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}), \dot{\delta}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) + \mathfrak{i}\dot{\eta}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}})\right).$$

1) The score value $\mathfrak{S}_{\mathcal{B}}$ is interpreted as

$$\mathfrak{S}_{\mathcal{B}}\left(\dot{\check{T}}\right) = \frac{1}{4}\left(2 + \dot{\delta}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) + \dot{\eta}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) + \dot{\delta}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) + \dot{\eta}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}})\right), \quad \mathfrak{S}_{\mathcal{B}} \in [0,1].$$

2) The accuracy value $\mathcal{H}_{\mathcal{B}}$ is interpreted as

$$\mathcal{H}_{\mathcal{B}}\left(\dot{\check{T}}\right) = \frac{\dot{\delta}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) + \dot{\eta}^{+}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) + \dot{\delta}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}}) + \dot{\eta}^{-}_{\dot{\check{T}}}(\dot{\mathfrak{y}})}{4}, \qquad \mathcal{H}_{\mathcal{B}} \in [0,1].$$

Definition 3: [36] In the existence of two BCFNs $\dot{\check{T}}_1 = \left(\dot{\mathfrak{y}}, \left(\dot{\Gamma}^{+}_{\dot{\check{T}}_1}(\dot{\mathfrak{y}}), \dot{\Gamma}^{-}_{\dot{\check{T}}_1}(\dot{\mathfrak{y}})\right)\right)$ and $\dot{\check{T}}_2 = \left(\dot{\mathfrak{y}}, \left(\dot{\Gamma}^{+}_{\dot{\check{T}}_2}(\dot{\mathfrak{y}}), \dot{\Gamma}^{-}_{\dot{\check{T}}_2}(\dot{\mathfrak{y}})\right)\right)$.

1) If $\mathfrak{S}_{\mathcal{B}}\left(\dot{\check{T}}_1\right) < \mathfrak{S}_{\mathcal{B}}\left(\dot{\check{T}}_2\right)$, then $\dot{\check{T}}_1 < \dot{\check{T}}_2$;

2) If $\mathfrak{S}_{\mathcal{B}}\left(\dot{\breve{T}}_1\right) > \mathfrak{S}_{\mathcal{B}}\left(\dot{\breve{T}}_2\right)$, then $\dot{\breve{T}}_1 > \dot{\breve{T}}_2$;

3) If $\mathfrak{S}_{\mathcal{B}}\left(\dot{\breve{T}}_1\right) = \mathfrak{S}_{\mathcal{B}}\left(\dot{\breve{T}}_2\right)$, then

a. If $\mathcal{H}_{\mathcal{B}}\left(\dot{\breve{T}}_1\right) < \mathcal{H}_{\mathcal{B}}\left(\dot{\breve{T}}_2\right)$, then $\dot{\breve{T}}_1 < \dot{\breve{T}}_2$,

b. If $\mathcal{H}_{\mathcal{B}}\left(\dot{\breve{T}}_1\right) > \mathcal{H}_{\mathcal{B}}\left(\dot{\breve{T}}_2\right)$, then $\dot{\breve{T}}_1 > \dot{\breve{T}}_2$,

c. If $\mathcal{H}_{\mathcal{B}}\left(\dot{\breve{T}}_1\right) = \mathcal{H}_{\mathcal{B}}\left(\dot{\breve{T}}_2\right)$, then $\dot{\breve{T}}_1 = \dot{\breve{T}}_2$.

Definition 4: [36] In the existence of two BCFNs

$$\dot{\breve{T}}_1 = \left(\dot{\breve{\mathfrak{y}}}, \left(\dot{\breve{\Gamma}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}}), \dot{\breve{\Gamma}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\right)\right) = \left(\dot{\breve{\mathfrak{y}}}, \dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}}) + \mathrm{i}\,\dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}}), \dot{\breve{\delta}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}}) + \mathrm{i}\,\dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\right)$$

and

$$\dot{\breve{T}}_2 = \left(\dot{\breve{\mathfrak{y}}}, \left(\dot{\breve{\Gamma}}^{+}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}}), \dot{\breve{\Gamma}}^{-}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}})\right)\right) = \left(\dot{\breve{\mathfrak{y}}}, \dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}}) + \mathrm{i}\,\dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}}), \dot{\breve{\delta}}^{-}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}}) + \mathrm{i}\,\dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}})\right),$$

with $\alpha > 0$, then

1) $$\dot{\breve{T}}_1 \oplus \dot{\breve{T}}_2 = \left(\dot{\breve{\mathfrak{y}}}, \begin{pmatrix} \dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}}) + \dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}}) - \dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}}) + \mathrm{i}\left(\dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}}) + \dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}}) - \dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}})\right), \\ -\left(\dot{\breve{\delta}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\dot{\breve{\delta}}^{-}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}})\right) + \mathrm{i}\left(-\left(\dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}})\right)\right) \end{pmatrix}\right),$$

2) $$\dot{\breve{T}}_1 \otimes \dot{\breve{T}}_2 = \left(\dot{\breve{\mathfrak{y}}}, \begin{pmatrix} \dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}}) + \mathrm{i}\left(\dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}})\right), \\ \dot{\breve{\delta}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}}) \dot{\breve{+}} \delta^{-}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}}) + \dot{\breve{\delta}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\dot{\breve{\delta}}^{-}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}}) + \mathrm{i}\left(\dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}}) + \dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}}) + \dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_2}(\dot{\breve{\mathfrak{y}}})\right) \end{pmatrix}\right),$$

3) $$\alpha\dot{\breve{T}}_1 = \left(\dot{\breve{\mathfrak{y}}}, \left(1 - \left(1 - \dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\right)^{\alpha} + \mathrm{i}\left(1 - \left(1 - \dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\right)^{\alpha}\right), - \left|\dot{\breve{\delta}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\right|^{\alpha} + \mathrm{i}\left(-\left|\dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\right|^{\alpha}\right)\right)\right),$$

4) $$\dot{\breve{T}}_1^{\ \alpha} = \left(\tau, \left(\left(\dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\right)^{\alpha} + \mathrm{i}\left(\dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\right)^{\alpha}, -1 + \left(1 + \dot{\breve{\delta}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\right)^{\alpha} + \mathrm{i}\left(-1 + \left(1 + \dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\right)^{\alpha}\right)\right)\right).$$

Theorem 1: [36] In the existence of two BCFNs

$$\dot{\breve{T}}_1 = \left(\dot{\breve{\mathfrak{y}}}, \left(\dot{\breve{\Gamma}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}}), \dot{\breve{\Gamma}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\right)\right) = \left(\dot{\breve{\mathfrak{y}}}, \dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}}) + \mathrm{i}\dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}}), \dot{\breve{\delta}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}}) + \mathrm{i}\dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_1}(\dot{\breve{\mathfrak{y}}})\right)$$

and

$$\dot{\check{T}}_2 = \left(\check{\mathfrak{y}}, \left(\dot{\check{\Gamma}}^{+}_{\dot{\check{T}}_2}(\check{\mathfrak{y}}), \dot{\check{\Gamma}}^{-}_{\dot{\check{T}}_2}(\check{\mathfrak{y}})\right)\right) = \left(\check{\mathfrak{y}}, \dot{\check{\delta}}^{+}_{\dot{\check{T}}_2}(\check{\mathfrak{y}}) + i\check{\eta}^{+}_{\dot{\check{T}}_2}(\check{\mathfrak{y}}), \dot{\check{\delta}}^{-}_{\dot{\check{T}}_2}(\check{\mathfrak{y}}) + i\check{\eta}^{-}_{\dot{\check{T}}_2}(\check{\mathfrak{y}})\right),$$

with $\alpha, \alpha_1, \alpha_2 > 0$, then

1) $\dot{\check{T}}_1 \oplus \dot{\check{T}}_2 = \dot{\check{T}}_2 \oplus \dot{\check{T}}_1$,

2) $\dot{\check{T}}_1 \otimes \dot{\check{T}}_2 = \dot{\check{T}}_2 \otimes \dot{\check{T}}_2$,

3) $\alpha\left(\dot{\check{T}}_1 \oplus \dot{\check{T}}_2\right) = \alpha\dot{\check{T}}_1 \oplus \alpha\dot{\check{T}}_2$,

4) $\left(\dot{\check{T}}_1 \otimes \dot{\check{T}}_2\right)^{\alpha} = \dot{\check{T}}_1^{\ \alpha} \otimes \dot{\check{T}}_2^{\ \alpha}$,

5) $\alpha_1\dot{\check{T}}_1 \oplus \alpha_2\dot{\check{T}}_1 = (\alpha_1 + \alpha_2)\dot{\check{T}}_1$,

6) $\dot{\check{T}}_1^{\ \alpha_1} \otimes \dot{\check{T}}_1^{\ \alpha_2} = \dot{\check{T}}_1^{\ \alpha_1+\alpha_2}$,

7) $\left(\dot{\check{T}}_1^{\ \alpha_1}\right)^{\alpha_2} = \dot{\check{T}}_1^{\ \alpha_1\alpha_2}$.

3. Generalized bipolar complex fuzzy Heronian mean operators

Firstly, the conceptions of HM [33] and GHM [35] are revised in this portion. Secondly, the BCFS is an effective technique to describe the uncertainty in genuine DM procedures. Therefore, we expand the GHM into the environment of BCFSs to introduce BCFGHM and BCFWGHM operators for aggregating BCFNs. Moreover, the necessary properties and different special cases of the introduced operator are similarly discussed in this portion. For our convenience, the terms

$$\dot{\check{T}}_{\check{r}} = \left(\dot{\check{\Gamma}}^{+}_{\dot{\check{T}}_{\check{r}}}, \dot{\check{\Gamma}}^{-}_{\dot{\check{T}}_{\check{r}}}\right) = \left(\dot{\check{\delta}}^{+}_{\dot{\check{T}}_{\check{r}}} + i\dot{\check{\eta}}^{+}_{\dot{\check{T}}_{\check{r}}}, \dot{\check{\delta}}^{-}_{\dot{\check{T}}_{\check{r}}} + i\dot{\check{\eta}}^{-}_{\dot{\check{T}}_{\check{r}}}\right)\ (\check{r} = 1, 2, 3, \dots, n)$$

specified the collection of BCFNs in the rest of the article.

3.1. The HM and GHM

Following we revise the conception of HM [33] and GHM [35].

Definition 5: [40] In the existence of a family $\dot{\check{T}}_{\check{r}}\ (\check{r} = 1, 2, 3, \dots, n)$ of all real numbers greater than or equal to zero the HM is interpreted as

$$HM\left(\dot{\check{T}}_1, \dot{\check{T}}_2, \dot{\check{T}}_3, \dots, \dot{\check{T}}_n\right) = \frac{2}{n(n+1)}\sum_{\check{r}=1}^{n}\sum_{\check{s}=1}^{n}\sqrt{\dot{\check{T}}_{\check{r}}\dot{\check{T}}_{\check{s}}},$$

Definition 6: [42] In the existence of a family $\dot{\check{T}}_{\check{r}}\ (\check{r} = 1, 2, 3, \dots, n)$ of all real numbers greater than or equal to zero, with $\check{\mathfrak{p}}, \check{\mathfrak{q}} \geq 0$, then the GHM is interpreted as

$$GHM^{\check{p},\check{q}}\left(\dot{\check{T}}_1,\dot{\check{T}}_2,\dot{\check{T}}_3,\dots,\dot{\check{T}}_n\right)=\left(\frac{2}{n(n+1)}\sum_{\check{r}=1}^{n}\sum_{\check{s}=1}^{n}\dot{\check{T}}_{\check{r}}^{\check{p}}\dot{\check{T}}_{\check{s}}^{\check{q}}\right)^{\frac{1}{\check{p}+\check{q}}}.$$

3.2. The BCFGHM operator and its special cases

Following we propound BCFGHM operator.

Definition 7: A function $BCFGHM:\dot{\check{T}}^n\to\dot{\check{T}}$, with $\check{p},\ \check{q}\geq 0$ is demonstrated as

$$BCFGHM^{\check{p},\check{q}}\left(\dot{\check{T}}_1,\dot{\check{T}}_2,\dot{\check{T}}_3,\dots,\dot{\check{T}}_n\right)=\left(\frac{2}{n(n+1)}\bigoplus_{\check{r}=1}^{n}\bigoplus_{\check{s}=1}^{n}\left(\dot{\check{T}}_{\check{r}}^{\check{p}}\otimes\dot{\check{T}}_{\check{s}}^{\check{q}}\right)\right)^{\frac{1}{\check{p}+\check{q}}},$$

is said to be a BCFGHM operator.

Theorem 2: The aggregated value by employing the BCFGHM operator is a BCFN and interpreted as

$$BCFGHM^{\check{p},\check{q}}\left(\dot{\check{T}}_1,\dot{\check{T}}_2,\dot{\check{T}}_3,\dots,\dot{\check{T}}_n\right)$$

$$=\left(\begin{array}{c}\left(1-\left(\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(1-\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{+\check{p}}\dot{\check{\delta}}_{\dot{\check{T}}_{\check{s}}}^{+\check{q}}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{\check{p}+\check{q}}}\\ +i\left(1-\left(\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(1-\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{+\check{p}}\dot{\check{\eta}}_{\dot{\check{T}}_{\check{s}}}^{+\check{q}}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{\check{p}+\check{q}}}\\ -1+\left(1-\left(\left|\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(-1+\left(1+\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{-}\right)^{\check{p}}\left(1+\dot{\check{\delta}}_{\dot{\check{T}}_{\check{s}}}^{-}\right)^{\check{q}}\right)\right|^{\frac{2}{n(n+1)}}\right)\right)^{\frac{1}{\check{p}+\check{q}}}\\ +i\left(-1+\left(1-\left(\left|\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(-1+\left(1+\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{-}\right)^{\check{p}}\left(1+\dot{\check{\eta}}_{\dot{\check{T}}_{\check{s}}}^{-}\right)^{\check{q}}\right)\right|^{\frac{2}{n(n+1)}}\right)\right)^{\frac{1}{\check{p}+\check{q}}}\right)\end{array}\right).$$

Proof: By employing Definition (4), we have

$$\dot{\check{T}}_{\check{r}}^{\check{p}}=\left(\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{+\check{p}}+i\,\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{+\check{p}},-1+\left(1+\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{-}\right)^{\check{p}}+i\left(-1+\left(1+\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{-}\right)^{\check{p}}\right)\right),$$

$$\dot{\check{T}}_{\check{s}}^{\check{q}}=\left(\dot{\check{\delta}}_{\dot{\check{T}}_{\check{s}}}^{+\check{q}}+i\,\dot{\check{\eta}}_{\dot{\check{T}}_{\check{s}}}^{+\check{q}},-1+\left(1+\dot{\check{\delta}}_{\dot{\check{T}}_{\check{s}}}^{-}\right)^{\check{q}}+i\left(-1+\left(1+\dot{\check{\eta}}_{\dot{\check{T}}_{\check{s}}}^{-}\right)^{\check{q}}\right)\right),$$

and

$$\left(\dot{\check{T}}_{\check{r}}^{\check{\mathfrak{p}}} \otimes \dot{\check{T}}_{\check{s}}^{\check{\mathfrak{q}}}\right) = \left(\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{+\check{\mathfrak{p}}}\dot{\check{\delta}}_{\dot{\check{T}}_{\check{s}}}^{+\check{\mathfrak{q}}}, -1+\left(1+\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{-}\right)^{\check{\mathfrak{p}}}\left(1+\dot{\check{\delta}}_{\dot{\check{T}}_{\check{s}}}^{-}\right)^{\check{\mathfrak{q}}}\right),$$

then

$$\bigoplus_{\check{r}=1}^{n}\bigoplus_{\check{s}=1}^{n}\left(\dot{\check{T}}_{\check{r}}^{\check{\mathfrak{p}}} \otimes \dot{\check{T}}_{\check{s}}^{\check{\mathfrak{q}}}\right)$$

$$= \left(\begin{array}{c} 1-\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(1-\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{+\check{\mathfrak{p}}}\dot{\check{\delta}}_{\dot{\check{T}}_{\check{s}}}^{+\check{\mathfrak{q}}}\right)+i\left(1-\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(1-\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{+\check{\mathfrak{p}}}\dot{\check{\eta}}_{\dot{\check{T}}_{\check{s}}}^{+\check{\mathfrak{q}}}\right)\right) \\ -\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(-1+\left(1+\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{-}\right)^{\check{\mathfrak{p}}}\left(1+\dot{\check{\delta}}_{\dot{\check{T}}_{\check{s}}}^{-}\right)^{\check{\mathfrak{q}}}\right)+i\left(-\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(-1+\left(1+\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{-}\right)^{\check{\mathfrak{p}}}\left(1+\dot{\check{\eta}}_{\dot{\check{T}}_{\check{s}}}^{-}\right)^{\check{\mathfrak{q}}}\right)\right) \end{array}\right),$$

and

$$\frac{2}{n(n+1)}\bigoplus_{\check{r}=1}^{n}\bigoplus_{\check{s}=1}^{n}\left(\dot{\check{T}}_{\check{r}}^{\check{\mathfrak{p}}} \otimes \dot{\check{T}}_{\check{s}}^{\check{\mathfrak{q}}}\right)$$

$$= \left(\begin{array}{c} 1-\left(\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(1-\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{+\check{\mathfrak{p}}}\dot{\check{\delta}}_{\dot{\check{T}}_{\check{s}}}^{+\check{\mathfrak{q}}}\right)\right)^{\frac{2}{n(n+1)}} \\ +i\left(1-\left(\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(1-\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{+\check{\mathfrak{p}}}\dot{\check{\eta}}_{\dot{\check{T}}_{\check{s}}}^{+\check{\mathfrak{q}}}\right)\right)^{\frac{2}{n(n+1)}}\right) \\ -\left(\left|-\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(-1+\left(1+\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{-}\right)^{\check{\mathfrak{p}}}\left(1+\dot{\check{\delta}}_{\dot{\check{T}}_{\check{s}}}^{-}\right)^{\check{\mathfrak{q}}}\right)\right|^{\frac{2}{n(n+1)}}\right) \\ +i\left(-\left(\left|-\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(-1+\left(1+\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{-}\right)^{\check{\mathfrak{p}}}\left(1+\dot{\check{\delta}}_{\dot{\check{T}}_{\check{s}}}^{-}\right)^{\check{\mathfrak{q}}}\right)\right|^{\frac{2}{n(n+1)}}\right)\right) \end{array}\right).$$

Thus,

$$BCFGHM^{\mathfrak{p},\check{\mathfrak{q}}}\left(\dot{\check{T}}_1,\dot{\check{T}}_2,\dot{\check{T}}_3,\dots,\dot{\check{T}}_n\right)=\left(\frac{2}{n(n+1)}\bigoplus_{\check{r}=1}^{n}\bigoplus_{\check{s}=1}^{n}\left(\dot{\check{T}}_{\check{r}}^{\mathfrak{p}}\otimes\dot{\check{T}}_{\check{s}}^{\check{\mathfrak{q}}}\right)\right)^{\frac{1}{\mathfrak{p}+\check{\mathfrak{q}}}}$$

$$=\left(\begin{array}{c}\left(1-\left(\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(1-\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{+\mathfrak{p}}\dot{\check{\delta}}_{\dot{\check{T}}_{\check{s}}}^{+\check{\mathfrak{q}}}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{\mathfrak{p}+\check{\mathfrak{q}}}}\\ +i\left(1-\left(\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(1-\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{+\mathfrak{p}}\dot{\check{\eta}}_{\dot{\check{T}}_{\check{s}}}^{+\check{\mathfrak{q}}}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{\mathfrak{p}+\check{\mathfrak{q}}}}\\ -1+\left(1-\left(\left|\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(-1+\left(1+\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{-}\right)^{\mathfrak{p}}\left(1+\dot{\check{\delta}}_{\dot{\check{T}}_{\check{s}}}^{-}\right)^{\check{\mathfrak{q}}}\right)\right|^{\frac{2}{n(n+1)}}\right)\right)^{\frac{1}{\mathfrak{p}+\check{\mathfrak{q}}}}\\ +i\left(-1+\left(1-\left(\left|\prod_{\substack{\check{r}=1\\ \check{s}=1}}^{n}\left(-1+\left(1+\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{-}\right)^{\mathfrak{p}}\left(1+\dot{\check{\eta}}_{\dot{\check{T}}_{\check{s}}}^{-}\right)^{\check{\mathfrak{q}}}\right)\right|^{\frac{2}{n(n+1)}}\right)\right)^{\frac{1}{\mathfrak{p}+\check{\mathfrak{q}}}}\right)\end{array}\right).$$

The BCFGHM operator holds the following axioms

1) Idempotency: For a BCFN $\dot{\check{T}}_0$, if $\dot{\check{T}}_{\check{r}}=\dot{\check{T}}_0\ \forall\ \check{r}$, then

$$BCFGHM^{\mathfrak{p},\check{\mathfrak{q}}}\left(\dot{\check{T}}_1,\dot{\check{T}}_2,\dot{\check{T}}_3,\dots,\dot{\check{T}}_n\right)=\dot{\check{T}}_0.$$

2) Boundedness: For any collection of BCFNs $\dot{\check{T}}_{\check{r}}\ (\check{r}=1,2,3,\dots,n)$ if $\dot{\check{T}}_{\check{r}}^{\ *}=\max_{\dot{r}}\left\{\dot{\check{T}}_{\check{r}}\right\}$ and $\dot{\check{T}}_{\check{r}}^{\ \blacksquare}=\min_{\dot{r}}\left\{\dot{\check{T}}_{\check{r}}\right\}$, then

$$\min_{\dot{r}}\left\{\dot{\check{T}}_{\check{r}}\right\}\le BCFGHM^{\mathfrak{p},\check{\mathfrak{q}}}\left(\dot{\check{T}}_1,\dot{\check{T}}_2,\dot{\check{T}}_3,\dots,\dot{\check{T}}_n\right)\le\max_{\dot{r}}\left\{\dot{\check{T}}_{\check{r}}\right\}.$$

3) Monotonicity: For any two collections of BCFNs $\dot{\check{T}}_{\check{r}}$, $\dot{\check{T}}_{\check{r}}'(\check{r}=1,2,3,\dots,n)$ if $\dot{\check{T}}_{\check{r}}\le\dot{\check{T}}_{\check{r}}'$ for all $\check{r}$, i.e. $\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{+}\le\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{+\prime},\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{+}\le\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{+\prime}$, $\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{-}\le\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{-\prime},\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{-}\le\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{-\prime}$, then

$$BCFGHM^{\mathfrak{p},\check{\mathfrak{q}}}\left(\dot{\check{T}}_1,\dot{\check{T}}_2,\dot{\check{T}}_3,\dots,\dot{\check{T}}_n\right)\le BCFGHM^{\mathfrak{p},\check{\mathfrak{q}}}\left(\dot{\check{T}}_1',\dot{\check{T}}_2',\dot{\check{T}}_3',\dots,\dot{\check{T}}_n'\right).$$

By changing the values of the parameters $\breve{p}$ and $\breve{q}$, we obtain the following particular cases of the interpreted BCFGHM operator.

Case 1: By taking $\breve{p}=\breve{q}=\frac{1}{2}$, the interpreted BCFGHM converted BCF HM (BCFHM), demonstrated as

$$BCFHM^{\frac{1}{2},\frac{1}{2}}\left(\dot{\breve{T}}_1,\dot{\breve{T}}_2,\dot{\breve{T}}_3,\dots,\dot{\breve{T}}_n\right)$$

$$=\left(\begin{array}{c}
\left(1-\left(\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left(1-\sqrt{\dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_{\breve{r}}}\dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_{\breve{s}}}}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{\breve{p}+\breve{q}}} \\
+i\left(1-\left(\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left(1-\sqrt{\dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_{\breve{r}}}\dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_{\breve{s}}}}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{\breve{p}+\breve{q}}} \\
-1+\left(1-\left(-\left|\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left(-1+\sqrt{\left(1+\dot{\breve{\delta}}^{-}_{\dot{\breve{T}}_{\breve{r}}}\right)\left(1+\dot{\breve{\delta}}^{-}_{\dot{\breve{T}}_{\breve{s}}}\right)}\right)\right|^{\frac{2}{n(n+1)}}\right)\right)^{\frac{1}{\breve{p}+\breve{q}}} \\
+i\left(-1+\left(1-\left(\left|-\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left(-1+\sqrt{\left(1+\dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_{\breve{r}}}\right)\left(1+\dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_{\breve{s}}}\right)}\right)\right|^{\frac{2}{n(n+1)}}\right)\right)^{\frac{1}{\breve{p}+\breve{q}}}\right)
\end{array}\right).$$

Case 2: By considering $\breve{q}\to 0$, the BCFGHM operators converted to BCF generalized mean (BCFGM), which is demonstrated as

$$\lim_{\breve{\mathfrak{q}}\to 0} BCFGHM^{\breve{\mathfrak{p}},\breve{\mathfrak{q}}}\left(\dot{\breve{T}}_1,\dot{\breve{T}}_2,\dot{\breve{T}}_3,\ldots,\dot{\breve{T}}_n\right)=\left(\frac{1}{n}\bigoplus_{\breve{r}=1}^{n}\dot{\breve{T}}_{\breve{r}}^{\breve{\mathfrak{p}}}\right)^{\frac{1}{\breve{\mathfrak{p}}}}$$

$$=\left(\begin{array}{c}\left(1-\left(\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left(1-\dot{\breve{\delta}}_{\dot{\breve{T}}_{\breve{r}}}^{+\breve{\mathfrak{p}}}\right)\right)^{\frac{1}{n}}\right)^{\frac{1}{\breve{\mathfrak{p}}}}\\ +i\left(1-\left(\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left(1-\dot{\breve{\eta}}_{\dot{\breve{T}}_{\breve{r}}}^{+\breve{\mathfrak{p}}}\right)\right)^{\frac{1}{n}}\right)^{\frac{1}{\breve{\mathfrak{p}}}}\\ -1+\left(1-\left(\left|-\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left(-1+\left(1+\dot{\breve{\delta}}_{\dot{\breve{T}}_{\breve{r}}}^{-}\right)^{\breve{\mathfrak{p}}}\right)\right|^{\frac{1}{n}}\right)\right)^{\frac{1}{\breve{\mathfrak{p}}}}\\ +i\left(-1+\left(1-\left(\left|-\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left(-1+\left(1+\dot{\breve{\eta}}_{\dot{\breve{T}}_{\breve{r}}}^{-}\right)^{\breve{\mathfrak{p}}}\right)\right|^{\frac{1}{n}}\right)\right)^{\frac{1}{\breve{\mathfrak{p}}}}\right)\end{array}\right).$$

Case 3: By taking $\breve{\mathfrak{p}}\to 2$ and $\breve{\mathfrak{q}}\to 0$, the BCFGHM operator is converted to BCF square mean (BCFSM), which is displayed as

$$BCFGHM^{2,0}\left(\dot{\check{T}}_1,\dot{\check{T}}_2,\dot{\check{T}}_3,\ldots,\dot{\check{T}}_n\right)=\left(\frac{1}{n}\bigoplus_{\check{r}=1}^{n}\dot{\check{T}}_{\check{r}}^{2}\right)^{\frac{1}{2}}$$

$$=\left(\begin{array}{c}\left(1-\left(\prod_{\substack{\check{r}=1\\\check{s}=1}}^{n}\left(1-\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{+2}\right)\right)^{\frac{1}{n}}\right)^{\frac{1}{2}}\\+i\left(1-\left(\prod_{\substack{\check{r}=1\\\check{s}=1}}^{n}\left(1-\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{+2}\right)\right)^{\frac{1}{n}}\right)^{\frac{1}{2}}\\-1+\left(1-\left(\left|-\prod_{\substack{\check{r}=1\\\check{s}=1}}^{n}\left(-1+\left(1+\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{-}\right)^{2}\right)\right|^{\frac{1}{n}}\right)\right)^{\frac{1}{2}}\\+i\left(-1+\left(1-\left(\left|-\prod_{\substack{\check{r}=1\\\check{s}=1}}^{n}\left(-1+\left(1+\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{-}\right)^{2}\right)\right|^{\frac{1}{n}}\right)\right)^{\frac{1}{2}}\right)\end{array}\right).$$

Case 4: By taking $\check{\mathfrak{p}}\rightarrow 1$ and $\check{\mathfrak{q}}\rightarrow 0$, the BCFGHM operator is converted to BCF average (BCFA), which is displayed as

$$BCFGHM^{1,0}\left(\dot{\check{T}}_1,\dot{\check{T}}_2,\dot{\check{T}}_3,\ldots,\dot{\check{T}}_n\right)=\left(\frac{1}{n}\bigoplus_{\check{r}=1}^{n}\dot{\check{T}}_{\check{r}}\right)$$

$$=\left(\begin{array}{c}\left(1-\left(\prod_{\substack{\check{r}=1\\\check{s}=1}}^{n}\left(1-\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{+}\right)\right)^{\frac{1}{n}}\right)+i\left(1-\left(\prod_{\substack{\check{r}=1\\\check{s}=1}}^{n}\left(1-\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{+}\right)\right)^{\frac{1}{n}}\right)\\-\left(\left|-\prod_{\substack{\check{r}=1\\\check{s}=1}}^{n}\left(\dot{\check{\delta}}_{\dot{\check{T}}_{\check{r}}}^{-}\right)\right|^{\frac{1}{n}}\right)+i\left(-\left(\left|-\prod_{\substack{\check{r}=1\\\check{s}=1}}^{n}\left(\dot{\check{\eta}}_{\dot{\check{T}}_{\check{r}}}^{-}\right)\right|^{\frac{1}{n}}\right)\right)\end{array}\right),$$

$$= \begin{pmatrix} \left(1-\left(\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left(1-\dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_{\breve{r}}}\right)\right)^{\frac{1}{n}}\right)+i\left(1-\left(\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left(1-\dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_{\breve{r}}}\right)\right)^{\frac{1}{n}}\right) \\ -\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left|\dot{\breve{\delta}}^{-}_{\dot{\breve{T}}_{\breve{r}}}\right|^{\frac{1}{n}}+i\left(-\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left|\dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_{\breve{r}}}\right|^{\frac{1}{n}}\right) \end{pmatrix}.$$

Case 5: By taking $\breve{\mathfrak{p}}=\breve{\mathfrak{q}}=1$ then the BCFGHM operator is converted to BCF generalized interrelated square mean (BCFGISM) which is displayed as

$$BCFGHM^{1,1}\left(\dot{\breve{T}}_1,\dot{\breve{T}}_2,\dot{\breve{T}}_3,\dots,\dot{\breve{T}}_n\right)=\left(\frac{2}{n(n+1)}\bigoplus_{\breve{r}=1}^{n}\bigoplus_{\breve{s}=1}^{n}\left(\dot{\breve{T}}_{\breve{r}}\otimes\dot{\breve{T}}_{\breve{s}}\right)\right)^{\frac{1}{2}}$$

$$= \begin{pmatrix} \left(1-\left(\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left(1-\dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_{\breve{r}}}\dot{\breve{\delta}}^{+}_{\dot{\breve{T}}_{\breve{s}}}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{2}} \\ +i\left(1-\left(\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left(1-\dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_{\breve{r}}}\dot{\breve{\eta}}^{+}_{\dot{\breve{T}}_{\breve{s}}}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{2}} \\ -1+\left(1-\left(\left|-\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left(-1+\left(1+\dot{\breve{\delta}}^{-}_{\dot{\breve{T}}_{\breve{r}}}\right)\left(1+\dot{\breve{\delta}}^{-}_{\dot{\breve{T}}_{\breve{s}}}\right)\right)\right|^{\frac{2}{n(n+1)}}\right)\right)^{\frac{1}{2}} \\ +i\left(-1+\left(1-\left(\left|-\prod_{\substack{\breve{r}=1\\ \breve{s}=1}}^{n}\left(-1+\left(1+\dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_{\breve{r}}}\right)\left(1+\dot{\breve{\eta}}^{-}_{\dot{\breve{T}}_{\breve{s}}}\right)\right)\right|^{\frac{2}{n(n+1)}}\right)\right)^{\frac{1}{2}}\right) \end{pmatrix}.$$

Case 6: By taking $\breve{\mathfrak{p}}\to\infty,\breve{\mathfrak{q}}=0$, then the BCFGHM operator decreases to the following

$$\lim_{\breve{\mathfrak{p}}\to\infty} BCFGHM^{\breve{\mathfrak{p}},0}\left(\dot{\breve{T}}_1,\dot{\breve{T}}_2,\dot{\breve{T}}_3,\dots,\dot{\breve{T}}_n\right)=\left(\frac{1}{n}\bigoplus_{\breve{r}=1}^{n}\dot{\breve{T}}_{\breve{r}}^{\breve{\mathfrak{p}}}\right)^{\frac{1}{\breve{\mathfrak{p}}}}=\max_{\breve{r}}\left(\dot{\breve{T}}_{\breve{r}}\right)$$

Case 7: By taking $\breve{\mathfrak{p}}\to 0,\breve{\mathfrak{q}}=0$, then the BCFGHM operator decreases to the BCF geometric mean operator

$$\lim_{\breve{\mathfrak{p}}\to 0} BCFGHM^{\breve{\mathfrak{p}},0}\left(\dot{\check{T}}_1,\dot{\check{T}}_2,\dot{\check{T}}_3,\dots,\dot{\check{T}}_n\right)=\lim_{\breve{\mathfrak{p}}\to 0}\left(\frac{1}{n}\bigoplus_{\check{r}=1}^{n}\dot{\check{T}}_{\check{r}}^{\breve{\mathfrak{p}}}\right)^{\frac{1}{\breve{\mathfrak{p}}}}=\bigotimes_{\check{r}=1}^{n}\left(\dot{\check{T}}_{\check{r}}\right)^{\frac{1}{n}}=\left(\bigotimes_{\check{r}=1}^{n}\left(\dot{\check{T}}_{\check{r}}\right)\right)^{\frac{1}{n}}.$$

3.3. BCFWGHM operator

It is essential to contract the weighted structure of BCFGHM operators because, the aggregated values have their weights in many cases. Here, we establish the BCFWGHM operator.

Definition 8: A function $BCFWGHM: \dot{\check{T}}^{n} \to \dot{\check{T}}$, with $\breve{\mathfrak{p}},\ \breve{\mathfrak{q}} \geq 0$ is demonstrated as

$$BCFWGHM^{\breve{\mathfrak{p}},\breve{\mathfrak{q}}}\left(\dot{\check{T}}_1,\dot{\check{T}}_2,\dot{\check{T}}_3,\dots,\dot{\check{T}}_n\right)=\left(\frac{2}{n(n+1)}\bigoplus_{\check{r}=1}^{n}\bigoplus_{\breve{s}=1}^{n}\left(\dot{\check{T}}_{\check{r}}^{\tilde{\omega}_{\check{r}}\breve{\mathfrak{p}}}\otimes\dot{\check{T}}_{\breve{s}}^{\tilde{\omega}_{\breve{s}}\breve{\mathfrak{q}}}\right)\right)^{\frac{1}{\breve{\mathfrak{p}}+\breve{\mathfrak{q}}}},$$

is said to be a BCFWGHM operator. Where $\tilde{\omega}=\left(\tilde{\omega}_1,\tilde{\omega}_2,\dots,\tilde{\omega}_n\right)^{T}$ is a weight vector (WV) of $\dot{\check{T}}_{\check{r}}(\check{r}=1,2,3,\dots,n)$ with $\tilde{\omega}_{\check{r}}\geq 0$ and $\sum_{\check{r}=1}^{n}\tilde{\omega}_{\check{r}}=1$.

Theorem 3: The aggregated value by employing the BCFWGHM operator is a BCFN and interpreted as

$$BCFWGHM^{\breve{\mathfrak{p}},\breve{\mathfrak{q}}}\left(\dot{\check{T}}_1,\dot{\check{T}}_2,\dot{\check{T}}_3,\dots,\dot{\check{T}}_n\right)$$

$$=\left(\begin{array}{c}\left(1-\left(\prod_{\substack{\check{r}=1\\ \breve{s}=1}}^{n}\left(1-\dot{\delta}_{\dot{\check{T}}_{\check{r}}}^{+\tilde{\omega}_{\check{r}}\breve{\mathfrak{p}}}\dot{\delta}_{\dot{\check{T}}_{\breve{s}}}^{+\tilde{\omega}_{\breve{s}}\breve{\mathfrak{q}}}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{\breve{\mathfrak{p}}+\breve{\mathfrak{q}}}}\\ +i\left(1-\left(\prod_{\substack{\check{r}=1\\ \breve{s}=1}}^{n}\left(1-\dot{\eta}_{\dot{\check{T}}_{\check{r}}}^{+\tilde{\omega}_{\check{r}}\breve{\mathfrak{p}}}\dot{\eta}_{\dot{\check{T}}_{\breve{s}}}^{+\tilde{\omega}_{\breve{s}}\breve{\mathfrak{q}}}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{\breve{\mathfrak{p}}+\breve{\mathfrak{q}}}}\\ -1+\left(1-\left(\left|-\prod_{\substack{\check{r}=1\\ \breve{s}=1}}^{n}\left(-1+\left(1+\dot{\delta}_{\dot{\check{T}}_{\check{r}}}^{-}\right)^{\tilde{\omega}_{\check{r}}\breve{\mathfrak{p}}}\left(1+\dot{\delta}_{\dot{\check{T}}_{\breve{s}}}^{-}\right)^{\tilde{\omega}_{\breve{s}}\breve{\mathfrak{q}}}\right)\right|^{\frac{2}{n(n+1)}}\right)\right)^{\frac{1}{\breve{\mathfrak{p}}+\breve{\mathfrak{q}}}}\\ +i\left(-1+\left(1-\left(\left|-\prod_{\substack{\check{r}=1\\ \breve{s}=1}}^{n}\left(-1+\left(1+\dot{\eta}_{\dot{\check{T}}_{\check{r}}}^{-}\right)^{\tilde{\omega}_{\check{r}}\breve{\mathfrak{p}}}\left(1+\dot{\eta}_{\dot{\check{T}}_{\breve{s}}}^{-}\right)^{\tilde{\omega}_{\breve{s}}\breve{\mathfrak{q}}}\right)\right|^{\frac{2}{n(n+1)}}\right)\right)^{\frac{1}{\breve{\mathfrak{p}}+\breve{\mathfrak{q}}}}\right)\end{array}\right).$$

4. MADM model for BCFNs

We propound a MADM procedure based on the established BCFGHM, and BCFWGHM operators to cope with BCFN information in this portion.

Let m alternatives i.e. $\dot{\check{\mathcal{V}}}_{\check{s}} = \{\dot{\check{\mathcal{V}}}_1, \dot{\check{\mathcal{V}}}_2, \dots, \dot{\check{\mathcal{V}}}_m\}$ and n attributes i.e. $\dot{\check{\mathfrak{L}}}_{\check{r}} = \{\dot{\check{\mathfrak{L}}}_1, \dot{\check{\mathfrak{L}}}_2, \dots, \dot{\check{\mathfrak{L}}}_n\}$ with respect to WVs $\tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2, \dots, \tilde{\omega}_n)^T$ with $\tilde{\omega}_{\check{r}} \geq 0$ and $\sum_{\check{r}=1}^{n} \tilde{\omega}_{\check{r}} = 1$. Let $\dot{\check{\mathfrak{A}}} = (\dot{\check{\alpha}}_{\check{s}\check{r}})_{m\times n} = (\dot{\check{\Gamma}}^{+}_{\check{s}\check{r}}, \dot{\check{\Gamma}}^{-}_{\check{s}\check{r}})_{m\times n}$ be a BCF decision matrix, which contains BCFNs and every pair consists of PMG and NMG, where $\dot{\check{\Gamma}}^{+}_{\check{s}\check{r}}$ signifies the PMG that the alternatives $\dot{\check{\mathcal{V}}}_{\check{s}}$ satisfies the positive aspect of the attribute $\dot{\check{\mathfrak{L}}}_{\check{r}}$ provided by an expert and $\dot{\check{\Gamma}}^{-}_{\check{s}\check{r}}$ signifies the NMG that the alternatives $\dot{\check{\mathcal{V}}}_{\check{s}}$ satisfies the negative aspect of the attribute $\dot{\check{\mathfrak{L}}}_{\check{r}}$ provided by an expert. The DM procedure includes the following steps

Step 1: The expert provides his/her point of view in the shape of BCFNs and by using these we create a decision matrix.

Step 2: Frequently the attributes are split into two various categories, categories: Benefit kind and cost kind attributes. To retain the steadiness of the attribute values, we transform the decision matrix into a normalization matrix excluding if all attributes are of a similar sort. For this, we utilize the formula given as follows

$$\dot{\check{\alpha}}_{\check{s}\check{r}} = \left(\dot{\check{\Gamma}}^{+}_{\check{s}\check{r}}, \dot{\check{\Gamma}}^{-}_{\check{s}\check{r}}\right) = \begin{cases} \left(\dot{\check{\delta}}^{+}_{\check{s}\check{r}} + i\,\dot{\check{\eta}}^{+}_{\check{s}\check{r}}, \dot{\check{\delta}}^{-}_{\check{s}\check{r}} + i\dot{\check{\eta}}^{-}_{\check{s}\check{r}}\right), & \text{for benefit kind,} \\ \left(\dot{\check{\delta}}^{+}_{\check{s}\check{r}} + i\,\dot{\check{\eta}}^{+}_{\check{s}\check{r}}, \dot{\check{\delta}}^{-}_{\check{s}\check{r}} + i\dot{\check{\eta}}^{-}_{\check{s}\check{r}}\right)^{c}, & \text{for cost kin d,} \end{cases}$$

where, $\left(\dot{\check{\delta}}^{+}_{\check{s}\check{r}} + i\,\dot{\check{\eta}}^{+}_{\check{s}\check{r}}, \dot{\check{\delta}}^{-}_{\check{s}\check{r}} + i\dot{\check{\eta}}^{-}_{\check{s}\check{r}}\right)^{c}$ means the complement of $\left(\dot{\check{\delta}}^{+}_{\check{s}\check{r}} + i\,\dot{\check{\eta}}^{+}_{\check{s}\check{r}}, \dot{\check{\delta}}^{-}_{\check{s}\check{r}} + i\dot{\check{\eta}}^{-}_{\check{s}\check{r}}\right)$.

Step 3: Employ the interpreted BCFGHM and BCFWGHM operators to aggregate the BCFNs.

Step 4: Determine the score values by employing Definition (2 (1)) of the aggregated values acquired in the last step. If the score values of any two alternatives are the same then find the accuracy values by employing Definition (2 (2)).

Step 5: Position all the alternatives as per their score or accuracy values.

Step 6: The finest alternative is chosen in this step as per their ranking determined in the last step.

Step 7: End.

5. Descriptive example

Here, we illustrate the benefits and competencies of the interpreted MADM procedure through a descriptive example.

These days, our society is presently undermined because of various natural problems and the utilization of non-renewable energy sources. In this way, the advancement of novel energy generation projects are currently expanding rapidly. More extensive assessment techniques are expected to choose the appropriate projects, so we can perceive their solidarity and shortcoming, and set forward a few novel suggestions to accomplish the objective. Renewable energy, regularly referred to as hygienic energy, is produced from natural resources or cycles that are constantly recharged. For example, wind or sunlight is constant, irrespective of whether their availability depends on the time and climate. Since we have progressively imaginative and more affordable approaches to catch and maintain solar and wind power, renewables are turning into an additional significant energy source, representing more than one-eighth of the US generation. The development of renewables is occurring on both large and small scales from roof solar plates on houses that can offer energy back to the grid to massive offshore wind ranches. Indeed, even a few whole provincial communities depend on renewable energy. The selection of the finest renewable energy generation project (REGP) is necessary to save society from climate and weather changes, which is a DM problem. Here we choose the finest REGP by employing interpreted MADM producer.

We assume that there is an expert who has to choose the finest REGP in the shortlist 4 projects i.e., $\dot{\breve{\mathcal{V}}}_{\breve{s}}$ $(\breve{s} = 1, 2, 3, 4)$ which is assessed under the 4 attributes

1) $\dot{\breve{\mathfrak{L}}}_1 = internal\ rate\ of\ return$,
2) $\dot{\breve{\mathfrak{L}}}_2 = static\ investment\ return\ period$,
3) $\dot{\breve{\mathfrak{L}}}_3 = net\ present\ value$,
4) $\dot{\breve{\mathfrak{L}}}_4 = return\ on\ investment$.

The 4 shortlisted REGPs $\dot{\breve{\mathcal{V}}}_{\breve{s}}$ $(\breve{s} = 1, 2, 3, 4)$ are evaluated with BCFNs employing 4 attributes weights $\dot{\breve{\mathfrak{w}}} = (0.3.\, 0.15, 0.28, 0.27)$ by an expert.

Step 1: The expert delivers his/her opinion in the shape of BCFNs and constructs a BCF decision matrix which is displayed in Table 1.

Table 1. The data is given by an expert.

	$\dot{\breve{\mathfrak{L}}}_1$	$\dot{\breve{\mathfrak{L}}}_2$	$\dot{\breve{\mathfrak{L}}}_3$	$\dot{\breve{\mathfrak{L}}}_4$
$\dot{\breve{\mathcal{V}}}_1$	$\begin{pmatrix} 0.6 + i0.7, \\ -0.2 - i0.3 \end{pmatrix}$	$\begin{pmatrix} 0.9 + i0.99, \\ -0.7 - i0.46 \end{pmatrix}$	$\begin{pmatrix} 0.67 + i0.75, \\ -0.2 - i0.6 \end{pmatrix}$	$\begin{pmatrix} 0.55 + i0.77, \\ -0.54 - i0.45 \end{pmatrix}$
$\dot{\breve{\mathcal{V}}}_2$	$\begin{pmatrix} 0.33 + i0.2, \\ -0.27 - i0.4 \end{pmatrix}$	$\begin{pmatrix} 0.12 + i0.56, \\ -0.32 - i0.63 \end{pmatrix}$	$\begin{pmatrix} 0.4 + i0.25, \\ -0.7 - i0.6 \end{pmatrix}$	$\begin{pmatrix} 0.52 + i0.7, \\ -0.1 - i0.2 \end{pmatrix}$
$\dot{\breve{\mathcal{V}}}_3$	$\begin{pmatrix} 0.35 + i0.32, \\ -0.17 - i0.5 \end{pmatrix}$	$\begin{pmatrix} 0.22 + i0.46, \\ -0.23 - i0.36 \end{pmatrix}$	$\begin{pmatrix} 0.8 + i0.52, \\ -0.35 - i0.3 \end{pmatrix}$	$\begin{pmatrix} 0.25 + i0.35, \\ -0.2 - i0.4 \end{pmatrix}$
$\dot{\breve{\mathcal{V}}}_4$	$\begin{pmatrix} 0.53 + i0.23, \\ -0.71 - i0.5 \end{pmatrix}$	$\begin{pmatrix} 0.11 + i0.64, \\ -0.15 - i0.3 \end{pmatrix}$	$\begin{pmatrix} 0.55 + i0.29, \\ -0.39 - i0.71 \end{pmatrix}$	$\begin{pmatrix} 0.21 + i0.53, \\ -0.32 - i0.52 \end{pmatrix}$

Step 2: The information is of the same kind so no need for normalization.

Step 3: Employing the interpreted BCFGHM and BCFWGHM operators to aggregate the BCFNs which is revealed in Table 2.

Table 2. The aggregate values of each REGP.

Operators	$\dot{\breve{\mathcal{V}}}_1$	$\dot{\breve{\mathcal{V}}}_2$	$\dot{\breve{\mathcal{V}}}_3$	$\dot{\breve{\mathcal{V}}}_4$
BCFGHM	$\begin{pmatrix} 0.7778 + i0.9051 \\ -0.2855 - 0.3747 \end{pmatrix}$	$\begin{pmatrix} 0.419 + i0.54 \\ -0.2201 - 0.3552 \end{pmatrix}$	$\begin{pmatrix} 0.5401 + i0.3723 \\ -0.1606 - 0.3152 \end{pmatrix}$	$\begin{pmatrix} 0.4516 + i0.5135 \\ -0.2756 - 0.4217 \end{pmatrix}$
BCFWGHM	$\begin{pmatrix} 0.954 + i0.9822 \\ -0.0572 - 0.0758 \end{pmatrix}$	$\begin{pmatrix} 0.8276 + i0.8732 \\ -0.0428 - 0.0743 \end{pmatrix}$	$\begin{pmatrix} 0.8589 + i0.8634 \\ -0.0247 - 0.0592 \end{pmatrix}$	$\begin{pmatrix} 0.8257 + i0.8676 \\ -0.0606 - 0.0991 \end{pmatrix}$

Step 4: Determine the score values by employing Definition (2) of the aggregated values acquired in

the last step which is presented in Table 3.

Table 3. The score values of each REGP.

Operators	$\mathfrak{S}_{\mathcal{B}}\left(\dot{\breve{\mathcal{V}}}_1\right)$	$\mathfrak{S}_{\mathcal{B}}\left(\dot{\breve{\mathcal{V}}}_2\right)$	$\mathfrak{S}_{\mathcal{B}}\left(\dot{\breve{\mathcal{V}}}_3\right)$	$\mathfrak{S}_{\mathcal{B}}\left(\dot{\breve{\mathcal{V}}}_4\right)$
BCFGHM	0.7556	0.5959	0.6342	0.567
BCFWGHM	0.9508	0.8959	0.9096	0.8834

Step 5: Rank all the renewable energy generation projects $\dot{\breve{\mathcal{V}}}_{\breve{s}}$ $(\breve{s} = 1, 2, 3, 4)$ as per their score values in the last step. The ranking is demonstrated in Table 4.

Table 4. The Ordering of each REGP.

Operators	Ordering
BCFGHM	$\dot{\breve{\mathcal{V}}}_1 > \dot{\breve{\mathcal{V}}}_3 > \dot{\breve{\mathcal{V}}}_2 > \dot{\breve{\mathcal{V}}}_4$
BCFWGHM	$\dot{\breve{\mathcal{V}}}_1 > \dot{\breve{\mathcal{V}}}_3 > \dot{\breve{\mathcal{V}}}_2 > \dot{\breve{\mathcal{V}}}_4$

Step 6: From table 4, it is evident that both BCFGHM and BCFWGHM operators provide us that REGP is $\dot{\breve{\mathcal{V}}}_1$ is the best REGP among the all shortlisted 4 REGPs. The 2nd best ERGP is REGP is $\dot{\breve{\mathcal{V}}}_3$ among the all shortlisted 4 REGPs.

Step 7: End.

In the above example, we are considering the value of parameters $\breve{\mathfrak{p}}$ and $\breve{\mathfrak{q}}$ is 2. But for various values of both parameters, we can get various outcomes, and the sensitivity analysis of parameters $\breve{\mathfrak{p}}$ and $\breve{\mathfrak{q}}$ are given below.

In the following, we will see the effect of the parameters $\breve{\mathfrak{p}}$ and $\breve{\mathfrak{q}}$ on the DM outcomes depend on BCFGHM and BCFWGHM operators. For this, we will consider the above example (Selection of the best REGP) and the results for various values of $\breve{\mathfrak{p}}$ and $\breve{\mathfrak{q}}$ are demonstrated in Tables 5 and 6.

Table 5. The effect of the parameters $\breve{\mathfrak{p}}$ and $\breve{\mathfrak{q}}$ on the BCFGHM operator.

Values of $\breve{\mathfrak{p}}$ and $\breve{\mathfrak{q}}$	$\mathfrak{S}_{\mathcal{B}}\left(\dot{\breve{\mathcal{V}}}_1\right)$	$\mathfrak{S}_{\mathcal{B}}\left(\dot{\breve{\mathcal{V}}}_2\right)$	$\mathfrak{S}_{\mathcal{B}}\left(\dot{\breve{\mathcal{V}}}_3\right)$	$\mathfrak{S}_{\mathcal{B}}\left(\dot{\breve{\mathcal{V}}}_4\right)$	Ranking
$\breve{\mathfrak{p}} = 0, \breve{\mathfrak{q}} = 1$	0.846	0.692	0.728	0.664	$\dot{\breve{\mathcal{V}}}_1 > \dot{\breve{\mathcal{V}}}_3 > \dot{\breve{\mathcal{V}}}_2 > \dot{\breve{\mathcal{V}}}_4$
$\breve{\mathfrak{p}} = 0.5, \breve{\mathfrak{q}} = 0.5$	0.82	0.653	0.704	0.627	$\dot{\breve{\mathcal{V}}}_1 > \dot{\breve{\mathcal{V}}}_3 > \dot{\breve{\mathcal{V}}}_2 > \dot{\breve{\mathcal{V}}}_4$
$\breve{\mathfrak{p}} = 1, \breve{\mathfrak{q}} = 1$	0.781	0.61	0.658	0.582	$\dot{\breve{\mathcal{V}}}_1 > \dot{\breve{\mathcal{V}}}_3 > \dot{\breve{\mathcal{V}}}_2 > \dot{\breve{\mathcal{V}}}_4$
$\breve{\mathfrak{p}} = 0, \breve{\mathfrak{q}} = 2$	0.816	0.659	0.69	0.628	$\dot{\breve{\mathcal{V}}}_1 > \dot{\breve{\mathcal{V}}}_3 > \dot{\breve{\mathcal{V}}}_2 > \dot{\breve{\mathcal{V}}}_4$
$\breve{\mathfrak{p}} = 2, \breve{\mathfrak{q}} = 2$	0.756	0.596	0.634	0.567	$\dot{\breve{\mathcal{V}}}_1 > \dot{\breve{\mathcal{V}}}_3 > \dot{\breve{\mathcal{V}}}_2 > \dot{\breve{\mathcal{V}}}_4$
$\breve{\mathfrak{p}} = 10, \breve{\mathfrak{q}} = 10$	0.782	0.658	0.662	0.623	$\dot{\breve{\mathcal{V}}}_1 > \dot{\breve{\mathcal{V}}}_3 > \dot{\breve{\mathcal{V}}}_2 > \dot{\breve{\mathcal{V}}}_4$

Table 6. The effect of the parameters $\breve{\mathfrak{p}}$ and $\breve{\mathfrak{q}}$ on the BCFWGHM operator.

Values of $\breve{\mathfrak{p}}$ and $\breve{\mathfrak{q}}$	$\mathfrak{S}_{\mathcal{B}}(\dot{\breve{\mathcal{V}}}_1)$	$\mathfrak{S}_{\mathcal{B}}(\dot{\breve{\mathcal{V}}}_2)$	$\mathfrak{S}_{\mathcal{B}}(\dot{\breve{\mathcal{V}}}_3)$	$\mathfrak{S}_{\mathcal{B}}(\dot{\breve{\mathcal{V}}}_4)$	Ranking
$\breve{\mathfrak{p}} = 0, \breve{\mathfrak{q}} = 1$	0.983	0.986	0.99	0.982	$\dot{\breve{\mathcal{V}}}_3 > \dot{\breve{\mathcal{V}}}_2 > \dot{\breve{\mathcal{V}}}_1 > \dot{\breve{\mathcal{V}}}_4$
$\breve{\mathfrak{p}} = 0.5, \breve{\mathfrak{q}} = 0.5$	0.974	0.936	0.946	0.929	$\dot{\breve{\mathcal{V}}}_1 > \dot{\breve{\mathcal{V}}}_3 > \dot{\breve{\mathcal{V}}}_2 > \dot{\breve{\mathcal{V}}}_4$
$\breve{\mathfrak{p}} = 1, \breve{\mathfrak{q}} = 1$	0.963	0.916	0.928	0.906	$\dot{\breve{\mathcal{V}}}_1 > \dot{\breve{\mathcal{V}}}_3 > \dot{\breve{\mathcal{V}}}_2 > \dot{\breve{\mathcal{V}}}_4$
$\breve{\mathfrak{p}} = 0, \breve{\mathfrak{q}} = 2$	0.977	0.981	0.986	0.975	$\dot{\breve{\mathcal{V}}}_3 > \dot{\breve{\mathcal{V}}}_2 > \dot{\breve{\mathcal{V}}}_1 > \dot{\breve{\mathcal{V}}}_4$
$\breve{\mathfrak{p}} = 2, \breve{\mathfrak{q}} = 2$	0.951	0.896	0.91	0.883	$\dot{\breve{\mathcal{V}}}_1 > \dot{\breve{\mathcal{V}}}_3 > \dot{\breve{\mathcal{V}}}_2 > \dot{\breve{\mathcal{V}}}_4$
$\breve{\mathfrak{p}} = 10, \breve{\mathfrak{q}} = 10$	0.93	0.878	0.891	0.869	$\dot{\breve{\mathcal{V}}}_1 > \dot{\breve{\mathcal{V}}}_3 > \dot{\breve{\mathcal{V}}}_2 > \dot{\breve{\mathcal{V}}}_4$

By varying the values of parameters $\breve{\mathfrak{p}}$ and $\breve{\mathfrak{q}}$ in the BCFWGHM operators, we noted that the outcomes are changing, for example considering the value of $\breve{\mathfrak{p}} = 0$ and $\breve{\mathfrak{q}} = 1$ or 2, we achieve that $\dot{\breve{\mathcal{V}}}_3$ is the desirable alternative. This implies that by varying the values of parameters $\breve{\mathfrak{p}}$ and $\breve{\mathfrak{q}}$, one can get various outcomes. Thus, the proposed method is more flexible and gives the experts more options. The value of parameters depends on the choice of an expert and the given situation.

6. Comparative analysis

To demonstrate the usefulness of the propounded operators and MADM procedure interpreted in the above sections, we explain a similar descriptive example by competing with several present operators and MADM techniques based on various AOs under intuitionistic fuzzy (IF) setting, such as IF geometric Heronian mean (IFHM) AOs invented by Yu [45], picture fuzzy (PF) setting i.e. PF Heronian mean (PFHM) AOs in MADM is explored by Wei et al. [44], and bipolar fuzzy (BF) setting i.e. Dombi AOs explored by Jana et al. [7], and Hamacher AOs invented by Wei et al. [8], under BCF setting i.e. BM operator investigated by Mahmood et al. [39] and AOs initiated by Mahmood et al. [38]. In the descriptive example, the information is two-dimension with both positive and negative opinions of the expert.

Now by seeing the above-mentioned structures and their MADM procedure, one can easily notice that these present structures are not able to cope with such sort of information because the IFGHM AOs [45] can merely cope with the information which involves satisfaction and non-satisfaction opinion of the expert in a single dimension and can't overcome the data where two-dimensional and negative opinion are involved, likewise, the PFHM AOs [44] can merely cope with the information which involves the satisfaction, neutral and non-satisfaction opinion of the expert in a single dimension and can't overcome the data where two-dimensional and negative opinion are involved, and the Dombi AOs [7], Hamacher AOs [8] in the setting of BFS can merely cope with the information which involves the positive and negative opinion of the expert in a single dimension and can't overcome the data where two dimensions are involved. The BM operator and AOs in the setting of BCFN can handle this information and the required outcomes are presented in the Table 7 We depict the score values and positioning results of exhibited and current work in table 7 and their graphical depiction is introduced in Figure 1.

Table 7. The score values and ordering results of the propounded work and present work.

Operators	$\mathfrak{S}_{\mathcal{B}}(\dot{\mathcal{V}}_1)$	$\mathfrak{S}_{\mathcal{B}}(\dot{\mathcal{V}}_2)$	$\mathfrak{S}_{\mathcal{B}}(\dot{\mathcal{V}}_3)$	$\mathfrak{S}_{\mathcal{B}}(\dot{\mathcal{V}}_4)$	Ordering
IFHMA [45]	Crashed	Crashed	Crashed	Crashed	Crashed
PFHMA [44]	Crashed	Crashed	Crashed	Crashed	Crashed
BFHA [8]	Crashed	Crashed	Crashed	Crashed	Crashed
BFDA [7]	Crashed	Crashed	Crashed	Crashed	Crashed
BCFNWBM [39]	0.666	0.514	0.552	0.474	$\dot{\mathcal{V}}_1 > \dot{\mathcal{V}}_3 > \dot{\mathcal{V}}_2 > \dot{\mathcal{V}}_4$
BCFWAA [38]	0.693	0.539	0.574	0.481	$\dot{\mathcal{V}}_1 > \dot{\mathcal{V}}_3 > \dot{\mathcal{V}}_2 > \dot{\mathcal{V}}_4$
BCFWGA [38]	0.637	0.455	0.533	0.416	$\dot{\mathcal{V}}_1 > \dot{\mathcal{V}}_3 > \dot{\mathcal{V}}_2 > \dot{\mathcal{V}}_4$
BCFGHM	0.756	0.596	0.634	0.567	$\dot{\mathcal{V}}_1 > \dot{\mathcal{V}}_3 > \dot{\mathcal{V}}_2 > \dot{\mathcal{V}}_4$
BCFWGHM	0.951	0.896	0.91	0.883	$\dot{\mathcal{V}}_1 > \dot{\mathcal{V}}_3 > \dot{\mathcal{V}}_2 > \dot{\mathcal{V}}_4$

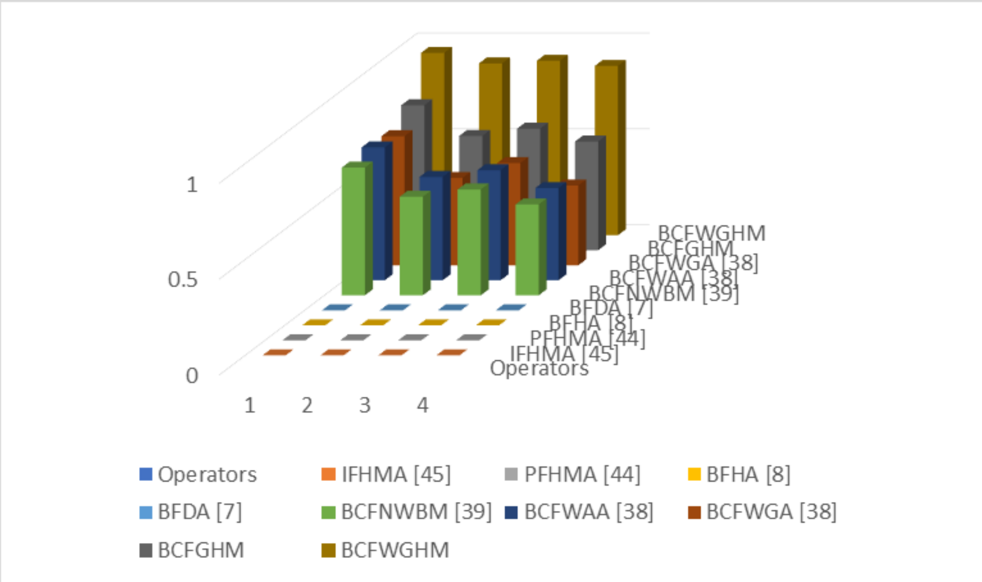

Figure 1. The graphical depiction of propounded and prevailing work.

The BCFNWBM operator investigated by Mahmood et al. [39] and BCFWAA, BCFWGA operators introduced by Mahmood et al. [38] are giving us the same outcomes as the proposed operators and method are giving for example $\dot{\mathcal{V}}_1$ is the finest alternative. But the BCFNWBM [39] is not capable of catching the relationship of the argument with itself. Thus, the BM operator can't provide an accurate outcome in every situation. The AOs [38] have no parameters and can just give us a single answer and are also not useful in every situation. But the proposed operators have both abilities, which include the investigated operators and method can catch the relationship between the attribute with itself and also have the parameters $\breve{\mathfrak{p}}$ and $\breve{\mathfrak{q}}$, to give various outcomes. This infers that the laid-out work is more precise and prevailing than the current work. By our invented MADM based on BCFGHM and BCFWGHM operators, we can solve the data stated in the setting BFS, CFS, and FS as follows

1) For the data in the setting BFS, we have to take the unreal part in PMG and NMG equal to zero in our propounded work.

2) For the data in the setting CFS, we have to ignore the NMG in our propounded work.

For the data in the setting FS, we have to ignore the NMG and take the unreal part in PMG equal to zero in our invented work.

7. Conclusions

With the development of the world, decision experts face more difficulties in DM dilemmas. To cope with these difficulties various scholars modified the structure of FS and presented BFS, CFS, etc. One of the most advance and effective structures of FS is BCFS. The BCFS set can tackle both sides of opinion along with extra fuzzy information. Therefore, in this article, we determined the advantages of the structure of BCFS, HM, and GHM operators, which give the AOs the ability of taking into account the interrelatedness among the parameters. Firstly, we initiated the BCF generalized Heronian mean (BCFGHM) operator. Then, we investigated a few of its particular cases by changing the values of the parameter to show its supremacy. To handle the weight vector, we propounded the BCFWGHM operator. Next, for solving genuine-life dilemmas, we established a MADM procedure based on the investigated BCFGHM, and BCFWGHM operator. To portray the practical use of established MADM, we presented a descriptive example (selection of the finest renewable energy generation project).

Through the established MADM approach we determined that $\dot{\breve{\mathcal{V}}}_1$ is the finest project. After that, we studied the effectiveness of the parameters $\breve{\mathfrak{p}}$ and $\breve{\mathfrak{q}}$, on the DM outcomes. Finally, we demonstrated the usefulness of the propounded operators and MADM procedure by comparing our invented work with several present operators and MADM techniques. In the result of the comparison of the propounded work with the prevailing work, we saw that the presented work is more advanced and generalized. By our invented MADM based on BCFGHM and BCFWGHM operators, we can solve the data stated in the setting BFS, CFS, and FS. The investigated method is not valid for tackling the data in the form of bipolar complex intuitionistic FS, BCF soft set, bipolar complex picture FS, etc.

In the future, we would like to study different articles such as Quasirung FS [46, 47], Q-rung Orthopair fuzzy frank AOs [48], PF N-soft set (PFN-SS) [49], complex dual hesitant FS [50], and CHFS [51] and will apply for the propounded work in these areas.

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Conflict of interest

About the publication of this manuscript, the authors declare that they have no conflicts of interest.

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