Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 8(1): 1770–1783.
DOI: 10.3934/math.2023091
Received: 04 September 2022
Revised: 17 October 2022
Accepted: 18 October 2022
Published: 24 October 2022

Research article

The fuzzy fractional acoustic waves model in terms of the Caputo-Fabrizio operator

Naveed Iqbal[1], **Imran Khan**[2], **Rasool Shah**[3] **and Kamsing Nonlaopon**[4,*]

[1] Department of Mathematics, College of Science, University of Ha'il, Ha'il 2440, Saudi Arabia
[2] Department of Mathematics and Statistics Bacha Khan University Charsadda, KP, 24420, Pakistan
[3] Department of mathematics, Abdul Wali Khan University Mardan Pakistan
[4] Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand

* **Correspondence:** Email: nkamsi@kku.ac.th; Tel:+66866421582.

Abstract: This paper proposes an analytical solution for a fractional fuzzy acoustic wave equation. Under the fractional Caputo-Fabrizio operator, we use the Laplace transformation and the iterative technique. In the present study, the achieved series type result was determined, and we approximated the estimated values of the suggested models. All three problems used two various fractional-order simulations between 0 and 1 to obtain the upper and lower portions of the fuzzy results. Since the exponential function is present, the fractional operator is non-singular and global. Due to its dynamic behaviors, it provides all fuzzy form solutions that happen between 0 and 1 at any level of fractional order. Because the fuzzy numbers return the solution in a fuzzy shape with upper and lower branches, the unknown quantity likewise incorporates fuzziness.

Keywords: iterative transform method; Caputo-Fabrizio operator; fuzzy fractional acoustic waves equation; approximate solution
Mathematics Subject Classification: 34A34, 33B15, 35A22, 35A20, 44A10

1. Introduction

The theory of fuzziness is an effective method designed to simulate uncertainties. As a result, several natural phenomena have been modeled using fuzzy notions [1, 2]. The fractional fuzzy differential equation is a widely utilized model in various research fields, including the evaluation of weapon systems, population modeling, electro hydraulics, and civil engineering modeling. As a result, in fuzzy calculus, the idea of the derivative is vital [3,4]. As a result, fuzzy fractional differential equations have received significant focus in engineering and mathematics [5–8]. The first is research by Agarwal et al.

on fuzzy fractional differential equations [1]. They introduced the Riemann-Liouville notion to study fractional fuzzy differential equations as part of the Hukuhara concept. We continue to live in a world of ambiguity and uncertainty, which is the nature of the real world [9, 10]. Numerous people are prone to distrust everything around them and question if it is for their benefit or that of others [11–14]. Since their observations are insufficient or incorrect, and they lack clarity. Assume we are in a situation where there is a great deal of incorrect information and uncertainty [15, 16]. We cannot respond to many legitimate inquiries based on inaccurate facts [17–19]. For scientists, this mindset and attitude of uncertainty are crucial. Rather than combating uncertainty, our objective should be to discover ways to comprehend and operate around it, since the evolution, resources, and way of life you desire are constantly changing [20–24].

Consider the following fuzzy fractional long wave equation investigated with the iterative transform method :

$$\frac{\partial^{\varpi}\tilde{\mu}}{\partial\psi^{\varpi}} + \frac{1}{2}\frac{\partial\tilde{\mu}^2}{\partial\varepsilon} - \frac{\partial}{\partial\psi}\left(\frac{\partial^2\tilde{\mu}}{\partial\varepsilon^2}\right) = 0, \quad 0 < \varepsilon \leq 1, \quad 0 < \varpi \leq 1, \quad \psi > 0, \tag{1.1}$$

with initial condition

$$\tilde{\mu}(\varepsilon, 0) = \varepsilon,$$

$$\frac{\partial^{\varpi}\tilde{\mu}}{\partial\psi^{\varpi}} + \frac{\partial\tilde{\mu}}{\partial\varepsilon} + \tilde{\mu}\frac{\partial\tilde{\mu}}{\partial\psi} - \frac{\partial}{\partial\psi}\left(\frac{\partial^2\tilde{\mu}}{\partial\varepsilon^2}\right) = 0, \quad 0 < \varepsilon \leq 1, \quad 0 < \varpi \leq 1, \quad \psi > 0, \tag{1.2}$$

with initial condition

$$\tilde{\mu}(\varepsilon, 0) = 3\alpha\text{sech}^2(\varpi\varepsilon), \quad \alpha > 0, \quad \varpi = \frac{1}{2}\sqrt{\frac{\alpha}{1+\alpha}},$$

$$\frac{\partial^{\varpi}\tilde{\mu}}{\partial\psi^{\varpi}} + \frac{\partial\tilde{\mu}}{\partial\varepsilon} - 2\frac{\partial}{\partial\psi}\left(\frac{\partial^2\tilde{\mu}}{\partial\varepsilon^2}\right) = 0, \quad 0 < \varepsilon \leq 1, \quad 0 < \varpi \leq 1, \quad \psi > 0, \tag{1.3}$$

with initial condition

$$\tilde{\mu}(\varepsilon, 0) = e^{-\varepsilon}$$

and

$$\frac{\partial^{\varpi}\tilde{\mu}(\varepsilon,\psi)}{\partial\psi^{\varpi}} + \frac{\partial^4\tilde{\mu}(\varepsilon,\psi)}{\partial\varepsilon^4} = 0, \quad 0 < \varepsilon \leq 1, \quad 0 < \varpi \leq 1, \quad \psi > 0, \tag{1.4}$$

with initial condition

$$\tilde{\mu}(\varepsilon, 0) = \sin\varepsilon. \tag{1.5}$$

Equation (1.1) is the nonlinear fractional order regularized long wave equation (RLWE), (1.2) is the nonlinear fractional general RLWE, and (1.3) and (1.4) are linear fractional RLWEs [25].

Benjamin Bona Mahony's equation (BBME) also recognized the regularized long wave (RLW) equation. This is an improved version of the Kortewega-de Vries equation (KdV), simulating long surface gravity waves with low amplitude propagating in a single direction in two dimensions. Examples of RLW equations in action include wave propagation in elastic rods with longitudinal dispersion, stress waves in compressed gas bubble mixtures, ion-acoustic plasma waves, rotational tube flows, and plasma magneto-hydrodynamic waves. The RLW equations are characterized as effective models for various major physical structures in engineering and physics [26–28]. RLW equations also generates numerous liquid flow issues where shocks or viscosity make diffusion a significant concern.

RLW equations represent any nonlinear wave diffusion problem, including dissipation. This dissipation may result from heat conduction, a chemical reaction, viscosity, thermal radiation, or mass diffusion, depending on the issue being modeled [29, 30].

In 2006, Daftardar-Gejji and Jafari [31] suggested a novel iterative technique that various authors have applied to get the numerical solutions of different classes of linear and nonlinear ordinary, partial and fractional order differential equations. Jafari et al. [32] employed a new iterative method to solve numerically different classes of fractional diffusion and fractional wave equations. It has also been applied by Bhalekar and Daftardar-Gejji to find numerical solutions to fractional evolution equations and fractional boundary value problems [33]. In this article, we propose a new iterative transform method to solve numerically fuzzy fractional acoustic wave equations. The fuzzy fractional acoustic wave equations have many applications in physical sciences; therefore, different graphs are presented to show the dynamics behavior of the equations. The solutions are obtained in a rather more straightforward way than other techniques [34–36].

The current study has been structured as follows: In Section 2, we give some basic notions of basic definitions of the Laplace transformation. In Section 3, we give an analysis of the suggested technique. In Section 4, numerical results and finally, the conclusion is presented in Section 5.

2. Preliminaries

Definition 2.1. [37–39] The Caputo-Fabrizio fractional fuzzy integral form with respect to ψ, with the fuzzy continuous term $\tilde{U}(\psi)$ on a $[0, b]$ subset of $\mathcal{R}$ is defined as

$$^{CF}I^{\varpi}\tilde{\mu}(\psi) = \frac{1-\varpi}{M(\varpi)}\tilde{\mu}(\varpi) + \frac{\varpi}{M(\varpi)}\int_0^{\psi}\tilde{\mu}(\Im)d\Im, \quad \varpi, \Im \in (0, \infty), \tag{2.1}$$

where $M(0) = M(1) = 1$.

The Caputo-Fabrizio non-integer order fuzzy integral is given as follows:

$$[^{CF}I^{\varpi}\tilde{\mu}(\psi)]_r = [I^{\varpi}\underline{\mu}_r(\psi), I^{\varpi}\overline{\mu}_r(\psi)], \quad 0 \le r \le 1, \tag{2.2}$$

where

$$^{CF}I^{\varpi}\underline{\mu}_r(\psi) = \frac{1-\varpi}{M(\varpi)}\underline{\mu}(\varpi) + \frac{\varpi}{M(\varpi)}\int_0^{\psi}\tilde{\mu}(\Im)d\Im, \quad \varpi, \Im \in (0, \infty),$$

and

$$^{CF}I^{\varpi}\overline{\mu}_r(\psi) = \frac{1-\varpi}{M(\varpi)}\overline{\mu}(\varpi) + \frac{\varpi}{M(\varpi)}\int_0^{\psi}\tilde{\mu}(\Im)d\Im, \quad \varpi, \Im \in (0, \infty).$$

Definition 2.2. [37–39] The same applies for operator $\tilde{\mu}(\psi) \in L^F[0, b] \cap C^F[0, b]$, as $\tilde{\mu}(\psi) = [\underline{\mu}(\psi), \overline{\mu}(\psi)]$, $0 \le r \le 1$ and $0 < \psi_0 < b$. CF is defined as the fractional Caputo-Fabrizio derivative in the sense of fuzzy logic:

$$[^{CF}D^{\varpi}\tilde{\mu}(\psi)]_r = [D^{\varpi}\underline{\mu}(\psi_0), D^{\varpi}\overline{\mu}(\psi_0)], \quad 0 < \varpi \le 1, \tag{2.3}$$

where

$$[^{CF}D^{\varpi}\underline{\mu}(\psi_0)] = \frac{M(\varpi)}{1-\varpi}\left[\int_0^{\psi}\underline{\mu}(\Im)' \exp(\frac{-\varpi(\psi-\Im)}{1-\varpi})d\Im\right]$$

and

$$[{}^{CF}D^{\varpi}\overline{\mu}(\psi_0)] = \frac{M(\varpi)}{1-\varpi}\left[\int_0^{\psi}\overline{\mu}(\Im)'\exp(\frac{-\varpi(\psi-\Im)}{1-\varpi})d\Im\right]$$

convergence or existence of the integral, and $m = \lceil\varpi\rceil + 1$. As ϖ lies in the interval $(0,1]$, $m = 1$.

Definition 2.3. The fuzzy Laplace transform function $\mathcal{F}(x)$ is defined as [37–39]

$$\mathcal{F}(x) = \pounds[f(x)] = \int_0^{\infty} e^{-x\psi}f(\psi)d\psi, \quad \psi > 0. \tag{2.4}$$

Definition 2.4. The Laplace transform of Caputo-Fabrizio is

$$\pounds[{}^{CF}D^{\varpi+n}\tilde{\mu}(\psi)] = \frac{\upsilon^{n+1}\tilde{\mu}(\upsilon) - \upsilon^n\tilde{\mu}(0) - \upsilon^{n-1}\tilde{\mu}'(0) - \cdots - \tilde{\mu}^n(0)}{\upsilon + \varpi(1-\upsilon)}.$$

Definition 2.5. [37–39] The Mittag-Leffler function $E_\beta(\psi)$ is defined by

$$E_\beta(\psi) = \sum_{n=0}^{\infty}\frac{\psi^n}{\Gamma(1+n\beta)}, \tag{2.5}$$

where $\beta > 0$.

Definition 2.6. [37–39] A mappings $k : \mathbb{R} \to [0,1]$, if the following criteria are met, it is named a fuzzy number:

(i) The value of k is continuous until it reaches its maximum;
(ii) $k\{\rho(y_1) + \rho(y_2)\} \geq \min\{k(y_1), k(y_2)\}$;
(iii) there exists $y_0 \in \mathbb{R}; k(y_0) = 1$, i.e., k is normal;
(iv) $cl\{y \in \mathbb{R}, k(y) > 0\}$ is bounded and continuous, where cl defines closured for the support of y.

These fuzzy numbers are collectively referred to as ε.

3. The general application of the proposed technique

Consider the fractional fuzzy partial differential equation

$$\pounds[{}^{CF}D_{\psi}^{\varpi}\tilde{\mu}(\varepsilon,\psi)] = \pounds\left[D_{\varepsilon}^2\tilde{\mu}(\varepsilon,\psi) + D_{\zeta}^2\tilde{\mu}(\varepsilon,\psi) + \widetilde{\kappa}(r)\mathcal{H}(\varepsilon,\psi)\right], \tag{3.1}$$

where $\varpi \in (0,1]$; the Laplace transformation used in (3.1) is

$$\frac{\upsilon\pounds\tilde{\mu}(\varepsilon,\psi) - \tilde{\mu}(\varepsilon,\zeta,0)}{\upsilon + \varpi(1-\upsilon)} = \pounds\left[D_{\varepsilon}^2\tilde{\mu}(\varepsilon,\psi) + D_{\zeta}^2\tilde{\mu}(\varepsilon,\psi) + \widetilde{\kappa}(r)\mathcal{H}(\varepsilon,\psi)\right]$$

On applying the initial fuzzy source, we achieve

$$\begin{aligned}\upsilon\pounds\tilde{\mu}(\varepsilon,\psi) &= g(\varepsilon,\zeta) + (\upsilon + \varpi(1-\upsilon))\pounds\left[D_{\varepsilon}^2\tilde{\mu}(\varepsilon,\psi) + D_{\zeta}^2\tilde{\mu}(\varepsilon,\psi) + \widetilde{\kappa}(r)\mathcal{H}(\varepsilon,\psi)\right],\\ \pounds\tilde{\mu}(\varepsilon,\psi) &= \frac{g(\varepsilon,\zeta)}{\upsilon} + \left(\frac{\upsilon + \varpi(1-\upsilon)}{\upsilon}\right)\pounds\left[D_{\varepsilon}^2\tilde{\mu}(\varepsilon,\psi) + D_{\zeta}^2\tilde{\mu}(\varepsilon,\psi) + \widetilde{\kappa}(r)\mathcal{H}(\varepsilon,\psi)\right].\end{aligned} \tag{3.2}$$

We write the result of $\tilde{\mu}(\varepsilon,\psi)=\sum_{n=0}^{\infty}\tilde{\mu}_n(\varepsilon,\psi)$; we get

$$\begin{aligned}\text{£}\sum_{n=0}^{\infty}\tilde{\mu}_n(\varepsilon,\psi)=\frac{g(\varepsilon,\zeta)}{\upsilon}+\left(\frac{\upsilon+\varpi(1-\upsilon)}{\upsilon}\right)\text{£}\Bigg[D_{\varepsilon}^{2}\sum_{n=0}^{\infty}\tilde{\mu}_n(\varepsilon,\psi)\\ +D_{\zeta}^{2}\sum_{n=0}^{\infty}\tilde{\mu}_n(\varepsilon,\psi)+\widetilde{\kappa}(r)\mathcal{H}(\varepsilon,\psi)\Bigg].\end{aligned}\tag{3.3}$$

We can write the following solutions:

$$\begin{aligned}\text{£}\tilde{\mu}_0(\varepsilon,\psi)&=\frac{g(\varepsilon,\zeta)}{\upsilon}+\left(\frac{\upsilon+\varpi(1-\upsilon)}{\upsilon}\right)\text{£}\left[\widetilde{\kappa}(r)\mathcal{H}(\varepsilon,\psi)\right],\\ \text{£}\tilde{\mu}_1(\varepsilon,\psi)&=\left(\frac{\upsilon+\varpi(1-\upsilon)}{\upsilon}\right)\text{£}\left[D_{\varepsilon}^{2}\tilde{\mu}_0(\varepsilon,\psi)+D_{\zeta}^{2}\tilde{\mu}_0(\varepsilon,\psi)\right],\\ \text{£}\tilde{\mu}_2(\varepsilon,\psi)&=\left(\frac{\upsilon+\varpi(1-\upsilon)}{\upsilon}\right)\text{£}\left[D_{\varepsilon}^{2}\tilde{\mu}_1(\varepsilon,\psi)+D_{\zeta}^{2}\tilde{\mu}_1(\varepsilon,\psi)\right],\\ &\vdots\\ \text{£}\tilde{\mu}_{n+1}(\varepsilon,\psi)&=\left(\frac{\upsilon+\varpi(1-\upsilon)}{\upsilon}\right)\text{£}\left[D_{\varepsilon}^{2}\tilde{\mu}_n(\varepsilon,\psi)+D_{\zeta}^{2}\tilde{\mu}_n(\varepsilon,\psi)\right].\end{aligned}\tag{3.4}$$

Applying the Laplace inverse transformation, we get

$$\begin{aligned}\underline{\mu}_0(\varepsilon,\psi)&=g(\varepsilon,\zeta)+\text{£}^{-1}\left[\left(\frac{\upsilon+\varpi(1-\upsilon)}{\upsilon}\right)\text{£}\left[\underline{\kappa}(r)\mathcal{H}(\varepsilon,\psi)\right]\right],\\ \overline{\mu}_0(\varepsilon,\psi)&=g(\varepsilon,\zeta)+\text{£}^{-1}\left[\left(\frac{\upsilon+\varpi(1-\upsilon)}{\upsilon}\right)\text{£}\left[\overline{\kappa}(r)\mathcal{H}(\varepsilon,\psi)\right]\right],\\ \underline{\mu}_1(\varepsilon,\psi)&=\text{£}^{-1}\left[\left(\frac{\upsilon+\varpi(1-\upsilon)}{\upsilon}\right)\text{£}\left[D_{\varepsilon}^{2}\underline{\mu}_0(\varepsilon,\psi)+D_{\zeta}^{2}\underline{\mu}_0(\varepsilon,\psi)\right]\right],\\ \overline{\mu}_1(\varepsilon,\psi)&=\text{£}^{-1}\left[\left(\frac{\upsilon+\varpi(1-\upsilon)}{\upsilon}\right)\text{£}\left[D_{\varepsilon}^{2}\overline{\mu}_0(\varepsilon,\psi)+D_{\zeta}^{2}\overline{\mu}_0(\varepsilon,\psi)\right]\right],\\ \underline{\mu}_2(\varepsilon,\psi)&=\text{£}^{-1}\left[\left(\frac{\upsilon+\varpi(1-\upsilon)}{\upsilon}\right)\text{£}\left[D_{\varepsilon}^{2}\underline{\mu}_1(\varepsilon,\psi)+D_{\zeta}^{2}\underline{\mu}_1(\varepsilon,\psi)\right]\right],\\ \overline{\mu}_2(\varepsilon,\psi)&=\text{£}^{-1}\left[\left(\frac{\upsilon+\varpi(1-\upsilon)}{\upsilon}\right)\text{£}\left[D_{\varepsilon}^{2}\overline{\mu}_1(\varepsilon,\psi)+D_{\zeta}^{2}\overline{\mu}_1(\varepsilon,\psi)\right]\right],\\ &\vdots\\ \underline{\mu}_{n+1}(\varepsilon,\psi)&=\text{£}^{-1}\left[\left(\frac{\upsilon+\varpi(1-\upsilon)}{\upsilon}\right)\text{£}\left[D_{\varepsilon}^{2}\underline{\mu}_n(\varepsilon,\psi)+D_{\zeta}^{2}\underline{\mu}_n(\varepsilon,\psi)\right]\right],\\ \overline{\mu}_{n+1}(\varepsilon,\psi)&=\text{£}^{-1}\left[\left(\frac{\upsilon+\varpi(1-\upsilon)}{\upsilon}\right)\text{£}\left[D_{\varepsilon}^{2}\overline{\mu}_n(\varepsilon,\psi)+D_{\zeta}^{2}\overline{\mu}_n(\varepsilon,\psi)\right]\right].\end{aligned}\tag{3.5}$$

Thus, the result is obtained as

$$\begin{aligned}\underline{\mu}(\varepsilon,\psi)&=\underline{\mu}_0(\varepsilon,\psi)+\underline{\mu}_1(\varepsilon,\psi)+\underline{\mu}_2(\varepsilon,\psi)+\cdots,\\ \overline{\mu}(\varepsilon,\psi)&=\overline{\mu}_0(\varepsilon,\psi)+\overline{\mu}_1(\varepsilon,\psi)+\overline{\mu}_2(\varepsilon,\psi)+\cdots.\end{aligned}\tag{3.6}$$

Equation (3.6) is the series form result.

4. Numerical solutions

Example 1. Consider the fractional fuzzy non-linear regularised long wave equation [40]

$$ {}^{CF}D_{\psi}^{\varpi}\tilde{\mu}(\varepsilon,\psi) + \frac{1}{2}\frac{\partial\tilde{\mu}^2}{\partial\varepsilon} - \frac{\partial\partial^2\tilde{\mu}}{\partial\psi\partial\varepsilon^2} = 0, \quad 0 < \varepsilon \leq 1, \quad 0 < \varpi \leq 1, \quad \psi > 0, \tag{4.1} $$

with the initial fuzzy condition

$$ \tilde{\mu}(\varepsilon, 0) = \widetilde{\kappa}\varepsilon. \tag{4.2} $$

Using the scheme of (4.1), we obtain

$$ \begin{aligned} &\underline{\mu}_0(\varepsilon,\psi) = \underline{\kappa}(r)\varepsilon, \quad \overline{\mu}_0(\varepsilon,\psi) = \overline{\kappa}(r)\varepsilon, \\ &\underline{\mu}_1(\varepsilon,\psi) = -\underline{\kappa}(r)\varepsilon\left\{1 + \varpi\psi - \varpi\right\}, \\ &\overline{\mu}_1(\varepsilon,\psi) = -\overline{\kappa}(r)\varepsilon\left\{1 + \varpi\psi - \varpi\right\}, \\ &\underline{\mu}_2(\varepsilon,\psi) = 2\underline{\kappa}(r)\varepsilon\left\{(1-\varpi)2\varpi\psi + (1-\varpi)^2 + \frac{\varpi^2\psi^2}{2}\right\}, \\ &\overline{\mu}_2(\varepsilon,\psi) = 2\overline{\kappa}(r)\varepsilon\left\{(1-\varpi)2\varpi\psi + (1-\varpi)^2 + \frac{\varpi^2\psi^2}{2}\right\}, \\ &\underline{\mu}_3(\varepsilon,\psi) = -4\underline{\kappa}(r)\varepsilon\left\{(1-\varpi)^2 3\varpi\psi + (1-\varpi)^3 + \frac{3\varpi^2(1-\varpi)\psi^2}{2} + \frac{\varpi^3\psi^3}{3!}\right\}, \\ &\overline{\mu}_3(\varepsilon,\psi) = -4\overline{\kappa}(r)\varepsilon\left\{(1-\varpi)^2 3\varpi\psi + (1-\varpi)^3 + \frac{3\varpi^2(1-\varpi)\psi^2}{2} + \frac{\varpi^3\psi^3}{3!}\right\}. \end{aligned} \tag{4.3} $$

The series type solution is achieved using (4.1) and we get

$$ \tilde{\mu}(\varepsilon,\psi) = \tilde{\mu}_0(\varepsilon,\psi) + \tilde{\mu}_1(\varepsilon,\psi) + \tilde{\mu}_2(\varepsilon,\psi) + \tilde{\mu}_3(\varepsilon,\psi) + \tilde{\mu}_4(\varepsilon,\psi) + \cdots. \tag{4.4} $$

The lower and upper type results are achieved as

$$ \begin{aligned} \underline{\mu}(\varepsilon,\psi) &= \underline{\mu}_0(\varepsilon,\psi) + \underline{\mu}_1(\varepsilon,\psi) + \underline{\mu}_2(\varepsilon,\psi) + \underline{\mu}_3(\varepsilon,\psi) + \underline{\mu}_4(\varepsilon,\psi) + \cdots, \\ \overline{\mu}(\varepsilon,\psi) &= \overline{\mu}_0(\varepsilon,\psi) + \overline{\mu}_1(\varepsilon,\psi) + \overline{\mu}_2(\varepsilon,\psi) + \overline{\mu}_3(\varepsilon,\psi) + \overline{\mu}_4(\varepsilon,\psi) + \cdots, \\ \underline{\mu}(\varepsilon,\psi) &= \underline{\kappa}(r)\varepsilon - \underline{\kappa}(r)\varepsilon\left\{1 + \varpi\psi - \varpi\right\} + 2\underline{\kappa}(r)\varepsilon\left\{(1-\varpi)2\varpi\psi + (1-\varpi)^2 + \frac{\varpi^2\psi^2}{2}\right\} \\ &\quad - 4\underline{\kappa}(r)\varepsilon\left\{(1-\varpi)^2 3\varpi\psi + (1-\varpi)^3 + \frac{3\varpi^2(1-\varpi)\psi^2}{2} + \frac{\varpi^3\psi^3}{3!}\right\} + \cdots, \\ \overline{\mu}(\varepsilon,\psi) &= \overline{\kappa}(r)\varepsilon - \overline{\kappa}(r)\varepsilon\left\{1 + \varpi\psi - \varpi\right\} + 2\overline{\kappa}(r)\varepsilon\left\{(1-\varpi)2\varpi\psi + (1-\varpi)^2 + \frac{\varpi^2\psi^2}{2}\right\} \\ &\quad - 4\overline{\kappa}(r)\varepsilon\left\{(1-\varpi)^2 3\varpi\psi + (1-\varpi)^3 + \frac{3\varpi^2(1-\varpi)\psi^2}{2} + \frac{\varpi^3\psi^3}{3!}\right\} + \cdots. \end{aligned} \tag{4.5} $$

The exact solution is

$$ \tilde{\mu}(\varepsilon,\psi) = \widetilde{\kappa}\frac{\varepsilon}{1+\psi}. \tag{4.6} $$

Figures 1 and 2 show that the effectiveness of multiple (upper and lower bound accuracy) surface graphs for Example 1 interacting with the fuzzy Caputo-Fabrizio operator and Laplace transformation is being exhibited in this investigation. Figure 1 represents a three-dimensional graph of the fuzzy solution at different fractional orders of ϖ of Example 1. Figure 2 represents the two-dimensional graph of the fuzzy solution at different fractional orders of ϖ of Example 1. The two similar color legends represent the upper and lower portions of the fuzzy solution, respectively.

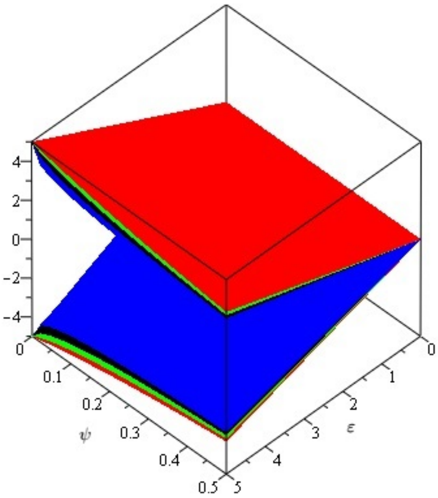

Figure 1. The fuzzy analytical solution graph of upper and lower branches of Example 1.

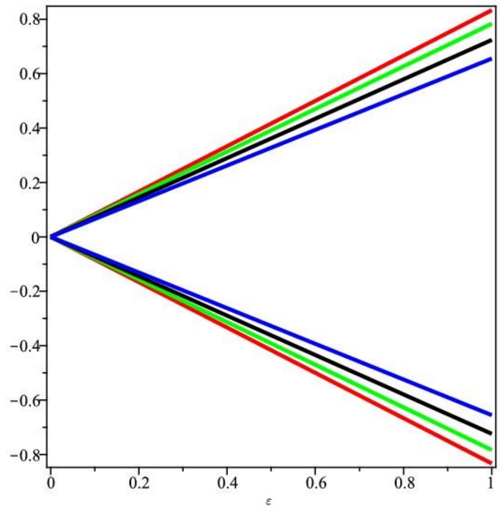

Figure 2. The fuzzy analytical result figure of lower and upper branches of Example 1.

Example 2. Consider the fractional fuzzy linear regularised long wave equation [40]

$$ {}^{CF}D_{\psi}^{\varpi}\tilde{\mu}(\varepsilon,\psi)+\frac{\partial\tilde{\mu}}{\partial\varepsilon}-2\frac{\partial\partial^2\tilde{\mu}}{\partial\psi\partial\varepsilon^2}=0, \quad 0<\varepsilon\leq 1, \quad 0<\varpi\leq 1, \quad \psi>0, \tag{4.7}$$

with the initial fuzzy condition

$$\tilde{\mu}(\varepsilon,0)=\widetilde{\kappa}e^{-\varepsilon}. \tag{4.8}$$

Using the scheme of (4.7), we obtain

$$\begin{aligned}
&\underline{\mu}_0(\varepsilon,\psi) = \underline{\kappa}(r)e^{-\varepsilon}, \ \ \overline{\mu}_0(\varepsilon,\psi) = \overline{\kappa}(r)e^{-\varepsilon},\\
&\underline{\mu}_1(\varepsilon,\psi) = \underline{\kappa}(r)e^{-\varepsilon}\{1+\varpi\psi-\varpi\}, \ \ \overline{\mu}_1(\varepsilon,\psi) = \overline{\kappa}(r)e^{-\varepsilon}\{1+\varpi\psi-\varpi\},\\
&\underline{\mu}_2(\varepsilon,\psi) = \underline{\kappa}(r)e^{-\varepsilon}\left\{(1-\varpi)2\varpi\psi+(1-\varpi)^2+\frac{\varpi^2\psi^2}{2}\right\},\\
&\overline{\mu}_2(\varepsilon,\psi) = \overline{\kappa}(r)e^{-\varepsilon}\left\{(1-\varpi)2\varpi\psi+(1-\varpi)^2+\frac{\varpi^2\psi^2}{2}\right\},\\
&\underline{\mu}_3(\varepsilon,\psi) = \underline{\kappa}(r)e^{-\varepsilon}\left\{(1-\varpi)^2 3\varpi\psi+(1-\varpi)^3+\frac{3\varpi^2(1-\varpi)\psi^2}{2}+\frac{\varpi^3\psi^3}{3!}\right\},\\
&\overline{\mu}_3(\varepsilon,\psi) = \overline{\kappa}(r)e^{-\varepsilon}\left\{(1-\varpi)^2 3\varpi\psi+(1-\varpi)^3+\frac{3\varpi^2(1-\varpi)\psi^2}{2}+\frac{\varpi^3\psi^3}{3!}\right\}.
\end{aligned} \tag{4.9}$$

The series type solution is achieved using (4.7) and we get

$$\tilde{\mu}(\varepsilon,\psi) = \tilde{\mu}_0(\varepsilon,\psi)+\tilde{\mu}_1(\varepsilon,\psi)+\tilde{\mu}_2(\varepsilon,\psi)+\tilde{\mu}_3(\varepsilon,\psi)+\tilde{\mu}_4(\varepsilon,\psi)+\cdots. \tag{4.10}$$

The lower and upper type results are achieved as

$$\begin{aligned}
\underline{\mu}(\varepsilon,\psi) &= \underline{\mu}_0(\varepsilon,\psi)+\underline{\mu}_1(\varepsilon,\psi)+\underline{\mu}_2(\varepsilon,\psi)+\underline{\mu}_3(\varepsilon,\psi)+\underline{\mu}_4(\varepsilon,\psi)+\cdots,\\
\overline{\mu}(\varepsilon,\psi) &= \overline{\mu}_0(\varepsilon,\psi)+\overline{\mu}_1(\varepsilon,\psi)+\overline{\mu}_2(\varepsilon,\psi)+\overline{\mu}_3(\varepsilon,\psi)+\overline{\mu}_4(\varepsilon,\psi)+\cdots,\\
\underline{\mu}(\varepsilon,\psi) &= \underline{\kappa}(r)e^{-\varepsilon}+\underline{\kappa}(r)e^{-\varepsilon}\{1+\varpi\psi-\varpi\}+\underline{\kappa}(r)e^{-\varepsilon}\left\{(1-\varpi)2\varpi\psi+(1-\varpi)^2+\frac{\varpi^2\psi^2}{2}\right\}\\
&\quad+\underline{\kappa}(r)e^{-\varepsilon}\left\{(1-\varpi)^2 3\varpi\psi+(1-\varpi)^3+\frac{3\varpi^2(1-\varpi)\psi^2}{2}+\frac{\varpi^3\psi^3}{3!}\right\}+\cdots,\\
\overline{\mu}(\varepsilon,\psi) &= \overline{\kappa}(r)e^{-\varepsilon}+\overline{\kappa}(r)e^{-\varepsilon}\{1+\varpi\psi-\varpi\}+\overline{\kappa}(r)e^{-\varepsilon}\left\{(1-\varpi)2\varpi\psi+(1-\varpi)^2+\frac{\varpi^2\psi^2}{2}\right\}\\
&\quad+\overline{\kappa}(r)e^{-\varepsilon}\left\{(1-\varpi)^2 3\varpi\psi+(1-\varpi)^3+\frac{3\varpi^2(1-\varpi)\psi^2}{2}+\frac{\varpi^3\psi^3}{3!}\right\}+\cdots.
\end{aligned} \tag{4.11}$$

The exact solution is

$$\tilde{\mu}(\varepsilon,\psi) = \widetilde{\kappa}e^{\psi-\varepsilon}. \tag{4.12}$$

Figures 3 and 4 show that the effectiveness of multiple (upper and lower bound accuracy) surface graphs, for Example, 2 interacting with the fuzzy Caputo-Fabrizio operator and Laplace transformation is being exhibited in this investigation. Figure 3 represents the three-dimensional graph of the fuzzy solution at different fractional orders of ϖ of Example 2. Figure 4 represents the two-dimensional graph of the fuzzy solution at different fractional orders of ϖ of Example 2. The two similar color legends represent the upper and lower portions of the fuzzy solution, respectively.

Example 3. Consider the fractional fuzzy regularised linear long wave equation [40]

$$^{CF}D_{\psi}^{\varpi}\tilde{\mu}(\varepsilon,\psi)+\frac{\partial^4\tilde{\mu}(\varepsilon,\psi)}{\partial\varepsilon^4} = 0, \quad 0<\varepsilon\leq 1, \quad 0<\varpi\leq 1, \quad \psi>0, \tag{4.13}$$

with the initial fuzzy condition

$$\tilde{\mu}(\varepsilon,0) = \sin\varepsilon. \tag{4.14}$$

Using the scheme of (4.13), we obtain

$$
\begin{aligned}
&\underline{\mu}_0(\varepsilon,\psi) = \underline{\kappa}(r)\sin\varepsilon, \quad \overline{\mu}_0(\varepsilon,\psi) = \overline{\kappa}(r)\sin\varepsilon,\\
&\underline{\mu}_1(\varepsilon,\psi) = -\underline{\kappa}(r)\sin\varepsilon\{1+\varpi\psi-\varpi\},\ \ \overline{\mu}_1(\varepsilon,\psi) = -\overline{\kappa}(r)\sin\varepsilon\{1+\varpi\psi-\varpi\},\\
&\underline{\mu}_2(\varepsilon,\psi) = \underline{\kappa}(r)\sin\varepsilon\left\{(1-\varpi)2\varpi\psi+(1-\varpi)^2+\frac{\varpi^2\psi^2}{2}\right\},\\
&\overline{\mu}_2(\varepsilon,\psi) = \overline{\kappa}(r)\sin\varepsilon\left\{(1-\varpi)2\varpi\psi+(1-\varpi)^2+\frac{\varpi^2\psi^2}{2}\right\},\\
&\underline{\mu}_3(\varepsilon,\psi) = -\underline{\kappa}(r)\sin\varepsilon\left\{(1-\varpi)^2 3\varpi\psi+(1-\varpi)^3+\frac{3\varpi^2(1-\varpi)\psi^2}{2}+\frac{\varpi^3\psi^3}{3!}\right\},\\
&\overline{\mu}_3(\varepsilon,\psi) = -\overline{\kappa}(r)\sin\varepsilon\left\{(1-\varpi)^2 3\varpi\psi+(1-\varpi)^3+\frac{3\varpi^2(1-\varpi)\psi^2}{2}+\frac{\varpi^3\psi^3}{3!}\right\}.
\end{aligned}
\tag{4.15}
$$

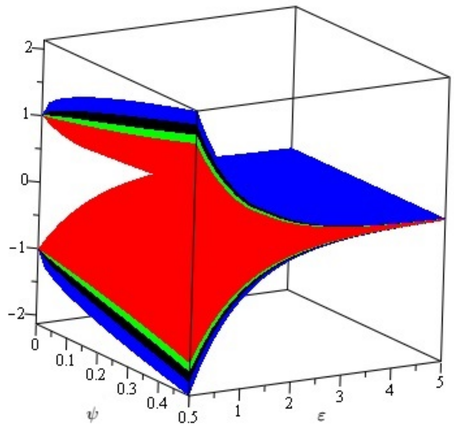

Figure 3. The fuzzy analytical solution graph of upper and lower branches of Example 2.

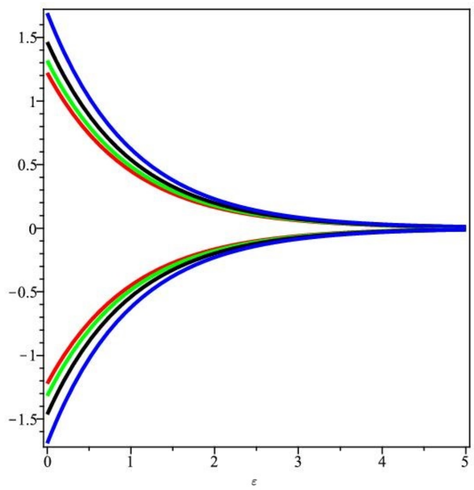

Figure 4. The fuzzy analytical result figure of lower and upper branches of Example 2.

The series type solution is achieved using (4.13) and we get

$$\tilde{\mu}(\varepsilon,\psi) = \tilde{\mu}_0(\varepsilon,\psi) + \tilde{\mu}_1(\varepsilon,\psi) + \tilde{\mu}_2(\varepsilon,\psi) + \tilde{\mu}_3(\varepsilon,\psi) + \tilde{\mu}_4(\varepsilon,\psi) + \cdots . \tag{4.16}$$

The lower and upper type results are achieved as

$$\underline{\mu}(\varepsilon,\psi) = \underline{\mu}_0(\varepsilon,\psi) + \underline{\mu}_1(\varepsilon,\psi) + \underline{\mu}_2(\varepsilon,\psi) + \underline{\mu}_3(\varepsilon,\psi) + \underline{\mu}_4(\varepsilon,\psi) + \cdots ,$$
$$\overline{\mu}(\varepsilon,\psi) = \overline{\mu}_0(\varepsilon,\psi) + \overline{\mu}_1(\varepsilon,\psi) + \overline{\mu}_2(\varepsilon,\psi) + \overline{\mu}_3(\varepsilon,\psi) + \overline{\mu}_4(\varepsilon,\psi) + \cdots ,$$

$$\begin{aligned}
\underline{\mu}(\varepsilon,\psi) =& \underline{\kappa}(r)\sin\varepsilon + \underline{\kappa}(r)\sin\varepsilon\{1+\varpi\psi-\varpi\} + \underline{\kappa}(r)\sin\varepsilon\left\{(1-\varpi)2\varpi\psi + (1-\varpi)^2 + \frac{\varpi^2\psi^2}{2}\right\} \\
&+ \underline{\kappa}(r)\sin\varepsilon\left\{(1-\varpi)^2 3\varpi\psi + (1-\varpi)^3 + \frac{3\varpi^2(1-\varpi)\psi^2}{2} + \frac{\varpi^3\psi^3}{3!}\right\} + \cdots , \\
\overline{\mu}(\varepsilon,\psi) =& \overline{\kappa}(r)\sin\varepsilon + \overline{\kappa}(r)\sin\varepsilon\{1+\varpi\psi-\varpi\} + \overline{\kappa}(r)\sin\varepsilon\left\{(1-\varpi)2\varpi\psi + (1-\varpi)^2 + \frac{\varpi^2\psi^2}{2}\right\} \\
&+ \overline{\kappa}(r)\sin\varepsilon\left\{(1-\varpi)^2 3\varpi\psi + (1-\varpi)^3 + \frac{3\varpi^2(1-\varpi)\psi^2}{2} + \frac{\varpi^3\psi^3}{3!}\right\} + \cdots .
\end{aligned} \tag{4.17}$$

The exact solution is

$$\tilde{\mu}(\varepsilon,\psi) = \widetilde{\kappa}\sin\varepsilon e^{-\psi}. \tag{4.18}$$

Figure 5 represents the three-dimensional graph of the fuzzy solution at different fractional orders of ϖ of Example 3. Figure 6 represents the two-dimensional graph of the fuzzy solution at different fractional orders of ϖ of Example 3. The two similar color legends represent the upper and lower portions of the fuzzy solution, respectively.

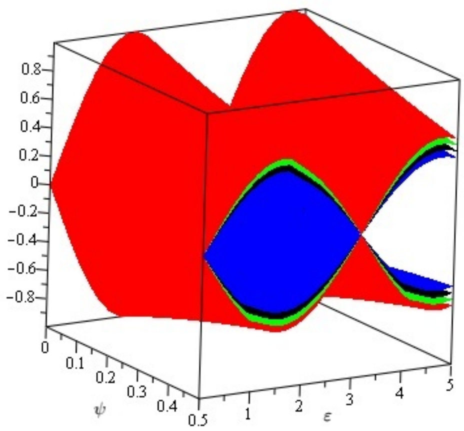

Figure 5. The fuzzy analytical solution graph of upper and lower branches of Example 3.

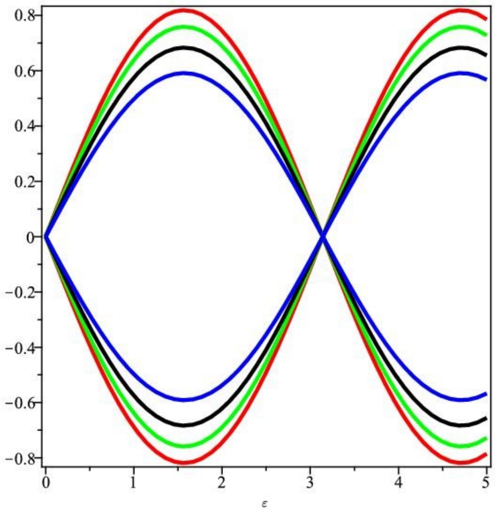

Figure 6. The fuzzy analytical result figure of lower and upper branches of Example 3.

5. Conclusions

The purpose of this study is to develop a semi-analytic solution to the fuzzy fractional acoustic waves equation utilizing Caputo-Fabrizio fractional derivatives. As a result, fuzzy operators are a more appropriate way to explain the physical phenomenon in this case. Considering the uncertainty in the initial conditions, we examined the acoustic waves equation in a fuzzy manner. Then, we obtained a parametric formulation of the suggested problem using a new iterative transform method. Our research has presented numerous illustrations to support the intended methodology and has achieved a parametric solution for each situation. In future research, this method can be applied to achieve analytical and approximate solutions of perturbed fractional differential equations under the uncertainty equipped with non-classical and integral boundary conditions in light of Caputo-Fabrizio.

Conflict of interest

The authors declare no conflict of interest.

References

1. T. Allahviranloo, *Fuzzy fractional differential equations*, Springer, Cham, **397** (2021), 127–192. https://doi.org/10.1007/978-3-030-51272-9_4

2. T. Allahviranloo, B. Ghanbari, On the fuzzy fractional differential equation with interval Atangana-Baleanu fractional derivative approach, *Chaos Soliton. Fract.*, **130** (2020), 109397. https://doi.org/10.1016/j.chaos.2019.109397

3. N. V. Hoa, H. Vu, T. M. Duc, Fuzzy fractional differential equations under Caputo-Katugampola fractional derivative approach, *Fuzzy Set. Syst.*, **375** (2019), 70–99. https://doi.org/10.1016/j.fss.2018.08.001

4. S. Chakraverty, S. Tapaswini, D. Behera, *Fuzzy arbitrary order system: Fuzzy fractional differential equations and applications*, John Wiley & Sons, 2016.

5. S. Rashid, M. K. Kaabar, A. Althobaiti, M. S. Alqurashi, Constructing analytical estimates of the fuzzy fractional-order Boussinesq model and their application in oceanography, *J. Ocean Eng. Sci.*, 2022. https://doi.org/10.1016/j.joes.2022.01.003

6. M. S. Alqurashi, S. Rashid, B. Kanwal, F. Jarad, S. K. Elagan, A novel formulation of the fuzzy hybrid transform for dealing nonlinear partial differential equations via fuzzy fractional derivative involving general order, *AIMS Math.*, **7** (2022), 14946–14974. https://doi.org/10.3934/math.2022819

7. M. S. Shagari, S. Rashid, F. Jarad, M. S. Mohamed, Interpolative contractions and intuitionistic fuzzy set-valued maps with applications, *AIMS Math.*, **7** (2022), 10744–10758. https://doi.org/10.3934/math.2022600

8. M. Al-Qurashi, M. S. Shagari, S. Rashid, Y. S. Hamed, M. S. Mohamed, Stability of intuitionistic fuzzy set-valued maps and solutions of integral inclusions, *AIMS Math.*, **7** (2022), 315–333. https://doi.org/10.3934/math.2022022

9. V. H. Ngo, Fuzzy fractional functional integral and differential equations, *Fuzzy Set. Syst.*, **280** (2015), 58–90. https://doi.org/10.1016/j.fss.2015.01.009

10. T. Allahviranloo, Z. Gouyandeh, A. Armand, A full fuzzy method for solving differential equation based on Taylor expansion, *J. Intell. Fuzzy Syst.*, **29** (2015), 1039–1055. https://doi.org/10.3233/IFS-151713

11. M. Das, A. Maiti, G. P. Samanta, Stability analysis of a prey-predator fractional order model incorporating prey refuge, *Ecol. Genet. Genomics*, **7** (2018), 33–46. https://doi.org/10.1016/j.egg.2018.05.001

12. M. Das, G. P. Samanta, A delayed fractional order food chain model with fear effect and prey refuge, *Math. Comput. Simulat.*, **178** (2020), 218–245. https://doi.org/10.1016/j.matcom.2020.06.015

13. M. Das, G. P. Samanta, A prey-predator fractional order model with fear effect and group defense, *Int. J. Dyn. Control*, **9** (2021), 334–349. https://doi.org/10.1007/s40435-020-00626-x

14. A. A. Alderremy, R. Shah, N. Iqbal, S. Aly, K. Nonlaopon, Fractional series solution construction for nonlinear fractional reaction-diffusion Brusselator model utilizing Laplace residual power series, *Symmetry*, **14** (2022), 1944. https://doi.org/10.3390/sym14091944

15. M. Chehlabi, T. Allahviranloo, Concreted solutions to fuzzy linear fractional differential equations, *Appl. Soft Comput.*, **44** (2016), 108–116. https://doi.org/10.1016/j.asoc.2016.03.011

16. N. V. Hoa, Fuzzy fractional functional differential equations under Caputo gH-differentiability, *Commun. Nonlinear Sci.*, **22** (2015), 1134–1157. https://doi.org/10.1016/j.cnsns.2014.08.006

17. M. Naeem, H. Rezazadeh, A. A. Khammash, S. Zaland, Analysis of the fuzzy fractional-order solitary wave solutions for the KdV equation in the sense of Caputo-Fabrizio derivative, *J. Math.*, **2022** (2022). https://doi.org/10.1155/2022/3688916

18. A. U. K. Niazi, N. Iqbal, F. Wannalookkhee, K. Nonlaopon, Controllability for fuzzy fractional evolution equations in credibility space, *Fractal Fract.*, **5** (2021), 112. https://doi.org/10.3390/fractalfract5030112

19. K. Nonlaopon, M. Naeem, A. M. Zidan, R. Shah, A. Alsanad, A. Gumaei, Numerical investigation of the time-fractional Whitham-Broer-Kaup equation involving without singular kernel operators, *Complexity*, **2021** (2021). https://doi.org/10.1155/2021/7979365

20. R. P. Agarwal, D. Baleanu, J. J. Nieto, D. F. M. Torres, Y. Zhou, A survey on fuzzy fractional differential and optimal control nonlocal evolution equations, *J. Comput. Appl. Math.*, **339** (2018), 3–29. https://doi.org/10.1016/j.cam.2017.09.039

21. H. V. Long, N. T. K. Son, H. T. T. Tam, The solvability of fuzzy fractional partial differential equations under Caputo gH-differentiability, *Fuzzy Set. Syst.*, **309** (2017), 35–63. https://doi.org/10.1016/j.fss.2016.06.018

22. S. Salahshour, T. Allahviranloo, S. Abbasbandy, Solving fuzzy fractional differential equations by fuzzy Laplace transforms, *Commun. Nonlinear Sci.*, **17** (2012), 1372–1381. https://doi.org/10.1016/j.cnsns.2011.07.005

23. A. S. Alshehry, M. Imran, A. Khan, W. Weera, Fractional view analysis of Kuramoto-Sivashinsky equations with non-singular kernel operators, *Symmetry*, **14** (2022), 1463. https://doi.org/10.3390/sym14071463

24. S. Mukhtar, R. Shah, S. Noor, The numerical investigation of a fractional-order multi-dimensional model of Navier-Stokes equation via novel techniques, *Symmetry*, **14** (2022), 1102. https://doi.org/10.3390/sym14061102

25. A. Goswami, J. Singh, D. Kumar, S. Gupta, An efficient analytical technique for fractional partial differential equations occurring in ion acoustic waves in plasma, *J. Ocean Eng. Sci.*, **4** (2019), 85–99. https://doi.org/10.1016/j.joes.2019.01.003

26. M. M. Al-Sawalha, A. Khan, O. Y. Ababneh, T. Botmart, Fractional view analysis of Kersten-Krasil shchik coupled KdV-mKdV systems with non-singular kernel derivatives, *AIMS Math.*, **7** (2022), 18334–18359. http://dx.doi.org/10.3934/math.20221010

27. A. S. Alshehry, N. Amir, N. Iqbal, K. Nonlaopon, On the solution of nonlinear fractional-order shock wave equation via analytical method, *AIMS Math.*, **7** (2022), 19325–19343. https://doi.org/10.3934/math.20221061

28. M. M. Al-Sawalha, A. S. Alshehry, K. Nonlaopon, O. Y. Ababneh, Approximate analytical solution of time-fractional vibration equation via reliable numerical algorithm, *AIMS Math.*, **7** (2022), 19739–19757. https://doi.org/10.3934/math.20221082

29. Y. Khan, R. Taghipour, M. Falahian, A. Nikkar, A new approach to modified regularized long wave equation, *Neural Comput. Appl.*, **23** (2013), 1335–1341. https://doi.org/10.1007/s00521-012-1077-0

30. C. Bota, B. Căruntu, Approximate analytical solutions of the regularized long wave equation using the optimal homotopy perturbation method, *The Scientific World J.*, **2014** (2014). https://doi.org/10.1155/2014/721865

31. V. D. Gejji, H. Jafari, An iterative method for solving nonlinear functional equation, *J. Math. Anal. Appl.*, **316** (2006), 753–763. https://doi.org/10.1016/j.jmaa.2005.05.009
32. H. Jafari, S. Seifi, A. Alipoor, M. Zabihi, An iterative method for solving linear and nonlinear fractional diffusion-wave equation, *J. Nonlinear Fract. Phenom. Sci. Eng.*, 2007.
33. S. Bhalekar, V. Daftardar-Gejji, Solving evolution equations using a new iterative method, *Numer. Method. Part. D. E.*, **26** (2010), 906–916. https://doi.org/10.1002/num.20463
34. V. N. Kovalnogov, R. V. Fedorov, D. A. Generalov, E. V. Tsvetova, T. E. Simos, C. Tsitouras, On a new family of Runge-Kutta-Nystrom pairs of orders, *Mathematics*, **10** (2022), 875. https://doi.org/10.3390/math10060875
35. R. Ye, P. Liu, K. Shi, B. Yan, State damping control: A novel simple method of rotor UAV with high performance, *IEEE Access*, **8** (2020) 214346–214357. https://doi.org/10.1109/ACCESS.2020.3040779
36. W. Dang, J. Guo, M. Liu, S. Liu, B. Yang, L. Yin, et al., A semi-supervised extreme learning machine algorithm based on the new weighted kernel for machine smell, *Appl. Sci.*, **12** (2022), 9213. https://doi.org/10.3390/app12189213
37. T. Sitthiwirattham, M. Arfan, K. Shah, A. Zeb, S. Djilali, S. Chasreechai, Semi-analytical solutions for fuzzy Caputo-Fabrizio fractional-order two-dimensional heat equation, *Fractal Fract.*, **5** (2021), 139. https://doi.org/10.3390/fractalfract5040139
38. M. Alesemi, N. Iqbal, M. S. Abdo, Novel investigation of fractional-order Cauchy-reaction diffusion equation involving Caputo-Fabrizio operator, *J. Function Space.*, **2022** (2022).
39. N. Harrouche, S. Momani, S. Hasan, M. Al-Smadi, Computational algorithm for solving drug pharmacokinetic model under uncertainty with nonsingular kernel type Caputo-Fabrizio fractional derivative, *Alex. Eng. J.*, **60** (2021), 4347–4362. https://doi.org/10.1016/j.aej.2021.03.016
40. P. Veeresha, D. G. Prakasha, J. Singh, A novel approach for nonlinear equations occurs in ion acoustic waves in plasma with Mittag-Leffler law, *Eng. Comput.*, **37** (2020). https://doi.org/10.1108/EC-09-2019-0438

AIMS Press