AIMS Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 8(1): 691–712.
DOI: 10.3934/math.2023033
Received: 19 July 2022
Revised: 21 September 2022
Accepted: 26 September 2022
Published: 11 October 2022

Research article

New delay-range-dependent exponential stability criterion and H_∞ performance for neutral-type nonlinear system with mixed time-varying delays

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Abstract: For a neutral system with mixed discrete, neutral and distributed interval time-varying delays and nonlinear uncertainties, the problem of exponential stability is investigated in this paper based on the H_∞ performance condition. The uncertainties are nonlinear time-varying parameter perturbations. By introducing a decomposition matrix technique, using Jensen's integral inequality, Peng-Park's integral inequality, Leibniz-Newton formula and Wirtinger-based integral inequality, utilization of a zero equation and the appropriate Lyapunov-Krasovskii functional, new delay-range-dependent sufficient conditions for the H_∞ performance with exponential stability of the system are presented in terms of linear matrix inequalities. Moreover, we present numerical examples that demonstrate exponential stability of the neutral system with mixed time-varying delays, and nonlinear uncertainties to show the advantages of our method.

Keywords: exponential stability; H_∞ performance; neutral system; time-varying delay
Mathematics Subject Classification: 93B36, 93C43, 93D23

1. Introduction

Neutral time-delay systems contain delays both in the state and in the derivatives of the state which can be found in various dynamic systems, such as chemical reactors, nuclear reactors, biological systems, economical systems, water pipes, population ecology, power systems, etc. [1–14]. On the other hand, nonlinear uncertainties are commonly encountered because it is very problematic to derive a certain mathematical model due to slowly varying parameters, environmental noise and so on. Stability criteria for time-delay systems are classified into two categories: delay-independent and delay-dependent. In general, the delay-dependent criteria are less conservative than the delay-independent ones, especially when the size of the delay is small. Therefore, many researchers have dedicated much effort to studying the delay-dependent stability criteria for neutral time-delay systems with nonlinear uncertainties in recent years; see, for instance, [1, 5, 6, 10, 15–18].

The stability analysis of neutral-type systems is considered with various inequality techniques and Lyapunov approaches, which are significant to reduce conservatism. Therefore, many inequality techniques have been applied in the published literature to estimate the upper bound of the time derivative of the introduced Lyapunov-Krasovskii functional (LKF). In [1], the new stability conditions for the neutral delay differential system are derived by applying Jensen's integral inequality. In order to reduce the conservatism, Wirtinger's integral inequality was introduced in [19]. The free weighting matrices were utilized with a new integral inequality lemma in [6] to achieve less conservative results.

As pointed out in [1, 9–11, 16, 17, 20], the exponential stability problem is also significant since it can determine the convergence rate of system states to equilibrium points. The problem of delay-dependent exponential stability criteria for neutral systems with nonlinear uncertainties have been investigated in [10, 11, 20]. Recently, many researchers have paid a lot of attention to the H_∞ control problem in time-delay systems [21–24]. Li and Hu [25] studied the problem of H_∞ control for neutral systems without nonlinear uncertainties. The H_∞ control for uncertain neutral systems have been reported in [26]. The problem of H_∞ performance for a neutral system with discrete, neutral and distributed time-varying delays and nonlinear uncertainties have been investigated in [19]. Their results are restricted on delay-independent criteria for neutral systems [25] or uncertain neutral systems without the condition of lower bounds of time-varying delays [19, 26].

Motivated by the above statement, in this paper, the problem of H_∞ performance and exponential stability analysis for a neutral system with interval discrete, neutral and distributed time-varying delays and nonlinear uncertainties are considered based on Jensen's integral inequality, the Wirtinger-based integral inequality, an extended Wirtinger's integral inequality, Peng-Park's integral inequality, the Leibniz-Newton formula, utilization of a zero equation, a decomposition matrix technique and the appropriate LKF. In the numerical part, we give some examples to present the effectiveness of the theorem. The main contributions and highlights of this paper are summarized in the following key points.

- We consider the problem of exponential stability for a neutral system with interval discrete, neutral and distributed time-varying delays and nonlinear uncertainties based on an H_∞ performance condition. It is noted that this work is the first study of the exponential stability and H_∞ performance for an uncertain neutral system with three (discrete, neutral, and distributed) interval time-varying delays.
- We construct the LKFs including single, double, and triple integral terms involving lower and upper bounds of time delays and use them to formulate a new delay-range-dependent stability criterion for a neutral system. In addition, the LKF consists of five new triple integral terms, i.e., $\frac{\lambda_2^2}{2}\int_{-\lambda_2}^{0}\int_{s}^{0}\int_{t+\theta}^{t} e^{2\kappa(u+\theta-t)}\dot{\varphi}^T(u)Z_{21}\dot{\varphi}(u)dud\theta ds$, $\lambda_2^2\int_{-\lambda_2}^{0}\int_{s}^{0}\int_{t+\theta}^{t} e^{2\kappa(u+\theta-t)}\dot{\varphi}^T(u)Z_{22}\dot{\varphi}(u)dud\theta ds$,

$\int_{-\lambda_2}^{0}\int_{s}^{0}\int_{t+\theta}^{t} e^{\kappa(u+\theta-t)}\dot{\varphi}^T(u)Z_{23}\dot{\varphi}(u)dud\theta ds$, $\lambda_1^2\int_{-\lambda_1}^{0}\int_{s}^{0}\int_{t+\theta}^{t} e^{2\kappa(u+\theta-t)}\dot{\varphi}^T(u)Z_{24}\dot{\varphi}(u)dud\theta ds$ and $\frac{(\lambda_2^2-\lambda_1^2)}{2}\int_{-\lambda_2}^{-\lambda_1}\int_{s}^{0}\int_{t+\theta}^{t} e^{2\kappa(u+\theta-t)}\dot{\varphi}^T(u)Z_{25}\dot{\varphi}(u)dud\theta ds$, that were not used in [25, 26].

- We apply tighter inequalities to improve the stability criterion, such as Jensen's integral inequality (Lemma 1) and extended single and double Wirtinger's integral inequalities (Lemmas 9 and 10). Using the above new LKFs and the lemmas leads to less conservatism of the obtained results than in published literature, as presented via numerical examples.
- We derive new delay-range-dependent sufficient conditions for the exponential stability with H_∞ performance (Theorem 1). Moreover, we obtain the improved delay-range-dependent exponential stability criterion of a neutral system with discrete, neutral and distributed time-varying delays, and nonlinear uncertainties. The proposed conditions are less conservative than the other references as shown in Theorem 1.
- We present numerical examples to demonstrate the feasibility and effectiveness of the theorem.

The outline of this work is structured as follows. In Section 2, we give the problem statement, definitions and lemmas. We discuss some results for a neutral system and their proofs in Section 3. In Section 4, we give two numerical examples to present the effectiveness of the obtained criterion. Section 5 shows the conclusion of our results.

Notations: $\mathbb{R}^n$ denotes the $n-$dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ real matrices. For a matrix A, $A > 0$ means that A is a symmetric positive definite matrix and $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote the minimum and maximum eigenvalues of A, respectively. The superscript "T" denotes matrix transposition. diag$\{\ldots\}$ denotes the block diagonal matrix. Sym$\{A\} = A + A^T$.

2. Problem formulation and preliminaries

We introduce the following neutral system with interval time-varying delays and nonlinear uncertainties of the form

$$\begin{aligned} \dot{\varphi}(t) &= A_1\varphi(t) + A_2\varphi(t-\lambda(t)) + A_3\dot{\varphi}(t-\sigma(t)) + A_4\int_{t-\rho(t)}^{t}\varphi(s)ds + Bw(t) \\ &\quad +\zeta_1(t,\varphi(t)) + \zeta_2(t,\varphi(t-\lambda(t))) + \zeta_3(t,\dot{\varphi}(t-\rho(t))), \quad t \geq 0; \\ \chi(t) &= C_1\varphi(t) + C_2\varphi(t-\lambda(t)) + Dw(t), \quad t \geq 0; \\ \varphi(t) &= \phi(t), \quad \forall t \in [-\max\{\lambda_2,\sigma_2,\rho_2\}, 0], \end{aligned} \tag{2.1}$$

where $\varphi(t) \in \mathbb{R}^n$ is the state of the system, $w(t) \in \mathbb{R}^p$ is the disturbance input which belongs to $L_2[0,\infty]$, $\chi(t) \in \mathbb{R}^q$ is the controlled output, $\phi(t)$ is the initial condition function that is continuously differentiable on $[-\max\{\lambda_2,\sigma_2,\rho_2\}, 0]$ with $\|\phi\| = \sup_{s\in[-\max\{\lambda_2,\sigma_2,\rho_2\},0]}\|\phi(s)\|$, A_1, A_2, A_3, A_4, B, C_1, C_2 and D are real constant matrices with appropriate dimensions and $\lambda(t)$, $\sigma(t)$ and $\rho(t)$ are time-varying discrete, neutral and distributed delays, respectively. The delays satisfy the following conditions:

$$0 \leq \lambda_1 \leq \lambda(t) \leq \lambda_2, \quad 0 \leq \dot{\lambda}(t) \leq \lambda_d, \tag{2.2}$$

$$0 \leq \sigma_1 \leq \sigma(t) \leq \sigma_2, \quad 0 \leq \dot{\sigma}(t) \leq \sigma_d, \tag{2.3}$$

$$0 \leq \rho_1 \leq \rho(t) \leq \rho_2, \quad 0 \leq \dot{\rho}(t) \leq \rho_d, \tag{2.4}$$

where $\sigma_1,\ \sigma_2,\ \sigma_d,\ \lambda_1,\ \lambda_2,\ \lambda_d,\ \rho_1,\ \rho_2$ and ρ_d are positive real constants and $\zeta_1(t,\varphi(t))$, $\zeta_2(t,\varphi(t-\lambda(t)))$ and $\zeta_3(t,\dot{\varphi}(t-\sigma(t)))$ are nonlinear uncertainties that are assumed to satisfy the following inequalities

$$\zeta_1^T(t,\varphi(t))\zeta_1(t,\varphi(t)) \leq \eta_1^2\varphi^T(t)\varphi(t), \tag{2.5}$$

$$\zeta_2^T(t,\varphi(t-\lambda(t)))\zeta_2(t,\varphi(t-\lambda(t))) \leq \eta_2^2\varphi^T(t-\lambda(t))\varphi(t-\lambda(t)), \tag{2.6}$$

$$\zeta_3^T(t,\dot{\varphi}(t-\sigma(t)))\zeta_3(t,\dot{\varphi}(t-\sigma(t))) \leq \eta_3\dot{\varphi}^T(t-\sigma(t))\dot{\varphi}(t-\sigma(t)), \tag{2.7}$$

where η_1, η_2 and η_3 are known positive real constants. We consider the Leibniz-Newton formula of the form

$$0 = \varphi(t) - \varphi(t-\lambda(t)) - \int_{t-\lambda(t)}^{t} \dot{\varphi}(s)ds. \tag{2.8}$$

In order to improve the discrete delay $\lambda(t)$ in (2.2), let us decompose the constant matrix A_2 as

$$A_2 = H_1 + H_2, \tag{2.9}$$

where $H_1, H_2 \in \mathbb{R}^{n\times n}$ are real constant matrices. By (2.8) and (2.9), System (2.1) can be represented in the form

$$\begin{aligned}\dot{\varphi}(t) \;=\;& [A_1+H_1+I]\varphi(t) + [H_2-I]\varphi(t-\lambda(t)) + A_3\dot{\varphi}(t-\sigma(t)) + A_4\int_{t-\rho(t)}^{t}\varphi(s)ds \\ &+Bw(t) + \zeta_1(t,\varphi(t)) + \zeta_2(t,\varphi(t-\lambda(t))) + \zeta_3(t,\dot{\varphi}(t-\sigma(t))) \\ &-[H_1+I]\int_{t-\lambda(t)}^{t}\dot{\varphi}(s)ds.\end{aligned} \tag{2.10}$$

Remark 1. *In System* (2.1)*, we assume that the delays in the discrete delay term and the distributed delay term are different but these two delay terms in [19] are the same.*

Definition 1. *[20] If there exist real positive scalars β and κ that satisfy*

$$\|\varphi(t,\phi)\| \leq \beta\|\phi\|e^{-\kappa t}, \quad \forall t \geq 0,$$

then System (2.1) *is exponentially stable.*

Definition 2. *[3] For a given real positive scalar δ, we say that System* (2.1) *is exponentially stable with the H_∞ performance level δ if the system is exponentially stable and also satisfies $\|\chi(t)\|_2 \leq \delta\|w(t)\|_2$, for all nonzero $w(t) \in L_2[0,\infty)$ under the zero initial condition.*

Lemma 1. *(Jensen's inequality [19]). For any positive definite symmetric matrix $W \in R^{n\times n}$, k_2 is a positive scalar and the vector function $\omega : [-k_2, 0] \to \mathbb{R}^n$ such that the integrals concerned are well defined; the following inequality holds:*

$$-k_2\int_{-k_2}^{0}\omega^T(s+t)W\omega(s+t)ds \leq -\left(\int_{-k_2}^{0}\omega(s+t)ds\right)^T W\left(\int_{-k_2}^{0}\omega(s+t)ds\right).$$

Lemma 2. *[2] For any positive definite symmetric matrix $W \in R^{n\times n}$, k_2 is a positive scalar and the vector function $\dot{\omega} : [-k_2, 0] \to R^n$ such that the integrals concerned are well defined; then,*

$$-\int_{-k_2}^{0}\int_{t+s}^{t} \dot{\omega}^T(u)W\dot{\omega}(u)duds \le \psi_1^T(t)\Omega_1\psi_1(t),$$

where

$$\psi_1(t) = \begin{bmatrix} \omega(t) \\ \frac{1}{k_2}\int_{t-k_2}^{t} \omega(s)ds \end{bmatrix}, \quad \Omega_1 = \begin{bmatrix} -2W & 2W \\ * & -2W \end{bmatrix}.$$

Lemma 3. *[27] For any positive definite symmetric matrix $W \in \mathbb{R}^{n\times n}$, $k_1 < k_2$ are positive scalars and the vector function $\dot{\omega} : [-k_2, -k_1] \to \mathbb{R}^n$ such that the integrals concerned are well defined; then,*

$$\begin{aligned} -k_2 \int_{t-k_2}^{t} \dot{\omega}^T(s)W\dot{\omega}(s)ds &\le -\psi_2^T(t)W\psi_2(t), \\ -\frac{(k_2^2-k_1^2)}{2}\int_{-k_2}^{-k_1}\int_{t+s}^{t} \omega^T(u)W\omega(u)duds &\le -\psi_3^T(t)W\psi_3(t), \end{aligned}$$

where

$$\psi_2(t) = \Big(\int_{t-k_2}^{t} \dot{\omega}(s)ds\Big), \quad \psi_3(t) = \Big(\int_{-k_2}^{-k_1}\int_{t+s}^{t} \omega(u)duds\Big).$$

Lemma 4. *[19] For any positive definite symmetric matrix $W \in \mathbb{R}^{n\times n}$, $k(t)$ is a time-varying delay with $0 < k_1 < k(t) < k_2$, $k_2 \in \mathbb{R}$ and the vector function $\omega : [-k_2, -k_1] \to \mathbb{R}^n$ such that the integrals concerned are well defined; then,*

$$-[k_2 - k_1]\int_{t-k_2}^{t-k_1} \omega^T(s)W\omega(s)ds \le -\psi_4^T(t)W\psi_4(t) - \psi_5^T(t)W\psi_5(t),$$

where

$$\psi_4(t) = \int_{t-k(t)}^{t-k_1} \omega(s)ds, \quad \psi_5(t) = \int_{t-k_2}^{t-k(t)} \omega(s)ds.$$

Lemma 5. *[19] For any constant matrices $Y_1,\ Y_2,\ Y_3 \in \mathbb{R}^{n\times n}$, $Y_1 \ge 0,\ Y_3 > 0$, $\begin{bmatrix} Y_1 & Y_2 \\ * & Y_3 \end{bmatrix} \ge 0$, $k(t)$ is a time-varying delay with $0 \le k_1 \le k(t) \le k_2$, $k_1,\ k_2 \in \mathbb{R}$ and the vector function $\dot{\omega} : [-k_2, -k_1] \to \mathbb{R}^n$ such that the integrals concerned are well defined; then,*

$$-(k_2 - k_1)\int_{t-k_2}^{t-k_1} \begin{bmatrix} \omega(s) \\ \dot{\omega}(s) \end{bmatrix}^T \begin{bmatrix} Y_1 & Y_2 \\ * & Y_3 \end{bmatrix} \begin{bmatrix} \omega(s) \\ \dot{\omega}(s) \end{bmatrix} ds \le \psi_6^T(t)\Omega_2\psi_6(t),$$

where

$$\psi_6(t) = \begin{bmatrix} \omega(t-k_1) \\ \omega(t-k(t)) \\ \omega(t-k_2) \\ \int_{t-k(t)}^{t-k_1} \omega(s)ds \\ \int_{t-k_2}^{t-k(t)} \omega(s)ds \end{bmatrix}, \quad \Omega_2 = \begin{bmatrix} -Y_3 & Y_3 & 0 & -Y_2^T & 0 \\ * & -Y_3 - Y_3^T & Y_3 & Y_2^T & -Y_2^T \\ * & * & -Y_3 & 0 & Y_2^T \\ * & * & * & -Y_1 & 0 \\ * & * & * & * & -Y_1 \end{bmatrix}.$$

Lemma 6. *[19] For any constant matrices W, $Y_i \in \mathbb{R}^{n\times n}$, $i = 4, 5, ..., 8$, k(t) is a time-varying delay with $0 \le k_1 \le k(t) \le k_2$, $k_2 \in \mathbb{R}$ and the vector function $\dot{\omega} : [-k_2, -k_1] \to \mathbb{R}^n$ such that the integrals concerned are well defined; then,*

$$-\int_{t-k_2}^{t-k_1} \dot{\omega}^T(s)W\dot{\omega}(s)ds \quad \le \quad \psi_7^T(t)\Omega_3\psi_7(t) + (k_2 - k_1)\psi_7^T(t)\Omega_4\psi_7(t),$$

where

$$\psi_7(t) = \begin{bmatrix} \omega(t-k_1) \\ \omega(t-k(t)) \\ \omega(t-k_2) \end{bmatrix}, \quad \Omega_3 = \begin{bmatrix} Y_4 + Y_4^T & -Y_4^T + Y_5 & 0 \\ * & Y_4 + Y_4^T - Y_5 - Y_5^T & -Y_4^T + Y_5 \\ * & * & -Y_5 - Y_5^T \end{bmatrix},$$

$$\Omega_4 = \begin{bmatrix} Y_6 & Y_7 & 0 \\ * & Y_6 + Y_8 & Y_7 \\ * & * & Y_8 \end{bmatrix}, \quad \begin{bmatrix} W & Y_4 & Y_5 \\ * & Y_6 & Y_7 \\ * & * & Y_8 \end{bmatrix} \ge 0.$$

Lemma 7. *(Wirtinger-based integral inequality [28]). For any positive definite symmetric matrix $W \in \mathbb{R}^{n\times n}$, $k_1 < k_2$ are positive scalars and the vector function $\dot{\omega} : [-k_2, -k_1] \to \mathbb{R}^n$ such that the integrals concerned are well defined; then,*

$$-(k_2 - k_1)\int_{t-k_2}^{t-k_1} \dot{\omega}^T(s)W\dot{\omega}(s)ds \le \psi_8^T(t)\Omega_5\psi_8(t),$$

where

$$\psi_8(t) = \begin{bmatrix} \omega(t-k_1) \\ \omega(t-k_2) \\ \frac{1}{k_2-k_1}\int_{t-k_2}^{t-k_1} \omega(s)ds \end{bmatrix}, \quad \Omega_5 = \begin{bmatrix} -4W & -2W & 6W \\ * & -4W & 6W \\ * & * & -12W \end{bmatrix}.$$

Lemma 8. *(Peng-Park's integral inequality [29]). If W and S are real constant matrices such that $\begin{bmatrix} W & S \\ * & W \end{bmatrix} \ge 0$, k(t) is a time-varying delay with $0 < k(t) < k_2$, $k_2 \in \mathbb{R}$ and the vector function $\dot{\omega}$: $[-k_2, 0] \to \mathbb{R}^n$ is well defined; then, the following inequality holds:*

$$-k_2\int_{t-k_2}^{t} \dot{\omega}^T(s)W\dot{\omega}(s)ds \le \psi_9^T(t)\Omega_6\psi_9(t),$$

where

$$\psi_9(t) = \begin{bmatrix} \omega(t) \\ \omega(t-k(t)) \\ \omega(t-k_2) \end{bmatrix}, \quad \Omega_6 = \begin{bmatrix} -W & W-S & S \\ * & -2W + S + S^T & W-S \\ * & * & -W \end{bmatrix}.$$

Lemma 9. *(An extended Wirtinger's integral inequality [30]). For any positive definite symmetric matrix $W \in \mathbb{R}^{n\times n}$, k_1 and k_2 are positive scalars and the vector function $\omega : [k_1, k_2] \to \mathbb{R}^n$ such that the integrals concerned are well defined; then,*

$$(k_2 - k_1)\int_{k_1}^{k_2} \omega^T(u)W\omega(u)du \ge {\Omega_7}^T W\Omega_7 + 3{\Omega_8}^T W\Omega_8 + 5{\Omega_9}^T W\Omega_9, \tag{2.11}$$

where

$$\Omega_7 = \int_{k_1}^{k_2} \omega(u)du,$$
$$\Omega_8 = \int_{k_1}^{k_2} \omega(u)du - \frac{2}{k_2-k_1}\int_{k_1}^{k_2} du \int_{k_1}^{u} \omega(r)dr,$$
$$\Omega_9 = \int_{k_1}^{k_2} \omega(u)du - \frac{6}{k_2-k_1}\int_{k_1}^{k_2} du \int_{k_1}^{u} \omega(r)dr + \frac{12}{(k_2-k_1)^2}\int_{k_1}^{k_2} du \int_{k_1}^{u} ds \int_{k_1}^{s} \omega(r)dr.$$

Lemma 10. *[31] For any positive definite symmetric matrix $W \in \mathbb{R}^{n\times n}$, k_1 and k_2 are positive scalars and the vector function $\dot{\omega} : [k_1, k_2] \to \mathbb{R}^n$ such that the integrals concerned are well defined; then,*

$$\int_{k_1}^{k_2}\int_{u}^{k_2} \dot{\omega}^T(s)W\dot{\omega}(s)dsdu \geq 2\Omega_{10}^T W\Omega_{10} + 4\Omega_{11}^T W\Omega_{11} + 6\Omega_{12}^T W\Omega_{12}, \tag{2.12}$$

where

$$\Omega_{10} = \omega(k_2) - \frac{1}{k_2-k_1}\int_{k_1}^{k_2} \omega(s)ds,$$
$$\Omega_{11} = \omega(k_2) + \frac{2}{k_2-k_1}\int_{k_1}^{k_2} \omega(s)ds - \frac{6}{(k_2-k_1)^2}\int_{k_1}^{k_2}\int_{u}^{k_2} \omega(s)dsdu,$$
$$\Omega_{12} = \omega(k_2) - \frac{3}{k_2-k_1}\int_{k_1}^{k_2} \omega(s)ds + \frac{24}{(k_2-k_1)^2}\int_{k_1}^{k_2}\int_{u}^{k_2} \omega(s)dsdu - \frac{60}{(k_2-k_1)^3}\int_{k_1}^{k_2}\int_{u}^{k_2}\int_{s}^{k_2} \omega(r)drdsdu.$$

3. Main results

We introduce the following notations for later use:

$$\sum = \left[\Sigma_{(i,j)}\right]_{27\times 27}, \tag{3.1}$$

where

$$\begin{aligned}\sum\nolimits_{1,1} = {} & 2Q_1^T I + 2Q_5^T I + 2Q_9^T H_1 + 2Q9^T I + \lambda_2^2 Z_{19} + \lambda_2 e^{-2\kappa\lambda_2}F_3 + e^{-2\kappa\lambda_2}F_1 + e^{-2\kappa\lambda_2}F_1^T + 2Z_1 I \\ & -4e^{-2\kappa\lambda_2}Z_{17} - e^{-2\kappa\lambda_2}Z_{18} + \lambda_2^2 G_1 + \lambda_2^2 G_4 - 2\lambda_1\lambda_2 G_4 + \lambda_1^2 G_4 - e^{-2\kappa\lambda_2}G_3 - \lambda_2^2 e^{-4\kappa\lambda_2}Z_{21} \\ & -\lambda_2^2 e^{-4\kappa\lambda_2}Z_{25} + 2\lambda_1\lambda_2 e^{-4\kappa\lambda_2}Z_{25} - \lambda_1^2 e^{-4\kappa\lambda_2}Z_{25} - 2\lambda_1^2 e^{-4\kappa\lambda_2}Z_{24} - 12e^{-4\kappa\lambda_2}Z_{23} + \epsilon_1\eta_1^2 I \\ & -2\lambda_2^2 e^{-4\kappa\lambda_2}Z_{22} + Z_2 + Z_3 + \lambda_2 Z_5 - \lambda_1 Z_5 + Z_7 + \rho_2^2 Z_8 + \rho_2^2 Z_9 + \rho_1^2 Z_{10} + \rho_2^2 Z_{11} + 2\kappa Z_1 \\ & -2\rho_1\rho_2 Z_{11} + \rho_1^2 Z_{11} + \lambda_2^2 Z_{12} + \lambda_2^2 Z_{13} + \lambda_1^2 Z_{14} + \lambda_2^2 Z_{15} - 2\lambda_1\lambda_2 Z_{15} + \lambda_1^2 Z_{15}, \\ \sum\nolimits_{1,2} = {} & Q_1^T I + Q_5^T I + A_1^T Q_{10} + IQ_2 + IQ_6 + H_1^T Q_{10} + Q_9^T H_2 - Q_9^T I + IQ_{10} + \lambda_2 e^{-2\kappa\lambda_2}F_4 \\ & -e^{-2\kappa\lambda_2}F_1^T + e^{-2\kappa\lambda_2}F_2 + e^{-2\kappa\lambda_2}Z_{18} - e^{-2\kappa\lambda_2}S + Z_1 I + e^{-2\kappa\lambda_2}G_3,\end{aligned}$$

$$
\begin{aligned}
\sum\nolimits_{1,3} &= Z_1 + IQ_4 + IQ_8 - Q_9^T + A_1^T Q_{12} + H_1^T Q_{12} + I^T Q_{12} + \lambda_2^2 G_2 + \lambda_2^2 G_5 - 2\lambda_1\lambda_2 G_5 + \lambda_1^2 G_5,\\
\sum\nolimits_{1,4} &= Q_1^T I + Q_5^T I + IQ_3 + IQ_7 - Q_9^T H_1 - Q_9^T I + A_1^T Q_{11} + H_1^T Q_{11} + IQ_{11} + Z_1 I,\\
\sum\nolimits_{1,5} &= -2e^{-2\kappa\lambda_2} Z_{17} + e^{-2\kappa\lambda_2} S, \quad \sum\nolimits_{1,7} = Q_9^T A_3, \quad \sum\nolimits_{1,9} = -e^{-2\kappa\lambda_2} G_2^T, \quad \sum\nolimits_{1,12} = \lambda_2 e^{-4\kappa\lambda_2} Z_{21},\\
\sum\nolimits_{1,13} &= \lambda_2 e^{-4\kappa\lambda_2} Z_{25} - \lambda_1 e^{-4\kappa\lambda_2} Z_{25}, \quad \sum\nolimits_{1,14} = 6e^{-2\kappa\lambda_2} Z_{17} + 12e^{-4\kappa\lambda_2} Z_{23} + 2\lambda_2^2 e^{-4\kappa\lambda_2} Z_{22},\\
\sum\nolimits_{1,15} &= 2\lambda_1^2 e^{-4\kappa\lambda_2} Z_{24}, \quad \sum\nolimits_{1,18} = -120e^{-4\kappa\lambda_2} Z_{23}, \quad \sum\nolimits_{1,19} = 360e^{-4\kappa\lambda_2} Z_{23}, \quad \sum\nolimits_{1,20} = Q_9^T A_4,\\
\sum\nolimits_{1,25} &= Q_9^T, \quad \sum\nolimits_{1,26} = Q_9^T, \quad \sum\nolimits_{1,27} = Q_9^T, \quad \sum\nolimits_{2,26} = Q_{10}^T, \quad \sum\nolimits_{2,27} = Q_{10}^T,\\
\sum\nolimits_{2,2} &= 2Q_2^T I + 2Q_6^T I + 2Q_{10}^T H_2 - 2Q_{10}^T I + \lambda_2 e^{-2\kappa\lambda_2} F_3 + \lambda_2 e^{-2\kappa\lambda_2} F_5 - \lambda_1 F_8 + e^{-2\kappa\lambda_2} F_1\\
&\quad -e^{-2\kappa\lambda_2} F_1^T - e^{-2\kappa\lambda_2} F_2 - e^{-2\kappa\lambda_2} F_2^T + \lambda_2 F_8 + \lambda_2 F_{10} - \lambda_1 F_{10} + e^{-2\kappa\lambda_2} F_6 + e^{-2\kappa\lambda_2} F_6^T\\
&\quad -e^{-2\kappa\lambda_2} F_7 - e^{-2\kappa\lambda_2} F_7^T - 2e^{-2\kappa\lambda_2} Z_{18} + e^{-2\kappa\lambda_2} S + e^{-2\kappa\lambda_2} S^T - e^{-2\kappa\lambda_2} G_3 - e^{-2\kappa\lambda_2} G_3^T\\
&\quad -e^{-2\kappa\lambda_2} G_6 - e^{-2\kappa\lambda_2} G_6^T - \lambda_2 e^{-2\kappa\lambda_2} Z_5 + \lambda_2\lambda_d e^{-2\kappa\lambda_2} Z_5 + \lambda_1 e^{-2\kappa\lambda_2} Z_5 - \lambda_1\lambda_d e^{-2\kappa\lambda_2} Z_5 + \epsilon_2\eta_2^2 I,\\
\sum\nolimits_{2,3} &= IQ_4 + IQ_8 - Q_{10}^T + H_2^T Q_{12} - IQ_{12}, \quad \sum\nolimits_{3,2} = Q_4^T I + Q_8^T I - Q_{10} - Q_{12}^T I + Q_{12}^T H_2,\\
\sum\nolimits_{2,4} &= Q_2^T I + Q_6^T I + IQ_3 + IQ_7 - Q_{10}^T H_1 - Q_{10}^T I - IQ_{11} + H_2^T Q_{11}, \quad \sum\nolimits_{3,26} = Q_{12}^T,\\
\sum\nolimits_{2,5} &= \lambda_2 e^{-2\kappa\lambda_2} F_4 - e^{-2\kappa\lambda_2} F_1^T + e^{-2\kappa\lambda_2} F_2 + \lambda_2 F_9 - \lambda_1 F_9 - e^{-2\kappa\lambda_2} F_6^T + e^{-2\kappa\lambda_2} F_7 + e^{-2\kappa\lambda_2} Z_{18}\\
&\quad -e^{-2\kappa\lambda_2} S + e^{-2\kappa\lambda_2} G_3 + e^{-2\kappa\lambda_2} G_6,\\
\sum\nolimits_{2,6} &= \lambda_2 F_9^T - \lambda_1 F_9^T - e^{-2\kappa\lambda_2} F_6 + e^{-2\kappa\lambda_2} F_7^T + e^{-2\kappa\lambda_2} G_6^T, \quad \sum\nolimits_{2,7} = Q_{10}^T A_3, \quad \sum\nolimits_{2,9} = e^{-2\kappa\lambda_2} G_2^T,\\
\sum\nolimits_{2,10} &= e^{-2\kappa\lambda_2} G_5^T, \quad \sum\nolimits_{2,11} = -e^{-2\kappa\lambda_2} G_2^T - e^{-2\kappa\lambda_2} G_5^T, \quad \sum\nolimits_{2,20} = Q_{10}^T A_4, \quad \sum\nolimits_{2,25} = Q_{10}^T,\\
\sum\nolimits_{3,3} &= -2Q_{12}^T + \lambda_2 Z_{16} + \lambda_2^2 Z_{17} + \lambda_2^2 Z_{18} + \lambda_2 Z_{20} - \lambda_1 Z_{20} + \lambda_2^2 G_3 + \lambda_2^2 G_6 - 2\lambda_1\lambda_2 G_6 + \lambda_1^2 G_6\\
&\quad + \frac{\lambda_2^4}{4} Z_{21} + \frac{\lambda_2^4}{2} Z_{22} + \frac{\lambda_2^2}{2} Z_{23} + \frac{\lambda_1^4}{2} Z_{24} + \frac{\lambda_2^4}{4} Z_{25} - \frac{\lambda_1^2\lambda_2^2}{2} Z_{25} + \frac{\lambda_1^4}{4} Z_{25},\\
\sum\nolimits_{3,4} &= Q_4^T I + Q_8^T I - Q_{11} - Q_{12}^T H_1 - Q_{12}^T I, \quad \sum\nolimits_{3,20} = Q_{12}^T A_4, \quad \sum\nolimits_{3,25} = Q_{12}^T,\\
\sum\nolimits_{4,4} &= 2Q_3^T I + 2Q_7^T I - 2Q_{11}^T H_1 - 2Q_{11}^T I, \quad \sum\nolimits_{4,7} = Q_{11}^T A_3, \quad \sum\nolimits_{4,20} = Q_{11}^T A_4,\\
\sum\nolimits_{4,25} &= Q_{11}^T, \quad \sum\nolimits_{4,26} = Q_{11}^T, \quad \sum\nolimits_{4,27} = Q_{11}^T, \quad \sum\nolimits_{3,27} = Q_{12}^T, \quad \sum\nolimits_{5,11} = e^{-2\kappa\lambda_2} G_2^T + e^{-2\kappa\lambda_2} G_5^T,\\
\sum\nolimits_{5,5} &= \lambda_2 e^{-2\kappa\lambda_2} F_5 - e^{-2\kappa\lambda_2} F_2 - e^{-2\kappa\lambda_2} F_2^T - 4e^{-2\kappa\lambda_2} Z_{17} + \lambda_2 F_{10} - \lambda_1 F_{10} - e^{-2\kappa\lambda_2} F_7 - e^{-2\kappa\lambda_2} F_7^T\\
&\quad -e^{-2\kappa\lambda_2} Z_{18} - e^{-2\kappa\lambda_2} G_3 - e^{-2\kappa\lambda_2} G_6 - e^{-2\kappa\lambda_2} Z_2 - e^{-2\kappa\lambda_2} Z_4,\\
\sum\nolimits_{5,14} &= 6e^{-2\kappa\lambda_2} Z_{17}, \quad \sum\nolimits_{6,2} = \lambda_2 F_9 - \lambda_1 F_9 - e^{-2\kappa\lambda_2} F_6^T + e^{-2\kappa\lambda_2} F_7 + e^{-2\kappa\lambda_2} G_6,\\
\sum\nolimits_{6,6} &= \lambda_2 F_8 - \lambda_1 F_8 + e^{-2\kappa\lambda_2} F_6 + e^{-2\kappa\lambda_2} F_6^T - e^{-2\kappa\lambda_2} G_6 - e^{-2\kappa\lambda_1} Z_3 + e^{-2\kappa\lambda_1} Z_4,\\
\sum\nolimits_{7,7} &= -\sigma_2 e^{-2\kappa\sigma_2} Z_6 + \sigma_2\sigma_d e^{-2\kappa\sigma_2} Z_6 + \sigma_1 e^{-2\kappa\sigma_2} Z_6 - \sigma_1\sigma_d e^{-2\kappa\sigma_2} Z_6 + \epsilon_3\eta_3^2 I,\\
\sum\nolimits_{8,8} &= \rho_d e^{-2\kappa\rho_2} Z_7 - e^{-2\kappa\rho_2} Z_7, \quad \sum\nolimits_{9,9} = -e^{-2\kappa\lambda_2} G_1 - e^{-2\kappa\lambda_2} Z_{13}, \quad \sum\nolimits_{6,10} = -e^{-2\kappa\lambda_2} G_5^T,\\
\sum\nolimits_{10,10} &= -e^{-2\kappa\lambda_2} G_4 - e^{-2\kappa\lambda_2} Z_{15}, \quad \sum\nolimits_{11,11} = -e^{-2\kappa\lambda_2} G_1 - e^{-2\kappa\lambda_2} G_4 - e^{-2\kappa\lambda_2} Z_{13} - e^{-2\kappa\lambda_2} Z_{15},
\end{aligned}
$$

$$
\begin{aligned}
\sum\nolimits_{12,12} &= -e^{-4\kappa\lambda_2}Z_{21}, \quad \sum\nolimits_{13,13} = -e^{-4\kappa\lambda_2}Z_{25},\\
\sum\nolimits_{14,14} &= -12e^{-2\kappa\lambda_2}Z_{17} - e^{-2\kappa\lambda_2}\lambda_2^2 Z_{12} - 9\lambda_2^2 e^{-2\kappa\lambda_2}Z_{19} - 72e^{-4\kappa\lambda_2}Z_{23} - 2\lambda_2^2 e^{-4\kappa\lambda_2}Z_{22},\\
\sum\nolimits_{14,16} &= 36\lambda_2 e^{-2\kappa\lambda_2}Z_{19}, \quad \sum\nolimits_{14,17} = 60\lambda_2 e^{-2\kappa\lambda_2}Z_{19}, \quad \sum\nolimits_{14,18} = 480e^{-4\kappa\lambda_2}Z_{23},\\
\sum\nolimits_{14,19} &= -1080e^{-4\kappa\lambda_2}Z_{23}, \quad \sum\nolimits_{15,15} = -2\lambda_1^2 e^{-4\kappa\lambda_2}Z_{24} - \lambda_1^2 e^{-2\kappa\lambda_1}Z_{14},\\
\sum\nolimits_{16,16} &= -192e^{-2\kappa\lambda_2}Z_{19}, \quad \sum\nolimits_{16,17} = -360e^{-2\kappa\lambda_2}Z_{19}, \quad \sum\nolimits_{17,17} = -720e^{-2\kappa\lambda_2}Z_{19},\\
\sum\nolimits_{18,18} &= -3600e^{-4\kappa\lambda_2}Z_{23}, \quad \sum\nolimits_{18,19} = 8640e^{-4\kappa\lambda_2}Z_{23}, \quad \sum\nolimits_{19,19} = -21600e^{-4\kappa\lambda_2}Z_{23},\\
\sum\nolimits_{20,20} &= -Z_9, \quad \sum\nolimits_{21,21} = -Z_8, \quad \sum\nolimits_{22,22} = -Z_{10}, \quad \sum\nolimits_{23,23} = -Z_9 - Z_{11},\\
\sum\nolimits_{24,24} &= -Z_{11}, \quad \sum\nolimits_{25,25} = -\epsilon_1 I, \quad \sum\nolimits_{26,26} = -\epsilon_2 I, \quad \sum\nolimits_{27,27} = -\epsilon_3 I,
\end{aligned}
$$

and the other terms are 0;

$$
\widehat{\sum} = \left[\widehat{\Sigma}_{(i,j)}\right]_{28\times 28}, \tag{3.2}
$$

where $\widehat{\Sigma}_{i,j} = \widehat{\Sigma}_{i,j}^T = \Sigma_{i,j}$, $i, j = 1, 2, 3, ..., 27$, except

$$
\begin{aligned}
\widehat{\sum}_{1,1} &= 2Q_1^T I + 2Q_5^T I + 2Q_9^T H_1 + 2Q_9^T I + \lambda_2^2 Z_{19} + \lambda_2 e^{-2\kappa\lambda_2}F_3 + e^{-2\kappa\lambda_2}F_1 + C_1^T C_1 + e^{-2\kappa\lambda_2}F_1^T\\
&\quad -4e^{-2\kappa\lambda_2}Z_{17} - e^{-2\kappa\lambda_2}Z_{18} + \lambda_2^2 G_1 + \lambda_2^2 G_4 - 2\lambda_1\lambda_2 G_4 + 2Z_1 I + \lambda_1^2 G_4 - e^{-2\kappa\lambda_2}G_3 + 2\kappa Z_1\\
&\quad -\lambda_2^2 e^{-4\kappa\lambda_2}Z21 - \lambda_2^2 e^{-4\kappa\lambda_2}Z_{25} + 2\lambda_1\lambda_2 e^{-4\kappa\lambda_2}Z_{25} - \lambda_1^2 e^{-4\kappa\lambda_2}Z_{25} - 2\lambda_1^2 e^{-4\kappa\lambda_2}Z_{24} + \epsilon_1\eta_1^2 I\\
&\quad -12e^{-4\kappa\lambda_2}Z_{23} - 2\lambda_2^2 e^{-4\kappa\lambda_2}Z_{22} + Z_2 + Z_3 + \lambda_2 Z_5 - \lambda_1 Z_5 + Z_7 + \rho_2^2 Z_8 + \rho_2^2 Z_9 + \rho_1^2 Z_{10}\\
&\quad +\rho_2^2 Z_{11} - 2\rho_1\rho_2 Z_{11} + \rho_1^2 Z_{11} + \lambda_2^2 Z_{12} + \lambda_2^2 Z_{13} + \lambda_1^2 Z_{14} + \lambda_2^2 Z_{15} - 2\lambda_1\lambda_2 Z_{15} + \lambda_1^2 Z_{15},\\
\widehat{\sum}_{1,2} &= Q_1^T I + Q_5^T I + A_1^T Q_{10} + IQ_2 + IQ_6 + H_1^T Q_{10} + Q_9^T H_2 - Q_9^T I + IQ_{10} + C_1^T C_2\\
&\quad +\lambda_2 e^{-2\kappa\lambda_2}F_4 - e^{-2\kappa\lambda_2}F_1^T + e^{-2\kappa\lambda_2}F_2 + e^{-2\kappa\lambda_2}Z_{18} - e^{-2\kappa\lambda_2}S + e^{-2\kappa\lambda_2}G_3 + Z_1 I,\\
\widehat{\sum}_{2,2} &= 2Q_2^T I + 2Q_6^T I + 2Q_{10}^T H_2 - 2Q_{10}^T I + \lambda_2 e^{-2\kappa\lambda_2}F_3 + \lambda_2 e^{-2\kappa\lambda_2}F_5 + e^{-2\kappa\lambda_2}F_1 - e^{-2\kappa\lambda_2}F_1^T\\
&\quad -e^{-2\kappa\lambda_2}F_2 - e^{-2\kappa\lambda_2}F_2^T + \lambda_2 F_8 + \lambda_2 F_{10} - \lambda_1 F_8 - \lambda_1 F_{10} + e^{-2\kappa\lambda_2}F_6 + e^{-2\kappa\lambda_2}F_6^T\\
&\quad -e^{-2\kappa\lambda_2}F_7 - e^{-2\kappa\lambda_2}F_7^T - 2e^{-2\kappa\lambda_2}Z_{18} + e^{-2\kappa\lambda_2}S + e^{-2\kappa\lambda_2}S^T - e^{-2\kappa\lambda_2}G3 - e^{-2\kappa\lambda_2}G_3^T\\
&\quad -e^{-2\kappa\lambda_2}G_6 - e^{-2\kappa\lambda_2}G_6^T - \lambda_2 e^{-2\kappa\lambda_2}Z_5 + \lambda_2\lambda_d e^{-2\kappa\lambda_2}Z_5 + \lambda_1 e^{-2\kappa\lambda_2}Z_5 - \lambda_1\lambda_d e^{-2\kappa\lambda_2}Z_5\\
&\quad +\epsilon_2\eta_2^2 I + C_2^T C_2, \quad \widehat{\sum}_{1,28} = Q_9^T B + C_1^T D,\\
\widehat{\sum}_{2,28} &= Q_{10}^T B + C_2^T D, \quad \widehat{\sum}_{3,28} = Q_{11}^T B, \quad \widehat{\sum}_{4,28} = Q_{12}^T B, \quad \widehat{\sum}_{28,28} = D^T D - \delta^2 I,
\end{aligned}
$$

and the other terms are 0.

Theorem 1. For a prescribed scalar $\delta > 0$, given positive scalars λ_2, σ_2, ρ_2, λ_d, σ_d and ρ_d, the System (2.1) is exponentially stable for a decay rate $\kappa > 0$ with the H_∞ performance δ; if $\|A_3\| + \eta_3 < 1$, there exist positive definite symmetric matrices Z_i, $i = 1, 2, ..., 25$, any appropriate dimensional matrices $S, Q_j, j = 1, 2, ..., 12$, $G_l, l = 1, 2, ...6$, and $F_k, k = 1, 2, ..., 10$, and positive real constants ε_n, $n = 1, 2, 3$, such that the following symmetric linear matrix inequalities hold

$$\begin{bmatrix} Z_{16} & F_1 & F_2 \\ * & F_3 & F_4 \\ * & * & F_5 \end{bmatrix} \geq 0, \tag{3.3}$$

$$\begin{bmatrix} Z_{20} & F_6 & F_7 \\ * & F_8 & F_9 \\ * & * & F_{10} \end{bmatrix} \geq 0, \tag{3.4}$$

$$\begin{bmatrix} Z_{18} & S \\ * & Z_{18} \end{bmatrix} \geq 0, \tag{3.5}$$

$$\begin{bmatrix} G_1 & G_2 \\ * & G_3 \end{bmatrix} \geq 0, \tag{3.6}$$

$$\begin{bmatrix} G_4 & G_5 \\ * & G_6 \end{bmatrix} \geq 0, \tag{3.7}$$

$$\widehat{\sum} < 0. \tag{3.8}$$

Proof. Under the condition of the theorem, we first show the exponential stability of System (2.10). Consider System (2.10) with $w(t) = 0$, that is,

$$\begin{aligned} \dot{\varphi}(t) &= [A_1 + H_1 + I]\varphi(t) + [H_2 - I]\varphi(t - \lambda(t)) + A_3\dot{\varphi}(t - \sigma(t)) + A_4 \int_{t-\rho(t)}^{t} \varphi(s)ds \\ &\quad + \zeta_1(t, \varphi(t)) + \zeta_2(t, \varphi(t - \lambda(t))) + \zeta_3(t, \dot{\varphi}(t - \sigma(t))) - [H_1 + I] \int_{t-\lambda(t)}^{t} \dot{\varphi}(s)ds. \end{aligned}$$

Construct an LKF candidate for the System (2.10) of the form

$$V(t) = \sum_{i=1}^{9} V_i(t), \tag{3.9}$$

where

$$V_1(t) = \varphi^T(t)Z_1\varphi(t) = \beta_1^T(t)I_0\Psi_1\beta_1(t),$$

wherein

$$\beta_1(t) = \begin{bmatrix} \varphi(t) \\ \varphi(t - \lambda(t)) \\ \int_{t-\lambda(t)}^{t} \dot{\varphi}(s)ds \\ \dot{\varphi}(t) \end{bmatrix}, \; I_0 = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \; \Psi_1 = \begin{bmatrix} Z_1 & 0 & 0 & 0 \\ Q_1 & Q_2 & Q_3 & Q_4 \\ Q_5 & Q_6 & Q_7 & Q_8 \\ Q_9 & Q_{10} & Q_{11} & Q_{12} \end{bmatrix},$$

$$V_2(t) = \int_{t-\lambda_2}^{t} e^{2\kappa(s-t)}\varphi^T(s)Z_2\varphi(s)ds + \int_{t-\lambda_1}^{t} e^{2\kappa(s-t)}\varphi^T(s)Z_3\varphi(s)ds + \int_{t-\lambda_2}^{t-\lambda_1} e^{2\kappa(s-t)}\varphi^T(s)Z_4\varphi(s)ds,$$

$$V_3(t) = (\lambda_2-\lambda_1)\int_{t-\lambda(t)}^{t} e^{2\kappa(s-t)}\varphi^T(s)Z_5\varphi(s)ds, \quad V_4(t) = (\sigma_2-\sigma_1)\int_{t-\sigma(t)}^{t} e^{2\kappa(s-t)}\dot{\varphi}^T(s)Z_6\dot{\varphi}(s)ds,$$

$$\begin{aligned}
V_5(t) &= \int_{t-\rho(t)}^{t} e^{2\kappa(s-t)}\varphi^T(s)Z_7\varphi(s)ds + \rho_2\int_{-\rho_2}^{0}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\varphi^T(\theta)Z_8\varphi(\theta)d\theta ds \\
&\quad +\rho_2\int_{-\rho_2}^{0}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\varphi^T(\theta)Z_9\varphi(\theta)d\theta ds + \rho_1\int_{-\rho_1}^{0}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\varphi^T(\theta)Z_{10}\varphi(\theta)d\theta ds \\
&\quad +(\rho_2-\rho_1)\int_{-\rho_2}^{-\rho_1}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\varphi^T(\theta)Z_{11}\varphi(\theta)d\theta ds, \\
V_6(t) &= \lambda_2\int_{-\lambda_2}^{0}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\varphi^T(\theta)Z_{12}\varphi(\theta)d\theta ds + \lambda_2\int_{-\lambda_2}^{0}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\varphi^T(\theta)Z_{13}\varphi(\theta)d\theta ds \\
&\quad +\lambda_1\int_{-\lambda_1}^{0}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\varphi^T(\theta)Z_{14}\varphi(\theta)d\theta ds + \lambda_2\int_{-\lambda_2}^{-\lambda_1}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\varphi^T(\theta)Z_{15}\varphi(\theta)d\theta \\
&\quad -\lambda_1\int_{-\lambda_2}^{-\lambda_1}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\varphi^T(\theta)Z_{15}\varphi(\theta)d\theta ds, \\
V_7(t) &= \int_{-\lambda_2}^{0}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\dot{\varphi}^T(\theta)Z_{16}\dot{\varphi}(\theta)d\theta ds + \lambda_2\int_{-\lambda_2}^{0}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\dot{\varphi}^T(\theta)Z_{17}\dot{\varphi}(\theta)d\theta ds \\
&\quad +\lambda_2\int_{-\lambda_2}^{0}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\dot{\varphi}^T(\theta)Z_{18}\dot{\varphi}(\theta)d\theta ds + \lambda_2\int_{-\lambda_2}^{0}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\varphi^T(\theta)Z_{19}\varphi(\theta)d\theta ds \\
&\quad +\int_{-\lambda_2}^{-\lambda_1}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\dot{\varphi}^T(\theta)Z_{20}\dot{\varphi}(\theta)d\theta ds, \\
V_8(t) &= \lambda_2\int_{-\lambda_2}^{0}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\begin{bmatrix}\varphi(\theta)\\ \dot{\varphi}(\theta)\end{bmatrix}^T\begin{bmatrix}G_1 & G_2\\ * & G_3\end{bmatrix}\begin{bmatrix}\varphi(\theta)\\ \dot{\varphi}(\theta)\end{bmatrix}d\theta ds \\
&\quad +(\lambda_2-\lambda_1)\int_{-\lambda_2}^{-\lambda_1}\int_{t+s}^{t} e^{2\kappa(\theta-t)}\begin{bmatrix}\varphi(\theta)\\ \dot{\varphi}(\theta)\end{bmatrix}^T\begin{bmatrix}G_4 & G_5\\ * & G_6\end{bmatrix}\begin{bmatrix}\varphi(\theta)\\ \dot{\varphi}(\theta)\end{bmatrix}d\theta ds, \\
V_9(t) &= \frac{\lambda_2^2}{2}\int_{-\lambda_2}^{0}\int_{s}^{0}\int_{t+\theta}^{t} e^{2\kappa(u+\theta-t)}\dot{\varphi}^T(u)Z_{21}\dot{\varphi}(u)dud\theta ds \\
&\quad +\lambda_2^2\int_{-\lambda_2}^{0}\int_{s}^{0}\int_{t+\theta}^{t} e^{2\kappa(u+\theta-t)}\dot{\varphi}^T(u)Z_{22}\dot{\varphi}(u)dud\theta ds \\
&\quad +\int_{-\lambda_2}^{0}\int_{s}^{0}\int_{t+\theta}^{t} e^{\kappa(u+\theta-t)}\dot{\varphi}^T(u)Z_{23}\dot{\varphi}(u)dud\theta ds \\
&\quad +\lambda_1^2\int_{-\lambda_1}^{0}\int_{s}^{0}\int_{t+\theta}^{t} e^{2\kappa(u+\theta-t)}\dot{\varphi}^T(u)Z_{24}\dot{\varphi}(u)dud\theta ds \\
&\quad +\frac{(\lambda_2^2-\lambda_1^2)}{2}\int_{-\lambda_2}^{-\lambda_1}\int_{s}^{0}\int_{t+\theta}^{t} e^{2\kappa(u+\theta-t)}\dot{\varphi}^T(u)Z_{25}\dot{\varphi}(u)dud\theta ds.
\end{aligned}$$

The time derivative of $V(t)$ along the solution of (2.10) is given by

$$\dot{V}(t) = \sum_{i=1}^{9}\dot{V}_i(t). \tag{3.10}$$

We compute $\dot{V}_1(t)$, $\dot{V}_2(t)$, $\dot{V}_3(t)$ and $\dot{V}_4(t)$ as

$$\dot{V}_1(t) = 2\begin{bmatrix} \varphi(t) \\ \varphi(t-\lambda(t)) \\ \int_{t-\lambda(t)}^{t}\dot{\varphi}(s)ds \\ \dot{\varphi}(t) \end{bmatrix}^T \begin{bmatrix} Z_1 & Q_1^T & Q_5^T & Q_9^T \\ 0 & Q_2^T & Q_6^T & Q_{10}^T \\ 0 & Q_3^T & Q_7^T & Q_{11}^T \\ 0 & Q_4^T & Q_8^T & Q_{12}^T \end{bmatrix} \begin{bmatrix} \dot{\varphi}(t)-\beta_2(t) \\ 0 \\ 0 \\ \beta_3(t) \end{bmatrix} + 2\kappa\varphi^T(t)Z_1\varphi(t) - 2\kappa V_1(t),$$

where

$$\begin{aligned}
\beta_2(t) &= I\varphi(t) - I\varphi(t-\lambda(t)) - I\int_{t-\lambda(t)}^{t}\dot{\varphi}(s)ds, \\
\beta_3(t) &= -\dot{\varphi}(t) + [A_1 + H_1 + I]\varphi(t) + [H_2 - I]\varphi(t-\lambda(t)) + A_3\dot{\varphi}(t-\sigma(t)) + A_4\int_{t-\rho(t)}^{t}\varphi(s)ds \\
&\quad +\zeta_1(t,\varphi(t)) + \zeta_2(t,\varphi(t-\lambda(t))) + \zeta_3(t,\dot{\varphi}(t-\sigma(t))) - [H_1 + I]\int_{t-\lambda(t)}^{t}\dot{\varphi}(s)ds,
\end{aligned}$$

$$\begin{aligned}
\dot{V}_2(t) &= \varphi^T(t)(Z_2 + Z_3)\varphi(t) - e^{-2\kappa\lambda_2}\varphi^T(t-\lambda_2)(Z_2 + Z_4)\varphi(t-\lambda_2) \\
&\quad -e^{-2\kappa\lambda_1}\varphi^T(t-\lambda_1)(Z_3 - Z_4)\varphi(t-\lambda_1) - 2\kappa V_2(t), \\
\dot{V}_3(t) &\leq (\lambda_2 - \lambda_1)\varphi^T(t)Z_5\varphi(t) - 2\kappa V_3(t) \\
&\quad -(\lambda_2 - \lambda_1)(1 - \lambda_d)e^{-2\kappa\lambda_2}\varphi^T(t-\lambda(t))Z_5\varphi(t-\lambda(t)), \\
\dot{V}_4(t) &\leq (\sigma_2 - \sigma_1)\dot{\varphi}^T(t)Z_6\dot{\varphi}(t) - 2\kappa V_4(t) \\
&\quad -(\sigma_2 - \sigma_1)(1 - \sigma_d)e^{-2\kappa\sigma_2}\dot{\varphi}^T(t-\sigma(t))Z_6\dot{\varphi}(t-\sigma(t)).
\end{aligned}$$

By Lemmas 3 and 4, we obtain $\dot{V}_5(t)$ and $\dot{V}_6(t)$ as follows

$$\begin{aligned}
\dot{V}_5(t) &\leq \varphi^T(t)Z_7\varphi(t) + \rho_2^2\varphi^T(t)Z_8\varphi(t) + \rho_2^2\varphi^T(t)Z_9\varphi(t) + \rho_1^2\varphi^T(t)Z_{10}\varphi(t) - 2\kappa V_5(t) \\
&\quad +(\rho_d - 1)e^{-2\kappa\rho_2}\varphi^T(t-\rho(t))Z_7\varphi(t-\rho(t)) + (\rho_2 - \rho_1)^2\varphi^T(t)Z_{11}\varphi(t) \\
&\quad -\Big(\int_{t-\rho_2}^{t}\varphi(s)ds\Big)^T Z_8\Big(\int_{t-\rho_2}^{t}\varphi(s)ds\Big) - \Big(\int_{t-\rho(t)}^{t}\varphi^T(s)ds\Big)Z_9\Big(\int_{t-\rho(t)}^{t}\varphi(s)ds\Big) \\
&\quad -\Big(\int_{t-\rho_2}^{t-\rho(t)}\varphi^T(s)ds\Big)Z_9\Big(\int_{t-\rho_2}^{t-\rho(t)}\varphi(s)ds\Big) - \Big(\int_{t-\rho_1}^{t}\varphi(s)ds\Big)^T Z_{10}\Big(\int_{t-\rho_1}^{t}\varphi(s)ds\Big) \\
&\quad -\Big(\int_{t-\rho(t)}^{t-\rho_1}\varphi^T(s)ds\Big)Z_{11}\Big(\int_{t-\rho(t)}^{t-\rho_1}\varphi(s)ds\Big) - \Big(\int_{t-\rho_2}^{t-\rho(t)}\varphi^T(s)ds\Big)Z_{11}\Big(\int_{t-\rho_2}^{t-\rho(t)}\varphi(s)ds\Big), \\
\dot{V}_6(t) &\leq \lambda_2^2\varphi^T(t)(Z_{12} + Z_{13})\varphi(t) + \lambda_1^2\varphi^T(t)Z_{14}\varphi(t) + (\lambda_2 - \lambda_1)^2\varphi^T(t)Z_{15}\varphi(t) - 2\kappa V_6(t) \\
&\quad -\lambda_2^2 e^{-2\kappa\lambda_2}\Big(\frac{1}{\lambda_2}\int_{t-\lambda_2}^{t}\varphi^T(s)ds\Big)Z_{12}\Big(\frac{1}{\lambda_2}\int_{t-\lambda_2}^{t}\varphi(s)ds\Big) \\
&\quad -e^{-2\kappa\lambda_2}\Big(\int_{t-\lambda(t)}^{t}\varphi^T(s)ds\Big)Z_{13}\Big(\int_{t-\lambda(t)}^{t}\varphi(s)ds\Big) \\
&\quad -e^{-2\kappa\lambda_2}\Big(\int_{t-\lambda(t)}^{t-\lambda_1}\varphi^T(s)ds\Big)Z_{15}\Big(\int_{t-\lambda(t)}^{t-\lambda_1}\varphi(s)ds\Big)
\end{aligned}$$

$$-e^{-2\kappa\lambda_2}\Big(\int_{t-\lambda_2}^{t-\lambda(t)}\varphi^T(s)ds\Big)(Z_{13}+Z_{15})\Big(\int_{t-\lambda_2}^{t-\lambda(t)}\varphi(s)ds\Big)$$
$$-\lambda_1^2 e^{-2\kappa\lambda_1}\Big(\frac{1}{\lambda_1}\int_{t-\lambda_1}^{t}\varphi^T(s)ds\Big)Z_{14}\Big(\frac{1}{\lambda_1}\int_{t-\lambda_1}^{t}\varphi(s)ds\Big).$$

Applying Lemmas 6–9, we obtain

$$\begin{aligned}\dot V_7(t) \ \le\ & \lambda_2\dot\varphi^T(t)Z_{16}\dot\varphi(t)+\lambda_2^2\dot\varphi^T(t)(Z_{17}+Z_{18})\dot\varphi(t)+e^{-2\kappa\lambda_2}\beta_4^T(t)\Psi_3\beta_4(t)+(\lambda_2-\lambda_1)\dot\varphi^T(t)Z_{20}\dot\varphi(t)\\ &+\lambda_2e^{-2\kappa\lambda_2}\beta_4^T(t)\Psi_2\beta_4(t)+\lambda_2^2\varphi^T(t)Z_{19}\varphi(t)+e^{-2\kappa\lambda_2}\beta_6^T(t)\Psi_4\beta_6(t)+(\lambda_2-\lambda_1)\beta_5^T(t)\Psi_5\beta_5(t)\\ &+e^{-2\kappa\lambda_2}\beta_5^T(t)\Psi_6\beta_5(t)+e^{-2\kappa\lambda_2}\beta_4^T(t)\Psi_7\beta_4(t)-\beta_7^T(t)\Psi_8\beta_7(t)-2\kappa V_7(t),\end{aligned}$$

where

$$\beta_4(t)=\begin{bmatrix}\varphi(t)\\ \varphi(t-\lambda(t))\\ \varphi(t-\lambda_2)\end{bmatrix},\beta_5(t)=\begin{bmatrix}\varphi(t-\lambda_1)\\ \varphi(t-\lambda(t))\\ \varphi(t-\lambda_2)\end{bmatrix},\beta_6(t)=\begin{bmatrix}\varphi(t)\\ \varphi(t-\lambda_2)\\ \frac{1}{\lambda_2}\int_{t-\lambda_2}^{t}\varphi(s)ds\end{bmatrix},$$

$$\beta_7(t)=\begin{bmatrix}\frac{1}{\lambda_2}\int_{t-\lambda_2}^{t}\varphi(s)ds\\ \frac{1}{\lambda_2}\int_{t-\lambda_2}^{t}\int_{t-\lambda_2}^{u}\varphi(\theta)d\theta du\\ \frac{1}{\lambda_2{}^2}\int_{t-\lambda_2}^{t}\int_{t-\lambda_2}^{u}\int_{t-\lambda_2}^{s}\varphi(\theta)d\theta dsdu\end{bmatrix},\Psi_2=\begin{bmatrix}F_3 & F_4 & 0\\ * & F_3+F_5 & F_4\\ * & * & F_5\end{bmatrix},$$

$$\Psi_3=\begin{bmatrix}F_1+F_1^T & -F_1^T+F_2 & 0\\ * & F_1+F_1^T-F_2-F_2^T & -F_1^T+F_2\\ * & * & -F_2-F_2^T\end{bmatrix},$$

$$\Psi_4=\begin{bmatrix}-4Z_{17} & -2Z_{17} & 6Z_{17}\\ * & -4Z_{17} & 6Z_{17}\\ * & * & -12Z_{17}\end{bmatrix},\Psi_5=\begin{bmatrix}F_8 & F_9 & 0\\ * & F_8+F_{10} & F_9\\ * & * & F_{10}\end{bmatrix},$$

$$\Psi_6=\begin{bmatrix}F_6+F_6^T & -F_6^T+F_7 & 0\\ * & F_6+F_6^T-F_7-F_7^T & -F_6^T+F_7\\ * & * & -F_7-F_7^T\end{bmatrix},$$

$$\Psi_7=\begin{bmatrix}-Z_{18} & Z_{18}-S & S\\ * & -2Z_{18}+S+S^T & Z_{18}-S\\ * & * & -Z_{18}\end{bmatrix},$$

$$\Psi_8=\begin{bmatrix}-9\lambda_2^2e^{-2\kappa\lambda_2}Z_{19} & -36\lambda_2e^{-2\kappa\lambda_2}Z_{19} & -60\lambda_2e^{-2\kappa\lambda_2}Z_{19}\\ 36\lambda_2e^{-2\kappa\lambda_2}Z_{19} & -192e^{-2\kappa\lambda_2}Z_{19} & 360e^{-2\kappa\lambda_2}Z_{19}\\ -60\lambda_2e^{-2\kappa\lambda_2}Z_{19} & 360e^{-2\kappa\lambda_2}Z_{19} & -720e^{-2\kappa\lambda_2}Z_{19}\end{bmatrix}.$$

From Lemma 5, we compute $\dot V_8(t)$ as

$$\begin{aligned}\dot V_8(t) \ \le\ & \lambda_2^2\begin{bmatrix}\varphi(t)\\ \dot\varphi(t)\end{bmatrix}^T\begin{bmatrix}G_1 & G_2\\ * & G_3\end{bmatrix}\begin{bmatrix}\varphi(t)\\ \dot\varphi(t)\end{bmatrix}+(\lambda_2-\lambda_1)^2\begin{bmatrix}\varphi(t)\\ \dot\varphi(t)\end{bmatrix}^T\begin{bmatrix}G_4 & G_5\\ * & G_6\end{bmatrix}\begin{bmatrix}\varphi(t)\\ \dot\varphi(t)\end{bmatrix}\\ &+e^{-2\kappa\lambda_2}\beta_8^T(t)\Psi_9\beta_8(t)+e^{-2\kappa\lambda_2}\beta_9^T(t)\Psi_{10}\beta_9(t)-2\kappa V_8(t),\end{aligned}$$

where

$$\beta_8(t) = \begin{bmatrix} \varphi(t) \\ \varphi(t-\lambda(t)) \\ \varphi(t-\lambda_2) \\ \int_{t-\lambda(t)}^{t} \varphi(s)ds \\ \int_{t-\lambda_2}^{t-\lambda(t)} \varphi(s)ds \end{bmatrix}, \beta_9(t) = \begin{bmatrix} \varphi(t-\lambda_1) \\ \varphi(t-\lambda(t)) \\ \varphi(t-\lambda_2) \\ \int_{t-\lambda(t)}^{t-\lambda_1} \varphi(s)ds \\ \int_{t-\lambda_2}^{t-\lambda(t)} \varphi(s)ds \end{bmatrix},$$

$$\Psi_9 = \begin{bmatrix} -G_3 & G_3 & 0 & -G_2^T & 0 \\ * & -G_3-G_3^T & G_3 & G_2^T & -G_2^T \\ * & * & -G_3 & 0 & G_2^T \\ * & * & * & -G_1 & 0 \\ * & * & * & * & -G_1 \end{bmatrix},$$

$$\Psi_{10} = \begin{bmatrix} -G_6 & G_6 & 0 & -G_5^T & 0 \\ * & -G_6-G_6^T & G_6 & G_5^T & -G_5^T \\ * & * & -G_6 & 0 & G_5^T \\ * & * & * & -G_4 & 0 \\ * & * & * & * & -G_4 \end{bmatrix}.$$

Using Lemma 2, Lemma 3 and Lemma 10, $\dot{V}_9(t)$ can be estimated as follows

$$\begin{aligned} \dot{V}_9(t) \leq\ & \frac{\lambda_2^4}{4}\dot{\varphi}^T(t)Z_{21}\dot{\varphi}(t) + \frac{\lambda_2^4}{2}\dot{\varphi}^T(t)Z_{22}\dot{\varphi}(t) + \frac{\lambda_2^2}{2}\dot{\varphi}^T(t)Z_{23}\dot{\varphi}(t) + \frac{\lambda_1^4}{2}\dot{\varphi}^T(t)Z_{24}\dot{\varphi}(t) - 2\kappa V_9(t) \\ & + \frac{(\lambda_2^2-\lambda_1^2)^2}{4}\dot{\varphi}^T(t)Z_{25}\dot{\varphi}(t) - e^{-4\kappa\lambda_2}\beta_{10}^T(t)Z_{25}\beta_{10}(t) + e^{-2\kappa\lambda_2}\beta_{11}^T(t)\Psi_{11}\beta_{11}(t) \\ & - e^{-4\kappa\lambda_2}\Big(\lambda_2\varphi(t) - \int_{t-\lambda_2}^{t}\varphi(s)ds\Big)^T Z_{21}\Big(\lambda_2\varphi(t) - \int_{t-\lambda_2}^{t}\varphi(s)ds\Big) \\ & + {\lambda_2}^2 e^{-4\kappa\lambda_2}\begin{bmatrix} \varphi(t) \\ \frac{1}{\lambda_2}\int_{t-\lambda_2}^{t}\varphi(s)ds \end{bmatrix}^T \begin{bmatrix} -2Z_{22} & 2Z_{22} \\ * & -2Z_{22} \end{bmatrix} \begin{bmatrix} \varphi(t) \\ \frac{1}{\lambda_2}\int_{t-\lambda_2}^{t}\varphi(s)ds \end{bmatrix} \\ & + {\lambda_1}^2 e^{-4\kappa\lambda_2}\begin{bmatrix} \varphi(t) \\ \frac{1}{\lambda_1}\int_{t-\lambda_1}^{t}\varphi(s)ds \end{bmatrix}^T \begin{bmatrix} -2Z_{24} & 2Z_{24} \\ * & -2Z_{24} \end{bmatrix} \begin{bmatrix} \varphi(t) \\ \frac{1}{\lambda_1}\int_{t-\lambda_1}^{t}\varphi(s)ds \end{bmatrix}, \end{aligned}$$

where

$$\beta_{10}(t) = (\lambda_2-\lambda_1)\varphi(t) - \int_{t-\lambda_2}^{t-\lambda_1}\varphi(s)ds,\ \beta_{11}(t) = \begin{bmatrix} \varphi(t) \\ \frac{1}{\lambda_2}\int_{t-\lambda_2}^{t}\varphi(s)ds \\ \frac{1}{\lambda_2^2}\int_{t-\lambda_2}^{t}\int_{u}^{t}\varphi(s)dsdu \\ \frac{1}{\lambda_2^3}\int_{t-\lambda_2}^{t}\int_{u}^{t}\int_{s}^{t}\varphi(\theta)d\theta dsdu \end{bmatrix},$$

$$\Psi_{11} = e^{-4\kappa\lambda_2}\begin{bmatrix} -12Z_{23} & 12Z_{23} & -120Z_{23} & 360Z_{23} \\ 12Z_{23} & -72Z_{23} & 480Z_{23} & -1080Z_{23} \\ -120Z_{23} & 480Z_{23} & -3600Z_{23} & 8640Z_{23} \\ 360Z_{23} & -1080Z_{23} & 8640Z_{23} & -21600Z_{23} \end{bmatrix}.$$

From (2.5)–(2.7), for any scalars ε_1, ε_2 and ε_3 that are positive real constants, it can be determinded that the following inequalities hold:

$$\varepsilon_1({\eta_1}^2\varphi^T(t)\varphi(t) - \zeta_1^T(t,\varphi(t))\zeta_1(t,\varphi(t))) \geq 0, \tag{3.11}$$

$$\varepsilon_2({\eta_2}^2\varphi^T(t-\lambda(t))\varphi(t-\lambda(t)) - \zeta_2^T(t,\varphi(t-\lambda(t)))\zeta_2(t,\varphi(t-\lambda(t)))) \geq 0, \tag{3.12}$$

$$\varepsilon_3({\eta_3}^3\dot{\varphi}^T(t-\sigma(t))\dot{\varphi}(t-\sigma(t)) - \zeta_3^T(t,\dot{\varphi}(t-\sigma(t)))\zeta_3(t,\dot{\varphi}(t-\sigma(t)))) \geq 0. \tag{3.13}$$

According to (3.10)–(3.13), it is straightforward to see that

$$\dot{V}(t) + 2\kappa V(t) \leq \xi^T(t)\Sigma\xi(t),$$

where

$$\begin{aligned}\xi(t) = {} & [\varphi(t), \varphi(t-\lambda(t)), \dot{\varphi}(t), \int_{t-\lambda(t)}^{t}\dot{\varphi}(s)ds, \varphi(t-\lambda_2), \varphi(t-\lambda_1), \dot{\varphi}(t-\sigma(t)), \dot{\varphi}(t-\rho(t)), \int_{t-\lambda(t)}^{t}\varphi(s)ds, \\ & \int_{t-\lambda(t)}^{t-\lambda_1}\varphi(s)ds, \int_{t-\lambda_2}^{t-\lambda(t)}\varphi(s)ds, \int_{t-\lambda_2}^{t}\varphi(s)ds, \int_{t-\lambda_2}^{t-\lambda_1}\varphi(s)ds, \frac{1}{\lambda_2}\int_{t-\lambda_2}^{t}\varphi(s)ds, \frac{1}{\lambda_1}\int_{t-\lambda_1}^{t}\varphi(s)ds, \\ & \frac{1}{\lambda_2}\int_{t-\lambda_2}^{t}\int_{t-\lambda_2}^{u}\varphi(\theta)d\theta du, \frac{1}{\lambda_2^2}\int_{t-\lambda_2}^{t}\int_{t-\lambda_2}^{u}\int_{t-\lambda_2}^{s}\varphi(\theta)d\theta dsdu, \frac{1}{\lambda_2^2}\int_{t-\lambda_2}^{t}\int_{u}^{t}\varphi(s)dsdu, \\ & \frac{1}{\lambda_2^3}\int_{t-\lambda_2}^{t}\int_{u}^{t}\int_{s}^{t}\varphi(\theta)d\theta dsdu, \int_{t-\rho(t)}^{t}\varphi(s)ds, \int_{t-\rho_2}^{t}\varphi(s)ds, \int_{t-\rho_1}^{t}\varphi(s)ds, \int_{t-\rho_2}^{t-\rho(t)}\varphi(s)ds, \\ & \int_{t-\rho(t)}^{t-\rho_1}\varphi(s)ds, \zeta_1(t,\varphi(t)), \zeta_2(t,\varphi(t-\lambda(t))), \zeta_3(t,\dot{\varphi}(t-\sigma(t)))].\end{aligned}$$

If Conditions (3.3)–(3.8) and $\Sigma < 0$ hold, then

$$\dot{V}(t) + 2\kappa V(t) \leq 0, \quad \forall t \in R^+. \tag{3.14}$$

So, we have

$$\|\varphi(t,\phi)\| \leq M\|\phi\|e^{-\kappa t}, \quad t \in R^+,$$

where $M, \kappa \in R^+$. This means that System (2.1) with $w(t) = 0$ is exponentially stable. Next, we shall establish the H_∞ performance of System (2.1) under the zero initial condition. We now introduce

$$J(t) = \int_0^t [\chi^T(s)\chi(s) - \delta^2 w^T(s)w(s)]ds, \quad t > 0. \tag{3.15}$$

Under the zero initial condition, (3.15) becomes

$$\begin{aligned}J(t) &= \int_0^t [\chi^T(s)\chi(s) - \delta^2 w^T(s)w(s)]ds \\ &= \int_0^t [\chi^T(s)\chi(s) - \delta^2 w^T(s)w(s)]ds + \int_0^t \dot{V}(s)ds - V(t) + V(0) \\ &= \int_0^t [\chi^T(s)\chi(s) - \delta^2 w^T(s)w(s) + \dot{V}(s)]ds - V(t)\end{aligned}$$

$$\leq \int_0^t [\chi^T(s)\chi(s) - \delta^2 w^T(s)w(s) + \dot{V}(s)]ds \tag{3.16}$$

where $V(t)$ is defined in (3.9). After some algebraic manipulations, we obtain

$$\chi^T(t)\chi(t) - \delta^2 w^T(t)w(t) + \dot{V}(t) \leq \hat{\xi}^T(t)\widehat{\sum}\hat{\xi}(t) \tag{3.17}$$

where

$$\begin{aligned}
\hat{\xi}(t) = {} & [\varphi(t), \varphi(t-\lambda(t)), \dot{\varphi}(t), \int_{t-\lambda(t)}^t \dot{\varphi}(s)ds, \varphi(t-\lambda_2), \varphi(t-\lambda_1), \dot{\varphi}(t-\sigma(t)), \dot{\varphi}(t-\rho(t)), \int_{t-\lambda(t)}^t \varphi(s)ds, \\
& \int_{t-\lambda(t)}^{t-\lambda_1} \varphi(s)ds, \int_{t-\lambda_2}^{t-\lambda(t)} \varphi(s)ds, \int_{t-\lambda_2}^{t} \varphi(s)ds, \int_{t-\lambda_2}^{t-\lambda_1} \varphi(s)ds, \frac{1}{\lambda_2}\int_{t-\lambda_2}^{t} \varphi(s)ds, \frac{1}{\lambda_1}\int_{t-\lambda_1}^{t} \varphi(s)ds, \\
& \frac{1}{\lambda_2}\int_{t-\lambda_2}^{t}\int_{t-\lambda_2}^{u} \varphi(\theta)d\theta du, \frac{1}{\lambda_2^2}\int_{t-\lambda_2}^{t}\int_{t-\lambda_2}^{u}\int_{t-\lambda_2}^{s} \varphi(\theta)d\theta dsdu, \frac{1}{\lambda_2^2}\int_{t-\lambda_2}^{t}\int_{u}^{t} \varphi(s)dsdu, \\
& \frac{1}{\lambda_2^3}\int_{t-\lambda_2}^{t}\int_{u}^{t}\int_{s}^{t} \varphi(\theta)d\theta dsdu, \int_{t-\rho(t)}^{t} \varphi(s)ds, \int_{t-\rho_2}^{t} \varphi(s)ds, \int_{t-\rho_1}^{t} \varphi(s)ds, \int_{t-\rho_2}^{t-\rho(t)} \varphi(s)ds, \\
& \int_{t-\rho(t)}^{t-\rho_1} \varphi(s)ds, \zeta_1(t,\varphi(t)), \zeta_2(t,\varphi(t-\lambda(t))), \zeta_3(t,\dot{\varphi}(t-\sigma(t))), w(t)].
\end{aligned}$$

We can verify that the condition (3.8) guarantees $\chi^T(s)\chi(s) - \delta^2 w^T(s)w(s) + \dot{V}(t) \leq 0$. Therefore, $J(t) < 0$, which implies that $\|\chi(t)\|_2 \leq \delta\|w(t)\|_2$ for any nonzero $w(t) \in L_2[0,\infty)$. The proof of the theorem is complete.

4. Numerical examples

Example 1. Consider the uncertain neutral system (2.1) with the following parameters:

$$\begin{aligned}
A_1 &= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}, \\
B &= \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.
\end{aligned}$$

Decompose a matrix $A_2 = H_1 + H_2$, where

$$H_1 = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}. \tag{4.1}$$

Let the interval discrete time-varying delay be $\lambda(t) = |cos(t)|$ and the interval neutral and distributed time-varying delays be $\sigma(t) = \rho(t) = \sin^2(0.6t)$ for $t \in [-1, 0]$. By solving the linear matrix inequality (3.8) in Theorem 1, the maximum allowable upper bounds of ρ_2 for Example 1 are listed in Table 1 for various values of λ_2 and σ_2. We can see in Table 1 that the upper bound of the distributed delay ρ_2 has an effect on λ_2. For any given λ_2, σ_2 decreases as ρ_2 increases. Table 2 presents the maximum allowable upper bounds of λ_2 for Example 1 with different values of κ and σ_2. It shows that all of the conditions stated in Theorem 1 have been satisfied; hence, System (2.1) with the above given parameters has exponential stability with H_∞ performance.

For the initial condition $\phi(t) = [0.5 \quad 1]^T$, $\zeta_1(t, \varphi(t)) = \eta_1\varphi(t)sin(\varphi(t))$, $\zeta_2(t, \varphi(t - \lambda(t))) = \eta_2 sin(\varphi(t - \lambda(t)))e^{-2.3\varphi(t-\lambda(t))}cos(\varphi(t - \lambda(t)))$ and $\zeta_3(t, \dot{\varphi}(t - \rho(t))) = \eta_3\dot{\varphi}(t - \rho(t))cos(t)$. Figure 1 shows the trajectories of the solution $\varphi^T(t) = [\varphi_1(t), \varphi_2(t)]$ of the neutral system (2.1) with mixed time-varying delays and nonlinear uncertainties.

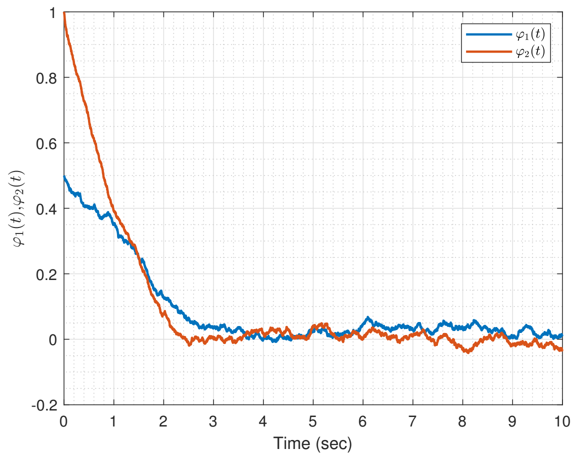

Figure 1. Trajectories of $\varphi_1(t)$ and $\varphi_2(t)$ of System (2.1) in Example 1.

Table 1. Maximum allowable upper bounds of ρ_2 for Example 1 with different values of λ_2 and σ_2 when $\kappa = 1$, $\delta = 2$, $\eta_1 = \eta_2 = \eta_3 = 0.5$, $\lambda_1 = \sigma_1 = \rho_1 = 1$, $\lambda_d = \sigma_d = 0.9$ and $\rho_d = 0.8$.

λ_2	$\sigma_2 = 2.0$	$\sigma_2 = 3.0$	$\sigma_2 = 5.0$	$\sigma_2 = 7.0$
2.0	20.3004	20.0273	20.0252	19.0005
3.0	19.6857	19.5352	18.9964	18.8596
5.0	19.0086	18.9835	18.9793	17.9930
7.0	17.4794	17.4794	16.4491	7.5000

Table 2. Maximum allowable upper bounds of λ_2 for Example 1 with different values of λ_2 and σ_2 when $\delta = 1$, $\eta_1 = \eta_2 = \eta_3 = 0.5$, $\lambda_1 = \rho_1 = \sigma_1 = 1$, $\rho_2 = 2$, $\lambda_d = \sigma_d = 0.9$ and $\rho_d = 0.8$.

κ	$\sigma_2 = 2.0$	$\sigma_2 = 3.0$	$\sigma_2 = 5.0$	$\sigma_2 = 7.0$
0.3	25.5703	24.9307	24.5997	24.2467
0.5	16.4537	16.2211	14.6854	13.9659
0.7	11.5230	11.4199	10.5807	9.6449
0.9	9.0099	8.6875	7.9842	7.2311

Example 2. Consider the following neutral system with $w(t) = 0$, $C_1 = C_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$:

$$\begin{aligned}\dot{\varphi}(t) &= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}\varphi(t) + \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix}\varphi(t-\lambda(t)) + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}\dot{\varphi}(t-\sigma(t)) + \zeta_1(t,\varphi(t)) \\ &\quad + \zeta_2(t,\varphi(t-\lambda(t))) + \zeta_3(t,\dot{\varphi}(t-\rho(t))) + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}\int_{t-\rho(t)}^{t}\varphi(s)ds \end{aligned} \tag{4.2}$$

when $\eta_1 = 0.1$, $\eta_2 = \eta_3 = 0.05$, $\lambda_d = 0.7$, $\rho_1 = 0.3$, $\rho_2 = \rho_d = 0.4$ $\sigma_1 = 0.3$, $\sigma_2 = 0.5$ and $\sigma_d = 0.1$. We separate a matrix A_2 as $A_2 = H_1 + H_2$, where

$$H_1 = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}.$$

Table 3 shows a comparison of the upper bounds for the exponential stability of System (2.1) by different methods. It can be concluded that our results are less conservative than those in [11]. Figure 2 demonstrates the trajectories of the solution $\varphi_1(t)$ and $\varphi_2(t)$ of the uncertain neutral system (2.1) with mixed time-varying delays when $w(t) = 0$.

Table 3. Maximum allowable upper bounds of λ_2 for Example 2 with different values of κ and λ_1.

Method	$\kappa = 0.1$		$\kappa = 0.3$		$\kappa = 0.5$	
λ_1	0.2	1.0	0.2	1.0	0.2	1.0
[11]	67.21	64.05	26.05	25.07	15.40	14.37
This Paper	67.89	67.61	28.23	26.90	16.50	15.55

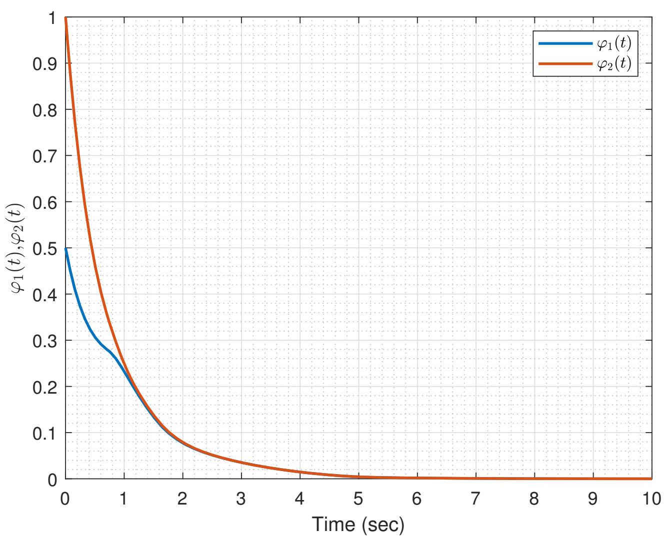

Figure 2. Trajectories of $\varphi_1(t)$ and $\varphi_2(t)$ of System (2.1) with $w(t) = 0$ in Example 2.

Example 3. Consider the System (2.1) with the following parameters:

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$A_4 = C_1 = C_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad w(t) = 0.$$

Decompose a matrix $A_2 = H_1 + H_2$, where

$$H_1 = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}.$$

Using the Matlab LMI toolbox, we obtain the maximum allowable upper bounds of λ_2, which are listed in Table 4. Table 4 describes the maximum allowable upper bounds of delays that guarantee the asymptotic stability of System (2.1) when $\kappa = 0$. It is clear that the obtained results in this study are better than those in [7, 10, 32–34].

Table 4. Maximum allowable upper bounds of λ_2 for Example 3 with different values of λ_d when $\lambda_1 = \sigma_1 = \rho_1 = 0$, $\sigma_2 = 1$, $\eta_1 = 0.1$ and $\eta_2 = \eta_3 = 0.05$.

	λ_d	0.5	0.9	1.1	Unknown
σ_d=0.6	[7]	-	-	-	3.9563
	[32]	-	-	-	4.6235
	[33]	-	-	-	4.9423
	[34]	-	-	-	8.7375
	This paper	-	-	-	8.8193
σ_d=0	[10]	8.975	8.820	-	-
	[33]	9.646	9.225	-	-
	[32]	9.975	9.756	9.685	-
	[34]	-	-	-	9.7967
	This paper	10.125	10.034	9.987	9.8023

Remark 2. The less conservatism of Theorem 1 benefits from the construction of new LKFs with the application of Jensen's integral inequality (Lemma 1), Peng-Park's integral inequality (Lemma 8) and extended Wirtinger's integral inequalities (Lemmas 9 and 10). These allowed our maximum delay to be greater than those in [7, 10, 11, 32–34] as shown in Tables 3 and 4.

5. Conclusions

In this article, the problem of exponential stability and H_∞ performance with mixed discrete, neutral and distributed interval time-varying delays and nonlinear uncertainties has been studied. To obtain delay-range-dependent sufficient conditions that can be achieved in the form of linear matrix inequalities for the H_∞ performance with exponential stability of the system, we have introduced an appropriate LKF and applied a decomposition matrix technique, the Leibniz-Newton formula, a zero equation, Peng-Park's integral inequality, Jensen's integral inequality and the Wirtinger-based integral inequality. Numerical examples have been provided to verify the effectiveness of the presented results, showing that our results are better than the existing results.

Acknowledgments

This work was supported by the Research and Graduate Studies Program, Khon Kaen University (Research Program, grant number RP65-8-004).

Conflict of interest

The authors declare that there are no conflicts of interest regarding the publication of this manuscript.

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