Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 7(9): 17286–17312.
DOI: 10.3934/math.2022952
Received: 25 March 2022
Revised: 07 June 2022
Accepted: 07 June 2022
Published: 25 July 2022

Research article

Entropy based extended TOPOSIS method for MCDM problem with fuzzy credibility numbers

Talha Midrar[1], **Saifullah Khan**[1], **Saleem Abdullah**[1] **and Thongchai Botmart**[2,*]

[1] Department of Mathematics, Faculty of Physical and Numerical Science, Abdul Wali Khan University Mardan, Mardan 23200, Pakistan

[2] Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand

* **Correspondence:** Email: thongbo@kku.ac.th.

Abstract: Due to the vagueness and uncertainty of human cognition/judgments as related to complicated decision-making problems, existing fuzzy decision-making approaches merely signal fuzzy assessment values and lack degrees/levels of credibility for the fuzzy assessment values in alternatives over attributes. As a result, the fuzzy evaluative value's credibility degree highlights its significance and importance in the fuzzy decision-making problem. To improve the degrees/levels of credibility of fuzzy evaluation values, the fuzzy assessment values should be tightly linked to their credibility measures, which would result in more abundant and reliable assessment information. The major goal of this research was to describe new procedures for credible fuzzy numbers based on the Dombi t-norm and Dombi t-conorm. Dombi operations can benefit from the operational parameter's best tractability. These operations are more generalized for credibility fuzzy numbers. Furthermore, using the basic operational laws of Dombi t-norm and Dombi t-conorm, we develop a series of fuzzy credibility Dombi aggregation operators, like the fuzzy credibility Dombi geometric aggregation operator, fuzzy credibility Dombi ordered geometric aggregation operator and fuzzy credibility Dombi hybrid geometric aggregation operator. To handle this sort of decision-making problem, an extended TOPSIS (technique for order of preference by similarity to ideal solution) is proposed. Finally, we present an example, along with a discussion of the comparative results to check the accuracy and validation of the proposed methods, to confirm that their results are credible and feasible.

Keywords: fuzzy credibility numbers; Dombi operational laws for FCNs; aggregation operators; TOPSIS method; decision-making

1. Introduction

A newly defined model is not accessible in fuzzy theory unless it overcomes the shortcomings of various previously specified theories and models. Routine mathematics is not always feasible because of the ambiguity and uncertainty that plague many daily issues. Different methods such as the use of the hypothesis of probability, rough set hypothesis and fuzzy set hypotheses have been suggested as alternative approaches to address such challenges. Unfortunately, the majority of alternative mathematics approaches has its own set of flaws and disadvantages. Most words, like authentic, excellent, great and renowned are not measurable and, in reality are ambiguous. The definitions of adjectives like wonderful, best, renowned, etc., differ from person to person. To handle such ambiguous and uncertain information, Zadeh [1] initiated a study of possibility based on the participation function, which assigns an enrollment grade in [0, 1] called a fuzzy set. However, the fuzzy set failed to describe the uncertainty with only the degree of membership of the element and cannot handle the complex problems. Atanassov [2, 3] established the generalized notion when he added the degree of non-membership to the concept of a fuzzy set and created a new concept called an intuitionistic fuzzy set (IFS). An IFS is defined by assigning two values from the range $[0, 1]$, named the membership degree μ and non-membership degree ν, under the condition $\mu + \nu \leq 1$ for all elements of the working universe. It has been frequently employed in a variety of fields, including decision-making, risk analysis, marketing and forecasting. The IFS theory has its own limitations in transmitting information on the degree of membership and degree of non-membership of an object. In this case, IFS theory has failed to describe the uncertainty in daily-life applications. Therefore, Yager [4, 5] developed the concept of a Pythagorean fuzzy set (PyFS) as an extension of the IFS under the condition $\mu^2 + \nu^2 \leq 1$. Due to the vagueness and uncertainty of human cognitions for sophisticated decision-making situations, existing fuzzy decision-making approaches merely signal fuzzy assessment values and lack a degree of credibility regarding the use of fuzzy assessment values in the appropriate assessment of alternatives over attributes. In fact, the majority of decision-making issues involve human subjective judgments and hazy assessments in uncertain and ambiguous situations. As a result, when decision-makers are more familiar with some traits but not so much with others, they may assign fuzzy assessment values with the help of some degrees of credibility with respect to distinct attributes. In 2002, Liu and Liu developed the expected value of fuzzy variable and fuzzy expected value models [6]. Liu [7] developed a survey of credibility theory. Sufficient and necessary conditions for credibility measure have been developed by Li and Liu [8]. A fuzzy group decision-making model based on credibility theory and gray relative degree was developed by Rao and Peng [9].

Multi-attribute decision-making (MADM) is a method for decision-making in which the best alternative is to derived from the optimal decision in comparison to the finite possibilities given a collection of many attributes. The MADM approach is gaining popularity among specialists since it can be used in a variety of fields, including operations research, engineering technology and management sciences [10–12]. Aggregation operators (AOs) play a critical role in solving MADM challenges by integrating data into a single useful form. Decision-makers employ various assessment techniques, like crisp numbers or interval numbers, to rate the attributes in real-world decision-making. Since several AOs in a fuzzy information were introduced to handle decision-making issues. To define and discuss various properties using fuzzy sets, Song et al. [13]

explained its idea and discussed the main operational laws about fuzzy sets. The induced generalized AOs and its application for solving decision-making issues using fuzzy sets were introduced by Merigo and Gil-Lafuente [14]. Fuzzy information AOs are suitable and relevant in attractive research areas, and are given strong consideration by researchers. Zhou and Chen [15] created a new distance measure for pythagorean fuzzy sets (PFSs) as well as several new similarity measures. Mohagheghi et al. [16] looked at several novel AOs in PFSs and how they could be used in multi-criteria group decision-making (MCGDM) situations. The Hamacher T-conorm and T-norm, which are extensions of the algebraic and Einstein T-conorm and T-norm, are more valid and flexible [17]. The value of studying AOs using Hamacher operations, as well as their applications to MADM issues, is considerable. Ejegwa [18] established a max-min composition for PFSs and applied it in the field of study placement based on academic ability. For MADM, Tan et al. [19] developed hesitant fuzzy Hamacher AOs. Senapati and Yager [20] developed Fermatean fuzzy AOs. The Dombi t-norm and t-conorm constitute a priority of variability, and were introduced by Dombi in 1982 [21]. Liu et al. applied Dombi operations to IFSs and presented the multi-attribute group decision-making (MAGDM) problem using the Dombi Bonferroni mean operator in the context of intuitionistic fuzzy information [22]. The idea of bringing the MAGDM issue together to form single-valued neutrosophic Dombi AOs was introduced in [23].

1.1. Relative work

The MADM approach is a method for decision-making in which the best alternative is to derived from the optimal decision in comparison to a finite possibilities given a collection of many attributes. There are three main steps in MADM; particularly, the decision process finds the optimal selection of alternatives. The MADM process begins with the structure of the decision model, which is used to formulate data information for each alternative based on criteria defined by each decision expert. The second step then begins with the data information of all alternatives based on defined criteria represented by each decision expert's decision matrix and this decision matrix will be normalized if needed. The final step of the decision process finalizes the decision for optimal selection of the alternative. In the case of a group decision, the decision expert needs operators to aggregate the multi-expert opinion to the single and collective decision information. To deal with this problem of aggregating the information of MADM, different AOs were developed to aggregate the information. The weighted average and weighted geometric are the most frequently used AOs in MADM problems. These AOs have been intensely deliberated for different decision-making problems addressed by various scholars [24–28]. The concept of weighted AOs for the fuzzy credibility number (FCN) and Cubic numbers, as well as their decision-making approach for slope design schemes was developed by Ye et al. [29, 30]. Hwang and Yoon [31] presented the TOPSIS to deal multi-attribute decision-making problems. In [32], Chen presented the technique for order of preference by similarity to ideal solution (TOPSIS) method using a fuzzy set environment to solve the decision-making problems. He [33], introduced a Dombi hesitant fuzzy information AO based typhoon disaster assessment. Shi and Ye [34] expanded Dombi operations to neutrosophic cubic sets for MADM problems. Chen et al. [35] introduced k-means clustering for the aggregation of hesitant fuzzy linguistic term sets (HFLTS) possibility distributions. In 2019, Chen et al. [36] introduced the idea of fostering linguistic decision-making under the conditions of uncertainty, a proportional interval type-2 hesitant fuzzy TOPSIS approach based on Hamacher AOs and optimization models.

1.2. Motivation of the study

Due to the vagueness and uncertainty of human cognitions/judgments as related to complicated decision-making problems, existing fuzzy decision-making approaches merely signal fuzzy assessment values and lack degrees/levels of credibility for the fuzzy assessment values in alternatives over attributes. As a result, the fuzzy evaluative value's credibility degree highlights its significance and importance in the fuzzy decision-making problem. To improve the degrees/levels of credibility of fuzzy evaluation values, the fuzzy assessment values should be tightly linked to their credibility measures, which results in more abundant and reliable assessment information. In some research methodology frameworks, for example, each expert/reviewer is required, for a manuscript to give his/her overall assessment and related credibility degree from 0 to 10. In this case, the expert can choose any value of 5 that is equivalent to the fuzzy assessment value of 0.5, as well as 6, which is equivalent to the credibility degree of 0.6, owing to some lack of the expert's knowledge. So, it is obvious that the fuzzy assessment value of 0.6 is more closely related to the credibility (correct) degree of 0.5 in the pair of fuzzy values (0.5, 0.6), which is needed to enhance the credibility degree of his/her overall assessment of the manuscript. Under this condition, they have fuzzy strategic assets as well, but they always decide their credibility degrees to maintain the credibility levels/degrees of the fuzzy evaluation values because human decisions in uncertain and unspecified circumstances are never completely credible and correct. In an unclear context, the credibility measure of a fuzzy evaluation value must be closely tied to the fuzzy evaluation value, which leads to more available and accurate assessment. As a result, this study was purpose to develop the concept of a FCN, which is a new interpretation of the fuzzy concept in which a pair of fuzzy numbers represents both a fuzzy value and a credibility degree. The flexibility and emergence of Dombi operations is a motivation. The Dombi operation is applied to FCNs so as to make the information more real and credible. This method not only solves the MAGDM problem with FCNs, but it also makes the decision process more credible and effective. Finding unknown weight vectors for decision-makers or criteria is a critical issue. To address this issue, the entropy measure was used to find the unknown weight vectors for decision-makers. The TOPSIS procedure was used to rank the alternatives.

1.3. Contribution of study

Considering the above discussion, we are obliged that the FCNs has an operative consistency to reveal the debatable and possible items that appear in real-life problems. The newly proposed FCN concept, which is based on the hybrid information of the fuzzy values and the degree of credibility, can make the information expression more credible and reasonable. FCN operations and score functions, as well as fuzzy credibility Dombi weighted geometric (FCDWG), fuzzy credibility Dombi ordered weighted geometric (FCDOWG) and fuzzy credibility Dombi hybrid weighted geometric (FCDHWG) operators, can be useful mathematical tools for modelling MADM problems with FCNs. The proposed method not only solves the MADM problem with FCNs, but it also improves the decision-making process credibility and effectiveness. This paper is organized as follows. Preliminaries are presented in the 2nd section. In the 3rd section of the given paper, we define the Dombi operational for FCNs and some basic properties. In the 4th section of the given paper, we design the FCDWG operators by using the Dombi t-norm and t-conorm. In the 5th section, we propose the measure of the generalized distance and weighted generalized distance for FCNs. In the

6th section of the given paper, illustrative example based on an extended TOPSIS method. In Section 7, a comparative analysis is presented to demonstrate the effectiveness of the proposed method. Finally, in Section 8, the conclusion and future work are discussed.

2. Materials and methods

In this section of the article, the basic concepts of fuzzy sets and FCNs are presented, which will be useful in certain studies.

Let W be the non-empty set. Then, the fuzzy set L in W is defined as follows [1]:

$$L = \{(w, \iota_n(w)|w \in W\},$$

where in the above equation, $\iota_n(w)$ denotes the degree of the membership; it also belongs to [0,1]. However, the fuzzy set fails to describe the uncertainty with only the degree of membership of the element, and it cannot handle the complex problems. Atanassov [2] defined the concept of the IFS in which both the degree of membership and degree of non-membership are discussed; it is defined as follows.

Let W be the non-empty set. Then, the IFS L in W is defined as follows [2]:

$$L = \{(w, \iota_n(w), \kappa_n(w)|w \in W\},$$

where $\iota_n(w)$ and $\kappa_n(w)$ represent the degree of membership and degree of non-membership respectively, with the condition $0 \leq \iota_n(w) + \kappa_n(w) \leq 1$.

Let W be the non-empty set. Then, the FCNs on W are defined as [29]

$$L = \{(w, \iota_n(w), \kappa_n(w)|w \in W\}$$

for all $\iota_n : W \to [0, 1]$ and $\kappa_n : W \to [0, 1]$, which denote the degree of member and the degree of credibility related to $\iota_n(w)$, respectively. Then, the pair $(w, \iota_n(w), \kappa_n(w))$ is called the FCNs.

Let $L_1 = (\iota_1, \kappa_1)$ and $L_2 = (\iota_1, \kappa_1)$ be two FCNs. Then, their relations are defined as follows [29]:

(1) $L_1 \supseteq L_2 \Leftrightarrow \iota_1 \geq \iota_1, \kappa_1 \geq \kappa_1$;

(2) $L_1 = L_2 \Leftrightarrow L_1 \supseteq L_2$ and $L_2 \supseteq L_1$;

(3) $L_1 \cup L_2 = (\iota_1 \vee \iota_1, \kappa_1 \vee \kappa_1)$;

(4) $L_1 \cap L_2 = (\iota_1 \wedge \iota_1, \kappa_1 \wedge \kappa_1)$;

(5) $(L_1)^c = (1 - \iota_1, 1 - \kappa_1)$.

To compare two FCNs, the score function is defined as follows [29]:

$$E(L_n) = [\iota_n\kappa_n + (\iota_n + \kappa_n)/2]/2, \text{ For } E(L_n) \in [0, 1].$$

Hence, the ranking relationship of the two FCNs is defined as follows:

(1) If $E(L_1) > E(L_2)$, then $L_1 > L_2$.

(2) If $E(L_1) = E(L_2)$, then $L_1 = L_2$.

2.1. Dombi operators

The terms t-norm and t-conorm are used in fuzzy set theory to construct a generalized union and the intersection of the fuzzy sets [37]. The definitions and explanations of t-norm and t-conorm have been provided by Roychowdhury and Wang [38]. Deschrijver et al. [37] proposed a generalized union and a generalized intersection of IFSs based on a t-norm and t-conorm. Dombi [21], developed the concept of a newly triangular norm, which is presented as the Dombi t-norm and t-conorm. These showed good flexibility with the operational parameters. Until now, Dombi operations have not been extended to aggregate FCNs. Hence, the Dombi product and Dombi sum are the special cases of t-norm and t-conorm, respectively, which are given in the following definition.

Let ι and κ be any two real numbers. Then, the Dombi t-norm and Dombi t-conorm are defined as follows [21]:

$$\begin{aligned} \iota \otimes \kappa &= Dom(\iota, \kappa) = \frac{1}{1 + \{(\frac{1-\iota}{\iota})^{\sigma} + (\frac{1-\kappa}{\kappa})^{\sigma}\}^{1/\sigma}}, \\ \iota \oplus \kappa &= Dom^{\mathbb{C}}(\iota, \kappa) = 1 - \frac{1}{1 + \{(\frac{\iota}{1-\iota})^{\sigma} + (\frac{\kappa}{1-\kappa})^{\sigma}\}^{1/\sigma}}, \end{aligned}$$

where $\sigma \geq 1$ and $(\iota, \kappa) \in [0, 1] * [[0, 1]$.

3. Dombi operations of FCNs

This section provides the new work on FCNs. First, we define the Dombi operational laws for FCNs with the help of the above expression that also discusses some basic properties on the basis of Dombi operational laws, we define the weighted geometric operators to aggregate the fuzzy credibility information.

Let $L_1 = (\iota_1, \kappa_1)$ and $L_2 = (\iota_2, \kappa_2)$ be two FCNs, where $\sigma \geq 1$, is an operational parameter and ς is any scalar greater than 0. Then, the operational laws of FCNs are defined as follows:

(1) $L_1 \oplus L_2 = \left\langle 1 - \frac{1}{1+\{(\frac{\iota_1}{1-\iota_1})^{\sigma}+(\frac{\iota_2}{1-\iota_2})^{\sigma}\}^{1/\sigma}}, 1 - \frac{1}{1+\{(\frac{1-\kappa_1}{\kappa_1})^{\sigma}+(\frac{1-\kappa_2}{\kappa_2})^{\sigma}\}^{1/\sigma}} \right\rangle$;

(2) $L_1 \otimes L_2 = \left\langle \frac{1}{1+\{(\frac{1-\iota_1}{\iota_1})^{\sigma}+(\frac{1-\iota_2}{\iota_2})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{(\frac{\kappa_1}{1-\kappa_1})^{\sigma}+(\frac{\kappa_2}{1-\kappa_2})^{\sigma}\}^{1/\sigma}} \right\rangle$;

(3) $\varsigma . L_1 = \left\langle 1 - \frac{1}{1+\{\varsigma(\frac{\iota_1}{1-\iota_1})^{\sigma}\}^{1/\sigma}}, 1 - \frac{1}{1+\{\varsigma(\frac{1-\kappa_1}{\kappa_1})^{\sigma}\}^{1/\sigma}} \right\rangle$;

(4) $L_1^{\varsigma} = \left\langle \frac{1}{1+\{\varsigma(\frac{1-\iota_1}{\iota_1})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{\varsigma(\frac{\kappa_1}{1-\kappa_1})^{\sigma}\}^{1/\sigma}} \right\rangle$.

Example 1. *Let* $L_1 = (0.68, 0.49)$ *and* $L_2 = (0.67, 0.50)$ *be two FCNs, and let* $\sigma = 2$ *be the Dombi t-norm parameter, which is a natural number and* $\varsigma = 3$ *be any scalar greater than* 0. *Now, we shall apply Dombi operational laws to the FCNs, as follows:*

$$\begin{aligned} L_1 \oplus L_2 &= \left\langle 1 - \frac{1}{1 + \{(\frac{0.68}{1-0.68})^2 + (\frac{0.67}{1-0.67})^2\}^{1/2}}, 1 - \frac{1}{1 + \{(\frac{1-0.49}{0.49})^2 + (\frac{1-0.50}{0.50})^2\}^{1/2}} \right\rangle \\ &= \langle 0.746\,13, 0.590\,73 \rangle, \end{aligned}$$

$$L_1 \otimes L_2 = \left\langle \frac{1}{1 + \{(\frac{1-0.68}{0.68})^2 + (\frac{1-0.67}{0.67})^2\}^{1/2}}, \frac{1}{1 + \{(\frac{0.49}{1-0.49})^2 + (\frac{0.50}{1-0.50})^2\}^{1/2}} \right\rangle$$

$$= \langle 0.594\,81, 0.418\,98\rangle,$$

$$\begin{aligned}\varsigma.L_1 &= \left\langle 1-\frac{1}{1+\{3(\frac{0.68}{1-0.68})^2\}^{1/2}}, 1-\frac{1}{1+\{3(\frac{1-0.49}{0.49})^2\}^{1/2}}\right\rangle\\ &= \langle 0.786\,35, 0.643\,21\rangle,\end{aligned}$$

$$\begin{aligned}L_1^{\varsigma} &= \left\langle \frac{1}{1+\{3(\frac{1-0.68}{0.68})^2\}^{1/2}}, \frac{1}{1+\{3(\frac{0.68}{1-0.68})^2\}^{1/2}}\right\rangle\\ &= \langle 0.550\,94, 0.213\,65\rangle.\end{aligned}$$

Theorem 1. *Let L =(ι,κ), $L_1=(\iota_1,\kappa_1)$ and $L_2=(\iota_2,\kappa_2)$ be three FCNs. Then, we have the following equations:*

(1) $L_1\oplus L_2 = L_2\oplus L_1$,
(2) $L_1\otimes L_2 = L_2\otimes L_1$,
(3) $\varsigma(L_1\oplus L_2) = \varsigma L_1\oplus\varsigma L_2, \varsigma>0$,
(4) $(L_1\otimes L_2)^{\varsigma} = L_1^{\varsigma}\otimes L_2^{\varsigma}$,
(5) $\varsigma_1 L\oplus\varsigma_2 L = (\varsigma_1\oplus\varsigma_2)L$,
(6) $L^{\varsigma_1}\otimes L^{\varsigma_2} = L^{(\varsigma_1\otimes\varsigma_2)}$.

Proof. For these three FCNs L, L_1 and L_2 where $\varsigma,\varsigma_1,\varsigma_2>0$, we obtain

$$\begin{aligned}L_1\oplus L_2 =& \left\langle 1-\frac{1}{1+\{(\frac{\iota_1}{1-\iota_1})^{\sigma}+(\frac{\iota_2}{1-\iota_2})^{\sigma}\}^{1/\sigma}}, 1-\frac{1}{1+\{(\frac{1-\kappa_1}{\kappa_1})^{\mathfrak{I}}+(\frac{1-\kappa_2}{\kappa_2})^{\sigma}\}^{1/\sigma}}\right\rangle\\ &\left\langle 1-\frac{1}{1+\{(\frac{\iota_2}{1-\iota_2})^{\sigma}+(\frac{\iota_1}{1-\iota_1})^{\sigma}\}^{1/\sigma}}, 1-\frac{1}{1+\{(\frac{1-\kappa_2}{\kappa_2})^{\sigma}+(\frac{1-\kappa_1}{\kappa_1})^{\sigma}\}^{1/\sigma}}\right\rangle\\ &= L_2\oplus L_1,\end{aligned}$$

$$\begin{aligned}L_1\otimes L_2 &= \left\langle \frac{1}{1+\{(\frac{1-\iota_1}{\iota_1})^{\sigma}+(\frac{1-\iota_2}{\iota_2})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{(\frac{\kappa_1}{1-\kappa_1})^{\sigma}+(\frac{\kappa_2}{1-\kappa_2})^{\sigma}\}^{1/\sigma}}\right\rangle\\ &= \left\langle \frac{1}{1+\{(\frac{1-\iota_2}{\iota_2})^{\sigma}+(\frac{1-\iota_1}{\iota_1})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{(\frac{\kappa_2}{1-\kappa_2})^{\sigma}+(\frac{\kappa_1}{1-\kappa_1})^{\sigma}\}^{1/\sigma}}\right\rangle\\ &= L_2\otimes L_1.\end{aligned}$$

Let $m = 1-\frac{1}{1+\{(\frac{\iota_1}{1-\iota_1})^{\mathfrak{I}}+(\frac{\iota_2}{1-\iota_2})^{\sigma}\}^{1/\sigma}}$; then, we have $\frac{m}{1-m} = \{(\frac{\iota_1}{1-\iota_1})^{\sigma}+(\frac{\iota_2}{1-\iota_2})^{\sigma}\}^{1/\sigma}$. Therefore, $(\frac{m}{1-m})^{\sigma} = (\frac{\iota_1}{1-\iota_1})^{\sigma}+(\frac{\iota_2}{1-\iota_2})^{\sigma}$. Using the above terms, we get

$$\begin{aligned}\varsigma(L_1\oplus L_2) =& \varsigma\left\langle 1-\frac{1}{1+\{(\frac{\iota_1}{1-\iota_1})^{\sigma}+(\frac{\iota_2}{1-\iota_2})^{\sigma}\}^{1/\sigma}}, 1-\frac{1}{1+\{(\frac{1-\kappa_1}{\kappa_1})^{\sigma}+(\frac{1-\kappa_2}{\kappa_2})^{\sigma}\}^{1/\sigma}}\right\rangle\\ =& \left\langle 1-\frac{1}{1+\{\varsigma(\frac{\iota_1}{1-\iota_1})^{\sigma}+\varsigma(\frac{\iota_2}{1-\iota_2})^{\sigma}\}^{1/\sigma}}, 1-\frac{1}{1+\{\varsigma(\frac{1-\kappa_1}{\kappa_1})^{\sigma}+\varsigma(\frac{1-\kappa_2}{\kappa_2})^{\sigma}\}^{1/\sigma}}\right\rangle\\ \varsigma L_1\oplus\varsigma L_2 =& \left\langle 1-\frac{1}{1+\{\varsigma(\frac{\iota_1}{1-\iota_1})^{\sigma}\}^{1/\sigma}}, 1-\frac{1}{1+\{\varsigma(\frac{1-\kappa_1}{\kappa_1})^{\sigma}\}^{1/\sigma}}\right\rangle\\ &\oplus\left\langle 1-\frac{1}{1+\{\varsigma(\frac{\iota_2}{1-\iota_2})^{\sigma}\}^{1/\sigma}}, 1-\frac{1}{1+\{\varsigma(\frac{1-\kappa_2}{\kappa_2})^{\sigma}\}^{1/\sigma}}\right\rangle\\ =& \left\langle 1-\frac{1}{1+\{\varsigma(\frac{\iota_1}{1-\iota_1})^{\sigma}+\varsigma(\frac{\iota_2}{1-\iota_2})^{\sigma}\}^{1/\sigma}}, 1-\frac{1}{1+\{\varsigma(\frac{1-\kappa_1}{\kappa_1})^{\sigma}+\varsigma(\frac{1-\kappa_2}{\kappa_2})^{\sigma}\}^{1/\sigma}}\right\rangle\\ &= \varsigma(L_1\oplus L_2),\end{aligned}$$

$$(L_1 \otimes L_2)^{\varsigma} = \left\langle \frac{1}{1+\{(\frac{\iota_1}{1-\iota_1})^{\sigma}+(\frac{\iota_2}{1-\iota_2})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{(\frac{1-\kappa_1}{\kappa_1})^{\sigma}+(\frac{1-\kappa_2}{\kappa_2})^{\sigma}\}^{1/\sigma}} \right\rangle^{\varsigma}$$
$$= \left\langle \frac{1}{1+\{\varsigma(\frac{\iota_1}{1-\iota_1})^{\sigma}+\varsigma(\frac{\iota_2}{1-\iota_2})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{\varsigma(\frac{1-\kappa_1}{\kappa_1})^{\sigma}+\varsigma(\frac{1-\kappa_2}{\kappa_2})^{\sigma}\}^{1/\sigma}} \right\rangle$$
$$= \left\langle \frac{1}{1+\{\varsigma(\frac{\iota_1}{1-\iota_1})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{\varsigma(\frac{1-\kappa_1}{\kappa_1})^{\sigma}\}^{1/\sigma}} \right\rangle \otimes \left\langle \frac{1}{1+\{\varsigma(\frac{\iota_2}{1-\iota_2})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{\varsigma(\frac{1-\kappa_2}{\kappa_2})^{\sigma}\}^{1/\sigma}} \right\rangle$$
$$= L_1^{\varsigma} \otimes L_2^{\varsigma},$$

$$\varsigma_1 L \oplus \varsigma_2 L = \left\langle 1 - \frac{1}{1+\{\varsigma_1(\frac{\iota}{1-\iota})^{\sigma}\}^{1/\sigma}}, 1 - \frac{1}{1+\{\varsigma_1(\frac{1-\kappa}{\kappa})^{\sigma}\}^{1/\sigma}} \right\rangle$$
$$\oplus \left\langle 1 - \frac{1}{1+\{\varsigma_2(\frac{\iota}{1-\iota})^{\sigma}\}^{1/\sigma}}, 1 - \frac{1}{1+\{\varsigma_2(\frac{1-\kappa}{\kappa})^{\sigma}\}^{1/\sigma}} \right\rangle$$
$$= \left\langle 1 - \frac{1}{1+\{(\varsigma_1+\varsigma_2)(\frac{\iota}{1-\iota})^{\sigma}\}^{1/\sigma}}, 1 - \frac{1}{1+\{(\varsigma_1+\varsigma_2)(\frac{1-\kappa}{\kappa})^{\sigma}\}^{1/\sigma}} \right\rangle$$
$$= (\varsigma_1 \oplus \varsigma_2)L,$$

$$L^{\varsigma_1} \otimes L^{\varsigma_2} = \left\langle \frac{1}{1+\{\varsigma_1(\frac{1-\iota}{\iota})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{\varsigma_1(\frac{\kappa}{1-\kappa})^{\sigma}\}^{1/\sigma}} \right\rangle$$
$$\otimes \left\langle \frac{1}{1+\{\varsigma_2(\frac{1-\iota}{\iota})\sigma\}^{1/\sigma}}, \frac{1}{1+\{\varsigma_2(\frac{\kappa}{1-\kappa})^{\sigma}\}^{1/\sigma}} \right\rangle$$
$$= \left\langle \frac{1}{1+\{(\varsigma_1+\varsigma_2)(\frac{1-\iota}{\iota})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{(\varsigma_1+\varsigma_2)(\frac{\kappa}{1-\kappa})^{\sigma}\}^{1/\sigma}} \right\rangle$$
$$= L^{(\varsigma_1+\varsigma_2)}.$$

□

4. Dombi weighted geometric operators for FCNs

This section of the given paper provides a the detailed discussion of newly designed fuzzy credibility Dombi geometric AOs, which have been derived by using the Dombi operational laws of FCNs. First, we define the FCDWG operators and some basic properties in detail. Next, we define the FCDOWG operator, which means that we have to find the score and accuracy function of the FCNs; then, we have to apply our proposed operator, i.e., the FCDOWG operator, to aggregate the FCNs to select the best option. Lastly, we define the FCDHWG operator with the help of Dombi operational laws. With this operator, here, we first multiply the associated weighted vectors and the given alternatives; then, our new proposed operator (FCDHWG) is applied to select the best option. Now, we respectively define these operators as follows.

Definition 1. *Let $L_N = (\iota_N, \kappa_N)$ for all $(N = 1, 2, ..., n)$ represent FCNs. Thus, the FCDWG operator is defined as*

$$FCDWG_{\rho}(L_1, L_2, ..., L_n) = \overset{n}{\underset{N=1}{\oplus}} (\rho_N L_N),$$

where the weight vector $\rho = (\rho_1, \rho_2, ..., \rho_n)^T$ of ρ_N for all $(N = 1, 2, ..., n)$, with $\rho_N > 0$ and $\sum_{N=1}^{n} \rho_N = 1$. With this, we will prove the following theorem.

Theorem 2. *Let $L_N = (\iota_N, \kappa_N)$ for all $(N = 1, 2, 3...n)$, represent FCNs. Then, the aggregated value for* *the FCDWG operator is also an FCN:*

$$FCDWG_{\rho}(L_1, L_2, ..., L_n) = \overset{n}{\underset{N=1}{\otimes}} (\rho_N L_N)$$
$$= \left\langle \frac{1}{1+\{\sum_{N=1}^{n} \rho_N (\frac{\iota_N}{1-\iota_N})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{\sum_{N=1}^{n} \rho_N (\frac{1-\kappa_N}{\kappa_N})^{\sigma}\}^{1/\sigma}} \right\rangle,$$

where $\rho_N = (\rho_1, \rho_2, ..., \rho_n)^T$ *is the weight of* L_n*, with* $0 \le \rho_N \le 1$ *and* $\sum_{N=1}^{n} \rho_N = 1$.

Proof. This theorem can be proved by using the mathematical induction method.

For $n = 2$, on the basis of Dombi operations for FCNs, we have the following results:

$$L_1 \otimes L_2 = (\iota_1, \kappa_1) \otimes (\iota_2, \kappa_2),$$

$$L_1 \otimes L_2 = \left\langle \frac{1}{1+\{\rho_1(\frac{\iota_1}{1-\iota_1})^{\sigma}+\rho_2(\frac{\iota_2}{1-\iota_2})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{\rho_1(\frac{1-\kappa_1}{\kappa_1})^{\sigma}+\rho_2(\frac{1-\kappa_2}{\kappa_2})^{\sigma}\}^{1/\sigma}} \right\rangle$$
$$= \left\langle \frac{1}{1+\{\sum_{N=1}^{2} \rho_N (\frac{\iota_N}{1-\nu_N})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{\sum_{N=1}^{2} \rho_N (\frac{1-\kappa_N}{\kappa_N})^{\sigma}\}^{1/\sigma}} \right\rangle,$$

which is valid for $n = 2$.

For $n = k$, we have

$$FCDWG_{\rho}(L_1, L_2, ..., L_k) = \overset{k}{\underset{N=1}{\otimes}} (\rho_N L_N)$$
$$= \left\langle \frac{1}{1+\{\sum_{N=1}^{k} \rho_N (\frac{\iota_N}{1-\iota_N})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{\sum_{N=1}^{k} \rho_N (\frac{1-\kappa_N}{\kappa_N})^{\sigma}\}^{1/\sigma}} \right\rangle .$$

For $n = k + 1$, we have

$$FCDWG_{\rho}(L_1, L_2, ..., L_{k+1}) = \overset{k}{\underset{N=1}{\otimes}} (\rho_N L_N) \otimes (\rho_{k+1} L_{k+1})$$
$$= \left[\begin{array}{c} \left\langle \frac{1}{1+\{\sum_{N=1}^{k} \rho_N (\frac{\iota_N}{1-\iota_N})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{\sum_{N=1}^{k} \rho_N (\frac{1-\kappa_N}{\kappa_N})^{\sigma}\}^{1/\sigma}} \right\rangle \\ \otimes \left\langle 1 - \frac{1}{1+\{\rho_{k+1}(\frac{\iota_{k+1}}{1-\iota_{k+1}})^{\Im}\}^{1/\Im}}, 1 - \frac{1}{1+\{\rho_{k+1}(\frac{1-\kappa_{k+1}}{\kappa_{k+1}})^{\Im}\}^{1/\Im}} \right\rangle \end{array} \right],$$
$$= \left\langle \frac{1}{1+\{\sum_{N=1}^{k+1} \rho_N (\frac{\iota_N}{1-\iota_N})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{\sum_{N=1}^{k+1} \rho_N (\frac{1-\kappa_N}{\kappa_N})^{\sigma}\}^{1/\sigma}} \right\rangle,$$

which is true for $n = k + 1$. Therefore, from the above proof, it is obvious that it is true for any n. □

In the next theorem, we discuss some characteristics of the FCDWG operator and its properties.

Theorem 3. *(Idempotent) Let us consider a collection* $L_N = (\iota_N, \kappa_N)$*, where* $(N = 1, 2, 3...n)$ *denote the FCNs. When* $L_N = L = (\iota, \kappa)$*, the* $FCDWG(L_1, L_2, ..., L_n) = L$.

Proof. $L_N = (\iota_N, \kappa_N) = L$, where $(N = 1, 2, 3...n)$. Then, we have

$$FCDWG_{\rho}(L_1, L_2, ..., L_n) = \underset{N=1}{\overset{n}{\otimes}}(\rho_N L_N)$$
$$= \left\langle \frac{1}{1+\{\sum\limits_{N=1}^{n}\rho_N(\frac{\iota_N}{1-\iota_N})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{\sum\limits_{N=1}^{n}\rho_N(\frac{1-\kappa_N}{\kappa_N})^{\sigma}\}^{1/\sigma}} \right\rangle$$
$$= \left\langle \frac{1}{1+\{(\frac{\iota}{1-\iota})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{\sum\limits_{N=1}^{n}\rho_N(\frac{1-\kappa}{\kappa})^{\sigma}\}^{1/\sigma}} \right\rangle$$
$$= \left\langle \frac{1}{1+\frac{\iota}{1-\iota}}, \frac{1}{1+\frac{1-\kappa}{\kappa}} \right\rangle = (\iota, \kappa) = L.$$

Thus, $FCDWG_{\rho}(L_1, L_2, ..., L_n) = L$ holds. □

Theorem 4. *(Boundednes) Suppose* $L_N = (\iota_N, \kappa_N)$ $(N = 1, 2, 3...n)$ *be a collection of FCNs; then, a set* $L^- = (\min_N(\iota_N), \min_N(\kappa_N))$ *and* $L^+ = (\max_N(\iota_N), \max_N(\kappa_N))$ *are the minimum and maximum FCNs, respectively. Then, we have*

$$L^- \leq FCDWG_{\rho}(L_1, L_2, ..., L_n) \leq L^+.$$

Proof. If $L^- = (\min_N(\iota_N), \min_N(\kappa_N))$ and $L^+ = (\max_N(\iota_N), \max_N(\kappa_N))$ are the minimum and maximum FCNs, respectively, then $L^- \leq L_N \leq L^+$ can hold. Hence, we have the subsequent inequalities:

$$\begin{array}{l} \frac{1}{1+\{\sum\limits_{N=1}^{n}\rho_N(\frac{\iota^-}{1-\iota^-})^{\sigma}\}^{1/\sigma}} \leq \frac{1}{1+\{\sum\limits_{N=1}^{n}\rho_N(\frac{\iota}{1-\iota})^{\sigma}\}^{1/\sigma}} \leq \frac{1}{1+\{\sum\limits_{N=1}^{n}\rho_N(\frac{\iota^+}{1-\iota^+})^{\sigma}\}^{1/\sigma}}, \\ \frac{1}{1+\{\sum\limits_{N=1}^{n}\rho_N(\frac{1-\kappa^-}{\kappa^-})^{\sigma}\}^{1/\sigma}} \leq \frac{1}{1+\{\sum\limits_{N=1}^{n}\rho_N(\frac{1-\kappa}{\kappa})^{\sigma}\}^{1/\sigma}} \leq \frac{1}{1+\{\sum\limits_{N=1}^{n}\rho_N(\frac{1-\kappa^+}{\kappa^+})^{\sigma}\}^{1/\sigma}} \end{array}.$$

Therefore, $L^- \leq FCDWG_{\rho}(L_1, L_2, ..., L_n) \leq L^+$. □

Theorem 5. *(Monotonicity) Let* $L_N = (\iota_N, \kappa_N)$*, where* $(N = 1, 2, 3...n)$ *is a number of FCNs if* $L_N \leq L_N^*$ $\forall$ *N. Then,*

$$FCDWG_{\rho}(L_1, L_2, ..., L_n) \leq FCDWG_{\rho}(L_1^*, L_2^*, ..., L_n^*).$$

Proof. If $L_N \leq L_N^*$, $\underset{N=1}{\overset{n}{\otimes}}(\rho_N L_N) \leq \underset{N=1}{\overset{n}{\otimes}}(\rho_N L_N^*)$ can hold. Then, there is

$$FCDWG_{\rho}(L_1, L_2, ..., L_n) \leq FCDWG_{\rho}(L_1^*, L_2^*, ..., L_n^*).$$

□

Definition 2. *Let* $L_N = (\iota_N, \kappa_N)$ *be a collection of FCNs. Then, the FCDOWG operator is defined as*

$$FCDOWG_{\rho}(L_1, L_2, ..., L_n) = \underset{N=1}{\overset{n}{\otimes}}(\rho_N L_N),$$

where $\rho = (\rho_1, \rho_2, ..., \rho_n)^T$ *is the weight vector of* L_N*, with* $0 \leq \rho_N \leq 1$*; also,* $\sum\limits_{N=1}^{n}\rho_N = 1$*, where the permutation* $(\varepsilon(1), \varepsilon(2), ..., \varepsilon(n))$ *applies for* $L_{\varepsilon(N-1)} \geq L_{\varepsilon(N)}$.

Theorem 6. *Let $L_N = (\iota_N, \kappa_N)$ be a family of FCNs. Then, the aggregated value of the FCDOWG is still an FCN, which is as follows:*

$$FCDOWG_\rho(L_1, L_2, ..., L_n) = \overset{n}{\underset{N=1}{\otimes}}(\rho_N L_N)$$
$$= \left\langle \frac{1}{1+\{\sum\limits_{N=1}^{n}\rho_N(\frac{\iota_{\varepsilon(N)}}{1-\iota_{\varepsilon(N)}})^\sigma\}^{1/\sigma}}, \frac{1}{1+\{\sum\limits_{N=1}^{n}\rho_N(\frac{1-\kappa_{\varepsilon(N)}}{\kappa_{\varepsilon(N)}})^\sigma\}^{1/\sigma}} \right\rangle,$$

where $\rho = (\rho_1, \rho_2, ..., \rho_n)^T$ is the weight vector of L_N, with $0 \le \rho_N \le 1$ and $\sum\limits_{N=1}^{n}\rho_N = 1$.

Proof. By the similar proof process of Theorem 2 one easily verifies the above equation, which is omitted here. □

Definition 3. *Let $L_N = (\iota_N, \kappa_N)$, where $(N = 1, 2, 3...n)$ is a collection of FCNs. Then, the FCDHWG operator can be defined as follows:*

$$FCDHWG_\rho(L_1, L_2, ..., L_n) = \overset{n}{\underset{N=1}{\otimes}}(\rho_N L_{N(\varepsilon)})$$
$$= \left\langle \frac{1}{1+\{\sum\limits_{N=1}^{n}\rho_N(\frac{\iota_{\varepsilon(N)}}{1-\iota_{\varepsilon(N)}})^\sigma\}^{1/\sigma}}, \frac{1}{1+\{\sum\limits_{N=1}^{n}\rho_N(\frac{1-\kappa_{\varepsilon(N)}}{\kappa_{\varepsilon(N)}})^\sigma\}^{1/\sigma}} \right\rangle ,$$

where $L_{N(\varepsilon)}$ represents the N^{th} largest weighted credibility fuzzy values $L_j(L_N = n\rho_N L_N, N = 1, 2, ..., n)$, and $\rho = (\rho_1, \rho_2, ..., \rho_n)^T$ represents the weight vector of L_N with $0 \le \rho_N \le 1$ and $\sum\limits_{N=1}^{n}\rho_N = 1$,where n is the balancing coefficient.

Theorem 7. *Let $L_N = (\iota_N, \kappa_N)$ be a family of FCNs. Then, the aggregated value of the FCDHWG is still an FCN, which is as follows:*

$$FCDHWG_\rho(L_1, L_2, ..., L_n) = \overset{n}{\underset{N=1}{\otimes}}(\rho_N L_{N(\varepsilon)})$$
$$= \left\langle \frac{1}{1+\{\sum\limits_{N=1}^{n}\rho_N(\frac{\iota_{\varepsilon(N)}}{1-\iota_{\varepsilon(N)}})^\sigma\}^{1/\sigma}}, \frac{1}{1+\{\sum\limits_{N=1}^{n}\rho_N(\frac{1-\kappa_{\varepsilon(N)}}{\kappa_{\varepsilon(N)}})^\sigma\}^{1/\sigma}} \right\rangle ,$$

where $L_{N(\varepsilon)}$ represents the largest permutation values from the collection of FCNs, i.e., $L_{N(\varepsilon)} = n\rho_N L_N, N = 1, 2, ..., n)$ and $\rho = (\rho_1, \rho_2, ..., \rho_n)^T$ is the weight vector of L_N with $0 \le \rho_N \le 1$ and $\sum\limits_{N=1}^{n}\rho_N = 1$, where n is the balancing coefficient.

Proof. By the similar proof process of Theorem 2 one easily verifies the above equation, which is omitted here. □

5. Entropy measure for credibility fuzzy sets

In this portion, we purpose the measure of generalized distance and weighted generalized distance for credibility fuzzy sets. Subsequently, by utilizing the measure of generalized distance, we propose the credibility fuzzy entropy measure for FCNs to measure the fuzziness of FCNs.

5.1. Distance measure for fuzzy credibility sets

Definition 4. *Suppose $L_x = (\iota_{L_x}, \kappa_{L_x})$ and $G_x = (\iota_{G_x}, \kappa_{G_x})$ be two FCNs. Then, the generalized distance measure (GDM) between any two FCNs for any $\phi > 0 \in \mathbb{R}$ is define as*

$$d_g\left(L_x, G_x\right) = \left(\frac{1}{2n}\sum_{x=1}^{n}\left(\left|(\iota_{L_x})^2 - (\iota_{G_x})^2\right|^{\phi} + \left|(\kappa_{L_x})^2 - (\kappa_{G_x})^2\right|^{\phi}\right)\right)^{\frac{1}{\phi}}.$$

Definition 5. *Suppose $L_x = (\iota_{L_x}, \kappa_{L_x})$ and $G_x = (\iota_{G_x}, \kappa_{G_x})$ be two FCNs. Then, the weighted GDM between any two FCNs for any $\phi > 0 \in \mathbb{R}$ is define as*

$$d_{wg}\left(L_x, G_x\right) = \left(\frac{1}{2n}\sum_{x=1}^{n}\omega_x\left(\left|(\iota_{L_x})^2 - (\iota_{G_x})^2\right|^{\phi} + \left|(\kappa_{L_x})^2 - (\kappa_{G_x})^2\right|^{\phi}\right)\right)^{\frac{1}{\phi}},$$

where $\omega_x(x = 1, 2, ..., n)$ denotes the weight vector with the condition that $\omega_x \geq 0$ and $\sum_{x=1}^{n}\omega_x = 1$.

Definition 6. *Let $L_x = \{\iota_{L_x}(w), \kappa_{L_x}(w)\}$ be a set of $FCNs$ $(x = 1, 2)$. Then, the GDM defined in Definition 4 is reduced as follows:*

$$d_{wg}(L_1, L_2) = \left(\frac{1}{2}\left(\left|(\iota_{L_1})^2 - (\iota_{L_2})^2\right|^{\phi} + \left|(\kappa_{L_1})^2 - (\kappa_{L_2})^2\right|^{\phi}\right)\right)^{\frac{1}{\phi}}, \phi > 0\,(\in \mathbb{R}).$$

For any two $L_1, L_2 \in FCN$, the following properties must be satisfied by the given GDMs.

(1) $0 \leq d(L_1, L_2) \leq 1$,
(2) $d(L_1, L_2) = 1 \Leftrightarrow L_1 = L_2$,
(3) $d(L_1, L_2) = d(L_2, L_1)$.

5.2. Entropy measure for fuzzy credibility sets

In this portion, we propose an entropy measure for FCNs based on the distance measure by using the idea of Guo and Song [39].

Definition 7. *Let $L = \{L_1, L_2, ..., L_n\}$ be a fuzzy credibility sets (FCS s), where $L_x = \{\iota_{L_x}(w), \kappa_{L_x}(w)\}$ is a set of FCNs for $x = 1, 2, ..., n$. Then, the entropy measure is defined for FCNs as follows:*

$$E(L) = \frac{1}{n}\sum_{x=1}^{n}\left[\left\{1 - \left(d\left(L_x, L_x^c\right)\right)\right\}\frac{1 + (v_{L_x})^2}{2}\right],$$

where $v_{L_x} = \sqrt[1]{1 - \iota_{\omega}^2(n) - \kappa_{\omega}^2(n)}$ is the degree of hesitancy function.

5.3. MAGDM problems for fuzzy credibility numbers

In this real-world situation, the importance of decision-making issues grows as the social atmosphere warms. As a result, an expert's ability to reach a reasonable and skillful conclusion is adversely affected in this situation. Group decision-making processes in real-world situations depend on the feedback of a group of skilled professionals to arrive at an effective approach. As a result, MAGDM has a described skill and regulatory system to improve and evaluate competing criteria in all areas of decision-making that is used to achieve the most effective and practical decision-making outcomes. We also explored and suggest an extended TOPSIS method using FCNs. The problems of MAGDM can also be addressed in decision matrix form, where the columns and rows represents the alternatives and criteria/attributes, respectively. Thus, the decision matrix is represented by $\mathrm{D}_{n\times m}$. A set $\{Y_1, Y_2, ..., Y_n\}$ is considered, which represents n alternatives; another set $\{c_1, c_2, ..., c_m\}$ represents m criteria/attributes. Suppose $D^{(k)} = \left[L_{ij}^{(k)}\right]_{n\times m} = \left\langle \iota_{L_{ij}}^{(k)}, \kappa_{L_{ij}}^{(k)} \right\rangle_{n\times m,}$ $(k \in 1, 2, ..., e)$ denotes the fuzzy credibility decision matrix.

$$D_{n\times m}^{(k)} = \begin{array}{c} \\ Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{array} \left(\begin{array}{ccccc} c_1 & c_2 & c_2 & \cdots & c_m \\ \left(\iota_{L_{11}}^{(k)}, \kappa_{L_{11}}^{(k)}\right) & \left(\iota_{L_{12}}^{(k)}, \kappa_{L_{12}}^{(k)}\right) & \left(\iota_{L_{13}}^{(k)}, \kappa_{L_{13}}^{(k)}\right) & \cdots & \left(\iota_{L_{1m}}^{(k)}, \kappa_{L_{1m}}^{(k)}\right) \\ \left(\iota_{L_{21}}^{(k)}, \kappa_{L_{21}}^{(k)}\right) & \left(\iota_{L_{22}}^{(k)}, \kappa_{L_{22}}^{(k)}\right) & \left(\iota_{L_{23}}^{(k)}, \kappa_{L_{23}}^{(k)}\right) & \cdots & \left(\iota_{L_{2m}}^{(k)}, \kappa_{L_{2m}}^{(k)}\right) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \left(\iota_{L_{n1}}^{(k)}, \kappa_{L_{n1}}^{(k)}\right) & \left(\iota_{L_{n2}}^{(k)}, \kappa_{L_{n2}}^{(k)}\right) & \left(\iota_{L_{n3}}^{(k)}, \kappa_{L_{n3}}^{(k)}\right) & \cdots & \left(\iota_{L_{nm}}^{(k)}, \kappa_{L_{nm}}^{(k)}\right) \end{array} \right).$$

It should be noted that in the context of decision-making, all of the data about the weights of decision-makers and criteria are unknown.

5.4. TOPSIS method

This method comprises five parts. In the first part, we normalize the data. In the second part, we need to compute the weight of the decision-maker. In the third part, we compute the attribute weight. In the fourth part, we calculate the weight of the criteria by utilizing the proposed entropy measure. In the final part, we determine the ranking method on the basis of the degree of similarity to the ideal solution with positive ideal solutions (PISs) and negative ideal solutions (NISs).

For the proof of the FCN MAGDM problem, we utilized the TOPSIS method, for which the following steps of the procedure were developed.

Step-1 In this step, we have to represent the data in the form of a matrix with alternatives and criteria in the form of FCNs.

Step-2 Normalization of the decision metrics $(N)^k$. Only two types of attributes exist for the MAGDM problem; one is the benefit and the other one is cost.

$$(N)^k = \{\left(\iota_{k_{ij}}, \kappa_{k_{ij}}\right)\left(\kappa_{k_{ij}}, \iota_{k_{ij}}\right) \begin{array}{c} \text{for benefit,} \\ \text{for cost.} \end{array}$$

Step-3 This step consists of five steps, which are given below.

(a). The opinion of each single decision matrix is closer to the group decision ideal solution (GDIS), and as a result, the computation of the best group ideal solution (GIS) is done by taking the average of all of the outlooks of each single decision matrix. Here, in this step, we take the fuzzy credibility weighted average of the decision values of the alternatives corresponding to the criteria, which is given by the decision-makers, considering equal weights of the decision-makers at the initial stage, as follows:

$$GDIS=\begin{pmatrix} GDIS_{11} & GDIS_{12} & \cdots & GDIS_{1n} \\ GDIS_{21} & GDIS_{22} & \cdots & GDIS_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ GDIS_{m1} & GDIS_{m2} & \cdots & GDIS_{mn} \end{pmatrix},$$

where

$$\begin{aligned} GDIS_{ij} &= \sum_{k=1}^{\varsigma} \frac{1}{\varsigma} N_{ij}^{(k)} \\ &= \left\{ \frac{1}{1+\sum_{k=1}^{\varsigma}\{\varsigma(\frac{\iota_{ij}^{(k)}}{1-\iota_{ij}^{(k)}})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\sum_{k=1}^{\varsigma}\{\varsigma(\frac{1-\kappa_{ij}^{(k)}}{\kappa_{ij}^{(k)}})^{\sigma}\}^{1/\sigma}} \right\}. \end{aligned}$$

(b). Compute the group right ideal solution (GRIS) and left ideal solution (GLIS) as follows:

$$GRIS=\begin{pmatrix} GRIS_{11} & GRIS_{12} & \cdots & GRIS_{1n} \\ GRIS_{21} & GRIS_{22} & \cdots & GRIS_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ GRIS_{m1} & GRIS_{m2} & \cdots & GRIS_{mn} \end{pmatrix},$$

where

$$GRIS_{ij}=\left\{\left(N_{ij}^{(k)}\right):\max k\left[sc\left(N_{ij}^{(k)}\right)\right]\right\},$$

and

$$GLIS=\begin{pmatrix} GLIS_{11} & GLIS_{12} & \cdots & GLIS_{1n} \\ GLIS_{21} & GLIS_{22} & \cdots & GLIS_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ GLIS_{m1} & GLIS_{m2} & \cdots & GLIS_{mn} \end{pmatrix},$$

where

$$GLIS_{ij}=\left\{\left(N_{ij}^{(k)}\right):\min k\left[sc\left(N_{ij}^{(k)}\right)\right]\right\}.$$

(c). In this step, we compute the distance by applying Definition 4 of the decision matrix $N_{ij}^{(k)}$ to $GDIS$, $GRIS$ and $GLIS$. The distances are shown symbolically as $DGDIS$, $DGRIS$ and $DGLIS$, respectively, where

$$DGDIS_i^{(k)}=\left(\frac{1}{2n}\sum_{u=1}^{n}\left(\left|(\iota_{N_{ij}^k})^2-(\iota_{I_{ij}})^2\right|^{\phi}+\left|(\kappa_{N_{ij}^k})^2-(\kappa_{I_{ij}})^2\right|^{\phi}\right)\right)^{\frac{1}{\phi}},$$

$$DGRIS_i^{(k)}=\left(\frac{1}{2n}\sum_{u=1}^{n}\left(\left|(\iota_{N_{ij}^k})^2-(\iota_{R_{ij}})^2\right|^{\phi}+\left|(\kappa_{N_{ij}^k})^2-(\kappa_{R_{ij}})^2\right|^{\phi}\right)\right)^{\frac{1}{\phi}},$$

$$DGLIS_i^{(k)} = \left(\frac{1}{2n}\sum_{u=1}^{n}\left(\left|(\iota_{N_{ij}^k})^2-(\iota_{L_{ij}})^2\right|^{\phi}+\left|(\kappa_{N_{ij}^k})^2-(\kappa_{L_{ij}})^2\right|^{\phi}\right)\right)^{\frac{1}{\phi}},$$

for $i = 1, 2, ..., m$ and $k = 1, 2, ..., e$.

(d). In this step, we compute the closeness indices (CI) by following the model proposed by Yue [40].

$$CI^{(k)} = \frac{\sum_{i=1}^{m} DGRIS_i^{(k)} + \sum_{i=1}^{m} DGLIS_i^{(k)}}{\sum_{i=1}^{m} DGDIS_i^{(k)} + \sum_{i=1}^{m} DGRIS_i^{(k)} + \sum_{i=1}^{m} DGLIS_i^{(k)}}$$

for $k = 1, 2, ..., e$.

(e). In this step, we calculate the decision matrix weights, as follows:

$$w^{(k)} = \frac{CI^{(k)}}{\sum_{k=1}^{e} CI^{(k)}}.$$

Step-4 In this step, we discuss the following three parts.

(a). In this step, the weights of attributes are computed by means of the proposed credibility fuzzy entropy measure; the revised GDIS (R_vGDIS) is calculated for this as follows:

$$\begin{aligned} R_vGDIS_{ij} &= \sum_{k=1}^{e} w^{(k)} N_{ij}^{(k)} \\ &= \left\{\frac{1}{1+\sum_{k=1}^{e}\{w^{(k)}(\frac{\iota_{ij}^{(k)}}{1-\iota_{ij}^{(k)}})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\sum_{k=1}^{e}\{w^{(k)}(\frac{1-\kappa_{ij}^{(k)}}{\kappa_{ij}^{(k)}})^{\sigma}\}^{1/\sigma}}\right\}. \end{aligned}$$

(b). In this step, using Definition 7, each attribute is computed in accordance with the credibility fuzzy entropy measure, which follows as

$$EM_j = E\left(R_vDGIS_{1j}, R_vDGIS_{2j}, ..., R_vDGIS_{mj}\right), \quad j = 1, 2, ..., n.$$

(c). In this step, we compute the attribute weights, which follows as

$$AW_j = \frac{1 - EM_j}{n - \sum_{u=1}^{n} EM_j}, j = 1, 2, ..., n.$$

Step-5 This step also comprises the following four steps.

(a). In this step, the attributes weight vector is used; the calculation of the weighted normalized decision matrix is as follows:

$$DM(N)_{ij}^{(k)} = \sum_{j=1}^{n} AW_j N_{ij}^{(k)}$$

$$= \left\{\frac{1}{1+\{AW_j(\frac{\iota_{ij}}{1-\iota_{ij}})^{\sigma}\}^{1/\sigma}}, \frac{1}{1+\{AW_j(\frac{1-\kappa_{ij}}{\kappa_{ij}})^{\sigma}\}^{1/\sigma}}\right\},$$

for each $k = 1, 2, ..., e$.

(b). In this step, the weighted normalized decision-matrix $DM(N)_{ij}^{(k)}$, which computes the $PIS^{(k)}$ and $NIS^{(k)}$ for each DM_k is utilized as follows:

$$PIS_{ij} = \left\{\left(DM(N)_{ij}^{(k)}\right) : \max k\left[sc\left(DM(N)_{ij}^{(k)}\right)\right]\right\}, (j = 1, 2, ..., n),$$

$$NIS_{ij} = \left\{\left(DM(N)_{ij}^{(k)}\right) : \min k\left[sc\left(DM(N)_{ij}^{(k)}\right)\right]\right\}, (j = 1, 2, ..., n).$$

(c). In this step, we compute the weighted distances for the weighted $DM(N)^{(k)}$ from $PIS^{(k)}$ and $NIS^{(k)}$, as follows:

$$DIS_i^{+k} = \left(\frac{1}{2n}\sum_{j=1}^{n} AW_j\left(\left|(\iota_{DM(N)^k})^2 - (\iota_{PIS^k})^2\right|^{\phi} + \left|(\kappa_{DM(N)^k})^2 - (\kappa_{PIS^k})^2\right|^{\phi}\right)\right)^{\frac{1}{\phi}},$$

and

$$DIS_i^{-k} = \left(\frac{1}{2n}\sum_{j=1}^{n} AW_j\left(\left|(\iota_{DM(N)^k})^2 - (\iota_{NIS^k})^2\right|^{\phi} + \left|(\kappa_{DM(N)^k})^2 - (\kappa_{NIS^k})^2\right|^{\phi}\right)\right)^{\frac{1}{\phi}}$$

$\forall\ i = 1, 2, ..., m.$

(d). In this step, For each DM$_k$, the revised closeness indices (RCIs) of the alternatives are calculated as follows:

$$RCI_i^k = \frac{DIS_i^{-k}}{DIS_i^{+k} + DIS_i^{-k}}.$$

Step-6 In this step, the computed weights of decision-makers $w^{(k)}$ final revised closeness indices are aggregated to obtain the final revised closeness index ($FRCI_i$) corresponding to each alternative, as follows:

$$FRCI_i = \sum_{k=1}^{e} w^k.RCI_i^k.$$

The calculated FRCI value is ranked by descending order, where the finest alternative has the largest value.

6. Illustrative example based on extended TOPSIS method

Here, the steps of the proposed method were applied to the numerical example, which has been discussed above. We demonstrate the characteristics and advantages of the proposed technique by performing comparison of the offered technique and the existing technique using the FCNs.

Example 2. *This example was adopted from [41, 44]. This section provides a numerical illustration of the technique proposed in this study. Assume a company intends to adopt an enterprise resource planning (ERP) system. The first stage is to identify an operation team* DM_1, DM_2 *and*

DM_3comprised of the chief information officer and two senior members from a relevant department. After gathering all available information about ERP vendors and systems, the operation team selects seven different ERP systems Y_i $(i = 1, 2, \cdots, 7)$ as applicants. To assist with this decision-making, the company hires some external professional organizations (or experts). To analyze the alternatives, the project manager chooses six attributes, which are Function, Technology, Strategic fitness, Vendor ability, Vendor financial status and Vendor reputation, represented by c_1, c_2, c_3, c_4, c_5 and c_6, respectively. The decision-making committee is required to utilize FCNs to express the best option.

Step-1 In this step of the proposed method, we represent the data in the form of a matrix with alternatives and criteria, which is expressed as the expert's information (DM_1, DM_2, DM_3) in the form of FCNs; it is represented as follows.

Table 1. Expert information.

	c_1	c_2	c_3	c_4	c_5	c_6
DM_1						
Y_1	$(0.84, 0.34)$	$(0.43, 0.39)$	$(0.67, 0.5)$	$(0.31, 0.21)$	$(0.4, 0.78)$	$(0.3, 0.71)$
Y_2	$(0.6, 0.11)$	$(0.23, 0.35)$	$(0.72, 0.31)$	$(0.11, 0.25)$	$(0.53, 0.59)$	$(0.41, 0.82)$
Y_3	$(0.79, 0.19)$	$(0.11, 0.21)$	$(0.71, 0.41)$	$(0.34, 0.25)$	$(0.39, 0.91)$	$(0.13, 0.51)$
Y_4	$(0.63, 0.51)$	$(0.49, 0.33)$	$(0.61, 0.43)$	$(0.49, 0.37)$	$(0.13, 0.42)$	$(0.45, 0.59)$
Y_5	$(0.57, 0.36)$	$(0.5, 0.15)$	$(0.7, 0.32)$	$(0.33, 0.44)$	$(0.29, 0.6)$	$(0.4, 0.65)$
Y_6	$(0.4, 0.39)$	$(0.78, 0.91)$	$(0.3, 0.13)$	$(0.71, 0.51)$	$(0.84, 0.43)$	$(0.67, 0.31)$
Y_7	$(0.53, 0.13)$	$(0.59, 0.42)$	$(0.41, 0.45)$	$(0.82, 0.59)$	$(0.34, 0.39)$	$(0.5, 0.21)$
DM_2						
Y_1	$(0.61, 0.15)$	$(0.16, 0.35)$	$(0.61, 0.35)$	$(0.55, 0.17)$	$(0.53, 0.62)$	$(0.47, 0.74)$
Y_2	$(0.66, 0.11)$	$(0.43, 0.23)$	$(0.93, 0.08)$	$(0.02, 0.06)$	$(0.51, 0.77)$	$(0.09, 0.99)$
Y_3	$(0.88, 0.09)$	$(0.05, 0.06)$	$(0.56, 0.17)$	$(0.43, 0.13)$	$(0.07, 0.89)$	$(0.44, 0.61)$
Y_4	$(0.59, 0.32)$	$(0.24, 0.48)$	$(0.68, 0.53)$	$(0.34, 0.21)$	$(0.34, 0.51)$	$(0.39, 0.61)$
Y_5	$(0.71, 0.31)$	$(0.35, 0.41)$	$(0.73, 0.44)$	$(0.22, 0.49)$	$(0.24, 0.69)$	$(0.21, 0.74)$
Y_6	$(0.53, 0.07)$	$(0.62, 0.89)$	$(0.47, 0.44)$	$(0.74, 0.61)$	$(0.61, 0.16)$	$(0.61, 0.55)$
Y_7	$(0.51, 0.34)$	$(0.77, 0.51)$	$(0.09, 0.39)$	$(0.99, 0.61)$	$(0.15, 0.35)$	$(0.35, 0.17)$
DM_3						
Y_1	$(0.85, 0.25)$	$(0.14, 0.23)$	$(0.78, 0.38)$	$(0.29, 0.39)$	$(0.15, 0.88)$	$(0.18, 0.83)$
Y_2	$(0.94, 0.04)$	$(0.39, 0.19)$	$(0.63, 0.18)$	$(0.48, 0.49)$	$(0.07, 0.61)$	$(0.35, 0.56)$
Y_3	$(0.73, 0.13)$	$(0.19, 0.39)$	$(0.87, 0.35)$	$(0.41, 0.13)$	$(0.46, 0.88)$	$(0.18, 0.81)$
Y_4	$(0.82, 0.12)$	$(0.55, 0.21)$	$(0.53, 0.33)$	$(0.46, 0.23)$	$(0.43, 0.63)$	$(0.47, 0.51)$
Y_5	$(0.61, 0.33)$	$(0.28, 0.41)$	$(0.74, 0.34)$	$(0.37, 0.32)$	$(0.29, 0.63)$	$(0.14, 0.65)$
Y_6	$(0.15, 0.46)$	$(0.88, 0.88)$	$(0.18, 0.18)$	$(0.83, 0.81)$	$(0.85, 0.14)$	$(0.78, 0.29)$
Y_7	$(0.07, 0.43)$	$(0.61, 0.63)$	$(0.35, 0.47)$	$(0.56, 0.51)$	$(0.25, 0.23)$	$(0.38, 0.39)$

Step-2 In this step, we normalize the data.

Table 2. Normalized expert information DM_1.

NDM_1	c_1	c_2	c_3	c_4	c_5	c_6
Y_1	$(0.84, 0.34)$	$(0.39, 0.43)$	$(0.67, 0.5)$	$(0.21, 0.31)$	$(0.4, 0.78)$	$(0.71, 0.3)$
Y_2	$(0.6, 0.11)$	$(0.35, 0.23)$	$(0.72, 0.31)$	$(0.25, 0.11)$	$(0.53, 0.59)$	$(0.82, 0.41)$
Y_3	$(0.79, 0.19)$	$(0.21, 0.11)$	$(0.71, 0.41)$	$(0.25, 0.34)$	$(0.39, 0.91)$	$(0.51, 0.13)$
Y_4	$(0.63, 0.51)$	$(0.33, 0.49)$	$(0.61, 0.43)$	$(0.37, 0.49)$	$(0.13, 0.42)$	$(0.59, 0.45)$
Y_5	$(0.57, 0.36)$	$(0.15, 0.5)$	$(0.7, 0.32)$	$(0.44, 0.33)$	$(0.29, 0.6)$	$(0.65, 0.4)$
Y_6	$(0.4, 0.39)$	$(0.91, 0.78)$	$(0.3, 0.13)$	$(0.51, 0.71)$	$(0.84, 0.43)$	$(0.31, 0.67)$
Y_7	$(0.53, 0.13)$	$(0.42, 0.59)$	$(0.41, 0.45)$	$(0.59, 0.82)$	$(0.34, 0.39)$	$(0.21, 0.5)$

Table 3. Normalized expert information DM_2.

NDM_2	c_1	c_2	c_3	c_4	c_5	c_6
Y_1	$(0.61, 0.15)$	$(0.35, 0.16)$	$(0.61, 0.35)$	$(0.17, 0.55)$	$(0.53, 0.62)$	$(0.74, 0.47)$
Y_2	$(0.66, 0.11)$	$(0.23, 0.43)$	$(0.93, 0.08)$	$(0.06, 0.02)$	$(0.51, 0.77)$	$(0.99, 0.09)$
Y_3	$(0.88, 0.09)$	$(0.06, 0.05)$	$(0.56, 0.17)$	$(0.13, 0.43)$	$(0.07, 0.89)$	$(0.61, 0.44)$
Y_4	$(0.59, 0.32)$	$(0.48, 0.24)$	$(0.68, 0.53)$	$(0.21, 0.34)$	$(0.34, 0.51)$	$(0.61, 0.39)$
Y_5	$(0.71, 0.31)$	$(0.41, 0.35)$	$(0.73, 0.44)$	$(0.49, 0.22)$	$(0.24, 0.69)$	$(0.74, 0.21)$
Y_6	$(0.53, 0.07)$	$(0.89, 0.62)$	$(0.47, 0.44)$	$(0.61, 0.74)$	$(0.61, 0.16)$	$(0.55, 0.61)$
Y_7	$(0.51, 0.34)$	$(0.51, 0.77)$	$(0.09, 0.39)$	$(0.61, 0.99)$	$(0.15, 0.35)$	$(0.17, 0.35)$

Table 4. Normalized expert information DM_3.

NDM_3	c_1	c_2	c_3	c_4	c_5	c_6
Y_1	$(0.85, 0.25)$	$(0.23, 0.14)$	$(0.78, 0.38)$	$(0.39, 0.29)$	$(0.15, 0.88)$	$(0.83, 0.18)$
Y_2	$(0.94, 0.04)$	$(0.19, 0.39)$	$(0.63, 0.18)$	$(0.49, 0.48)$	$(0.07, 0.61)$	$(0.56, 0.35)$
Y_3	$(0.73, 0.13)$	$(0.39, 0.19)$	$(0.87, 0.35)$	$(0.13, 0.41)$	$(0.46, 0.88)$	$(0.81, 0.18)$
Y_4	$(0.82, 0.12)$	$(0.21, 0.55)$	$(0.53, 0.33)$	$(0.23, 0.46)$	$(0.43, 0.63)$	$(0.51, 0.47)$
Y_5	$(0.61, 0.33)$	$(0.41, 0.28)$	$(0.74, 0.34)$	$(0.32, 0.37)$	$(0.29, 0.63)$	$(0.65, 0.14)$
Y_6	$(0.15, 0.46)$	$(0.88, 0.88)$	$(0.18, 0.18)$	$(0.81, 0.83)$	$(0.85, 0.14)$	$(0.29, 0.78)$
Y_7	$(0.07, 0.43)$	$(0.63, 0.61)$	$(0.35, 0.47)$	$(0.51, 0.56)$	$(0.25, 0.23)$	$(0.39, 0.38)$

Step-3 Here, the GDIS is calculated as shown in the following table.

Table 5. Group decision ideal solution (GDIS).

	c_1	c_2	c_3	c_4	c_5	c_6
Y_1	(.7151, .7306)	(.2951, .6854)	(.6695, .5730)	(.2140, .5593)	(.2251, .1725)	(.7503, .6318)
Y_2	(.6721, .9160)	(.2321, .6264)	(.7097, .7734)	(.0977, .6502)	(.1148, .3040)	(.6797, .6619)
Y_3	(.7846, .8543)	(.0966, .8650)	(.6585, .6574)	(.1485, .6014)	(.1142, .1042)	(.5952, .6762)
Y_4	(.6491, .6009)	(.2820, .5219)	(.5927, .5454)	(.2457, .5559)	(.1962, .4493)	(.5625, .5591)
Y_5	(.6181, .6650)	(.2234, .5906)	(.7221, .6224)	(.3919, .6795)	(.2696, .3527)	(.6737, .7018)
Y_6	(.2261, .6186)	(.8923, .1725)	(.2483, .6762)	(.5952, .2195)	(.7151, .6854)	(.3369, .2836)
Y_7	(.1148, .6517)	(.4929, .3040)	(.1430, .5691)	(.5625, .0171)	(.2054, .6611)	(.2140, .5730)

Table 6. Group decision right ideal solution (GDRIS).

GDRIS	c_1	c_2	c_3	c_4	c_5	c_6
Y_1	(0.84, 0.34)	(0.39, 0.43)	(0.67, 0.5)	(0.17, 0.55)	(0.53, 0.62)	(0.83, 0.18)
Y_2	(0.94, 0.04)	(0.04, 0.23)	(0.72, 0.31)	(0.49, 0.48)	(0.51, 0.77)	(0.99, 0.09)
Y_3	(0.79, 0.19)	(0.19, 0.39)	(0.87, 0.35)	(0.25, 0.34)	(0.46, 0.88)	(0.51, 0.13)
Y_4	(0.63, 0.51)	(0.51, 0.33)	(0.68, 0.53)	(0.37, 0.49)	(0.43, 0.63)	(0.51, 0.47)
Y_5	(0.71, 0.31)	(0.31, 0.41)	(0.73, 0.44)	(0.44, 0.33)	(0.29, 0.63)	(0.65, 0.14)
Y_6	(0.4, 0.39)	(0.39, 0.88)	(0.47, 0.44)	(0.81, 0.83)	(0.84, 0.43)	(0.31, 0.67)
Y_7	(0.51, 0.34)	(0.51, 0.77)	(0.41, 0.45)	(0.61, 0.99)	(0.34, 0.39)	(0.17, 0.35)

Table 7. Group decision left ideal solution (GDLIS).

GDLIS	c_1	c_2	c_3	c_4	c_5	c_6
Y_1	(0.61, 0.15)	(0.23, 0.14)	(0.61, 0.35)	(0.21, 0.31)	(0.15, 0.88)	(0.83, 0.18)
Y_2	(0.6, 0.11)	(0.19, 0.39)	(0.63, 0.18)	(0.06, 0.02)	(0.07, 0.61)	(0.99, 0.09)
Y_3	(0.73, 0.13)	(0.06, 0.05)	(0.56, 0.17)	(0.13, 0.41)	(0.07, 0.89)	(0.51, 0.13)
Y_4	(0.82, 0.12)	(0.48, 0.24)	(0.53, 0.33)	(0.21, 0.34)	(0.13, 0.42)	(0.51, 0.47)
Y_5	(0.57, 0.36)	(0.15, 0.5)	(0.7, 0.32)	(0.49, 0.22)	(0.29, 0.6)	(0.65, 0.14)
Y_6	(0.53, 0.07)	(0.89, 0.92)	(0.18, 0.18)	(0.51, 0.71)	(0.61, 0.16)	(0.31, 0.67)
Y_7	(0.07, 0.43)	(0.42, 0.95)	(0.09, 0.39)	(0.51, 0.56)	(0.25, 0.23)	(0.17, 0.35)

Table 8. Distance results for GDIS/GDRIS/GDLIS.

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7
Distance GDIS							
DM_1	0.25771	0.34885	0.41348	0.06499	0.20148	0.29823	0.27191
DM_2	0.24810	0.42043	0.40733	0.13837	0.24996	0.28916	0.35805
DM_3	0.32907	0.35792	0.42571	0.15981	0.23807	0.38675	0.19851
Distance GDRIS							
DM_1	0.10273	0.19063	0.10345	0.08861	0.08277	0.16286	0.12753
DM_2	0.11843	0.21565	0.15857	0.10250	0.05908	0.1884	0.06667
DM_3	0.18577	0.4857	0.10007	0.13627	0.06748	0.13389	0.22532
Distance GDLIS							
DM_1	0.14808	0.14046	0.08845	0.13863	0.04613	0.13781	0.15763
DM_2	0.16383	0.16849	0.09240	0.13212	0.09630	0.10184	0.22362
DM_3	0.12666	0.26464	0.18898	0.12275	0.08413	0.22384	0.08300

Table 9. CI results.

CI_1	CI_2	CI_3
0.42152	0.42392	0.45284

Table 10. Weight of DMs.

w_1	w_2	w_3
0.32467	0.32652	0.34880

Step-4

Table 11. R_vGDIS.

R_vGDIS	c_1	c_2	c_3	c_4	c_5	c_6
Y_1	$(.717, .938)$	$(.293, .909)$	$(.670, .802)$	$(.215, .788)$	$(.223, .201)$	$(.751, .886)$
Y_2	$(.673, .994)$	$(.231, .859)$	$(.707, .959)$	$(.098, .877)$	$(.113, .402)$	$(.676, .889)$
Y_3	$(.783, .985)$	$(.097, .987)$	$(.660, .886)$	$(.148, .833)$	$(.115, .115)$	$(.596, .903)$
Y_4	$(.650, .834)$	$(.279, .734)$	$(.591, .770)$	$(.245, .780)$	$(.196, .630)$	$(.561, .784)$
Y_5	$(.617, .891)$	$(.223, .822)$	$(.722, .855)$	$(.390, .903)$	$(.270, .479)$	$(.673, .920)$
Y_6	$(.223, .849)$	$(.892, .201)$	$(.246, .903)$	$(.596, .269)$	$(.717, .909)$	$(.335, .365)$
Y_7	$(.113, .880)$	$(.493, .402)$	$(.144, .784)$	$(.561, .017)$	$(.206, .890)$	$(.215, .802)$

Table 12. Attribute weights (AWs).

AW_1	AW_2	AW_3	AW_4	AW_5	AW_6
0.1904	0.1589	0.1777	0.1604	0.1477	0.1649

Step-5

Table 13. Weighted normalized expert information DM_1.

	c_1	c_2	c_3	c_4	c_5	c_6
Y_1	$(.923, .910)$	$(.615, .892)$	$(.828, .849)$	$(.398, .932)$	$(.634, .656)$	$(.857, .933)$
Y_2	$(.774, .977)$	$(.574, .954)$	$(.859, .926)$	$(.454, .980)$	$(.745, .825)$	$(.918, .897)$
Y_3	$(.896, .957)$	$(.400, .980)$	$(.853, .890)$	$(.454, .923)$	$(.624, .401)$	$(.719, .975)$
Y_4	$(.796, .834)$	$(.552, .867)$	$(.787, .881)$	$(.594, .866)$	$(.279, .903)$	$(.779, .881)$
Y_5	$(.752, .903)$	$(.306, .862)$	$(.846, .922)$	$(.662, .926)$	$(.515, .818)$	$(.820, .900)$
Y_6	$(.604, .891)$	$(.962, .639)$	$(.504, .974)$	$(.722, .718)$	$(.931, .899)$	$(.525, .749)$
Y_7	$(.721, .972)$	$(.644, .813)$	$(.622, .873)$	$(.782, .577)$	$(.572, .913)$	$(.395, .858)$

Table 14. Weighted normalized expert information DM_2.

	c_1	c_2	c_3	c_4	c_5	c_6
Y_1	$(.781, .967)$	$(.574, .970)$	$(.787, .912)$	$(.338, .836)$	$(.745, .805)$	$(.875, .872)$
Y_2	$(.816, .977)$	$(.428, .892)$	$(.969, .987)$	$(.137, .996)$	$(.730, .669)$	$(.995, .983)$
Y_3	$(.943, .981)$	$(.138, .991)$	$(.751, .964)$	$(.271, .892)$	$(.163, .455)$	$(.793, .885)$
Y_4	$(.767, .917)$	$(.698, .952)$	$(.834, .833)$	$(.398, .923)$	$(.572, .866)$	$(.793, .903)$
Y_5	$(.848, .921)$	$(.635, .912)$	$(.865, .877)$	$(.705, .956)$	$(.451, .752)$	$(.875, .958)$
Y_6	$(.721, .985)$	$(.953, .794)$	$(.677, .877)$	$(.796, .686)$	$(.802, .972)$	$(.750, .794)$
Y_7	$(.704, .910)$	$(.723, .652)$	$(.190, .890)$	$(.796, .059)$	$(.314, .926)$	$(.335, .918)$

Table 15. Weighted normalized expert information DM_3.

	c_1	c_2	c_3	c_4	c_5	c_6
Y_1	$(.928, .940)$	$(.428, .974)$	$(.893, .901)$	$(.614, .938)$	$(.314, .480)$	$(.923, .965)$
Y_2	$(.972, .992)$	$(.370, .907)$	$(.801, .962)$	$(.705, .871)$	$(.163, .812)$	$(.758, .918)$
Y_3	$(.861, .972)$	$(.615, .964)$	$(.940, .912)$	$(.271, .899)$	$(.689, .480)$	$(.913, .965)$
Y_4	$(.912, .974)$	$(.400, .837)$	$(.727, .919)$	$(.427, .879)$	$(.662, .799)$	$(.719, .872)$
Y_5	$(.781, .914)$	$(.635, .941)$	$(.871, .916)$	$(.540, .913)$	$(.515, .799)$	$(.820, .973)$
Y_6	$(.287, .860)$	$(.948, .461)$	$(.342, .962)$	$(.914, .560)$	$(.936, .976)$	$(.501, .631)$
Y_7	$(.147, .874)$	$(.810, .800)$	$(.560, .863)$	$(.722, .830)$	$(.464, .957)$	$(.611, .909)$

Table 16. PIS results.

PIS	c_1	c_2	c_3	c_4	c_5	c_6
PIS_1	$(0.90, 0.96)$	$(0.96, 0.64)$	$(0.86, 0.93)$	$(0.66, 0.93)$	$(0.93, 0.90)$	$(0.92, 0.90)$
PIS_2	$(0.94, 0.98)$	$(0.95, 0.79)$	$(0.97, 0.98)$	$(0.71, 0.96)$	$(0.80, 0.97)$	$(0.99, 0.98)$
PIS_3	$(0.97, 0.99)$	$(0.81, 0.80)$	$(0.94, 0.91)$	$(0.71, 0.87)$	$(0.94, 0.98)$	$(0.92, 0.97)$

Table 17. NIS results.

NIS						
NIS_1	$(0.60, 0.89)$	$(0.31, 0.86)$	$(0.50, 0.97)$	$(0.40, 0.93)$	$(0.62, 0.40)$	$(0.40, 0.86)$
NIS_2	$(0.70, 0.91)$	$(0.70, 0.95)$	$(0.19, 0.90)$	$(0.80, 0.06)$	$(0.16, 0.46)$	$(0.34, 0.92)$
NIS_3	$(0.14, 0.87)$	$(0.40, 0.84)$	$(0.34, 0.96)$	$(0.27, 0.90)$	$(0.31, 0.48)$	$(0.50, 0.63)$

Table 18. Revised CI.

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7
RCI_1	0.5195	0.5493	0.4020	0.4793	0.4594	0.5621	0.4553
RCI_2	0.5861	0.6440	0.4820	0.5997	0.6168	0.5752	0.3041
RCI_3	0.5654	0.5665	0.5970	0.5610	0.5902	0.4915	0.4297

Step-6

Table 19. FRCI results.

$FRCI$	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7
	0.55356	0.58608	0.49402	0.5466	0.5556	0.5421	0.3974

Here, the value of Y_2 is large; hence, it is the best alternative.

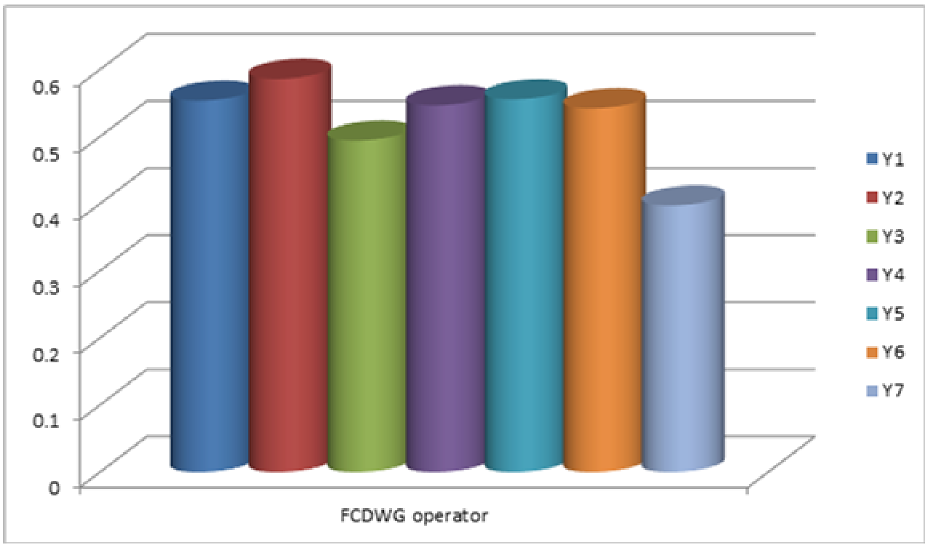

Figure 1. Graphical representation of proposed method.

7. Comparative analysis

A comparative analysis was done to demonstrate the advantage of the proposed TOPSIS-based methodology over existing methods [42, 43] in the context of solving MAGDM problems. It is clear from the following table, thus the highest value of the following result are in the same position. So, as a consequence, the proposed TOPSIS-based methodology is more accurate, feasible and effective as a tool to solve MAGDM problems with completely unknown information among decision-makers, as well as unknown criteria.

Table 20. Comparison of FRCI results.

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7
Existing Method [42]	0.4948	0.6094	0.4424	0.4708	0.4654	0.4982	0.3191
Existing Method [43]	0.1123	0.5598	0.0946	0.1615	0.3443	0.3145	0.0913
FCDWG (Proposed)	0.55356	0.58608	0.49402	0.5466	0.5556	0.5421	0.3974

Here, in this table, we present a comparison of the results of the proposed technique.

Table 21. Ranking of alternatives.

AO	Ranking	Finest Alternative
Existing Method [42]	$Y_2 > Y_6 > Y_1 > Y_4 > Y_5 > Y_3 > Y_7$	Y_2
Existing Method [43]	$Y_2 > Y_5 > Y_6 > Y_4 > Y_1 > Y_3 > Y_7$	Y_2
Proposed Method	$Y_2 > Y_5 > Y_1 > Y_4 > Y_6 > Y_3 > Y_7$	Y_2

Consequently, the proposed method is accurate, effective and more generalizable as a tool to solve MAGDM problems.

8. Results and discussion

The results and outcome of our proposed work are that we have taken the data in the form of FCNs and used the Dombi t-norm and t-conorm basic operational laws to develop a series of geometric AOs. Furthermore, we have applied these AOs to MAGDM issues to select the best option from the given data. Also, we have developed and explained a stepwise algorithm in which some steps have been defined and also quantified through a numerical example. The advantages of our proposed method are summarized as follows:

(1) Our proposed method is scalable to meet the requirements of a variety of situations by adjusting its own parameters.

(2) The FCNs of our proposed method can reliably depict more general decision-making problems.

(3) With the aid of the TOPSIS, our proposed method applies the satisfaction level of the alternative to the ideal solutions to make the decision.

9. Conclusions

This paper introduced a new FCN notion, which is expressed by a couple of fuzzy values related to both the fuzzy argument and the degree of credibility in the real state of uncertainty and fuzziness, and it is based on a new extension of the fuzzy notion. Normally, there are many types of decision-making problems, but we have focused on only two types, i.e., MADM and MAGDM during any decision problems. But, we have taken the MAGDM problems and obtained the best results on the basis of our proposed work, which is also defined on the basis of the Dombi t-norm and t-conorm; as a result, we have obtained a series of AOs, i.e., FCDWG, FCDOWG and FCDHWG operators. Furthermore, we applied the TOPSIS procedure to MCGDM. Finally, a comparison with other methods was done to check the accuracy of our new work. And, a numerical example was solved and verified by using a known (existing) method; the results confirmed that the best results was achieved by our proposed work. Also, the main benefit of using the FCNs is that they provide a credible and accurate degree for the decision-making problems. Regarding future work, this study may be applied to many practical applications like the Einstein t-norm and t-conorm and Yager t-norm and t-conorm to select the best possible outcome using the FCNs.

Acknowledgments

This work was supported by the Higher Education Commission (HEC) of Pakistan under a National Research Program for Universities (NRPU) grant, Project no. 10701. Therefore, the authors are thankful to the NRPU and HEC for financial support.

Conflict of interest

All authors declare no conflicts of interest regarding this study.

References

1. L. A. Zadeh, Fuzzy sets, *Inf. Control*, **8** (1965), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X
2. K. T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Set. Syst.*, **20** (1986), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3
3. K. T. Atanassov, Intuitionistic fuzzy logics as tools for evaluation of data mining processes, *Knowl.-Based Syst.*, **80** (2015), 122–130. https://doi.org/10.1016/j.knosys.2015.01.015
4. R. R. Yager, Pythagorean membership grades in multicriteria decision making, *IEEE T. Fuzzy Syst.*, **22** (2013), 958–965. http://doi.org/10.1109/TFUZZ.2013.2278989
5. R. R. Yager, A. M. Abbasov, Pythagorean membership grades, complex numbers, and decision making, *Int. J. Intel. Syst.*, **28** (2013), 436–452. https://doi.org/10.1002/int.21584
6. B. D. Liu, Y. K. Liu, Expected value of fuzzy variable and fuzzy expected value models, *IEEE T. Fuzzy Syst.*, **10** (2002), 445–450. https://doi.org/10.1109/TFUZZ.2002.800692

7. B. D. Liu, A survey of credibility theory, *Fuzzy Optim. Decis. Making*, **5** (2006), 387–408. https://doi.org/10.1007/s10700-006-0016-x
8. X. Li, B. D. Liu, A sufficient and necessary condition for credibility measures, *Int. J. Uncertain. Fuzz.*, **14** (2006), 527–535. https://doi.org/10.1142/S0218488506004175
9. C. J. Rao, J. Peng, Fuzzy group decision making model based on credibility theory and gray relative degree, *Int. J. Inf. Tech. Decis.*, **8** (2009), 515–527. https://doi.org/10.1142/S0219622009003533
10. C. L. Hwang, J. M. Lin, *Group decision making under multiple criteria: Methods and applications*, Berlin, Heidelberg: Springer, 2012. https://doi.org/10.1007/978-3-642-61580-1
11. Z. H. Xu, *Uncertain multi-attribute decision making: Methods and applications*, Berlin, Heidelberg: Springer, 2015. https://doi.org/10.1007/978-3-662-45640-8
12. J. Q. Wang, J. Ren, Approach to group decision-making with different forms of preference information, *J. Syst. Eng. Electron.*, **12** (2005), 2057–2059.
13. Q. Song, A. Kandel, M. Schneider, Parameterized fuzzy operators in fuzzy decision making, *Int. J. Intell. Syst.*, **18** (2003), 971–987. https://doi.org/10.1002/int.10124
14. J. M. Merigó, A. M. Gil-Lafuente, Fuzzy induced generalized aggregation operators and its application in multi-person decision making, *Expert. Syst. Appl.*, **38** (2011), 9761–9772. https://doi.org/10.1016/j.eswa.2011.02.023
15. F. Zhou, T. Y. Chen, A novel distance measure for pythagorean fuzzy sets and its applications to the technique for order preference by similarity to ideal solutions, *Int. J. Comput. Int. Sys.*, **12** (2019), 955–969. https://doi.org/10.2991/ijcis.d.190820.001
16. V. Mohagheghi, S. M. Mousavi, B. Vahdani, Enhancing decision-making flexibility by introducing a new last aggregation evaluating approach based on multi-criteria group decision making and Pythagorean fuzzy sets, *Appl. Soft Comput.*, **61** (2017), 527–535. https://doi.org/10.1016/j.asoc.2017.08.003
17. G. Beliakov, A. Pradera, A. T. Calvo, *Aggregation functions: A guide for practitioners*, Berlin, Heidelberg: Springer, 2007. https://doi.org/10.1007/978-3-540-73721-6
18. P. A. Ejegwa, Pythagorean fuzzy set and its application in career placements based on academic performance using max–min–max composition, *Complex Intell. Syst.*, **5** (2019), 165–175. https://doi.org/10.1007/s40747-019-0091-6
19. C. Q. Tan, W. T. Yi, X. H. Chen, Hesitant fuzzy Hamacher aggregation operators for multicriteria decision making, *Appl. Soft Comput.*, **26** (2015), 325–349. https://doi.org/10.1016/j.asoc.2014.10.007
20. T. Senapati, R. R. Yager, Some new operations over Fermatean fuzzy numbers and application of Fermatean fuzzy WPM in multiple criteria decision making, *Informatica*, **30** (2019), 391–412. https://doi.org/10.15388/Informatica.2019.211
21. J. Dombi, A general class of fuzzy operators, the DeMorgan class of fuzzy operators and fuzziness measures induced by fuzzy operators, *Fuzzy Set. Syst.*, **8** (1982), 149–163. https://doi.org/10.1016/0165-0114(82)90005-7

22. P. D. Liu, J. L. Liu, S. M. Chen, Some intuitionistic fuzzy Dombi Bonferroni mean operators and their application to multi-attribute group decision making, *JORS*, **69** (2017), 1–24. https://doi.org/10.1057/s41274-017-0190-y

23. J. Q. Chen, J. Ye, Some single-valued neutrosophic Dombi weighted aggregation operators for multiple attribute decision-making, *Symmetry*, **9** (2017), 82. https://doi.org/10.3390/sym9060082

24. Z. S. Xu, Q. L. Da, The ordered weighted geometric averaging operators, *Int. J. Intell. Syst.*, **17** (2002), 709–716. https://doi.org/10.1002/int.10045

25. H. Garg, Some picture fuzzy aggregation operators and their applications to multicriteria decision-making, *Arab. J. Sci. Eng.*, **42** (2017), 5275–5290. https://doi.org/10.1007/s13369-017-2625-9

26. G. W. Wei, Extension of TOPSIS method for 2-tuple linguistic multiple attribute group decision making with incomplete weight information, *Knowl. Inf. Syst.*, **25** (2010), 623–634. https://doi.org/10.1007/s10115-009-0258-3

27. Z. S. Xu, R. R. Yager, Some geometric aggregation operators based on intuitionistic fuzzy sets, *Int. J. Gen. Syst.*, **35** (2006), 417–433. https://doi.org/10.1080/03081070600574353

28. M. R. Seikh, U. Mandal, Intuitionistic fuzzy Dombi aggregation operators and their application to multiple attribute decision-making, *Granul. Comput.*, **6** (2021), 473—488. https://doi.org/10.1007/s41066-019-00209-y

29. J. Ye, J. M. Song, S. G. Du, R. Yong, Weighted aggregation operators of fuzzy credibility numbers and their decision-making approach for slope design schemes, *Comput. Appl. Math.*, **40** (2021), 155. https://doi.org/10.1007/s40314-021-01539-x

30. J. Ye, S. G. Du, R. Yong, F. W. Zhang, Weighted aggregation operators of fuzzy credibility cubic numbers and their decision making strategy for slope design schemes, *Curr. Chin. Comput. Sci.*, **1** (2020), 28–34. https://doi.org/10.2174/2665997201999200717165743

31. C. L. Hwang, K. Yoon, *Methods for multiple attribute decision making*, Berlin, Heidelberg: Springer, 1981.

32. C. T. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment, *Fuzzy Set. Syst.*, **114** (2000), 1–9. https://doi.org/10.1016/S0165-0114(97)00377-1

33. X. R. He, Typhoon disaster assessment based on Dombi hesitant fuzzy information aggregation operators, *Nat. Hazards*, **90** (2018), 1153–1175. https://doi.org/10.1007/s11069-017-3091-0

34. L. L. Shi, J. Ye, Dombi aggregation operators of neutrosophic cubic sets for multiple attribute decision-making, *Algorithms*, **11** (2018), 29. https://doi.org/10.3390/a11030029

35. Z. S. Chen, X. Zhang, W. Pedrycz, X. J. Wang, K. S. Chin, L. Martínez, *K*-means clustering for the aggregation of HFLTS possibility distributions: *N*-two-stage algorithmic paradigm, *Knowl.-Based Syst.*, **227** (2021), 107230. https://doi.org/10.1016/j.knosys.2021.107230

36. Z. S. Chen, Y. Yang, X. J. Wang, K. S. Chin, K. L. Tsui, Fostering linguistic decision-making under uncertainty: A proportional interval type-2 hesitant fuzzy TOPSIS approach based on Hamacher aggregation operators and andness optimization models, *Inf. Sci.*, **500** (2019), 229–258. https://doi.org/10.1016/j.ins.2019.05.074

37. G. Deschrijver, C. Cornelis, E. E. Kerre, On the representation of intuitionistic fuzzy t-norms and t-conorms, *IEEE T. Fuzzy Syst.*, **12** (2004), 45–61. https://doi.org/10.1109/TFUZZ.2003.822678

38. S. Roychowdhury, B. H. Wang, On generalized Hamacher families of triangular operators, *Int. J. Approx. Reason.*, **19** (1998), 419–439. https://doi.org/10.1016/S0888-613X(98)10018-X

39. K. H. Guo, Q. Song, On the entropy for Atanassov's intuitionistic fuzzy sets: An interpretation from the perspective of amount of knowledge, *Appl. Soft Comput.*, **24** (2014), 328–340. https://doi.org/10.1016/j.asoc.2014.07.006

40. Z. L. Yue, An extended TOPSIS for determining weights of decision makers with interval numbers, *Knowl.-Based Syst.*, **24** (2011), 146–153. https://doi.org/10.1016/j.knosys.2010.07.014

41. G. W. Wei, Picture fuzzy aggregation operators and their application to multiple attribute decision making, *J. Intell. Fuzzy Syst.*, **33** (2017), 713–724. https://doi.org/10.3233/JIFS-161798

42. X. L. Zhang, Z. S. Xu, Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets, *Int. J. Intell. Syst.*, **29** (2014), 1061–1078. https://doi.org/10.1002/int.21676

43. X. L. Zhang, A novel approach based on similarity measure for Pythagorean fuzzy multiple criteria group decision making, *Int. J. Intell. Syst.*, **31** (2016), 593–611. https://doi.org/10.1002/int.21796

44. X. W. Liao, Y. Li, B. Lu, A model for selecting an ERP system based on linguistic information processing, *Inf. Syst.*, **32** (2007), 1005–1017. https://doi.org/10.1016/j.is.2006.10.005