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Research article

Existence of fixed point results in neutrosophic metric-like spaces

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Abstract: In this article, we introduced the concept of neutrosophic metric-like spaces and obtained some fixed point results in the sense of neutrosophic metric-like spaces. Our results are improvements of recent results in the existing literature. For the validity of these results some non-trivial examples are imparted.

Keywords: metric-like space; fuzzy metric space; neutrosophic metric-like space; unique fixed point
Mathematics Subject Classification: 47H10, 54H25

1. Introduction

The notion of fuzzy sets was given by Zadeh [9], and this gave a new direction to this research field. A large number of researchers doing work in this direction due to its wide range of applications in science. In this connectedness, Kramosil and Michalek [10] initiated the concept of fuzzy metric spaces by generalizing the notion of probabilistic metric spaces to fuzzy metric spaces. George and Veeramani [11] derived a Hausdorff topology initiated by fuzzy metric to modify the notion of fuzzy metric spaces. Fixed point theory enriched with many generalizations and playing an important role to find the existence of solution. Garbiec [12] displayed the fuzzy version of Banach contraction principle in fuzzy metric spaces. In [13,14], a great job has been done by authors in extending contraction results. In recent times, Harandi [7] originated the notion of metric-like spaces, which generalized the concept of metric spaces in beautiful manners. In this connectedness, Shukla and Abbas [8] generalized the notion of metric-like spaces and introduced fuzzy metric-like spaces. The approach of intuitionistic fuzzy metric spaces was tossed by Park in [2]. Kirişci and Simsek [1] generalized the approach of intuitionistic fuzzy metric spaces and tossed the approach of neutrosophic metric spaces. Simsek, and Kirişci [5] and Sowndrarajan et al. [6] proved some fixed point results in the setting of neutrosophic metric spaces. In [3,4,15–17] proved several fixed point results for contractive mappings.

In this article, we introduced the notion of neutrosophic metric-like spaces and established some fixed point results with non-trivial examples.

2. Preliminaries

First, we give some basic definitions that are helpful for readers to understand main section.

Definition 2.1. [8] A 3-tuple $(\beta, \psi, *)$ is said to be a fuzzy metric-like space if $\beta \neq \Phi$ is a random set, $*$ is a continuous t-norm and ψ is a fuzzy set on $\beta \times \beta \times (0, \infty)$ meet the points below for all $\pi, \lambda, \mu \in \beta, t, s > 0$:

(FL1) $\psi(\pi, \lambda, t) > 0$;
(FL2) If $\psi(\pi, \lambda, t) = 1$, then $\pi = \lambda$;
(FL3) $\psi(\pi, \lambda, t) = \psi(\lambda, \pi, t)$;
(FL4) $\psi(\pi, \mu, t + s) \geq \psi(\pi, \lambda, t) * \psi(\lambda, \mu, s)$;
(FL5) $\psi(\pi, \lambda, \cdot): (0, +\infty) \to [0,1]$ is continuous.

Example 2.2. [8] Let $\beta = \mathbb{R}^+$, $k \in \mathbb{R}^+$ and $m > 0$. Define continuous t-norm by $g * h = gh$ and the fuzzy set ψ on $\beta \times \beta \times (0, +\infty)$ by

$$\psi(\pi, \lambda, t) = \frac{kt}{kt + m(\max\{\pi, \lambda\})}, for\ all\ \pi, \lambda \in \beta, t > 0.$$

Then $(\beta, \psi, *)$ is a fuzzy metric-like space.

Definition 2.3. [1] Suppose $\beta \neq \emptyset$, assume a six tuple $(\beta, \psi, \varphi, þ, *, \circ)$ where $*$ is a continuous t-norm, $\circ$ is a continuous t-conorm, ψ, φ and þ neutrosophic sets on $\beta \times \beta \times (0, +\infty)$. If $(\beta, \psi, \varphi, þ, *, \circ)$ meet the below circumstances for all $\pi, \lambda, \mu \in \beta$ and $t, s > 0$:

(NS1) $\psi(\pi, \lambda, t) + \varphi(\pi, \lambda, t) + þ(\pi, \lambda, t) \leq 3$,
(NS2) $0 \leq \psi(\pi, \lambda, t) \leq 1$,
(NS3) $\psi(\pi, \lambda, t) = 1 \iff \pi = \lambda$,
(NS4) $\psi(\pi, \lambda, t) = \psi(\lambda, \pi, t)$,
(NS5) $\psi\big(\pi, \mu, (t + s)\big) \geq \psi(\pi, \lambda, t) * \psi(\lambda, \mu, s)$,
(NS6) $\psi(\pi, \lambda, \cdot): [0, +\infty) \to [0,1]$ is a continuous,
(NS7) $\lim_{t \to +\infty} \psi(\pi, \lambda, t) = 1$,
(NS8) $0 \leq \varphi(\pi, \lambda, t) \leq 1$,
(NS9) $\varphi(\pi, \lambda, t) = 0 \iff \pi = \lambda$,
(NS10) $\varphi(\pi, \lambda, t) = \varphi(\lambda, \pi, t)$,
(NS11) $\varphi\big(\pi, \mu, (t + s)\big) \leq \varphi(\pi, \lambda, t) \circ \varphi(\lambda, \mu, s)$,
(NS12) $\varphi(\pi, \lambda, \cdot): [0, +\infty) \to [0,1]$ is a continuous,
(NS13) $\lim_{t \to +\infty} \varphi(\pi, \lambda, t) = 0$,

(NS14) $0 \leq \text{þ}(\pi, \lambda, t) \leq 1$,
(NS15) $\text{þ}(\pi, \lambda, t) = 0 \iff \pi = \lambda$,
(NS16) $\text{þ}(\pi, \lambda, t) = \text{þ}(\lambda, \pi, t)$,
(NS17) $\text{þ}(\pi, \mu, (t+s)) \leq \text{þ}(\pi, \lambda, t) \circ \text{þ}(\lambda, \mu, s)$,
(NS18) $\text{þ}(\pi, \lambda, \cdot): [0, +\infty) \to [0,1]$ is a continuous,
(NS19) $\lim_{t \to +\infty} \text{þ}(\pi, \lambda, t) = 0$,
(NS20) If $t \leq 0$ then $\psi(\pi, \lambda, t) = 0, \varphi(\pi, \lambda, t) = 1, \text{þ}(\pi, \lambda, t) = 1$.

Then, $(\beta, \psi, \varphi, \text{þ})$ is a neutrosophic metric on β and $(\beta, \psi, \varphi, \text{þ}, *, \circ)$ be a neutrosophic metric space.

3. Main results

In this section, we introduce the concept of neutrosophic metric-like spaces and prove some fixed point results.

Definition 3.1. Suppose $\beta \neq \emptyset$, assume a six tuple $(\beta, \psi, \varphi, \text{þ}, *, \circ)$ where $*$ is a continuous t-norm, $\circ$ is a continuous t-conorm, ψ, φ and þ neutrosophic sets on $\beta \times \beta \times (0, +\infty)$. If $(\beta, \psi, \varphi, \text{þ}, *, \circ)$ meets the below circumstances for all $\pi, \lambda, \mu \in \beta$ and $t, s > 0$:

(NL1) $\psi(\pi, \lambda, t) + \varphi(\pi, \lambda, t) + \text{þ}(\pi, \lambda, t) \leq 3$,
(NL2) $0 \leq \psi(\pi, \lambda, t) \leq 1$,
(NL3) $\psi(\pi, \lambda, t) = 1$ implies $\pi = \lambda$,
(NL4) $\psi(\pi, \lambda, t) = \psi(\lambda, \pi, t)$,
(NL5) $\psi(\pi, \mu, t+s) \geq \psi(\pi, \lambda, t) * \psi(\lambda, \mu, s)$,
(NL6) $\psi(\pi, \lambda, \cdot): [0, +\infty) \to [0,1]$ is a continuous,
(NL7) $\lim_{t \to +\infty} \psi(\pi, \lambda, t) = 1$,
(NL8) $0 \leq \varphi(\pi, \lambda, t) \leq 1$,
(NL9) $\varphi(\pi, \lambda, t) = 0$ implies $\pi = \lambda$,
(NL10) $\varphi(\pi, \lambda, t) = \varphi(\lambda, \pi, t)$,
(NL11) $\varphi(\pi, \mu, (t+s)) \leq \varphi(\pi, \lambda, t) \circ \varphi(\lambda, \mu, s)$,
(NL12) $\varphi(\pi, \lambda, \cdot): [0, +\infty) \to [0,1]$ is a continuous,
(NL13) $\lim_{t \to +\infty} \varphi(\pi, \lambda, t) = 0$,
(NL14) $0 \leq \text{þ}(\pi, \lambda, t) \leq 1$,
(NL15) $\text{þ}(\pi, \lambda, t) = 0$ implies $\pi = \lambda$,
(NL16) $\text{þ}(\pi, \lambda, t) = \text{þ}(\lambda, \pi, t)$,
(NL17) $\text{þ}(\pi, \mu, (t+s)) \leq \text{þ}(\pi, \lambda, t) \circ \text{þ}(\lambda, \mu, s)$,
(NL18) $\text{þ}(\pi, \lambda, \cdot): [0, +\infty) \to [0,1]$ is a continuous,
(NL19) $\lim_{t \to +\infty} \text{þ}(\pi, \lambda, t) = 0$,
(NL20) If $t \leq 0$ then $\psi(\pi, \lambda, t) = 0, \varphi(\pi, \lambda, t) = 1, \text{þ}(\pi, \lambda, t) = 1$.

Then, $(\beta, \psi, \varphi, \text{þ})$ be a neutrosophic metric-like on β and $(\beta, \psi, \varphi, \text{þ}, *, \circ)$ be a neutrosophic metric-like space.

Remark 3.2. In the above definition, a set β is endowed a neutrosophic metric-like space with a continuous t-norm $(*)$ and continuous t-conorm $(\circ)$. A neutrosophic metric space does not satisfy the (NL3), (NL9) and (NL15) conditions of neutrosophic metric-like space, that is, the self-distance may not be equal to 1 and 0, i.e., $\psi(\pi, \pi, t) \neq 1, \varphi(\pi, \pi, t) \neq 0$ and $\text{þ}(\pi, \pi, t) \neq 0$ for all $t > 0$, for some or may be for all $\pi \in \beta$. But, all other conditions are the same.

Proposition 3.3. Let (β,σ) be any metric-like space. Then $(\beta,\psi,\varphi,þ,*,\circ)$ is a neutrosophic metric-like space, where '*' is defined $g*h=gh$ and '∘' is defined by $g\circ h=\max\{g,h\}$ and NSs $\psi,\varphi$and þ are given by

$$\psi(\pi,\lambda,t)=\frac{kt^n}{kt^n+m\sigma(\pi,\lambda)}\ for\ all\ \pi,\lambda\in\beta,t>0,$$

$$\varphi(\pi,\lambda,t)=\frac{m\sigma(\pi,\lambda)}{kt^n+m\sigma(\pi,\lambda)}\ for\ all\ \pi,\lambda\in\beta,t>0,$$

$$þ(\pi,\lambda,t)=\frac{m\sigma(\pi,\lambda)}{kt^n}\ for\ all\ \pi,\lambda\in\beta,t>0.$$

Where, $k\in\mathbb{R}^+,m>0$ and $n\geq 1$.

Remark 3.4. Note that the above proposition also holds for continuous t-norm $g*h=\min\{g,h\}$ and continuous t-conorm $g\circ h=\max\{g,h\}$.

Remark 3.5. The proposition (3.3) shows that every metric-like space induces a neutrosophic metric-like space. For $k=n=m=1$ the induced neutrosophic metric-like space $(\beta,\psi,\varphi,þ,*,\circ)$ is called the standard neutrosophic metric-like space, where $k\in\mathbb{R}^+$

$$\psi(\pi,\lambda,t)=\frac{t}{t+\sigma(\pi,\lambda)}\ for\ all\ \pi,\lambda\in\beta,t>0,$$

$$\varphi(\pi,\lambda,t)=\frac{\sigma(\pi,\lambda)}{t+\sigma(\pi,\lambda)}\ for\ all\ \pi,\lambda\in\beta,t>0,$$

$$þ(\pi,\lambda,t)=\frac{\sigma(\pi,\lambda)}{t}\ for\ all\ \pi,\lambda\in\beta,t>0.$$

Example 3.6. Let $\beta=\mathbb{R}^+$, $k\in\mathbb{R}^+$and $m>0$. Define * by $g*h=gh$ and ∘ by $g\circ h=\max\{g,h\}$ and neutrosophic sets ψ,φ and þ in $\beta\times\beta\times(0,+\infty)$ by

$$\psi(\pi,\lambda,t)=\frac{kt}{kt+m(\max\{\pi,\lambda\})}\ for\ all\ \pi,\lambda\in\beta,t>0,$$

$$\varphi(\pi,\lambda,t)=\frac{m(\max\{\pi,\lambda\})}{kt+m(\max\{\pi,\lambda\})}\ for\ all\ \pi,\lambda\in\beta,t>0,$$

$$þ(\pi,\lambda,t)=\frac{m(\max\{\pi,\lambda\})}{kt}\ for\ all\ \pi,\lambda\in\beta,t>0.$$

Then, since $\sigma(\pi,\lambda)=\max\{\pi,\lambda\}\ for\ all\ \pi,\lambda\in\beta$ is a metric-like space on β. Therefore, by proposition (3.2) $(\beta,\psi,\varphi,þ,*,\circ)$ is a neutrosophic metric-like space, but it is not a neutrosophic metric space.

As,

$$\psi(\pi,\pi,t)=\frac{kt}{kt+m\pi}\neq 1\ for\ all\ \pi,\lambda\in\beta,t>0,$$

$$\varphi(\pi,\pi,t)=\frac{m\pi}{kt+m\pi}\neq 0\ for\ all\ \pi,\lambda\in\beta,t>0,$$

$$þ(\pi,\pi,t)=\frac{m\pi}{kt}\neq 0\ for\ all\ \pi,\lambda\in\beta,t>0.$$

Definition 3.7. A sequence $\{\pi_n\}$ is neutrosophic metric-like space $(\beta,\psi,\varphi,þ,*,\circ)$ is said to be convergent to $\pi\in\beta$ if

$$\lim_{n\to+\infty}\psi(\pi_n,\pi,t)=\psi(\pi,\pi,t)\ for\ all\ t>0,$$

$$\lim_{n\to+\infty}\varphi(\pi_n,\pi,t)=\varphi(\pi,\pi,t)\ for\ all\ t>0,$$

and

$$\lim_{n\to+\infty}þ(\pi_n,\pi,t)=þ(\pi,\pi,t)\ for\ all\ t>0.$$

Definition 3.8. A sequence $\{\pi_n\}$ in a neutrosophic metric-like space $(\beta,\psi,\varphi,þ,*,\circ)$ is said to be Cauchy sequence if

$$\lim_{n\to+\infty}\psi\left(\pi_n,\pi_{n+p},t\right),$$

$$\lim_{n\to+\infty}\varphi\left(\pi_n,\pi_{n+p},t\right),$$

and

$$\lim_{n\to+\infty}þ\left(\pi_n,\pi_{n+p},t\right)$$

for all $t\geq 0,p\geq 1$ exist and is finite.

Definition 3.9. A neutrosophic metric-like space $(\beta,\psi,\varphi,þ,*,\circ)$ is said to be complete if every Cauchy sequence $\{\pi_n\}$ in β converge to some $\pi\in\beta$ such that

$$\lim_{n\to+\infty}\psi(\pi_n,\pi,t)=\psi(\pi,\pi,t)=\lim_{n\to+\infty}\psi(\pi_n,\pi_{n+p},t)\text{ for all } t\geq 0,p\geq 1,$$

$$\lim_{n\to+\infty}\varphi(\pi_n,\pi,t)=\varphi(\pi,\pi,t)=\lim_{n\to+\infty}\varphi\left(\pi_n,\pi_{n+p},t\right)\text{ for all } t\geq 0,p\geq 1,$$

and

$$\lim_{n\to+\infty}þ(\pi_n,\pi,t)=þ(\pi,\pi,t)=\lim_{n\to+\infty}þ(\pi_n,\pi_{n+p},t)\text{ for all } t\geq 0,p\geq 1.$$

Remark 3.10. In neutrosophic metric-like space, the limit of a convergent sequence may not be unique for instance, for a neutrosophic metric-like space $(\beta,\psi,\varphi,þ,*,\circ)$ given in proposition (3.3) with $\sigma(\pi,\lambda)=\max\{\pi,\lambda\}$ and $n=k=m=1$. Define a sequence $\{\pi_n\}$ in β by $\pi_n=1-$

$\frac{1}{n}, for\ all\ n \in \mathbb{N}$. If $\pi \geq 1$ then

$$\lim_{n\to+\infty} \psi(\pi_n, \pi, t) = \lim_{n\to+\infty} \frac{t}{t + \max\{\pi_n, \pi\}} = \frac{t}{t + \max\{\pi, \pi\}} = \psi(\pi, \pi, t)\ for\ all\ t > 0,$$

$$\lim_{n\to+\infty} \varphi(\pi_n, \pi, t) = \lim_{n\to+\infty} \frac{\max\{\pi_n, \pi\}}{t + \max\{\pi_n, \pi\}} = \frac{\max\{\pi, \pi\}}{t + \max\{\pi, \pi\}} = \varphi(\pi, \pi, t)\ for\ all\ t > 0,$$

$$\lim_{n\to+\infty} þ(\pi_n, \pi, t) = \lim_{n\to+\infty} \frac{\max\{\pi_n, \pi\}}{t} = \frac{\max\{\pi, \pi\}}{t} = þ(\pi, \pi, t)\ for\ all\ t > 0.$$

Therefore, the sequence $\{\pi_n\}$ converge to all $\pi \in \beta$ with $\pi \geq 1$.

Remark 3.11. In an neutrosophic metric-like space, a convergent sequence may not be Cauchy. Assume a neutrosophic metric-like space $(\beta, \psi, \varphi, þ, *, \circ)$ given in above Remark 3.10. Define a sequence $\{\pi_n\}$ in β by $\pi_n = 1 + (-1)^n, for\ all\ n \in \mathbb{N}$. If $\pi \geq 2$, then

$$\lim_{n\to+\infty} \psi(\pi_n, \pi, t) = \lim_{n\to+\infty} \frac{t}{t + \max\{\pi_n, \pi\}} = \frac{t}{t + \max\{\pi, \pi\}} = \psi(\pi, \pi, t)\ for\ allt > 0,$$

$$\lim_{n\to+\infty} \varphi(\pi_n, \pi, t) = \lim_{n\to+\infty} \frac{\max\{\pi_n, \pi\}}{t + \max\{\pi_n, \pi\}} = \frac{\max\{\pi, \pi\}}{t + \max\{\pi, \pi\}} = \varphi(\pi, \pi, t)\ for\ allt > 0,$$

$$\lim_{n\to+\infty} þ(\pi_n, \pi, t) = \lim_{n\to+\infty} \frac{\max\{\pi_n, \pi\}}{t} = \frac{\max\{\pi, \pi\}}{t} = þ(\pi, \pi, t)\ for\ allt > 0.$$

Therefore, a sequence $\{\pi_n\}$ converge to all $\pi \in \beta$ with $\pi \geq 2$, but it is not a Cauchy sequence as $\lim_{n\to+\infty} \psi(\pi_n, \pi_{n+p}, t)$, $\lim_{n\to+\infty} \varphi(\pi_n, \pi_{n+p}, t)$ and $\lim_{n\to+\infty} þ(\pi_n, \pi_{n+p}, t)$ does not exist.

Theorem 3.12. Let $(\beta, \psi, \varphi, þ, *, \circ)$ be a complete neutrosophic metric-like space such that

$$\lim_{t\to+\infty} \psi(\pi, \lambda, t) = 1, \lim_{t\to+\infty} \varphi(\pi, \lambda, t) = 0 \text{ and } \lim_{t\to+\infty} \varphi(\pi, \lambda, t) = 0$$

for all $\pi, \lambda \in \beta, t > 0$ and $\mathcal{F}: \beta \to \beta$ be a mapping satisfying the conditions

$$\psi(\mathcal{F}\pi, \mathcal{F}\lambda, \alpha t) \geq \psi(\pi, \lambda, t),\ \varphi(\mathcal{F}\pi, \mathcal{F}\lambda, \alpha t) \leq \varphi(\pi, \lambda, t) \text{ and } þ(\mathcal{F}\pi, \mathcal{F}\lambda, \alpha t) \leq þ(\pi, \lambda, t), \quad (1)$$

for all $\pi, \lambda \in \beta, t > 0$, where $\alpha \in (0, 1)$. Then $\mathcal{F}$ has a unique fixed point $w \in \beta$ and

$$\psi(w, w, t) = 1, \varphi(w, w, t) = 0 \text{ and } þ(w, w, t) = 0\ for\ all\ t > 0.$$

Proof. Let $(\beta, \psi, \varphi, þ, *, \circ)$ be a complete neutrosophic metric-like space. For an arbitrary $\pi_0 \in \beta$, define a sequence $\{\pi_n\}$ in β by

$$\pi_1 = \mathcal{F}\pi_0,\ \pi_2 = \mathcal{F}^2\pi_0 = \mathcal{F}\pi_1, \ldots, \pi_n = \mathcal{F}^n\pi_0 = \mathcal{F}\pi_{n-1} \text{ for all } n \in \mathbb{N}.$$

If $\pi_n = \pi_{n-1}$ for some $n \in \mathbb{N}$ then π_n is a fixed point of $\mathcal{F}$. We assume that $\pi_n \neq \pi_{n-1}$ for all $n \in \mathbb{N}$. For $t > 0$ and $n \in \mathbb{N}$, we get from (1) that

$$\psi(\pi_n, \pi_{n+1}, t) \geq \psi(\pi_{n+1}, \pi_n, \alpha t) = \psi(\mathcal{F}\pi_n, \mathcal{F}\pi_{n-1}, \alpha t) \geq \psi(\pi_n, \pi_{n-1}, t),$$

$$\varphi(\pi_n, \pi_{n+1}, t) \leq \varphi(\pi_{n+1}, \pi_n, \alpha t) = \varphi(\mathcal{F}\pi_n, \mathcal{F}\pi_{n-1}, \alpha t) \leq \varphi(\pi_n, \pi_{n-1}, t)$$

and

$$\text{þ}(\pi_n, \pi_{n+1}, t) \leq \text{þ}(\pi_{n+1}, \pi_n, \alpha t) = \text{þ}(\mathcal{F}\pi_n, \mathcal{F}\pi_{n-1}, \alpha t) \leq \text{þ}(\pi_n, \pi_{n-1}, t),$$

for all $n \in \mathbb{N}$ and $t > 0$. Therefore, by applying the above expression, we can deduce that

$$\psi(\pi_{n+1}, \pi_n, t) \geq \psi(\pi_{n+1}, \pi_n, \alpha t) = \psi(\mathcal{F}\pi_n, \mathcal{F}\pi_{n-1}, \alpha t) \geq \psi(\pi_n, \pi_{n-1}, t)$$
$$= \psi(\mathcal{F}\pi_{n-1}, \mathcal{F}\pi_{n-2}, t) \geq \psi\left(\pi_{n-1}, \pi_{n-2}, \frac{t}{\alpha}\right) \geq \cdots \geq \psi\left(\pi_1, \pi_0, \frac{t}{\alpha^n}\right), \quad (2)$$

$$\varphi(\pi_{n+1}, \pi_n, t) \leq \varphi(\pi_{n+1}, \pi_n, \alpha t) = \varphi(\mathcal{F}\pi_n, \mathcal{F}\pi_{n-1}, \alpha t) \leq \varphi(\pi_n, \pi_{n-1}, t)$$
$$= \varphi(\mathcal{F}\pi_{n-1}, \mathcal{F}\pi_{n-2}, t) \leq \varphi\left(\pi_{n-1}, \pi_{n-2}, \frac{t}{\alpha}\right) \leq \cdots \leq \varphi\left(\pi_1, \pi_0, \frac{t}{\alpha^n}\right) \quad (3)$$

and

$$\text{þ}(\pi_{n+1}, \pi_n, t) \leq \text{þ}(\pi_{n+1}, \pi_n, \alpha t) = \text{þ}(\mathcal{F}\pi_n, \mathcal{F}\pi_{n-1}, \alpha t) \leq \text{þ}(\pi_n, \pi_{n-1}, t)$$
$$= \text{þ}(\mathcal{F}\pi_{n-1}, \mathcal{F}\pi_{n-2}, t) \leq \text{þ}\left(\pi_{n-1}, \pi_{n-2}, \frac{t}{\alpha}\right) \leq \cdots \leq \text{þ}\left(\pi_1, \pi_0, \frac{t}{\alpha^n}\right) \quad (4)$$

for all $n \in \mathbb{N}$, $p \geq 1$ and t>0. Thus, we have

$$\psi(\pi_n, \pi_{n+p}, t) \geq \psi\left(\pi_n, \pi_{n+1}, \frac{t}{2}\right) * \psi\left(\pi_{n+1}, \pi_{n+p}, \frac{t}{2}\right),$$

$$\varphi(\pi_n, \pi_{n+p}, t) \leq \varphi\left(\pi_n, \pi_{n+1}, \frac{t}{2}\right) \circ \varphi\left(\pi_{n+1}, \pi_{n+p}, \frac{t}{2}\right)$$

and

$$\text{þ}(\pi_n, \pi_{n+p}, t) \leq \text{þ}\left(\pi_n, \pi_{n+1}, \frac{t}{2}\right) \circ \text{þ}\left(\pi_{n+1}, \pi_{n+p}, \frac{t}{2}\right).$$

Continuing in this way, we get

$$\psi(\pi_n, \pi_{n+p}, t) \geq \psi\left(\pi_n, \pi_{n+1}, \frac{t}{2}\right) * \psi\left(\pi_{n+1}, \pi_{n+2}, \frac{t}{2^2}\right) * \cdots * \psi\left(\pi_{n+p-1}, \pi_{n+p}, \frac{t}{2^{p-1}}\right)$$

and

$$\varphi(\pi_n, \pi_{n+p}, t) \leq \varphi\left(\pi_n, \pi_{n+1}, \frac{t}{2}\right) \circ \varphi\left(\pi_{n+1}, \pi_{n+2}, \frac{t}{2^2}\right) \circ \cdots \circ \varphi\left(\pi_{n+p-1}, \pi_{n+p}, \frac{t}{2^{p-1}}\right),$$

and

$$\text{þ}(\pi_n, \pi_{n+p}, t) \leq \text{þ}\left(\pi_n, \pi_{n+1}, \frac{t}{2}\right) \circ \text{þ}\left(\pi_{n+1}, \pi_{n+2}, \frac{t}{2^2}\right) \circ \cdots \circ \text{þ}\left(\pi_{n+p-1}, \pi_{n+p}, \frac{t}{2^{p-1}}\right).$$

Using (2)–(4) in the above inequality, we deduce

$$\psi(\pi_n,\pi_{n+p},t) \geq \psi\left(\pi_0,\pi_1,\frac{t}{2\alpha^n}\right) * \psi\left(\pi_0,\pi_1,\frac{t}{2^2\alpha^{n+1}}\right) * \cdots * \psi\left(\pi_0,\pi_1,\frac{t}{2^{p-1}\alpha^{n+p-1}}\right), \tag{5}$$

$$\varphi(\pi_n,\pi_{n+p},t) \leq \varphi\left(\pi_0,\pi_1,\frac{t}{2\alpha^n}\right) \circ \varphi\left(\pi_0,\pi_1,\frac{t}{2^2\alpha^{n+1}}\right) \circ \cdots \circ \varphi\left(\pi_0,\pi_1,\frac{t}{2^{p-1}\alpha^{n+p-1}}\right), \tag{6}$$

and

$$\text{þ}(\pi_n,\pi_{n+p},t) \leq \text{þ}\left(\pi_0,\pi_1,\frac{t}{2\alpha^n}\right) \circ \text{þ}\left(\pi_0,\pi_1,\frac{t}{2^2\alpha^{n+1}}\right) \circ \cdots \circ \text{þ}\left(\pi_0,\pi_1,\frac{t}{2^{p-1}\alpha^{n+p-1}}\right). \tag{7}$$

We know that $\lim_{n\to+\infty}\psi(\pi,\lambda,t) = 1, s \lim_{n\to+\infty}\varphi(\pi,\lambda,t) = 0$, for all $\pi,\lambda \in \beta$ and $t > 0, \alpha \in (0,1)$. So, from (5)–(7) we deduce that

$$\lim_{n\to+\infty}\psi(\pi_n,\pi_{n+p},t) = 1 * 1 * \cdots * 1 = 1, for\ all\ t > 0, p \geq 1,$$

$$\lim_{n\to+\infty}\varphi(\pi_n,\pi_{n+p},t) = 0 \circ 0 \circ \cdots \circ 0 = 0, for\ all\ t > 0, p \geq 1,$$

and

$$\lim_{n\to+\infty}\text{þ}(\pi_n,\pi_{n+p},t) = 0 \circ 0 \circ \cdots \circ 0 = 0, for\ all\ t > 0, p \geq 1,$$

Hence, $\{\pi_n\}$ is a Cauchy sequence. The hypothesis of completeness of the neutrosophic metric-like space $(\beta,\psi,\varphi,\text{þ},*,\circ)$ ensures that there exists $w \in \beta$ such that

$$\lim_{n\to+\infty}\psi(\pi_n,w,t) = \lim_{n\to+\infty}\psi(\pi_n,\pi_{n+p},t) = \psi(w,w,t) = 1, for\ all\ t > 0, p \geq 1, \tag{8}$$

$$\lim_{n\to+\infty}\varphi(\pi_n,w,t) = \lim_{n\to+\infty}\varphi(\pi_n,\pi_{n+p},t) = \varphi(w,w,t) = 0, for\ all\ t > 0, p \geq 1, \tag{9}$$

and

$$\lim_{n\to+\infty}\text{þ}(\pi_n,w,t) = \lim_{n\to+\infty}\text{þ}(\pi_n,\pi_{n+p},t) = \text{þ}(w,w,t) = 0, for\ all\ t > 0, p \geq 1. \tag{10}$$

Now, we derive that $w \in \beta$ is a fixed point of $\mathcal{F}$. We have

$$\psi(w,\mathcal{F}w,t) \geq \psi\left(w,\pi_{n+1},\frac{t}{2}\right) * \psi\left(\pi_{n+1},\mathcal{F}w,\frac{t}{2}\right), for\ all\ t > 0,$$

$$= \psi\left(w,\pi_{n+1},\frac{t}{2}\right) * \psi\left(\mathcal{F}\pi_n,\mathcal{F}w,\frac{t}{2}\right) \geq \psi\left(w,\pi_{n+1},\frac{t}{2}\right) * \psi\left(\pi_n,w,\frac{t}{2\alpha}\right),$$

$$\varphi(w,\mathcal{F}w,t) \leq \varphi\left(w,\pi_{n+1},\frac{t}{2}\right) \circ \varphi\left(\pi_{n+1},\mathcal{F}w,\frac{t}{2}\right), for\ all\ t > 0,$$

$$= \varphi\left(w,\pi_{n+1},\frac{t}{2}\right) \circ \varphi\left(\mathcal{F}\pi_n,\mathcal{F}w,\frac{t}{2}\right) \leq \varphi\left(w,\pi_{n+1},\frac{t}{2}\right) \circ \varphi\left(\pi_n,w,\frac{t}{2\alpha}\right)$$

and

$$þ(w,\mathcal{F}w,t) \le þ\left(w,\pi_{n+1},\frac{t}{2}\right) \circ þ\left(\pi_{n+1},\mathcal{F}w,\frac{t}{2}\right), for\ all\ t > 0,$$

$$= þ\left(w,\pi_{n+1},\frac{t}{2}\right) \circ þ\left(\mathcal{F}\pi_n,\mathcal{F}w,\frac{t}{2}\right) \le þ\left(w,\pi_{n+1},\frac{t}{2}\right) \circ þ\left(\pi_n,w,\frac{t}{2\alpha}\right).$$

Taking limit as $n \to +\infty$, and by (8)–(10), we get

$$\psi(w,\mathcal{F}w,t) = 1 * 1 = 1,$$

$$\varphi(w,\mathcal{F}w,t) = 0 \circ 0 = 0,$$

and

$$þ(w,\mathcal{F}w,t) = 0 \circ 0 = 0.$$

Therefore, w is a fixed point of $\mathcal{F}$,

$$\psi(w,w,t) = 1, \varphi(w,w,t) = 0 \text{ and } þ(w,w,t) = 0, for\ all\ t > 0.$$

Now, we investigate the uniqueness of fixed point. For this, assume that v and w are two fixed points of $\mathcal{F}$, then by (1), we have

$$\psi(w,v,t) = \psi(\mathcal{F}w,\mathcal{F}v,t) \ge \psi\left(w,v,\frac{t}{\alpha}\right),$$

$$\psi(w,v,t) \ge \psi\left(w,v,\frac{t}{\alpha}\right), for\ all\ t > 0,$$

$$\varphi(w,v,t) = \varphi(\mathcal{F}w,\mathcal{F}v,t) \le \varphi\left(w,v,\frac{t}{\alpha}\right),$$

$$\varphi(w,v,t) \le \varphi\left(w,v,\frac{t}{\alpha}\right), for\ all\ t > 0,$$

and

$$þ(w,v,t) = þ(\mathcal{F}w,\mathcal{F}v,t) \le þ\left(w,v,\frac{t}{\alpha}\right),$$

$$þ(w,v,t) \le þ\left(w,v,\frac{t}{\alpha}\right), for\ all\ t > 0.$$

We obtain

$$\psi(w,v,t) \ge \psi\left(w,v,\frac{t}{\alpha^n}\right), for\ all\ n \in \mathbb{N},$$

$$\varphi(w,v,t) \le \varphi\left(w,v,\frac{t}{\alpha^n}\right), for\ all\ n \in \mathbb{N},$$

and

$$þ(w,v,t) \le þ\left(w,v,\frac{t}{\alpha^n}\right), for\ all\ n \in \mathbb{N}.$$

Taking limit as $n \to +\infty$ and using the fact $\lim_{t\to+\infty} \psi(\pi,\lambda,t) = 1$ and $\lim_{t\to+\infty} \varphi(\pi,\lambda,t) =$

0 and $\lim_{t\to+\infty} \text{þ}(\pi,\lambda,t) = 0$, so $w = v$, hence the fixed point is unique.

Example 3.13. Let $\beta = [0,1]$ and the continuous t-norm and continuous t-conorm respectively defined as $g * h = gh$ and $g \circ h = \max\{g,h\}$. Also, ψ, φ and þ are defined as

$$\psi(\pi,\lambda,t) = \frac{t}{t+\max\{\pi,\lambda\}} \; for\ all\ \pi,\lambda \in \beta, t > 0,$$

$$\varphi(\pi,\lambda,t) = \frac{\max\{\pi,\lambda\}}{t+\max\{\pi,\lambda\}} \; for\ all\ \pi,\lambda \in \beta, t > 0,$$

$$\text{þ}(\pi,\lambda,t) = \frac{\max\{\pi,\lambda\}}{t} \; for\ all\ \pi,\lambda \in \beta, t > 0.$$

Then $(\beta,\psi,\varphi,\text{þ},*,\circ)$ be a complete neutrosophic metric-like space. Define $\mathcal{F}:\beta\to\beta$ by

$$\mathcal{F}\pi = \begin{cases} 0, & \pi \in \left[0,\frac{1}{2}\right] \\ \frac{\pi}{8}, & \pi \in \left(\frac{1}{2},1\right]. \end{cases}$$

Now,

$$\lim_{t\to+\infty} \psi(\pi,\lambda,t) = \lim_{t\to+\infty} \frac{t}{t+\max\{\pi,\lambda\}} = 1,$$

$$\lim_{t\to+\infty} \varphi(\pi,\lambda,t) = \lim_{t\to+\infty} \frac{\max\{\pi,\lambda\}}{t+\max\{\pi,\lambda\}} = 0,$$

$$\lim_{t\to+\infty} \text{þ}(\pi,\lambda,t) = \lim_{t\to+\infty} \frac{\max\{\pi,\lambda\}}{t} = 0.$$

For $\alpha \in \left[\frac{1}{2},1\right)$, we have four cases:

Case1. If $\pi,\lambda \in \left[0,\frac{1}{2}\right]$, then $\mathcal{F}\pi = \mathcal{F}\lambda = 0$.

Case 2. If $\pi \in \left[0,\frac{1}{2}\right]$ and $\lambda \in \left(\frac{1}{2},1\right]$, then $\mathcal{F}\pi = 0$ and $\mathcal{F}\lambda = \frac{\lambda}{8}$.

Case 3. If $\pi,\lambda \in \left(\frac{1}{2},1\right]$, then $\mathcal{F}\pi = \frac{\pi}{8}$ and $\mathcal{F}\lambda = \frac{\lambda}{8}$.

Case4. If $\pi \in \left(\frac{1}{2},1\right]$ and $\lambda \in \left[0,\frac{1}{2}\right]$, then $\mathcal{F}\pi = \frac{\pi}{8}$ and $\mathcal{F}\lambda = 0$.

From all 4 cases, we obtain that

$$\psi(\mathcal{F}\pi,\mathcal{F}\lambda,\alpha t) \ge \psi(\pi,\lambda,t),$$

$$\varphi(\mathcal{F}\pi,\mathcal{F}\lambda,\alpha t) \le \varphi(\pi,\lambda,t),$$

$$\text{þ}(\mathcal{F}\pi,\mathcal{F}\lambda,\alpha t) \le \text{þ}(\pi,\lambda,t).$$

Hence all conditions of Theorem 3.12 are satisfied and 0 is the unique fixed point of $\mathcal{F}$. Also,

$$\psi(w,w,t)=\psi(0,0,t)=1, for\ all\ t>0,$$

$$\varphi(w,w,t)=\varphi(0,0,t)=0, for\ all\ t>0,$$

$$\text{þ}(w,w,t)=\text{þ}(0,0,t)=0, for\ all\ t>0.$$

Definition 3.14. Let $(\beta,\psi,\varphi,\text{þ},*,\circ)$ be an neutrosophic metric-like space. A mapping $\mathcal{F}:\beta\to\beta$ is said to be neutrosophic metric-like contractive if $k\in(0,1)$ such that

$$\frac{1}{\psi(\mathcal{F}\pi,\mathcal{F}\lambda,t)}-1\le k\left[\frac{1}{\psi(\pi,\lambda,t)}-1\right],\ \varphi(\mathcal{F}\pi,\mathcal{F}\lambda,t)\le k\varphi(\pi,\lambda,t) \text{ and } \text{þ}(\mathcal{F}\pi,\mathcal{F}\lambda,t)\le k\text{þ}(\pi,\lambda,t) \quad (11)$$

for all $\pi,\lambda\in\beta$ and $t>0$. Here, k is called the neutrosophic metric-like contractive constant of $\mathcal{F}$.

Theorem 3.15. Let $(\beta,\psi,\varphi,\text{þ},*,\circ)$ be a complete neutrosophic metric-like space and $\mathcal{F}:\beta\to\beta$ be a neutrosophic metric-like contractive mapping with a neutrosophic metric-like contractive constant k, then $\mathcal{F}$ has a unique fixed point $w\in\beta$ so that $\psi(w,w,t)=1$, $\varphi(w,w,t)=0$ and $\text{þ}(w,w,t)=0$, for all $t>0$.

Proof. Let $(\beta,\psi,\varphi,\text{þ},*,\circ)$ be a complete neutrosophic metric-like space. For an arbitrary $\pi_0\in\beta$, define a sequence $\{\pi_n\}$ in β by

$$\pi_1=\mathcal{F}\pi_0,\ \pi_2=\mathcal{F}^2\pi_0=\mathcal{F}\pi_1,\dots,\pi_n=\mathcal{F}^n\pi_0=\mathcal{F}\pi_{n-1} \text{ for all } n\in\mathbb{N}.$$

If $\pi_n=\pi_{n-1}$ for some $n\in\mathbb{N}$, then π_n is a fixed point of $\mathcal{F}$. We assume that $\pi_n\ne\pi_{n-1}$ for all $n\in\mathbb{N}$. For $t>0$ and $n\in\mathbb{N}$, we get from (11)

$$\frac{1}{\psi(\pi_n,\pi_{n+1},t)}-1=\frac{1}{\psi(\mathcal{F}\pi_{n-1},\mathcal{F}\pi_n,t)}-1\le k\left[\frac{1}{\psi(\pi_{n-1},\pi_n,t)}-1\right].$$

We have

$$\begin{aligned}\frac{1}{\psi(\pi_n,\pi_{n+1},t)}&\le\frac{k}{\psi(\pi_{n-1},\pi_n,t)}+(1-k), for\ all\ t>0,\\ &=\frac{k}{\psi(\mathcal{F}\pi_{n-2},\mathcal{F}\pi_{n-1},t)}+(1-k)\\ &\le\frac{k^2}{\psi(\pi_{n-2},\pi_{n-1},t)}+k(1-k)+(1-k).\end{aligned}$$

Continuing in this way, we get

$$\begin{aligned}\frac{1}{\psi(\pi_n,\pi_{n+1},t)}&\le\frac{k^n}{\psi(\pi_0,\pi_1,t)}+k^{n-1}(1-k)+k^{n-2}(1-k)+\cdots+k(1-k)+(1-k)\\ &\le\frac{k^n}{\psi(\pi_0,\pi_1,t)}+(k^{n-1}+k^{n-2}+\cdots+1)(1-k)\\ &\le\frac{k^n}{\psi(\pi_0,\pi_1,t)}+(1-k^n).\end{aligned}$$

We have

$$\frac{1}{\frac{k^n}{\psi(\pi_0,\pi_1,t)}+(1-k^n)} \leq \psi(\pi_n,\pi_{n+1},t),\ for\ all\ t>0, n\in\mathbb{N}. \tag{12}$$

Now,

$$\varphi(\pi_n,\pi_{n+1},t)=\varphi(\mathcal{F}\pi_{n-1},\mathcal{F}\pi_n,t)$$
$$\leq k\varphi(\pi_{n-1},\pi_n,t)=k\varphi(\mathcal{F}\pi_{n-2},\mathcal{F}\pi_{n-1},t)$$
$$\leq k^2\varphi(\pi_{n-2},\pi_{n-1},t)\leq\cdots\leq k^n\varphi(\pi_0,\pi_1,t) \tag{13}$$

and

$$þ(\pi_n,\pi_{n+1},t)=þ(\mathcal{F}\pi_{n-1},\mathcal{F}\pi_n,t)$$
$$\leq kþ(\pi_{n-1},\pi_n,t)=kþ(\mathcal{F}\pi_{n-2},\mathcal{F}\pi_{n-1},t)$$
$$\leq k^2þ(\pi_{n-2},\pi_{n-1},t)\leq\cdots\leq k^nþ(\pi_0,\pi_1,t). \tag{14}$$

Now, for $p\geq 1$ and $n\in\mathbb{N}$, we have

$$\psi(\pi_n,\pi_{n+p},t)\geq\psi\left(\pi_n,\pi_{n+1},\frac{t}{2}\right)*\psi\left(\pi_{n+1},\pi_{n+p},\frac{t}{2}\right)$$
$$\geq\psi\left(\pi_n,\pi_{n+1},\frac{t}{2}\right)*\psi\left(\pi_{n+1},\pi_{n+2},\frac{t}{2^2}\right)*\psi\left(\pi_{n+2},\pi_{n+p},\frac{t}{2^2}\right).$$

Continuing in this way, we get

$$\psi(\pi_n,\pi_{n+p},t)\geq\psi\left(\pi_n,\pi_{n+1},\frac{t}{2}\right)*\psi\left(\pi_{n+1},\pi_{n+2},\frac{t}{2^2}\right)*\cdots*\psi\left(\pi_{n+p-1},\pi_{n+p},\frac{t}{2^{p-1}}\right),$$

$$\varphi(\pi_n,\pi_{n+p},t)\leq\varphi\left(\pi_n,\pi_{n+1},\frac{t}{2}\right)\circ\varphi\left(\pi_{n+1},\pi_{n+p},\frac{t}{2}\right)$$
$$\leq\varphi\left(\pi_n,\pi_{n+1},\frac{t}{2}\right)\circ\varphi\left(\pi_{n+1},\pi_{n+2},\frac{t}{2^2}\right)\circ\varphi\left(\pi_{n+2},\pi_{n+p},\frac{t}{2^2}\right)$$

and

$$þ(\pi_n,\pi_{n+p},t)\leq þ\left(\pi_n,\pi_{n+1},\frac{t}{2}\right)\circ þ\left(\pi_{n+1},\pi_{n+p},\frac{t}{2}\right)$$
$$\leq þ\left(\pi_n,\pi_{n+1},\frac{t}{2}\right)\circ þ\left(\pi_{n+1},\pi_{n+2},\frac{t}{2^2}\right)\circ þ\left(\pi_{n+2},\pi_{n+p},\frac{t}{2^2}\right).$$

Continuing in this way, we get

$$þ(\pi_n,\pi_{n+p},t)\leq þ\left(\pi_n,\pi_{n+1},\frac{t}{2}\right)\circ þ\left(\pi_{n+1},\pi_{n+2},\frac{t}{2^2}\right)\circ\cdots\circ þ\left(\pi_{n+p-1},\pi_{n+p},\frac{t}{2^{p-1}}\right).$$

By using (12)–(14) in the above inequality, we have

$$\psi(\pi_n,\pi_{n+p},t) \geq \frac{1}{\frac{k^n}{\psi\left(\pi_0,\pi_1,\frac{t}{2}\right)}+(1-k^n)} * \frac{1}{\frac{k^{n+1}}{\psi\left(\pi_0,\pi_1,\frac{t}{2^2}\right)}+(1-k^{n+1})} * \cdots * \frac{1}{\frac{k^{n+p-1}}{\psi\left(\pi_0,\pi_1,\frac{t}{2^{p-1}}\right)}+(1-k^{n+p-1})}$$

$$\geq \frac{1}{\frac{k^n}{\psi\left(\pi_0,\pi_1,\frac{t}{2}\right)}+1} * \frac{1}{\frac{k^{n+1}}{\psi\left(\pi_0,\pi_1,\frac{t}{2^2}\right)}+1} * \cdots * \frac{1}{\frac{k^{n+p-1}}{\psi\left(\pi_0,\pi_1,\frac{t}{2^{p-1}}\right)}+1},$$

$$\varphi(\pi_n,\pi_{n+p},t) \leq k^n\varphi\left(\pi_0,\pi_1,\frac{t}{2}\right) \circ k^{n+1}\varphi\left(\pi_1,\pi_2,\frac{t}{2^2}\right) \circ \cdots \circ k^{n+p-1}\varphi\left(\pi_0,\pi_1,\frac{t}{2^{p-1}}\right),$$

and

$$þ(\pi_n,\pi_{n+p},t) \leq k^n þ\left(\pi_0,\pi_1,\frac{t}{2}\right) \circ k^{n+1}þ\left(\pi_1,\pi_2,\frac{t}{2^2}\right) \circ \cdots \circ k^{n+p-1}þ\left(\pi_0,\pi_1,\frac{t}{2^{p-1}}\right).$$

Here, $k \in (0,1)$, we deduce from the above expression that

$$\lim_{n\to+\infty} \psi(\pi_n,\pi_{n+p},t) = 1 \text{ for all } t>0, p\geq 1,$$

$$\lim_{n\to+\infty} \varphi(\pi_n,\pi_{n+p},t) = 0 \text{ for all } t>0, p\geq 1,$$

and

$$\lim_{n\to+\infty} þ(\pi_n,\pi_{n+p},t) = 0 \text{ for all } t>0, p\geq 1.$$

Therefore, $\{\pi_n\}$ is a Cauchy sequence in $(\beta,\psi,\varphi,þ,*,\circ)$. By the completeness of $(\beta,\psi,\varphi,þ,*,\circ)$. There is $w \in \beta$, such that

$$\lim_{n\to+\infty} \psi(\pi_n,w,t) = \lim_{n\to+\infty} \psi(\pi_n,\pi_{n+p},t) = \lim_{n\to+\infty} \psi(w,w,t) = 1, \textit{for all } t>0, p\geq 1. \quad (15)$$

$$\lim_{n\to+\infty} \varphi(\pi_n,w,t) = \lim_{n\to+\infty} \varphi(\pi_n,\pi_{n+p},t) = \lim_{n\to+\infty} \varphi(w,w,t) = 0, \textit{for all } t>0, p\geq 1. \quad (16)$$

and

$$\lim_{n\to+\infty} þ(\pi_n,w,t) = \lim_{n\to+\infty} þ(\pi_n,\pi_{n+p},t) = \lim_{n\to+\infty} þ(w,w,t) = 0, \textit{for all } t>0, p\geq 1. \quad (17)$$

Now, we prove that w is a fixed point for $\mathcal{F}$. For this, we obtain from (11) that

$$\frac{1}{\psi(\mathcal{F}\pi_n,\mathcal{F}w,t)} - 1 \leq k\left[\frac{1}{\psi(\pi_n,w,t)} - 1\right] = \frac{k}{\psi(\pi_n,w,t)} - k,$$

$$\frac{1}{\frac{k}{\psi(\pi_n, w, t)} + 1 - k} \leq \psi(\mathcal{F}\pi_n, \mathcal{F}w, t).$$

Using the above inequality, we obtain

$$\psi(w, \mathcal{F}w, t) \geq \psi\left(w, \pi_{n+1}, \frac{t}{2}\right) * \psi\left(\pi_{n+1}, \mathcal{F}w, \frac{t}{2}\right)$$

$$= \psi\left(w, \pi_{n+1}, \frac{t}{2}\right) * \psi\left(\mathcal{F}\pi_n, \mathcal{F}w, \frac{t}{2}\right) \geq \psi\left(w, \pi_{n+1}, \frac{t}{2}\right) * \frac{1}{\frac{k}{\psi\left(\pi_n, w, \frac{t}{2}\right)} + 1 - k},$$

$$\varphi(w, \mathcal{F}w, t) \leq \varphi\left(w, \pi_{n+1}, \frac{t}{2}\right) \circ \varphi\left(\pi_{n+1}, \mathcal{F}w, \frac{t}{2}\right)$$

$$= \varphi\left(w, \pi_{n+1}, \frac{t}{2}\right) \circ \varphi\left(\mathcal{F}\pi_n, \mathcal{F}w, \frac{t}{2}\right) \leq \varphi\left(w, \pi_{n+1}, \frac{t}{2}\right) \circ k\varphi\left(\pi_n, w, \frac{t}{2}\right),$$

and

$$þ(w, \mathcal{F}w, t) \leq þ\left(w, \pi_{n+1}, \frac{t}{2}\right) \circ þ\left(\pi_{n+1}, \mathcal{F}w, \frac{t}{2}\right)$$

$$= þ\left(w, \pi_{n+1}, \frac{t}{2}\right) \circ þ\left(\mathcal{F}\pi_n, \mathcal{F}w, \frac{t}{2}\right) \leq þ\left(w, \pi_{n+1}, \frac{t}{2}\right) \circ kþ\left(\pi_n, w, \frac{t}{2}\right)$$

Taking limit as $n \to +\infty$ and using (15)–(17) in the above expression, we get $\psi(w, \mathcal{F}w, t) = 1$, $\varphi(w, \mathcal{F}w, t) = 0$ and $þ(w, \mathcal{F}w, t) = 0$, that is, $\mathcal{F}w = w$. Therefore, w is a fixed point of $\mathcal{F}$ and $\psi(w, w, t) = 1$, $\varphi(w, w, t) = 0$ and $þ(w, w, t) = 0$ for all $t > 0$.

Now, we investigate the uniqueness of the fixed point w of $\mathcal{F}$. Let v be another fixed point of $\mathcal{F}$, such that $\psi(w, v, \mathrm{t}) \neq 1, \varphi(w, v, t) \neq 0$ and $þ(w, v, t) \neq 0$ for some $t > 0$. It follows from (11) that

$$\frac{1}{\psi(w, v, t)} - 1 = \frac{1}{\psi(\mathcal{F}w, \mathcal{F}v, t)} - 1 \leq k\left[\frac{1}{\psi(w, v, t)} - 1\right] < \frac{1}{\psi(w, v, t)} - 1,$$

$$\varphi(w, v, t) = \varphi(\mathcal{F}w, \mathcal{F}v, t) \leq k\varphi(w, v, t) < \varphi(w, v, t),$$

and

$$þ(w, v, t) = þ(\mathcal{F}w, \mathcal{F}v, t) \leq kþ(w, v, t) < þ(w, v, t),$$

a contradiction.

Therefore, we must have $\psi(w, v, t) = 1, \varphi(w, v, t) = 0$ and $þ(w, v, t) = 0$, for all $t > 0$, and hence $w = v$.

Corollary 3.16. Let $(\beta, \psi, \varphi, þ, *, \circ)$ be a complete neutrosophic metric-like space and $\mathcal{F}: \beta \to \beta$ be a mapping satisfying

$$\frac{1}{\psi(\mathcal{F}^n\pi,\mathcal{F}^n\lambda,t)}-1\leq k\left[\frac{1}{\psi(\pi,\lambda,t)}-1\right],$$

$$\varphi(\mathcal{F}^n\pi,\mathcal{F}^n\lambda,t)\leq k\varphi(\pi,\lambda,t),$$

and

$$þ(\mathcal{F}^n\pi,\mathcal{F}^n\lambda,t)\leq kþ(\pi,\lambda,t)$$

for some $n\in\mathbb{N}, for\ all\ \pi,\lambda\in\beta, t>0$, where $0<k<1$. Then $\mathcal{F}$ has a unique fixed point $w\in\beta$ and $\psi(w,w,t)=1, \varphi(w,w,t)=0$ and $þ(w,w,t)=0\ for\ all\ t>0$.

Proof. $w\in\beta$ is the unique fixed point of $\mathcal{F}^n$ by using Theorem 3.15, and $\psi(w,w,t)=1$, $\varphi(w,w,t)=0$ and $þ(w,w,t)=0\ for\ all\ t>0$. $\mathcal{F}w$ is also a fixed point of $\mathcal{F}^n$ as $\mathcal{F}^n(\mathcal{F}w)=\mathcal{F}w$ and from Theorem 3.15, $\mathcal{F}w=w$, w is the unique fixed point, since the unique fixed point of $\mathcal{F}$ is also the unique fixed point of $\mathcal{F}^n$.

Example 3.17. Let $\beta=[0,2]$ and the continuous t-norm and continuous t-conorm respectively defined as $g*h=gh$ and $g\circ h=\max\{g,h\}$, given ψ,φ and þ as

$$\psi(\pi,\lambda,t)=\frac{t}{t+\max\{\pi,\lambda\}}\ for\ all\ \pi,\lambda\in\beta,t>0,$$

$$\varphi(\pi,\lambda,t)=\frac{\max\{\pi,\lambda\}}{t+\max\{\pi,\lambda\}}\ for\ all\ \pi,\lambda\in\beta,t>0,$$

$$þ(\pi,\lambda,t)=\frac{\max\{\pi,\lambda\}}{t}\ for\ all\ \pi,\lambda\in\beta,t>0.$$

for all $\pi,\lambda\in\beta$ and $t>0$. Then $(\beta,\psi,\varphi,þ,*,\circ)$ is a complete neutrosophic metric-like space. Define $\mathcal{F}:\beta\to\beta$ as

$$\mathcal{F}\pi=\begin{cases}0, & \pi=1\\ \frac{\pi}{3}, & \pi\in[0,1)\\ \frac{\pi}{7}, & \pi\in(1,2]\end{cases}.$$

Then we have 8 cases:

Case 1. If $\pi=\lambda=1$, then $\mathcal{F}\pi=\mathcal{F}\lambda=0$.

Case 2. If $\pi=1$ and $\lambda\in[0,1)$, then $\mathcal{F}\pi=0$ and $\mathcal{F}\lambda=\frac{\lambda}{3}$.

Case 3. If $\pi=1$ and $\lambda\in(1,2]$, then $\mathcal{F}\pi=0$ and $\mathcal{F}\lambda=\frac{\lambda}{7}$.

Case 4. If $\pi\in[0,1)$ and $\lambda\in(1,2]$, then $\mathcal{F}\pi=\frac{\pi}{3}$ and $\mathcal{F}\lambda=\frac{\lambda}{7}$.

Case 5. If $\pi\in[0,1)$ and $\lambda\in[0,1)$, then $\mathcal{F}\pi=\frac{\pi}{3}$ and $\mathcal{F}\lambda=\frac{\lambda}{3}$.

Case 6. If $\pi\in[0,1)$ and $\lambda=1$, then $\mathcal{F}\pi=\frac{\pi}{3}$ and $\mathcal{F}\lambda=0$.

Case 7. If $\pi\in(1,2]$ and $\lambda=1$, then $\mathcal{F}\pi=\frac{\pi}{7}$ and $\mathcal{F}\lambda=0$.

Case 8. If $\pi \in (1,2]$ and $\lambda \in (1,2]$, then $\mathcal{F}\pi = \frac{\pi}{7}$ and $\mathcal{F}\lambda = \frac{\lambda}{7}$.

All above cases satisfy the neutrosophic metric-like contraction:

$$\frac{1}{\psi(\mathcal{F}\pi, \mathcal{F}\lambda, t)} - 1 \le k\left[\frac{1}{\psi(\pi, \lambda, t)} - 1\right],$$

$$\varphi(\mathcal{F}\pi, \mathcal{F}\lambda, t) \le k\varphi(\pi, \lambda, t),$$

$$þ(\mathcal{F}\pi, \mathcal{F}\lambda, t) \le kþ(\pi, \lambda, t)$$

with $k \in \left[\frac{1}{2}, 1\right)$ the neutrosophic metric-like contractive constant. Henc $\mathcal{F}$ is a neutrosophic metric-like contractive mapping with $k \in \left[\frac{1}{2}, 1\right)$. All conditions of Theorem 3.15 are satisfied. Also, 0 is the unique fixed point of $\mathcal{F}$ and $\psi(0,0,t) = 1, \varphi(0,0,t) = 0$ and $þ(0,0,t) = 0, for\ all\ t > 0$.

Theorem 3.18. Let $(\beta, \psi, \varphi, þ, *, \circ)$ be a complete neutrosophic metric-like space and $\mathcal{F}: \beta \to \beta$ be a NML contractive mapping with an neutrosophic metric-like space contractive constant k. Suppose that their exist $w \in \beta$, such that $\psi(w, \mathcal{F}w, t) \ge \psi(\pi, \mathcal{F}\pi, t)$, $\varphi(w, \mathcal{F}w, t) \le \varphi(\pi, \mathcal{F}\pi, t)$ and $þ(w, \mathcal{F}w, t) \le þ(\pi, \mathcal{F}\pi, t)$ for all $\pi \in \beta$ and $t > 0$, we claim that $\psi(w, \mathcal{F}w, t) = 1$, $\varphi(w, \mathcal{F}w, t) = 0$ and $þ(w, \mathcal{F}w, t) = 0$ for all $w \in \beta$ and $t > 0$, then $\mathcal{F}$ has a unique fixed point $w \in \beta$ so that $\psi(w, w, t) = 1$, $\varphi(w, w, t) = 0$ and $\varphi(w, w, t) = 0$ for all $t > 0$.

Proof. Let $\psi_\pi(t) = \psi(\pi, \mathcal{F}\pi, t)$, $\varphi_\pi(t) = \varphi(\pi, \mathcal{F}\pi, t)$ and $þ_\pi(t) = þ(\pi, \mathcal{F}\pi, t)$ for all $\pi \in \beta$ and $t > 0$. Then by the assumption $\psi_w(t) \ge \psi_\pi(t)$, $\varphi_w(t) \le \varphi_\pi(t)$ and $þ_w(t) \le þ_\pi(t)$ for all $\pi \in \beta$ and $t > 0$. We claim that $\psi(w, \mathcal{F}w, t) = 1$, $\varphi(w, \mathcal{F}w, t) = 0$ and $þ(w, \mathcal{F}w, t) = 0$ for all $t > 0$. Indeed, if $\psi_w(t) = \psi(w, \mathcal{F}w, t) < 1$, $\varphi_w(t) = \varphi(w, \mathcal{F}w, t) > 0$ and $þ_w(t) = þ(w, \mathcal{F}w, t) > 0$ for some $t > 0$, then it fellows from (11) that

$$\frac{1}{\psi_{\mathcal{F}w}(t)} - 1 = \frac{1}{\psi(\mathcal{F}w, \mathcal{F}\,\mathcal{F}w, t)} - 1 \le k\left[\frac{1}{\psi(w, \mathcal{F}w, t)} - 1\right] = k\left[\frac{1}{\psi_w(t)} - 1\right] < \left[\frac{1}{\psi_w(t)} - 1\right],$$

$$\varphi_{\mathcal{F}w}(t) = \varphi(\mathcal{F}w, \mathcal{F}\,\mathcal{F}w, t) \le k[\varphi(w, \mathcal{F}w, t)] = k[\varphi_w(t)] < \varphi_w(t),$$

$$þ_{\mathcal{F}w}(t) = þ(\mathcal{F}w, \mathcal{F}\,\mathcal{F}w, t) \le k[þ(w, \mathcal{F}w, t)] = k[þ_w(t)] < þ_w(t).$$

That is, $\psi_w(t) \le \psi_{\mathcal{F}w}(t), \mathcal{F}w \in \beta$ a contradiction. Therefore, we have $\psi_\pi(t) = \psi(w, \mathcal{F}w\ t) = 1$, $\varphi_\pi(t) = \varphi(w, \mathcal{F}w\ t) = 0$ and $þ_\pi(t) = þ(w, \mathcal{F}w\ t) = 0$ for all $t > 0$, and so $\mathcal{F}w = w$. Following the similar argument as in Theorem 2.15, uniqueness of fixed point of $\mathcal{F}$ follows. If $\psi(w, w, t) < 1$, $\varphi(w, w, t) > 0$ and $þ(w, w, t) > 0$ for some $t > 0$, then from (11), we have

$$\frac{1}{\psi(w, w, t)} - 1 = \frac{1}{\psi(\mathcal{F}w, \mathcal{F}w, t)} - 1 \le k\left[\frac{1}{\psi(w, w, t)} - 1\right] < \left[\frac{1}{\psi(w, w, t)} - 1\right],$$

$$\varphi(w, w, t) = \varphi(\mathcal{F}w, \mathcal{F}w, t) \le k[\varphi(w, w, t)] < \varphi(w, w, t),$$

$$þ(w, w, t) = þ(\mathcal{F}w, \mathcal{F}w, t) \le k[þ(w, w, t)] < þ(w, w, t),$$

a contradiction. Therefore, $\psi(w, w, t) = 1, \varphi(w, w, t) = 0$ and $þ(w, w, t) = 0$.

Remark 3.19. In the above theorem it is shown that in an neutrosophic metric-like space, the self-neutrosophic distance of the fixed point of a neutrosophic metric-like contractive mapping with a neutrosophic metric-like contractive constant $k,$ is always 1, 0. That is, the degree of self-nearness of the fixed point of a neutrosophic metric-like contractive mapping is perfect.

4. Conclusions

In this manuscript, we introduced the concept of neutrosophic metric like spaces and established some properties. Also, we established several fixed point results with non-trivial examples. As is well known, in recent years, the study of metric fixed point theory has been widely researched because of this theory has a fundamental role in various areas of mathematics, science and economic studies. This work can be extended in different generalized structures like, neutrosophic partial metric like spaces, neutrosophic *b*-metric like spaces.

Data availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflict of interest

The authors declare that they have no competing interests regarding the publication of this paper.

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