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Research article

The travelling wave solutions of nonlinear evolution equations with both a dissipative term and a positive integer power term

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Abstract: The formula of solution to a nonlinear ODE with an undetermined coefficient and a positive integer power term of dependent variable have been obtained by the transformation of dependent variable and $(\frac{G'}{G})$-expansion method. The travelling wave reduction ODEs (perhaps, after integration and identical deformation) of a class of nonlinear evolution equations with a dissipative term and a positive integer power term of dependent variable that includes GKdV-Burgers equation, GKP-Burgers equation, GZK-Burgers equation, GBoussinesq equation and GKlein-Gordon equation, are all attributed to the same type of ODEs as the nonlinear ODE considered. The kink type of travelling wave solutions for these nonlinear evolution equations are obtained in terms of the formula of solution to the nonlinear ODE considered.

Keywords: the ODE with an undetermined coefficient and a positive integer power term; the formula of solution; $(\frac{G'}{G})$-expansion; GKP-Burgers equation; GZK-Burgers equation; GBoussinesq equation; GKP-Gordon equation; the kink type of travelling wave solutions
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1. Introduction

It is well known that nonlinear waves are governed by nonlinear evolution equations and some problems related to nonlinear equations are very difficult to handle in a general way. So far, there is no unified method that can deal with all types of nonlinear evolution equations. However, some special problems have been solved from time to time by methods suited to the special problems. In the present paper, we shall concentrate on a class of nonlinear evolution equations with both a dissipative term and a positive integer power term of dependent variable. These equations are all generalized form of important model equations in mathematical physics. Several examples of this class of nonlinear equations are listed below:

The generalized KdV-Burgers equation (GKdV-Burgers)

$$u_t = u_{xxx} - u^n u_x + a u_{xx}. \tag{1}$$

The generalized KP-Burgers equation (GKP-Burgers)

$$(u_t + u_{xxx} + a u_{xx} - b u^n u_x)_x + \sigma^2 u_{yy} = 0. \tag{2}$$

The generalized Zakharov-Kuznetsov-Burgers equation (GZK-Burgers)

$$u_t + \alpha u^n u_x + \beta u_{xxx} + \gamma(u_{yy} + u_{zz})_x + a u_{xx} = 0. \tag{3}$$

The generalized Boussinesq equation (GBoussinesq)

$$u_{tt} + c^2 u_{xx} - b(u^{n+1})_{xx} + a u_{xxt} + u_{xxxx} = 0. \tag{4}$$

General (m + 1)-dimension Klein-Gordon equation (GKlein-Gordon)

$$u_{tt} + c^2 \Delta_m u + a u_t + du + b u^{n+1} = 0, \tag{5}$$

where m is positive integer, and so on.

In particular, when $n = 1$ the Eqs (1)–(5) above become the famous model equations in mathematical physics, to be more precise, Eqs (1)–(5) become KdV-Burgers equation [1–2], KP-Burgers equation [3], ZKB equation [4], Boussinesq equation [5] and dissipative Klein-Gordon equation [6], respectively. Each of these model equations mentioned has been investigated extensively by many authors and methods during the past four decades or so. Especially, for GKdV-Burgers equation, GZK-Burgers equation, GBoussinesq equation, and GKlein-Gordon equation, some literatures [7–10] have obtained different types of soliton solutions by using HPM and tanh method etc. Nevertheless, as for the Eqs (1)–(5), which contains a dissipative term and a positive integer power term, to our knowledge, there are less systemic methods to solve them. Since Eqs (1)–(5) are all the generalized form of the important model equations, therefore the investigation of them is of great significance both mathematically and physically

In recent years, the development of mathematical physics furtherly provides us with more detailed methods to seek exact solutions of NLDEs, for example, Hirota bilinear method [11], $(\frac{G'}{G})$-expansion method [12–14], exp-function method [15], complex method [16–19], $\exp(-\psi(\xi))$-expansion method [20,21], the sub-ODE method [22] and so on. As one of the efficient computing techniques, there is the spectral collocation method for solving equations with a fractional integro-differential [23]. In addition, Ahmad El-Ajou also has suggested a new method that relies on a new fractional expansion in the Laplace transform space and residual power series method to construct exact solitary solutions to the nonlinear time-fractional dispersive PDEs in the literature [24].

These developments have helped to open up various new mathematical branches and have led to immense enrichment therein.

The main goal of this paper is to obtain travelling wave solutions of Eqs (1)–(5). Obviously the crucial problem is how to solve the travelling wave reduction ODEs of Eqs (1)–(5), since reduction ODEs contain an undetermined coefficient and a positive integer power term of dependent variable. In Section 2, the formula of solution as well as the undetermined coefficient of a nonlinear ODE with an undetermined coefficient and a positive integer power term of dependent variable has been obtained by using transformation of dependent variable and $(\frac{G'}{G})$-expansion method, we use a theorem to describe such a result briefly. In subsequent sections, from Section 3 to Section 7, we shall apply the theorem obtained in Section 2 to KdV-Burgers, GKP-Burgers, GZK-Burgers, GBoussinesq and GKlein-Gordon, respectively, to obtain the travelling wave solutions of these equations. In Section 8, the technique used in the paper is summarized briefly.

2. A nonlinear ODE with an undetermined coefficient and a positive integer power term of dependent variable

In this section, we considered the following ODE

$$F''(\xi)+aF'(\xi)+VF(\xi)-bF^{n+1}(\xi)=0, \tag{6}$$

where $n\geq 1$, a positive integer; a,b and V are constants, $b>0, V$ to be determined later. Our aim is to find out V such that Eq (6) has a positive solution. We shall demonstrate the following:

Theorem. *The ODE (6) admits a positive solution*

$$F(\xi)=\left[\frac{2(n+2)a^2}{(n+4)^2 b}\left(\frac{1}{1+\exp\left[\frac{an}{n+4}(\xi+\xi_0)\right]}\right)^2\right]^{\frac{1}{n}}, \ \xi_0=const. \tag{7}$$

Provided that

$$V=\frac{2(n+2)a^2}{(n+4)^2}. \tag{8}$$

Proof. Making transformation of dependent variable

$$F=u^{\frac{1}{n}}. \tag{9}$$

Equation (6) is converted into the nonlinear ODE for $u=u(\xi)$,

$$uu''+(\frac{1}{n}-1)u'^2+auu'+nVu^2-bnu^3=0. \tag{10}$$

We shall use the $(\frac{G'}{G})$-expansion method [12] to solve ODE (10).

First of all, considering the homogeneous balance between uu' (or u'^2) and u^3

$(2m+2=3m \Rightarrow m=2)$, we can suppose that the solution of Eq (10) is of the form

$$u(\xi)=A\left(\frac{G'}{G}\right)^2, \tag{11}$$

where $G=G(\xi)$ satisfies

$$G''+\lambda G'=0, \tag{12}$$

A and λ are constants to be determined.

Second, from (11) and using (12), it is derived that

$$\begin{aligned} uu'' &= 6A^2\left(\frac{G'}{G}\right)^6+10A^2\lambda\left(\frac{G'}{G}\right)^5+4A^2\lambda^2\left(\frac{G'}{G}\right)^4, \\ u'^2 &= 4A^2\left(\frac{G'}{G}\right)^6+8A^2\lambda\left(\frac{G'}{G}\right)^5+4A^2\lambda^2\left(\frac{G'}{G}\right)^4, \\ uu' &= -2A^2\lambda\left(\frac{G'}{G}\right)^5-2A^2\lambda\left(\frac{G'}{G}\right)^4, \\ u^3 &= A^3\left(\frac{G'}{G}\right)^5, u^2=A^2\left(\frac{G'}{G}\right)^4. \end{aligned} \tag{13}$$

Substituting (13) into the left hand side of Eq (6), collecting all terms with $\left(\frac{G'}{G}\right)^i (i=4,5,6)$ together and setting the coefficient of $\left(\frac{G'}{G}\right)^i (i=4,5,6)$ to zero, yields a set of algebraic equations for A, λ and V as follows

$$\begin{aligned} \left(\frac{G'}{G}\right)^6: &\quad 6A^2+(\frac{1}{n}-1)4A^2-bnA^3=0, \\ \left(\frac{G'}{G}\right)^5: &\quad 10A^2\lambda+(\frac{1}{n}-1)8A^2\lambda-2aA^2=0, \\ \left(\frac{G'}{G}\right)^4: &\quad 4A^2\lambda^2+(\frac{1}{n}-1)4A^2\lambda^2-2aA^2\lambda+nVA^2=0. \end{aligned} \tag{14}$$

Solving (14), yields

$$A=\frac{2(n+2)}{bn^2}, \lambda=\frac{an}{n+4}, V=\frac{2(n+2)}{(n+4)^2}a^2.$$

Third, when $\lambda=\frac{an}{n+4}$, Eq (12) admits a solution

$$G(\xi)=1+\exp\left[-\frac{an}{n+4}(\xi+\xi_0)\right], \xi_0=const.$$

Therefore

$$\frac{G'(\xi)}{G(\xi)}=-\frac{an}{n+4}\left(\frac{\exp\left[-\frac{an}{n+4}(\xi+\xi_0)\right]}{1+\exp\left[-\frac{an}{n+4}(\xi+\xi_0)\right]}\right)=-\frac{an}{n+4}\left(\frac{1}{1+\exp\left[\frac{an}{n+4}(\xi+\xi_0)\right]}\right), \tag{15}$$

substituting $A=\frac{2(n+2)}{bn^2}$ and $\left(\frac{G'}{G}\right)$ in (15) into (9), we obtain the solution of nonlinear ODE (10)

$$u(\xi)=\frac{2(n+2)a^2}{(n+4)^2 b}\left(\frac{1}{1+\exp\left[\frac{an}{n+4}(\xi+\xi_0)\right]}\right)^2, \tag{16}$$

provided

$$\xi=x-\frac{2(n+2)}{(n+4)^2}a^2 t, \xi_0=const.$$

Last of all, substituting (16) into (9), we have the solution of Eq (6), which is expressed by (7), provided $V=\frac{2(n+2)}{(n+4)^2}a^2$.

The discussion above completes the proof of the theorem.

3. Generalized KdV-Burgers equation

Introducing travelling wave variables

$$u(x,t)=u(\xi), \xi=x-Vt. \tag{17}$$

Equation (1) is converted into an ODE for $u=u(\xi)$

$$-Vu'=u'''-u^n u'+au''.$$

Integrating it with respect to ξ once, and taking constant of integration to zero, yields

$$u''(\xi)+au'(\xi)+Vu(\xi)-\frac{1}{n+1}u^{n+1}=0. \tag{18}$$

Comparing Eq (18) with Eq (6) in Section 2, it is found that Eq (18) belongs to the same type ODE as Eq (6). According to the theorem in Section 2, the travelling wave solution of Eq (1) is given by

$$u(x,t)=u(\xi)=\left[\frac{2(n+2)(n+1)a^2}{(n+4)^2}\right]^{\frac{1}{n}}\left(\frac{1}{1+\exp\left[\frac{an}{n+4}(\xi+\xi_0)\right]}\right)^{\frac{2}{n}}, \tag{19}$$

provided $V=\frac{2(n+2)}{(n+4)^2}a^2$, in view of (17)

$$\xi = x - \frac{2(n+2)}{(n+4)^2} a^2 t.$$

In particular, when $n=1$, (19) becomes

$$u(x,t) = \frac{12a^2}{25} \cdot \left(\frac{1}{1+\exp\left[\frac{1}{5}a(\xi+\xi_0)\right]} \right)^2, \xi = x - \frac{6}{25}a^2 t, \xi_0 = const, \tag{20}$$

which is the solution of KdV-Burgers equation

$$u_t = u_{xxx} - uu_x + au_{xx}. \tag{21}$$

When $n=2$, (19) becomes

$$u(x,t) = \frac{\sqrt{6}a}{3} \cdot \left(\frac{1}{1+\exp\left[\frac{a}{3}(\xi+\xi_0)\right]} \right), \xi = x - \frac{2}{9}a^2 t, \xi_0 = const, \tag{22}$$

which is the solution of mKdV-Burgers equation

$$u_t = u_{xxx} - u^2 u_x + au_{xx}. \tag{23}$$

4. Generalized KP-Burgers equation

Introducing travelling wave variables

$$u(x,y,t) = u(\xi), \xi = x + ly - Vt. \tag{24}$$

Equation (2) is converted into an ODE for $u = u(\xi)$,

$$(-Vu' + u''' + au'' - bu^n u')' + \sigma^2 l^2 u'' = 0.$$

Integrating it with respect to ξ twice, and taking the constants of integration to zero, yields

$$u'' + au' + (\sigma^2 l^2 - V)u - \frac{b}{n+1}u^{n+1} = 0. \tag{25}$$

Comparing Eq (25) with Eq (6) in Section 2, it is found that Eq (25) belongs to the same type ODE as Eq (6), according to the theorem in Section 2,

$$\sigma^2 l^2 - V = \frac{2(n+2)}{(n+4)^2}a^2, \quad \text{which implies } V = \sigma^2 l^2 - \frac{2(n+2)}{(n+4)^2}a^2. \tag{26}$$

The travelling solution of general KP-Burgers equation is given by

$$u(x,y,t)=u(\xi)=\left[\frac{2(n+2)(n+1)a^2}{(n+4)^2 b}\right]^{\frac{1}{n}}\left(\frac{1}{1+\exp\left[\frac{an}{n+4}(\xi+\xi_0)\right]}\right)^{\frac{2}{n}},\ \xi_0=const, \tag{27}$$
$$\xi=x+ly-\left[\sigma^2 l^2-\frac{2(n+2)}{(n+4)^2}a^2\right]t.$$

In particular, when $n=1$, (27) becomes

$$u(x,y,t)=\frac{12}{25}\cdot\frac{a^2}{b}\left(\frac{1}{1+\exp\left[\frac{a}{5}(\xi+\xi_0)\right]}\right)^2, \tag{28}$$
$$\xi=x+ly-\left(\sigma^2 l^2-\frac{6}{25}a^2\right)t,\ \xi_0=const,$$

which is the solution of KP-Burgers equation

$$(u_t+u_{xxx}+au_{xx}-buu_x)_x+\sigma^2 u_{yy}=0. \tag{29}$$

When $n=2$, (27) becomes

$$u(x,y,t)=\frac{a}{3}\sqrt{\frac{6}{b}}\cdot\left(\frac{1}{1+\exp\left[\frac{a}{5}(\xi+\xi_0)\right]}\right),\ \xi=x+ly-\left(\sigma^2 l^2-\frac{1}{6}a^2\right)t,\ \xi_0=const, \tag{30}$$

which is the solution of mKP-Burgers equation

$$(u_t+u_{xxx}+au_{xx}-bu^2u_x)_x+\sigma^2 u_{yy}=0. \tag{31}$$

5. Generalized ZK-Burgers equation

Introducing travelling wave variables

$$u(x,y,z,t)=u(\xi),\ \xi=lx+my+pz-Vt, \tag{32}$$

where $l^2+m^2+p^2=1$. Substituting (32) into (3), integrating with respect to ξ, and taking the constants of integration to zero, yields

$$-Vu+\frac{\alpha l}{n+1}u^{n+1}+[(\beta-\gamma)l^3+\gamma l]u''+al^2u'=0. \tag{33}$$

Suppose that $\alpha<0, (\beta-\gamma)l^2+\gamma>0$, Eq (33) can be rewritten as

$$u''+\frac{al}{(\beta-\gamma)l^2+\gamma}u'-\frac{V}{(\beta-\gamma)l^3+\gamma l}u+\frac{\alpha}{(n+1)[(\beta-\gamma)l^2+\gamma]}u^{n+1}=0. \tag{34}$$

Equation (34) belongs to the same type ODE as Eq (6). According to the theorem in Section 2,

$$-\frac{V}{(\beta-\gamma)l^3+\gamma l}=\frac{2(n+2)}{(n+4)^2}\cdot\frac{a^2l^2}{[(\beta-\gamma)l^2+\gamma]^2},$$

which implies

$$V=-\frac{2(n+2)a^2l^3}{(n+4)^2[(\beta-\gamma)l^2+\gamma]^2},$$

and the solution of Eq (3) is given by

$$u(x,y,z,t)=u(\xi)=\left[-\frac{2(n+2)(n+1)a^2l^2}{(n+4)^2[(\beta-\gamma)l^2+\gamma]\alpha}\cdot\left(\frac{1}{1+\exp\left(\frac{n}{n+4}\cdot\frac{al(\xi+\xi_0)}{(\beta-\gamma)l^2+\gamma}\right)}\right)^2\right]^{\frac{1}{n}}, \tag{35}$$

where $\xi=lx+my+pz+\dfrac{2(n+2)a^2l^3}{(n+4)^2[(\beta-\gamma)l^2+\gamma]}t,\ l^2+m^2+p^2=1,\ \xi_0=const.$

In particular, when $n=1$, (35) becomes

$$u(x,y,z,t)=u(\xi)=-\frac{12}{25}\frac{a^2l^2}{[(\beta-\gamma)l^2+\gamma]\alpha}\cdot\left(\frac{1}{1+\exp\left(\frac{1}{5}\frac{al(\xi+\xi_0)}{(\beta-\gamma)l^2+\gamma}\right)}\right)^2, \tag{36}$$

where $\xi=lx+my+pz+\dfrac{6}{25}\cdot\dfrac{a^2l^3}{(\beta-\gamma)l^2+\gamma}t,\ l^2+m^2+p^2=1,\ \xi_0=const.$ Then Eq (36) is a bounded solution of ZK-Burgers equation

$$u_t+\alpha uu_x+\beta u_{xxx}+\gamma(u_{yy}+u_{zz})_x+au_{xx}=0. \tag{37}$$

6. Generalized Boussinesq equation

Introducing travelling wave variables

$$u(x,t)=u(\xi),\ \xi=x-Vt, \tag{38}$$

thus the Eq (4) can be reduced to ODE for $u=u(\xi)$,

$$(V^2-c^2)u''-b(u^{n+1})''-aVu'''+u^{(4)}=0.$$

Integrating it with respect to ξ twice, and taking the constants of integration to zero, yields

$$u''-aVu'+(V^2-c^2)u-bu^{n+1}=0, \tag{39}$$

where Eqs (39) and (6) belong to the same type. According to the theorem in Section 2,

$$V^2 - c^2 = \frac{2(n+2)a^2V^2}{(n+4)^2},$$

which implies $V = \pm \dfrac{c}{\sqrt{1 - \dfrac{2(n+2)a^2}{(n+4)^2}}}$, and the travelling wave solution of the general Boussinesq equation (4) is given by

$$u(x,t) = u(\xi) = A^{\frac{1}{n}} \left(\frac{1}{1+\exp\left[B(\xi+\xi_0)\right]} \right)^{\frac{2}{n}}, \tag{40}$$

where variables $A = \dfrac{2(n+2)a^2c^2}{[(n+4)^2 - 2(n+2)a^2]b}$, $B = \mp \dfrac{acn}{\sqrt{(n+4)^2 - 2(n+2)a^2}}$, $\xi = x \mp \dfrac{c}{\sqrt{1 - \dfrac{2(n+2)a^2}{(n+4)^2}}} t$, and $\xi_0 = const.$

7. Generalized (m+1)-dimension Klein-Gordon equation

Introducing travelling wave variables

$$u(x_1, x_2, \ldots, x_m, t) = u(\xi), \xi = \sum_{i=1}^{m} \alpha_i x_i - Vt, \sum_{i=1}^{m} \alpha_i{}^2 = 1, \tag{41}$$

the Eq (5) can be reduced to the ODE for $u = u(\xi)$,

$$(V^2 - c^2)u'' + au' + du + bu^{n+1} = 0. \tag{42}$$

Let $V^2 < c^2, b > 0, d < 0$, the ODE (42) can be rewritten as

$$u'' - \frac{a}{c^2 - V^2}u' - \frac{d}{c^2 - V^2}u - \frac{b}{c^2 - V^2}u^{n+1} = 0, \tag{43}$$

where Eqs (43) and (6) belong to the same type. According to the theorem in Section 2,

$$-\frac{d}{c^2 - V^2} = \frac{2(n+2)}{(n+4)^2} \frac{a^2}{(c^2 - V^2)^2},$$

which implies

$$V = \pm \sqrt{\frac{2(n+2)a^2}{(n+4)^2 d} + c^2}, \quad c^2 - V^2 = -\frac{2(n+2)a^2}{(n+4)^2 d}$$

and the travelling wave solution of the general Klein-Gordon equation (5) is given by

$$u(x_1,x_2,\ldots,x_m,t)=u(\xi)=\left(-\frac{d}{b}\right)^{\frac{1}{n}}\cdot\left(\frac{1}{1+\exp\left[\frac{(n+4)nd}{2(n+2)a}(\xi+\xi_0)\right]}\right)^{\frac{2}{n}}, \tag{44}$$

$$\xi=\sum_{i=1}^{m}\alpha_i x_i\mp\sqrt{\frac{2(n+2)a^2}{(n+4)^2 d}+c^2}\,t,\ \sum_{i=1}^{m}\alpha_i{}^2=1,\ \xi_0=const.$$

In particular, when $n=1$, (44) becomes

$$u(x_1,x_2,\ldots,x_m,t)=u(\xi)=-\frac{d}{b}\cdot\left(\frac{1}{1+\exp\left[\frac{5}{6}\cdot\frac{d}{a}(\xi+\xi_0)\right]}\right)^{2}, \tag{45}$$

$$\xi=\sum_{i=1}^{m}\alpha_i x_i\mp\sqrt{\frac{6}{25}\cdot\frac{a^2}{d}+c^2}\,t,\ \sum_{i=1}^{m}\alpha_i{}^2=1,\ \xi_0=const.$$

Then (45) is a travelling wave solution of Klein-Gordon equation

$$u_{tt}-c^2\Delta_m u+au_t+du+bu^2=0. \tag{46}$$

For $V^2>c^2, b<0, d>0$, there are some similar conclusions, the same travelling wave solution of Eq (5) is given by

$$u(x_1,x_2,\ldots,x_m,t)=u(\xi)=\left(-\frac{d}{b}\right)^{\frac{1}{n}}\cdot\left(\frac{1}{1+\exp\left[\frac{(n+4)nd}{2(n+2)a}(\xi+\xi_0)\right]}\right)^{\frac{2}{n}}, \tag{47}$$

$$\xi=\sum_{i=1}^{m}\alpha_i x_i\mp\sqrt{\frac{2(n+2)a^2}{(n+4)^2 d}+c^2}\,t,\ \sum_{i=1}^{m}\alpha_i{}^2=1,\ \xi_0=const.$$

8. Conclusions

In this paper, the nonlinear ODE equation (6) with an undetermined coefficient and a positive integer power term of dependent variable were introduced and solved. The formula of solution (7) with (8) to the nonlinear ODE (6) have played an important role for finding travelling wave solutions of a class of complicated nonlinear evolution equations with a dissipative term and a positive integer power term of dependent variable. We have successfully illustrated our technique in detail with five important model equations. The results show that the present method performs extremely well in terms of directness efficiency, simplicity and reliability to deal with various differential equations in the applied sciences, especially containing a dissipative term and a positive integer power term. It is worthy of note that the formula of solution (7) with (8) to the nonlinear ODE (6) can be also applied to other similar nonlinear evolution equations in complexity and nonlinear science to obtain their travelling wave solutions. It is expected that other extended and improved $(\frac{G'}{G})$-expansion methods will be more applied to solve nonlinear differential equations involving a dissipative term and a positive integer power term of dependent variable.

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Conflict of interest

The authors declare no conflict of interest.

References

1. M. L. Wang, Exact solutions for a compound KdV-Burgers equation, *Phys. Lett. A*, **213** (1996), 279–287. https://doi.org/10.1016/0375-9601(96)00103-X
2. M. L. Wang, Exact solutions for the RLW-Burgers equation, *Math. Appl.*, **8** (1995), 51–55.
3. A. N. Dev, J. Sarma, M. K. Deha, A. P. Misra, A. C. Adhihary, Kadomtsev-Petviashvili Burgers equation in dusty negative ion plasmas: Evolution of dustion acoustic shocks, *Commun. Theor. Phys.*, **62** (2014), 875–880. https://doi.org/10.1088/0253-6102/62/6/16
4. A. R. Seadawy, Nonlinear wave solutions of the three dimensional Zakharov-Kuznetsov-Burgers equation in plasma, *Physica A*, **439** (2015), 124–131. https://doi.org/10.1016/j.physa.2015.07.025
5. A. Esfahani, Instability of the stationary solutions of generalized dissipative Boussinesq equation, *Appl. Math.*, **59** (2014), 345–358. https://doi.org/10.1007/s10492-014-0059-1
6. J. A. Gonzakz, A. Bellorin, L. E. Guerrero, Kink-soliton explosion in generalized Klein-Gordon equations, *Chaos Soliton. Fract.*, **33** (2007), 143–155. https://doi.org/10.1016/j.chaos.2006.10.047
7. M. T. Darvishi, F. Khani, S. Kheybari, A numerical solution of the KdV-Burgers' equation by spectral collocation method and Darvishi's preconditionings, *Int. J. Contemp. Math. Sci.*, **2** (2007), 1085–1095.
8. M. T. Darvishi, S. Arbabi, M. Najafi, A. M. Wazwazet, Traveling wave solution of a (2+1)-dimensional Zakharov-like equation by the first integral method and the tanh method, *Optik*, **127** (2016) , 6312–6321. https://doi.org/10.1016/j.ijleo.2016.04.033
9. M. T. Darvishi，M. Najafi，A. M. Wazwaz, Soliton solutions for Boussinesq-like equations with spatio-temporal dispersion, *Ocean Eng.*, **130** (2017), 228–240. https://doi.org/10.1016/j.oceaneng.2016.11.052
10. M. T. Darvishi, F. Khani, S. Hamedi-Nezhad, S. W. Ryu, New modification of the HPM for numerical solutions of the sine-Gordon and coupled sine-Gordon equations, *Int. J. Comput. Math.*, **87** (2010), 908–919. https://doi.org/10.1080/00207160802247596
11. M. Li, W. K. Hu, C. F. Wu, Rational solutions of the classical Boussinesq-Burgers system, *Nonlinear Dyn.*, **94** (2018), 1291–1302. https://doi.org/10.1007/s11071-018-4424-6

12. M. L. Wang, X. Z. Li, J. L. Zhang, The $(\frac{G'}{G})$-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics, *Phys. Lett. A*, **372** (2008), 417–423. https://doi.org/10.1016/j.physleta.2007.07.051
13. M. T. Darvishi, M. Najafi. Traveling wave solutions for the (3+1)-dimensional breaking soliton equation by $(\frac{G'}{G})$-expansion method and modified F-expansion method, *Int. J. Comput. Math. Sci.*, **7** (2011), 1100–1105. https://doi.org/10.5281/zenodo.1078723
14. L. Akinyemi, M. Mirzazadeh, K. Hosseini, Solitons and other solutions of perturbed nonlinear Biswas-Milovic equation with Kudryashov's law of refractive index, *Nonlinear Anal.-Model.*, **27** (2022), 479–495. https://doi.org/10.15388/namc.2022.27.26374
15. K. Hosseini, A. Zabihi, F. Samadani, R. Ansari, New explicit exact solutions of the unstable nonlinear Schrödinger's equation using the expa and hyperbolic function methods, *Opt. Quant. Electron.*, **50** (2018), 1–8. https://doi.org/10.1007/s11082-018-1350-2
16. W. Yuan, F. Meng, Y. Huang, Y. Wu, All traveling wave exact solutions of the variant Boussinesq equations, *Appl. Math. Comput.*, **268** (2015), 865–872. https://doi.org/10.1016/j.amc.2015.06.088
17. W. Yuan, F. Meng, J. Lin, Y. Wu, All meromorphic solutions of an ordinary differential equation and its applications, *Math. Meth. Appl. Sci.*, **39** (2016), 2083–2092. https://doi.org/10.1002/mma.3625
18. Y. Gu, W. Yuan, N. Aminakbari, J. M. Lin, Meromorphic solutions of some algebraic differential equations related Painlev´e equation IV and its applications, *Math. Meth. Appl. Sci.*, **41** (2018), 3832–3840. https://doi.org/10.1002/mma.4869
19. Y. Gu, C. Wu, X. Yao, W. Yuan, Characterizations of all real solutions for the KdV equation and WR, *Appl. Math. Lett.*, **107** (2020), 106446. https://doi.org/10.1016/j.aml.2020.106446
20. N. Kadkhoda, H. Jafari, Analytical solutions of the Gerdjikov-Ivanov equation by using $\exp(-\psi(\xi))$-expansion method, *Optik*, **139** (2017), 72–76. https://doi.org/10.1016/j.ijleo.2017.03.078
21. Y. Gu, N. Aminakbari, Two different systematic methods for constructing meromorphic exact solutions to the KdV-Sawada-Kotera equation, *AIMS Math.*, **5** (2020), 3990–4010. https://doi.org/10.3934/math.2020257
22. M. L. Wang, X. Z. Li, Various exact solutions of nonlinear Schrödinger equation with two nonlinear terms, *Chaos Soliton. Fract.*, **31** (2007) 594–601. https://doi.org/10.1016/j.chaos.2005.10.009
23. C. H. Wu, Z. Q. Wang, The spectral collocation method for solving a fractional integro-differential equation, *AIMS Math.*, **7** (2022) 9577–9587. https://doi.org/10.3934/math.2022532
24. A. El-Ajou, Adapting the Laplace transform to create solitary solutions for the nonlinear time-fractional dispersive PDEs via a new approach, *Eur. Phys. J. Plus*, **136** (2021), 229–251. https://doi.org/10.1140/epjp/s13360-020-01061-9