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Research article

Data-driven two-stage fuzzy random mixed integer optimization model for facility location problems under uncertain environment

Zhimin Liu[1,*], **Ripeng Huang**[2] **and Songtao Shao**[3]

[1] School of Mathematics Science, Liaocheng University, Shandong, China
[2] School of Mathematics and Finance, Chuzhou University, Chuzhou, China
[3] Shaanxi University of Science and Technology, Xian, China

* **Correspondence:** Email: lzmliuzhimin@163.com.

Abstract: This paper studies the problem of facility location in a hybrid uncertain environment with both randomness and fuzziness. We establish a data-driven two-stage fuzzy random mixed integer optimization model, by considering the uncertainty of transportation cost and customer demand. Given the complexity of the model, this paper based on particle swarm optimization (PSO), beetle antenna search algorithm (BAS) and interior point algorithm, a hybrid intelligent algorithm (HIA) is proposed to solve two-stage fuzzy random mixed integer optimization model, yielding the optimal facility location and maximal expected return of supply chain simultaneously. Finally, taking the supply chain of medical mask in Shanghai as an example, the influence of uncertainty on the location of processing factory was studied. We compare the HIA with hybrid PSO and hybrid genetic algorithm (GA), to validate the proposed algorithm based on the computational time and the convergence rate.

Keywords: two-stage fuzzy random mixed integer optimization; facility location; fuzzy random; hybrid intelligent algorithm
Mathematics Subject Classification: : 90B06, 90C90

1. Introduction

Facility location is one of the most critical and strategic issues in the design and management of supply chain network, and it has been a focus and has been studied by many scholars [1–7]. The objective of the facility location strategy is to determine which plants to open by identifying a group of potential plant locations to maximize supply chain profits. The importance of facility location lies in that once the location is selected and the distribution mode is determined, it will directly affect the service mode, quality, efficiency, and cost, thus effecting the profit and market competitiveness, and even determining the fate of enterprises. The improper allocation of the site has consequences that are challenging to make up through other management follow-up measures. Therefore, the study of the facility location has vast economic and social significance.

Facility location design and planning invariably need to entertain numerous uncertainties. Conventional facility location modeling tends to focus on the randomness aspect of uncertainty. For that matter, a number of stochastic modeling techniques have been developed to deal with uncertainty and have been successfully applied to facility location design [8–13]. Laporte et al. established a kind of capacity facility location problem with random demand by using stochastic integer linear programming, and proposed a branch cutting method to solve the problem [8]. Liu et al. established a two-stage mean-risk stochastic mixed integer optimization model to solve the location problem in a random environment [9]. Wang et al. studied a facility location problem with random customer demand and fixed server, and designed a heuristic algorithm to solve the problem [10]. Zadeh et al. established a mixed integer nonlinear programming model and a mixed integer linear programming model to solve the dynamic multi-commodity inventory and facility location problem [11]. Armas et al. studied the problem of uncapacitated facility location with random demand and service cost, and proposed a simheuristic algorithm to solve the problem [12].

In addition, with the continuous development of fuzzy theories such as fuzzy set [14, 15], possibility [16] and credibility [17], many scholars describe uncertain parameters with fuzzy set or fuzzy variable and give various fuzzy facility location models [18–23]. Choua and Shen proposed a new fuzzy multi-attribute decision-making method to solve the facility location selection problem [18]. Wang et al. proposed a two-stage fuzzy facility location problem with risk value, and solved it by using a hybrid genotype-phenotype-mutation-based binary particle swarm optimization algorithm [19]. Rezaei and Zarandi proposed a continuous facility location model with fuzzy methodology [20]. Paksoy et al. considered the fuzzy supply chain network design problem composed of multiple suppliers, manufacturers, distribution centers and retailers, and established a mixed integer linear programming model [21]. Wang et al. established a two-stage capacitated facility location model with fuzzy costs and demands, and designed a hybrid algorithm to solve the proposed facility location problem [22].

In practice, fuzziness and randomness may coexist in the facility location problem. In today's highly competitive market, product life cycles are getting shorter and shorter, which makes the historical data statistics less accurate. There may be inaccurate human perception, and the parameters of the real supply chain may embrace randomness and fuzziness at the same time. Moreover, the historical data for the statistics may not be sufficient, so the empirical fuzzy information can be incorporated into the initially available statistics. This means that fuzzy random parameters are more suitable for practical application. Fuzzy random theory provided an acceptable alternative to deal with such uncertainty [24, 25]. Therefore, this paper studies the facility location problem in fuzzy random environment.

Wen and Iwamura uses the (α,β)-cost minimization model under Hurwicz criterion to consider the facility location-allocation problem in random fuzzy environment [26]. Wang et al. developed a two stage mixed integer programming model in a multi-facility multi-retailer under a fuzzy random operating costs and demands environment, and a hybrid mutation based binary ant colony optimization algorithm was proposed to solve the model [27]. Wen and Kang studied the facility location-allocation problem and proposed a cost minimization model and chance maximization model with random fuzzy demands, and proposed a hybrid intelligent algorithm to solve the problem [28]. Uno et al. studied competitive facility location problem with fuzzy random demands [29]. Watada studied the facility location problem with fuzzy random demand and established a two-stage fuzzy random facility location model [30].

Clearly, consumer demand uncertainty leads to an uncertainty in the product quantity demanded at each stage of the supply chain. Most studies, as highlighted earlier, tend to focus on the facility location problem under the situation of a single product, manufacturer or retailer. In reality, however, many facility location problems are not of a unitary type. Furthermore, to the best of our knowledge, there is a dearth of research on the multi-supplier, multi-processing factory, multi-retailer, multi-consumer, multi-raw material, and multi-product facility location problem under fuzzy random demand and fuzzy random transportation cost. Thus, to fill this gap, in this paper, we consider the coordination mechanism of a multi-supplier, multi-processing factory, multi-retailer, multi-consumer, multi-raw material, and multiple product. Based on the above mechanism, we establish a two-stage fuzzy random mixed integer optimization model. In the first stage, the location variable is the decision variable in the supply chain network. These decision variables are determined according to different realisation of the scenarios in the second stage, hence the decision process is called "here and now". The quantity of transportation between each member in the supply chain network is the variable of the second stage, which is related to the scenario. Different from [9, 23], this paper studied facility location in a fuzzy random environment, while [9] only considered randomness in an uncertain environment and lacked empirical fuzzy information, and [23] only considered fuzziness in an uncertain environment and lacks random information. In addition, in order to improve customer satisfaction, we use historical data on sales and transportation costs, as well as empirical fuzzy information, to accurately estimate transportation costs and demand between nodes of the supply chain through a data-driven method, so as to ensure consumer demand and reduce overall operating costs of the supply chain.

Recognizing the complexity of the model, we find that the traditional algorithm cannot solve the two-stage fuzzy random mixed integer optimization problem. Therefore, another contribution of this study comes from the development of algorithms. In particular, we propose a HIA, whose idea is to add BAS to PSO. In other words, each particle in PSO is described as a beetle and searched. It has the advantage of avoiding particle falling into local solution in the process of iteration. Moreover, the algorithm has the advantages of fast search speed and high efficiency. In the iterative process, we apply the interior point algorithm to solve the second stage problem. The main contributions of this paper are summarized as follows:

- Considering the uncertainty of the supply chain, a data-driven two-stage fuzzy random mixed integer optimization model is established.
- HIA is proposed to solve two-stage fuzzy random mixed integer optimization model.
- We compare the HIA with hybrid PSO and hybrid GA, to validate the proposed algorithm based on the computational time and the convergence rate.
- Based on the numerical results, management insights are presented.

The rest of this paper is set as follows. Section 2 describes the mathematical preliminaries for the fuzzy random variable and establishes a two-stage fuzzy random mixed integer optimization model. Section 3 then presents a solution algorithm for the model. The computational results of a numerical example are presented and discussed in Section 4. Section 5 concludes with some future research directions.

2. Two-stage fuzzy random mixed integer optimization model

In this section, we establish a two-stage fuzzy random mixed integer optimization model under the uncertainty of customer demand and transportation cost. Firstly, we review some basic concepts of fuzzy random variables to help readers understand the proposed two-stage fuzzy random optimization model.

2.1. Fuzzy random variable

Kwakernaak [24] first proposed the concept of fuzzy random variables, and then this concept and related theories have made great progress [25, 31, 32]. From the point of view of optimization, Liu et al. gives a new definition of fuzzy random variable and its expected value operator [25]. It is proved to be a suitable basis for fuzzy random system optimization and numerical simulation.

Let $(\Theta, \mathscr{A}(\Theta), Pos)$ be a possibility space, where $\mathscr{A}(\Theta)$ is the power set of Θ, and X be a fuzzy variable defined on $(\Theta, \mathscr{A}(\Theta), Pos)$ whose membership function is μ_X. The possibility and credibility of the event $X \leq r$, r beings a real number, is expressed as follows:

$$Pos\{X \leq r\} = \sup_{t \leq r} \mu_X(t), \tag{2.1}$$

$$Cr\{X \leq r\} = \frac{1}{2}(\sup_{t \leq r} \mu_X(t) + 1 - \sup_{t > r} \mu_X(t)). \tag{2.2}$$

Let $(\Omega, \mathscr{B}, Pr)$ be a probability space, $\mathscr{F}^n$ be the set of $n-$ary fuzzy vectors defined in the possibility space, and then the concept of fuzzy random vector is defined as follows.

Definition 1. ([25]) A fuzzy random vector is a mapping $\zeta = (\zeta_1, \zeta_2, \cdots, \zeta_n) : \Omega \to \mathscr{F}^n$, such that for any closed subset $C \subset \Re^n$, $Pos\{\gamma | \zeta(\omega, \gamma) \in C\}$ is a $\mathscr{B}-$measurable function of $\omega \in \Omega$, i.e., for any $t \in [0, 1]$, we have

$$\{\omega \in \Omega | Pos\{\gamma | \zeta(\omega, \gamma) \in C\} \leq t\} \in \mathscr{B}.$$

In this paper, in order to represent the expected value of the second stage, we calculate the expected value of the second stage in the fuzzy random environment. In this regard, Liu et al. have obtained the next definition [25].

Definition 2. ([25]) Let ζ be a fuzzy random variable defined on the probability space $(\Omega, \mathscr{B}, Pr)$. The expected value of a fuzzy random variable ζ, $E(\zeta)$ is defined as

$$E[\zeta] = \int_{\Omega} [\int_0^{\infty} Cr\{\zeta(\omega) \geq r\} dr - \int_{-\infty}^{0} Cr\{\zeta(\omega) \leq r\} dr] Pr(d\omega), \tag{2.3}$$

where $Cr\{\cdot\}$ is the credibility measure, and $Pr\{\cdot\}$ is the probability measure.

2.2. Establish model

In this subsection, we establish a two-stage fuzzy random mixed integer optimization model under the uncertainty of customer demand and transportation cost. The structure of supply chain network is shown as Figure 1. In particular, we considered S suppliers, I processing factories, J retailers and K types of customers. A typical supplier, processing factory, retailer, and customer are represented by s, i, j and k respectively. The links in the supply chain network denote the transaction links. The supplier supplies V kinds of raw materials, the processing factory produces L kinds of products, and then transport them to the retailer, who meets the customer's fuzzy random demand. The objective of the company is to maximize the expected profit by choosing the optimal number of processing plants in the market area on the premise of meeting customer fuzzy random demand.

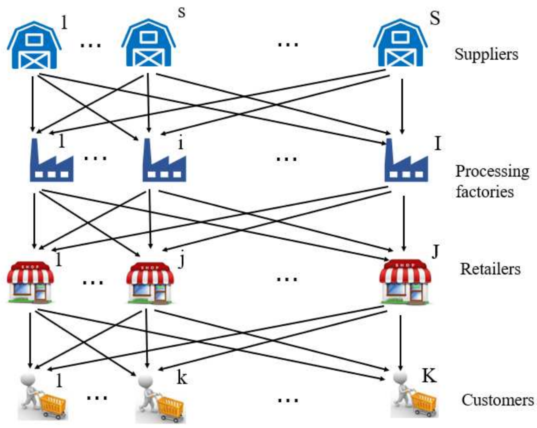

Figure 1. Network structure of supply chain.

For simplicity, we apply the following symbols for this model. All the vectors used in this paper are assumed to be column vectors.

2.2.1. Notation

Indices and set

s Suppliers index
i Processing factories index
j Retailers index
k Customers index
v Raw materials index
l Products index
S Set of the suppliers
I Set of the processing factories
J Set of the retailers
K Set of the customers
V Set of the raw materials
L Set of the products

Parameters

f_i Fixed cost of operating processing factory i
a_{vs} The ability of supplier s to provide raw material v
r_{vs} The cost of raw materials v provided by the supplier s
n_{vl} The quantity of raw material v required for processing product l

b_{il} The ability of processing factory i to produce product l
q_{il} The cost of producing product l in the processing factory i
τ_{lj} The ability of retailer j to sell product l
h_{lj} Retail price of unit product l of retailer j
$\tilde{t}_{vsi}$ Fuzzy random transportation cost of supplier s transporting raw material v to processing factory i
$\tilde{m}_{lij}$ Fuzzy random transportation cost of processing factory i transporting product l to retailer j
$\tilde{d}_{lk}$ Fuzzy random demand of consumer k for product l
ζ Fuzzy random demand and transportation cost vector $\zeta = (\tilde{t}_{vsi}, \tilde{m}_{lij}, \tilde{d}_{lk})$
$E(\cdot)$ Expectation

Variables

e_i Binary variables, open processing factory i, value 1, otherwise 0, a first-stage decision variable
x_{vsi} The quantity of raw materials v transported by the supplier s to the processing factory i, a second-stage decision variable
y_{lij} The quantity of product l transported by the processing factory i to the retailer j, a second-stage decision variable
z_{ljk} The quantity of product l transported by the retailer j to the consumer k, a second-stage decision variable

2.2.2. The model

Assumptions

1) Retailers provide products to meet customer needs.
2) Fuzzy random demand and transportation cost vector $\zeta = (\tilde{t}_{vsi}, \tilde{m}_{lij}, \tilde{d}_{lk})$ is defined from a probability space $(\Omega, \mathscr{B}, Pr)$ to a collection of fuzzy vectors on possibility space $(\Theta, \mathscr{A}(\Theta), Pos)$.
3) The probability space $(\Omega, \mathscr{B}, Pr)$ is finite.

Based on the above assumptions and description, we present a two-stage fuzzy random mixed integer optimization model as follows,

$$\begin{aligned} \max_{\mathbf{e}} \quad & \mathscr{Q}(\mathbf{e}) - \sum_{I} f_i e_i \\ \text{s.t.}\ & e_i \in \{0,1\}, \quad \forall i \in I. \end{aligned} \tag{2.4}$$

where $\mathbf{e} = (e_1, \cdots, e_I)$, $\mathscr{Q}(\mathbf{e}) = E[Q(\mathbf{e}, \zeta(\omega,\gamma))]$, and $Q(\mathbf{e}, \zeta(\omega,\gamma))$ is the optimal value of the second-stage problem

$$\begin{aligned} Q(\mathbf{e}, \zeta(\omega,\gamma)) = \max_{\mathbf{x},\mathbf{y},\mathbf{z}} \quad & \sum_{L}\sum_{J}(h_{lj}\sum_{K} z_{ljk}) - \sum_{I}\sum_{L}(q_{il}\sum_{J} y_{lij}) - \sum_{V}\sum_{S}(r_{vs}\sum_{I} x_{vsi}) \\ & - \sum_{L}\sum_{I}\sum_{J}\tilde{m}_{lij}(\omega,\gamma) y_{lij} - \sum_{V}\sum_{S}\sum_{I}\tilde{t}_{vsi}(\omega,\gamma) x_{vsi} \end{aligned} \tag{2.5}$$

$$\text{s.t.} \quad \sum_{I} x_{vsi} \le a_{vs} \ \ \forall v,\ s, \tag{2.6a}$$

$$\sum_{L}(n_{vl}\sum_{J} y_{lij}) \le \sum_{S} x_{vsi} \ \ \forall v,\ i, \tag{2.6b}$$

$$\sum_{J} y_{lij} \le e_i b_{il} \ \ \forall i,\ l, \tag{2.6c}$$

$$\sum_K z_{ljk} \leq \sum_I y_{lij} \ \ \forall l,\ j, \tag{2.6d}$$

$$\sum_K z_{ljk} \leq \tau_{lj} \ \ \forall l,\ j, \tag{2.6e}$$

$$\sum_J z_{ljk} = \tilde{d}_{lk}(\omega,\gamma) \ \ \forall l,\ k, \tag{2.6f}$$

$$x_{vsi}, y_{lij}, z_{ljk} \geq 0, \ \ \forall v,\ s,\ i,\ j,\ k,\ l. \tag{2.6g}$$

where, $\tilde{t}_{vsi}(\omega,\gamma), \tilde{m}_{lij}(\omega,\gamma)$, and $\tilde{d}_{lk}(\omega,\gamma)$ are the realizations of the fuzzy random transportation cost $\tilde{t}_{vsi}, \tilde{m}_{lij}$, and fuzzy random demand $\tilde{d}_{lk}$, respectively, for any $(\omega,\gamma) \in \Omega \times \Theta$.

In the two-stage fuzzy random mixed integer optimization model, the objective function (2.4) represents the expected profit maximization of the supply chain. The second-stage problem (2.5) consists of five parts. The first term is the revenue obtained by selling products, the second term is the production cost of the processing plant, the third term is the cost of raw materials, and the last two are the transportation costs. Constraints (2.6a) and (2.6c) represent the raw material constraints of suppliers and the production capacity constraints of processing plants, respectively. Constraints (2.6b) and (2.6d) represent the balance conditions of raw materials and products, respectively. Constraint (2.6e) represents the sales capacity constraints of retailers. Constraint (2.6f) ensures that products distributed by retailers to customers can meet customer needs. The final constraint (2.6g) guarantees the non-negativity of decision variables.

In model (2.4)–(2.6g), the two-stage process of fuzzy random supply chain problem is shown in Figure 2. The decision vector **e** is the first-stage decision which must be considered before the realizations of fuzzy random demand and transportation cost vector $\zeta(\omega,\gamma)$ coming out. In the second-stage, the fuzzy random demand and transportation cost $\zeta(\omega,\gamma)$ and **e** are known, the decision variable are **x**, **y**, **z**. The purpose is to determine the allocation mode and maximize the profit.

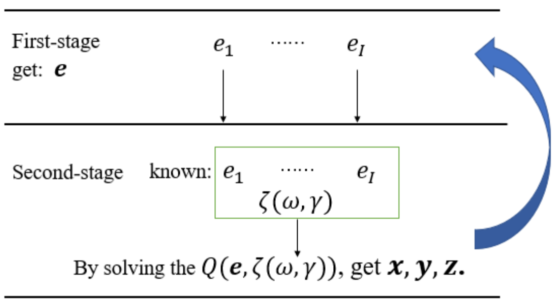

Figure 2. Two-stage process of fuzzy random supply chain problem.

3. Solution algorithm

This section focuses on the computation of the two-stage fuzzy random mixed integer optimization model (2.4) and (2.5). Firstly, we study the calculation of recourse function $\mathscr{Q}(\mathbf{e})$. To get $\mathscr{Q}(\mathbf{e})$, we need to compute $Q(\mathbf{e}, \zeta(\omega, \gamma))$ by solving the second-stage problem (2.5) of each (ω, γ). However, in general, for each ω, if $\zeta(\omega)$ is a continuous fuzzy vector, computing $\mathscr{Q}(\mathbf{e})$ becomes solving an infinite linear programming problem. In addition, if ω is also a continuous random variable, it will be more difficult to solve for $\mathscr{Q}(\mathbf{e})$. When fuzzy vectors are continuous, Wang et al. proposed a method to construct a discrete fuzzy vector for approximating continuous fuzzy vectors [27]. Without loss of generality, in the following discussion, we assume that the fuzzy random vector $\zeta(\omega, \gamma)$ is discrete. At the same time, from problem (2.5), we find that $\mathscr{Q}(\mathbf{e})$ cannot be expressed in an analytical way, and the traditional optimization algorithm cannot solve the two-stage fuzzy random mixed-integer programming (2.4) and (2.5).

The expected value of fuzzy random variable given in Definition 2 is not only applicable to the case of continuous fuzzy random variable, but also applicable to the case of discrete fuzzy random variable. Next, we use the expected value Definition (2.3) of fuzzy random vector and interior point algorithm to calculate $\mathscr{Q}(\mathbf{e})$.

Let fuzzy random vector $\zeta(\omega, \gamma)$ be a discrete one such that ω is a discrete random vector taking on a finite number of values ω_n with probability p_n, $n \in N$, where N is the set of scenarios, and for each n, $\zeta(\omega_n)$ is a discrete fuzzy vector taking on a finite number of values γ_{nm} with membership degree μ_{nm}, $m \in M$, i.e.

$$\zeta(\omega, \gamma) = \begin{cases} \zeta(\omega_1), & p_1 \\ \quad\vdots & \vdots \\ \zeta(\omega_n), & p_n \\ \quad\vdots & \vdots \\ \zeta(\omega_N), & p_N \end{cases}, \tag{3.1}$$

$$\mu(\zeta(\omega_n)) = \begin{cases} \mu_{n1}, & \zeta(\omega_n) = \gamma_{n1} \\ \quad\vdots & \quad\vdots \\ \mu_{nm}, & \zeta(\omega_n) = \gamma_{nm} \\ \quad\vdots & \quad\vdots \\ \mu_{nM}, & \zeta(\omega_n) = \gamma_{nM} \end{cases}, \tag{3.2}$$

here $\mu_{n1} \vee \mu_{n2} \vee \cdots \vee \mu_{nM} = 1$.

For each n, m and fixed $\mathbf{e}$, compute the $Q(\mathbf{e}, \gamma_{nm})$ through the interior point algorithm. Based on [33], using (2.1)–(2.3), we obtain

$$\mathscr{Q}(\mathbf{e}) = \sum_{n=1}^{N} p_n \mathscr{Q}(\mathbf{e}, \zeta(\omega_n)), \tag{3.3}$$

where

$$\mathscr{Q}(\mathbf{e}, \zeta(\omega_n)) = \sum_{m=1}^{M} p_{nm} Q(\mathbf{e}, \gamma_{nm}), \tag{3.4}$$

the corresponding weights p_{nm} are given in the form

$$p_{nm} = \frac{1}{2}(\max_{1\le t\le M}\{\mu_{nt}|Q(\mathbf{e},\gamma_{nt}) \le Q(\mathbf{e},\gamma_{nm})\} - \max_{1\le t\le M}\{\mu_{nt}|Q(\mathbf{e},\gamma_{nt}) < Q(\mathbf{e},\gamma_{nm})\})$$
$$+ \frac{1}{2}(\max_{1\le t\le M}\{\mu_{nt}|Q(\mathbf{e},\gamma_{nt}) \ge Q(\mathbf{e},\gamma_{nm})\} - \max_{1< t\le M}\{\mu_{nt}|Q(\mathbf{e},\gamma_{nt}) > Q(\mathbf{e},\gamma_{nm})\}),$$

and satisfy the following constraint

$$p_{nm} \ge 0, \quad \sum_{m=1}^{M} p_{nm} = 1, \quad n = 1, \cdots, N.$$

Medsker introduced many ideas for designing hybrid intelligent systems, which integrate a variety of intelligent algorithms to produce more powerful and effective algorithms [34]. In particular, Liu designed hybrid intelligent algorithms for solving fuzzy random chance-constrained programming and dependent-chance programming [35, 36]. Wang et al. proposed a hybrid mutation-based binary ant-colony optimization method to solve two-stage fuzzy random facility location model with recourse [27]. Wang and Watada proposed a hybrid modified PSO approach to the two-stage VaR-based fuzzy random facility location model with variable capacity [37].

Among many intelligent algorithms, PSO has the advantages of fast search speed, high efficiency and simple, but it is easy to fall into local optimum. In order to overcome this shortcoming, this paper considers adding BAS to PSO. The BAS is a new technology proposed by Jiang and Li to find the best solution [38]. In other words, each particle in PSO is described as a beetle and searched. The initial position and speed of beetle are the same as that of standard PSO. In the iteration process, the idea of BAS is added to avoid falling into local solution.

The updated formula of beetle swarm velocity vector is as follows:

$$\mathscr{V}_{bi'} = \delta^{k'}\vec{\mathbf{b}}sign(Fit(\mathbf{e}_r) - Fit(\mathbf{e}_l)), \tag{3.5}$$

$$\mathscr{V}_{i'}^{k'+1} = \mathscr{V}_{i'}^{k'} + c_1 \cdot rand \cdot (P_{di'}^{k'} - \mathbf{e}_{i'}^{k'}) + c_2 \cdot rand \cdot (P_{gi'}^{k'} - \mathbf{e}_{i'}^{k'}) + c_3 \cdot rand \cdot \mathscr{V}_{bi'}. \tag{3.6}$$

In (3.5), $\mathscr{V}_{bi'}$ represents the search behavior of beetle i', where $\vec{\mathbf{b}} = \frac{rands(I,1)}{\|rands(I,1)\|}$, $\delta^{k'}$ represents the step size of the $k' - th$ iteration, apply $sign(Fit(\mathbf{e}_r) - Fit(\mathbf{e}_l))$ to determine the odor intensity of the beetles' left and right antennae. $Fit(\mathbf{e}) = \mathscr{Q}(\mathbf{e}) - \sum_I f_i e_i$ is the fitness function. $\mathbf{e}_l = \mathbf{e}_{i'}^{k'} - d^{k'}\vec{\mathbf{b}}$, $\mathbf{e}_r = \mathbf{e}_{i'}^{k'} + d^{k'}\vec{\mathbf{b}}$, represents the antenna search behavior on the left and right sides of the beetle, respectively. d is the antenna perception length corresponding to the detection ability, and $d^{k'} = 0.95d^{k'-1} + 0.01$. In (3.6), $\mathscr{V}_{i'}^{k'+1}$ denotes the velocity of the i' particle after the $k' - th$ iteration, c_1, c_2, c_3 are learning rates, $P_{di'}^{k'}$ and $P_{gi'}^{k'}$ represent the personal best position and global best position of particle i' in the $k' - th$ iteration, respectively. Particle i' will compare the fitness function values on its left and right sides during each iteration, and compare the better values of both, which can be used to update the position of the particle swarm. HIA constructed by this method can well overcome the problems of poor stability and local optimum caused by PSO algorithm.

In order to improve the search efficiency of particles, avoid the expansion and divergence of the population, and avoid the blind search of particles in a wide range, we introduce the following boundary conditions:

$$(\mathscr{V}_{i'}^{k'+1})_i = \begin{cases} rand \cdot (\mathscr{V}_{max} - \mathscr{V}_{min}) + \mathscr{V}_{min}, (\mathscr{V}_{i'}^{k'+1})_i > \mathscr{V}_{max} \\ rand \cdot (\mathscr{V}_{max} - \mathscr{V}_{min}) + \mathscr{V}_{min}, (\mathscr{V}_{i'}^{k'+1})_i < \mathscr{V}_{min} \\ (\mathscr{V}_{i'}^{k'+1})_i, \qquad others \end{cases}. \tag{3.7}$$

In (3.7), $\mathscr{V}_{max}$ is the maximum velocity limit and $\mathscr{V}_{min}$ is the minimum velocity limit. Since the value and change of the particle in the state space are limited to the two values of 0 and 1, and the $(\mathscr{V}_{i'}^{k'+1})_i$ of each dimension of velocity represents the possibility that the value of position $(\mathbf{e}_{i'}^{k'+1})_i$ is 1. The location update equation [39] is expressed as follows:

$$\mathscr{S}(\mathscr{V}_{i'}^{k'+1}) = \frac{1}{1 + exp(-\mathscr{V}_{i'}^{k'+1})}, \tag{3.8}$$

$$(\mathbf{e}_{i'}^{k'+1})_i = \begin{cases} 1, & \mathscr{S}(\mathscr{V}_{i'}^{k'+1})_i > rand \\ 0, & others \end{cases}. \tag{3.9}$$

In (3.8), $\mathscr{S}(\mathscr{V}_{i'}^{k'+1})$ is a sigmoid function. In (3.9), $\mathbf{e}_{i'}^{k'+1}$ denotes the position of the i' particle after the $k' - th$ iteration.

Based on the above description, we give the detailed calculation process of the hybrid intelligent algorithm.

Algorithm 1: Hybrid intelligent algorithm (HIA)

Step 0. Initialize the particle swarm. The population size is I', maximum number of iterations T, $\mathscr{V}_{max} > \mathscr{V}_{min}$, $c_1 > 0, c_2 > 0, c_3 > 0, d > 0, \delta > 0$, $P_d^0 = ones(I', 1)$, $P_g^0 = eps$, $\mathbf{e}_{best} = ones(1, I)$, $k' = 0$, $i' = 1$.

Step 1. Initializes particle velocity and position. Randomly obtain the initial population of binary code $\mathbf{e}^0 = round(rand(I', I))$, $P = \mathbf{e}^0$, $\mathscr{V}^0 = rand(I', I) \cdot (\mathscr{V}_{max} - \mathscr{V}_{min}) + \mathscr{V}_{min}$.

Step 2. If $k' > T$, Stop. Return particle $\mathbf{e}_{best}$ as the optimal solution, and $Fit(\mathbf{e}_{best})$ as optimal value.

Step 3. If $i' > I'$, set $k' = k' + 1$, $i' = 1$, go to Step 2.

Step 4. Calculate the fitness $Fit(\mathbf{e}_{i'}^{k'})$ through the interior point algorithm and (3.3) and (3.4).

Step 5. Update the individual's optimal position and value. If $Fit(\mathbf{e}_{i'}^{k'}) > P_{di'}^{k'}$, $P_{i'} = \mathbf{e}_{i'}^{k'}$, $P_{di'}^{k'} = Fit(\mathbf{e}_{i'}^{k'})$, else $P_{di'}^{k'} = P_{di'}^{k'}$.

Step 6. Update the global optimal position and value. If $P_{di'}^{k'} > P_g^{k'}$, $\mathbf{e}_{best} = P_{i'}$, $P_g^{k'} = P_{di'}^{k'}$, else $P_g^{k'} = P_g^{k'}$.

Step 7. Update velocity particles by (3.5) and (3.6).

Step 8. Boundary condition treatment by (3.7).

Step 9. Update location particle by (3.8) and (3.9).

Step 10. Set $i' = i' + 1$, go to Step 3.

Theorem 1. The complexity of Algorithm 1 in solving model two-stage fuzzy random mixed integer optimization model (2.4) and (2.5) is $O(TI'MN \cdot D^{3.5^{log\frac{1}{\epsilon}}})$, where, D is the variable dimension in (2.5), $D = VSI + LIJ + LJK$, and ϵ is the tolerance of dual gap in the interior point algorithm.

Proof. According to the Algorithm 1 Step 0, we know that the maximum iteration is T. From Step 3 and Step 4 combined with (3.3) and (3.4), it can be seen that in each iteration process, to calculate the fitness function, the interior point algorithm needs to be applied $I'MN$ times to solve $Fit(\mathbf{e})$. According to [40], the complexity of interior point algorithm is $O(D^{3.5^{log\frac{1}{\epsilon}}})$. Therefore, the complexity of Algorithm 1 is $O(TI'MN \cdot D^{3.5^{log\frac{1}{\epsilon}}})$.

4. Numerical results

In this section, we apply the HIA to solve a numerical example and provide a discussion of results.

All the program codes are written on MATLAB R2014a using Lenovo computers running on Intel(R) Core(TM) i7-8565U CPU @ 1.80 GHz, 8.00-GB memory.

Throughout the computational experiments, the parameters in HIA are taken as: $I' = 10$, $\mathscr{V}_{max} = 10$, $\mathscr{V}_{min} = -10$, $c_1 = 1.5, c_2 = 1.5, c_3 = 1.0, d = 2.0, \delta = 0.5$. The stopping criterion for the algorithm is $T = 1000$.

In the respective tables of the numerical results, Fit denotes the total supply chain's profit, TI denotes the computing time in seconds. In what follows, we provide a description of the test problem.

Mask has played an important role in personal protection against COVID-19, which is mainly transmitted by droplets and contact. The COVID-19 outbreak has led to increased demand for face masks. Based on the sales of face masks in designated retail pharmacies under medical insurance, combined with the daily appointment demand data of street and neighborhood committees, this paper studied the location of a brand of mask processing factory in Shanghai and the distribution of masks. In the supply chain network, two suppliers provide raw materials, produce masks in four factories to be selected, and retail them in ten pharmacies, so as to meet the needs of consumers in four demand areas of Shanghai for daily medical surgical masks.

As showed in Figure 3, two triangles represent the specific location of the supplier, denoted as A1 and A2. The specific locations of the processing factories to be selected are marked with four squares and denoted as B1–B4. The specific locations of the ten pharmacies are marked with dots and denoted as C1-C10.The consumption demand area is marked by four minions, denoted as D1–D4. For the convenience of calculation, this paper only considers the unit raw material production unit mask, that is, $V = 1$, $L = 1$, $n_{vl} = 1$. In addition, the retail price of the uniform mask in pharmacies, $h_{lj} = 15$ CNY, $j = 1, ..., 10$.

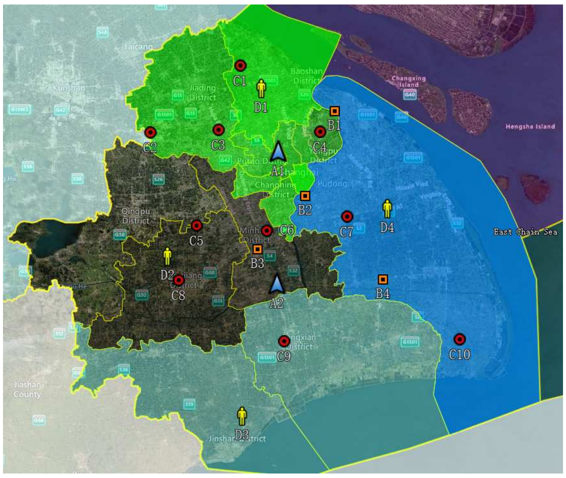

Figure 3. Network structure of supply chain.

The relevant parameters and fuzzy random transport costs and demands are given in Tables 1–4. The values of parameters related to suppliers, processing factories and retailers are shown in Table 1. According to reference [9], we estimate the fuzzy random transportation cost, and the specific data are shown in Tables 2 and 3. According to the market survey statistics, the fuzzy random daily demand of consumers is shown in Table 4.

Table 1. Sort results obtained by different methods.

Index	Suppliers		Processing		plants	Retailers	
s, i, j	a_{1s}	r_{1s}	b_{i1}	q_{i1}	f_i	h_{1j}	τ_{1j}
1	1450	6.6	1300	0.90	180	15	270
2	1300	6.4	1200	0.82	178	15	280
3	/	/	1250	0.85	175	15	300
4	/	/	1350	0.92	185	15	230
5	/	/	/	/	/	15	210
6	/	/	/	/	/	15	260
7	/	/	/	/	/	15	300
8	/	/	/	/	/	15	280
9	/	/	/	/	/	15	260
10	/	/	/	/	/	15	240

Table 2. Fuzzy random transportation cost of supplier transporting raw material to processing factory.

s, i	$(\gamma_{11})_{\tilde{t}_{1si}}$	$(\gamma_{12})_{\tilde{t}_{1si}}$	$(\gamma_{21})_{\tilde{t}_{1si}}$	$(\gamma_{22})_{\tilde{t}_{1si}}$
1, 1	[0.17, 0.19]	[0.19, 0.21]	[0.19, 0.21]	[0.17, 0.19]
1, 2	[0.12, 0.14]	[0.14, 0.16]	[0.13, 0.16]	[0.11, 0.13]
1, 3	[0.25, 0.27]	[0.27, 0.29]	[0.26, 0.28]	[0.24, 0.26]
1, 4	[0.40, 0.41]	[0.41, 0.43]	[0.42, 0.43]	[0.40, 0.42]
2, 1	[0.45, 0.47]	[0.47, 0.49]	[0.48, 0.49]	[0.46, 0.48]
2, 2	[0.25, 0.27]	[0.27, 0.29]	[0.26, 0.29]	[0.24, 0.26]
2, 3	[0.10, 0.13]	[0.13, 0.15]	[0.12, 0.15]	[0.10, 0.12]
2, 4	[0.27, 0.99]	[0, 29, 0.31]	[0.30, 0.32]	[0.28, 0.30]

Fuzzy random transportation cost and demand vector

$$\zeta(\omega, \gamma) = \begin{cases} \zeta(\omega_1), & p_1 = 0.4 \\ \zeta(\omega_2), & p_2 = 0.6 \end{cases},$$

the measure functions of the fuzzy vectors $\zeta(\omega_1)$ and $\zeta(\omega_2)$ are in the following forms:

$$\mu(\zeta(\omega_1)) = \begin{cases} 0.5, & \zeta(\omega_1) = \gamma_{11} \\ 1, & \zeta(\omega_2) = \gamma_{12} \end{cases},$$

$$\mu(\zeta(\omega_2)) = \begin{cases} 0.6, & \zeta(\omega_2) = \gamma_{21} \\ 1, & \zeta(\omega_2) = \gamma_{22} \end{cases},$$

here, $\gamma_{ij} \in [\underline{\gamma_{ij}}, \overline{\gamma_{ij}}], i, j = 1, 2$, and are related to $(\tilde{t}_{1si}, \tilde{m}_{1ij}, \tilde{d}_{1k})$, $\overline{\gamma_{ij}}$ and $\underline{\gamma_{ij}}$ represent the upper and lower bounds of the interval. The corresponding fuzzy random parameters are shown in Tables 2–4.

Table 3. Fuzzy random transportation cost of processing factory transporting product to retailer.

i, j	$(\gamma_{11})_{\tilde{m}_{1ij}}$	$(\gamma_{12})_{\tilde{m}_{1ij}}$	$(\gamma_{21})_{\tilde{m}_{1ij}}$	$(\gamma_{22})_{\tilde{m}_{1ij}}$
1, 1	[0.26,0.28]	[0.28,0.30]	[0.27,0.29]	[0.25,0.27]
1, 2	[0.46,0.47]	[0.47,0.49]	[0.48,0.50]	[0.46,0.48]
1, 3	[0.30,0.32]	[0.32,0.34]	[0.31,0.33]	[0.29,0.31]
1, 4	[0.08,0.10]	[0.10,0.12]	[0.11,0.13]	[0.09,0.11]
1, 5	[0.45,0.47]	[0.47,0.49]	[0.46,0.48]	[0.44,0.46]
1, 6	[0.36,0.38]	[0.38,0.40]	[0.39,0.41]	[0.37,0.39]
1, 7	[0.26,0.28]	[0.28,0.30]	[0.27,0.29]	[0.25,0.27]
1, 8	[0.58,0.60]	[0.60,0.62]	[0.60,0.62]	[0.58,0.60]
1, 9	[0.57,0.59]	[0.59,0.61]	[0.60,0.62]	[0.58,0.60]
1, 10	[0.63,0.65]	[0.65,0.67]	[0.64,0.66]	[0.62,0.64]
2, 1	[0.37,0.39]	[0.39,0.41]	[0.38,0.40]	[0.36,0.38]
2, 2	[0.40,0.42]	[0.42,0.44]	[0.43,0.45]	[0.41,0.43]
2, 3	[0.27,0.29]	[0.29,0.31]	[0.30,0.32]	[0.28,0.30]
2, 4	[0.17,0.19]	[0.19,0.21]	[0.20,0.22]	[0.18,0.20]
2, 5	[0.27,0.29]	[0.29,0.31]	[0.30,0.32]	[0.28,0.30]
2, 6	[0.12,0.15]	[0.15,0.17]	[0.15,0.17]	[0.13,0.15]
2, 7	[0.11,0.13]	[0.13,0.15]	[0.14,0.16]	[0.12,0.14]
2, 8	[0.37,0.39]	[0.39,0.41]	[0.38,0.40]	[0.36,0.38]
2, 9	[0.36,0.38]	[0.38,0.41]	[0.39,0.41]	[0.37,0.39]
2, 10	[0.52,0.54]	[0.54,0.56]	[0.53,0.55]	[0.51,0.53]
3, 1	[0.46,0.48]	[0.48,0.50]	[0.49,0.51]	[0.47,0.49]
3, 2	[0.40,0.42]	[0.42,0.44]	[0.43,0.45]	[0.41,0.43]
3, 3	[0.33,0.35]	[0.35,0.37]	[0.36,0.38]	[0.34,0.36]
3, 4	[0.34,0.36]	[0.36,0.38]	[0.37,0.39]	[0.35,0.37]
3, 5	[0.19,0.21]	[0.21,0.23]	[0.20,0.22]	[0.18,0.20]
3, 6	[0.06,0.08]	[0.08,0.10]	[0.09,0.10]	[0,07,0,09]
3, 7	[0.24,0.26]	[0.26,0.28]	[0.27,0.29]	[0.25,0.27]
3, 8	[0.21,0.23]	[0.23,0.25]	[0.24,0.26]	[0.22,0.24]
3, 9	[0.23,0.25]	[0.25,0.27]	[0.26,0.28]	[0.24,0.26]
3, 10	[0.54,0.56]	[0.56,0.58]	[0.55,0.57]	[0.53,0.55]
4, 1	[0.65,0.67]	[0.67,0.69]	[0.65,0.67]	[0.63,0.65]
4, 2	[0.68,0.70]	[0.70,0.72]	[0.69,0.71]	[0.67,0.69]
4, 3	[0.57,0.59]	[0.59,0.61]	[0.58,0.60]	[0.56,0.58]
4, 4	[0.41,0.43]	[0.43,0.45]	[0.40,0.42]	[0.38,0.40]
4, 5	[0.48,0.50]	[0.50,0.52]	[0.49,0.51]	[0.47,0.49]
4, 6	[0.33,0.35]	[0.35,0.37]	[0.35,0.37]	[0.33,0.35]
4, 7	[0.19,0.21]	[0.21,0.23]	[0.20,0.22]	[0.18,0.20]
4, 8	[0.51,0.53]	[0.53,0.55]	[0.52,0.54]	[0.50,0.52]
4, 9	[0.30,0.32]	[0.32,0.34]	[0.30,0.32]	[0.28,0.30]
4, 10	[0.22,0.24]	[0.24,0.26]	[0.23,0.25]	[0.21,0.23]

Table 4. Fuzzy random demand of consumer.

k	$(\gamma_{11})_{\tilde{d}_{1k}}$	$(\gamma_{12})_{\tilde{d}_{1k}}$	$(\gamma_{21})_{\tilde{d}_{1k}}$	$(\gamma_{22})_{\tilde{d}_{1k}}$
1	$[500, 525] - E[h_{1j}]$	$[525, 550] - E[h_{1j}]$	$[520, 550] - E[h_{1j}]$	$[500, 520] - E[h_{1j}]$
2	$[550, 575] - E[h_{1j}]$	$[575, 600] - E[h_{1j}]$	$[570, 590] - E[h_{1j}]$	$[550, 570] - E[h_{1j}]$
3	$[510, 520] - E[h_{1j}]$	$[520, 550] - E[h_{1j}]$	$[530, 545] - E[h_{1j}]$	$[500, 530] - E[h_{1j}]$
4	$[475, 500] - E[h_{1j}]$	$[500, 530] - E[h_{1j}]$	$[510, 525] - E[h_{1j}]$	$[480, 510] - E[h_{1j}]$

In order to solve the location allocation problem of medical surgical mask supply chain, for any feasible solution, we use Monte Carlo method to produce 1000 fuzzy random sampling points $\zeta_n(\omega, \gamma))(n = 1, ..., 1000)$ for fuzzy random simulation (such sample size is enough to simulate the expected value of fuzzy random).

We substitute the data in Table 1 and the generated sample points into the two-stage fuzzy random mixed integer optimization model (2.4) and (2.5), and apply the HIA to solve it. The numerical results are given in Tables 5–8 and Figures 4–6, respectively.

Table 5. Numerical optimal solutions and values of the example.

$e_1 = 0$	$e_2 = 1$	$e_3 = 1$
$e_4 = 0$	$x_{112} = 0.7692e + 03$	$x_{122} = 0.0500e + 03$
$x_{123} = 0.1250e + 03$	$y_{121} = 0.0639e + 03$	$y_{123} = 0.2253e + 03$
$y_{124} = 0.2300e + 03$	$y_{127} = 0.3000e + 03$	$y_{132} = 0.1653e + 03$
$y_{133} = 0.0747e + 03$	$y_{135} = 0.2100e + 03$	$y_{136} = 0.2600e + 03$
$y_{138} = 0.2800e + 03$	$y_{139} = 0.2600e + 03$	$z_{111} = 0.0156e + 03$
$z_{112} = 0.0172e + 03$	$z_{113} = 0.0157e + 03$	$z_{114} = 0.0153e + 03$
$z_{121} = 0.0404e + 03$	$z_{122} = 0.0467e + 03$	$z_{123} = 0.0400e + 03$
$z_{124} = 0.0382e + 03$	$z_{131} = 0.0743e + 03$	$z_{132} = 0.0809e + 03$
$z_{133} = 0.0736e + 03$	$z_{134} = 0.0712e + 03$	$z_{141} = 0.0570e + 03$
$z_{142} = 0.0616e + 03$	$z_{143} = 0.0565e + 03$	$z_{144} = 0.0549e + 03$
$z_{151} = 0.0521e + 03$	$z_{152} = 0.0562e + 03$	$z_{153} = 0.0516e + 03$
$z_{154} = 0.0502e + 03$	$z_{161} = 0.0644e + 03$	$z_{162} = 0.0701e + 03$
$z_{163} = 0.0637e + 03$	$z_{164} = 0.0618e + 03$	$z_{171} = 0.0742e + 03$
$z_{172} = 0.0813e + 03$	$z_{173} = 0.0734e + 03$	$z_{174} = 0.0711e + 03$
$z_{181} = 0.0693e + 03$	$z_{182} = 0.0754e + 03$	$z_{183} = 0.0687e + 03$
$z_{184} = 0.0666e + 03$	$z_{191} = 0.0644e + 03$	$z_{192} = 0.0697e + 03$
$z_{193} = 0.0638e + 03$	$z_{194} = 0.0620e + 03$	$Fit = 1.4967e + 04$

Figure 4 is the location-allocation supply chain network structure diagram, and the specific results are shown in Table 5. In order to meet the needs of customers, the processing factory chose B2 and B3 to produce surgical masks for medical use. At this time, the maximum profit of the supply chain is $1.4967e + 04$ CNY. Meanwhile, it can be seen from Figure 4 that the selected selected processing factories are evenly distributed, both of which are located in the relative centers of the whole demand market, which is conducive to distribution, reducing transportation costs and increasing profits of the supply chain.

In order to further evaluate the performance of our proposed HIA, we compared it with other discrete hybrid algorithms, such as binary PSO [39] and GA [41,42]. The numerical results are shown in Table 6 and Figure 5. The last column represents the relative error, which is defined by

$$Error = \frac{Optimal\ \ Fit - Fit}{Optimal\ \ Fit} \times 100\%.$$

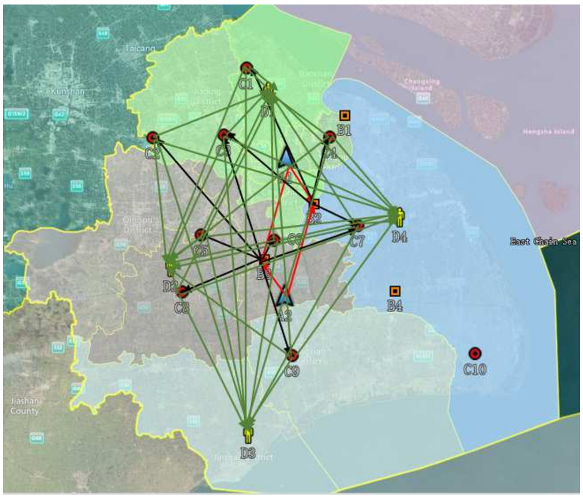

Figure 4. Location-allocation supply chain network structure diagram.

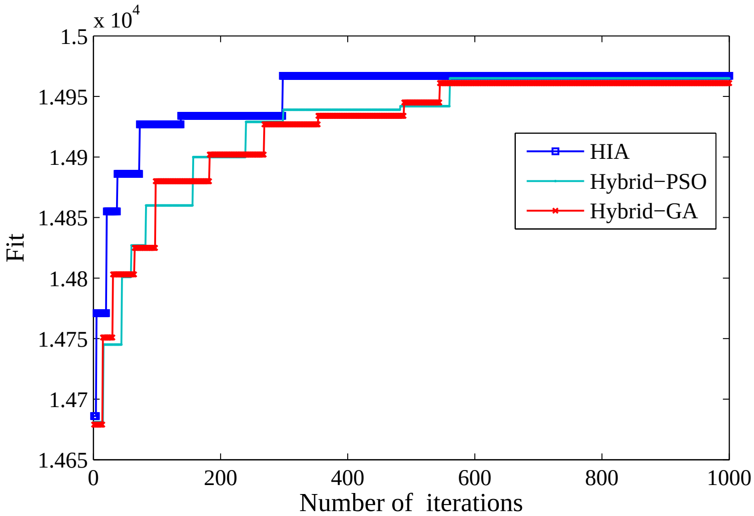

Figure 5. Comparisons of different algorithms.

Table 6. Comparisons of different algorithms.

Algorithm	$\mathbf{e}_{best}$	Fit	TI	
HIA	$(0, 1, 1, 0)$	$1.4967e + 04$	12636	0.00
Hybrid PSO	$(0, 1, 1, 0)$	$1.4965e + 04$	15378	0.01
Hybrid GA	$(0, 1, 1, 0)$	$1.4961e + 04$	18260	0.04

Remark 1. Throughout the computational experiments, the parameters in Hybrid GA are taken as: Population size NP=1000; Crossover probability Pc = 0.8; Mutation probability Pm = 0.05; Maximum genetic algebra G = 1000;

In Table 6 and Figure 5, we adopted three intelligent algorithms to solve the supply chain location-allocation problem, and obtained the same optimal solutions and similar optimal values. The relative error of the optimal value is less than 0.04%, which is caused by the Monte Carlo method random simulation. As can be seen from Figure 5, HIA algorithm has high search efficiency when dealing with problems. Convergence can be achieved in the first 300 iterations, while the other two algorithms can achieve convergence in the first 600 times, which fully demonstrates the efficient convergence of HIA algorithm. In addition, it can be seen from Table 6 that HIA algorithm has short computation time. The above analysis shows that HIA is more suitable for two-stage fuzzy random mixed integer optimization model.

In order to better demonstrate the performance of the HIA, the optimal solution under different parameters is obtained by setting different iteration numbers and population size. The numerical results are shown in Table 7.

Table 7. Results of HIA with different parameters.

System		Parameters			Results		
T	I'	c_1	c_2	c_3	$\mathbf{e}_{best}$	Fit	Error(%)
200	5	1.5	1.5	1.0	$(0, 1, 1, 0)$	$1.4934e + 04$	0.22
500	5	1.5	1.5	1.0	$(0, 1, 1, 0)$	$1.4967e + 04$	0.00
1000	5	1.5	1.5	1.0	$(0, 1, 1, 0)$	$1.4967e + 04$	0.00
5000	10	1.0	1.0	1.0	$(0, 1, 1, 0)$	$1.4967e + 04$	0.00
1000	10	1.0	1.5	1.0	$(1, 1, 1, 0)$	$1.4951e + 04$	0.11
1000	10	1.0	1.0	1.0	$(0, 1, 1, 0)$	$1.4957e + 04$	0.07
1000	10	0.5	0.5	0.5	$(1, 1, 1, 0)$	$1.4959e + 04$	0.05
1000	10	0.9	0.9	0.9	$(0, 1, 1, 0)$	$1.4962e + 04$	0.03
1000	100	1.5	1.0	1.0	$(0, 1, 1, 0)$	$1.4967e + 04$	0.00
1000	100	2.0	2.0	1.5	$(0, 1, 1, 0)$	$1.4954e + 04$	0.09

In Table 7, when we set different parameters, iteration numbers and population size in the HIA, the relative error is no more than 0.22%, which indicates that the HIA has strong robustness to parameters and can effectively solve the two-stage fuzzy random mixed integer optimization problem.

Figure 6 shows the comparison of the operational results of the HIA, Hybrid PSO and Hybrid GA to solve the problem of location-allocation. Obviously, as the sample data increases, the objective values obtained by the three intelligent algorithms tend to gradually tend to be determined values. In addition, the application of the method proposed in this paper to obtain supply chain benefits, significantly better than the other two intelligent algorithms. Overall, the HIA is better suited to solve the problem.

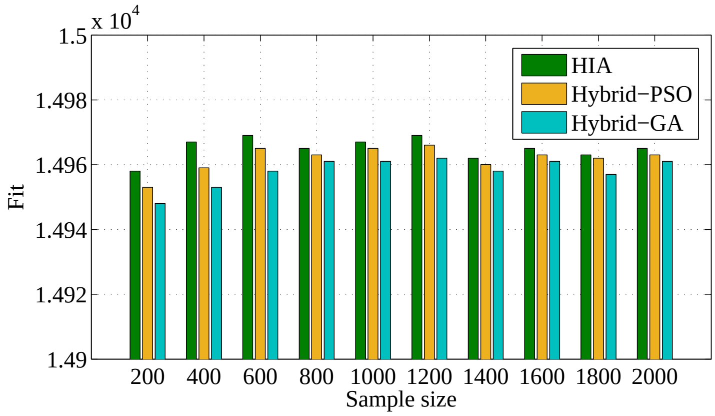

Figure 6. Comparison of the results of solving the location allocation problem with different methods under different sample sizes.

Table 8 shows the supply chain profit under fuzzy random, fuzzy, random and expected situations of cost demand. Table 8 suggests that the supply chain profit in the case of a fuzzy random cost demand is greater than random, fuzzy and expected, and the supply chain profit increased by 0.60%, 0.29%, 0.97%, respectively. The cost of ignoring the ambiguity of the cost demand in the selection decision is 89.00 CNY. The cost of ignoring the randomness of the cost demand in the selection decision is 43.00 CNY. The cost of ignoring the uncertainty of the cost demand in the selection decision is 144.00 CNY. This value is the motivation for fuzzy random programming, which assesses the value of knowing and using distributions on future outcomes. In some cases, more information might be available through more extensive forecasting, sampling, or exploration. In these cases, there are more fuzziness and randomness in the problem, so using fuzzy random programming modeling becomes more meaningful and practical.

Table 8. Supply chain profit with fuzzy random, fuzzy, random and expected demand.

$\zeta(\omega, \gamma)$	$\mathbf{e}_{best}$	Fit
Fuzzy random	$(0, 1, 1, 0)$	$1.4967e + 04$
Random	$(0, 1, 1, 0)$	$1.4878e + 04$
Fuzzy	$(0, 1, 1, 0)$	$1.4924e + 04$
Expected	$(0, 1, 1, 0)$	$1.4823e + 04$

5. Conclusions

In this paper, facility selection under hybrid uncertain environment with randomness and fuzziness is studied, and a two-stage fuzzy random mixed integer optimization model is established. Given the complexity of the model, based on PSO, BAS and interior point method, this paper proposes a HIA to solve the problem studied. Taking the supply chain of medical mask in Shanghai as an example, the influence of uncertainty on the location of processing plant was studied. By solving the problem, the optimal location of the processing factory and the optimal distribution of the masks were obtained, and the maximum expected profit was obtained. Finally, the HIA is compared with hybrid PSO and hybrid GA. The computational results suggest that HIA is better suited to a two-stage fuzzy random mixed integer optimization model.

Future research can consider the robustness and uncertainty of supply, demand in the supply chain simultaneously, to investigate the interaction among the various forms of uncertainty, and their effects on the supply chain system. We can also consider a single uncertainty, that is, applying a stochastic optimization model to deal with the randomness of facility location, or applying a fuzzy optimization model to deal with the fuzziness of facility location. We can also study the risks brought by supply chain uncertainty and carry out risk management.

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Conflict of interest

The authors declare that they have no competing interests.

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