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Research article

Sub-base local reduct in a family of sub-bases

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Abstract: This paper discusses sub-base local reducts in a family of sub-bases. Firstly, definitions of sub-base local consistent sets and sub-base local reducts are provided. Using the sub-base local discernibility matrix, a necessary and sufficient condition for sub-base local consistent sets is presented. Secondly, properties of the sub-base local core are studied. Finally, sub-base local discernibility Boolean matrices are defined, and the calculation method is given. Utilizing sub-base local discernibility Boolean matrices, an algorithm is devised to obtain sub-base local reducts.

Keywords: sub-base local reduct; sub-base local consistent set; sub-base local discernibility matrix; sub-base local discernibility Boolean matrix
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1. Introduction

Rough set theory, proposed by Pawlak [1], provides an approach to uncertainty management. In [2], the theoretical relationships connecting rough set theory and belief function theory were investigated, and their applications in knowledge representation and machine learning were researched. Covering rough sets [3], generalizations of the classical rough sets, have been proved to be suitable for discussing covering information systems. As a significant problem, the reduct problem has captured considerable attention of numerous scholars. Many methods were provided to find reducts of covering rough sets [4–8]. In addition, the invariant of separation in covering approximation spaces was concerned in [9].

Topology is a useful tool for investigating rough set theory and its applications. The interdependencies of topology and rough set theory were emphasized in [10]. The object of general topology is to study topological properties, namely, invariants of homeomorphism [11]. In the light of the properties of the topological rough membership function, sub-base reducts in a family of sub-bases were defined in [12]. To further research sub-base reducts in a family of sub-bases from the point of view of general topology, the concept of a minimal family of sub-bases was presented in [13]. By showing the relationship between reducts in covering information systems and minimal families of sub-bases, [13] provided an approach to deriving a minimal family of sub-bases. Moreover, minimal bases and minimal sub-bases were considered in [14, 15].

It is not hard to see that the above-cited works are focused on sub-base reducts on a given universal set. But some elements in the given universal set may be not important for specific problems. Motivated by that, this paper intents to discuss sub-base local reducts in a family of sub-bases, which has not been considered in the existing references. The main contributions are twofold. (i) The properties of sub-base local reducts in a family of sub-bases are investigated. (ii) The approach to finding sub-base local reducts in a family of sub-bases is provided, along with an algorithm for achieving it.

The remainder of this paper is organized as follows. Section 2 gives some basic information about sub-base local reducts. Section 3 illustrates how to obtain sub-base local reducts according to Boolean matrices. Section 4 has some concluding remarks.

2. Sub-base local reduct

Suppose $\mathscr{S}_i$ is a sub-base for finite topological space (X, τ_i) for $i = 1, 2, \cdots, n$, $\Delta = \{\mathscr{S}_1, \mathscr{S}_2, \cdots, \mathscr{S}_n\}$, and $\mathscr{S}_\Delta = \bigwedge_{i=1}^n \mathscr{S}_i = \{\bigcap_{i=1}^n S_i | S_i \in \mathscr{S}_i, i = 1, 2 \cdots, n\}$. Then $\mathscr{S}_\Delta$ is a sub-base for a topology τ_Δ of finite set X. Suppose $\mathscr{P}$ is a family of subsets of X. A minimal set containing x with respect to $\mathscr{P}$ is denoted by $N_{\mathscr{P}}(x) = \bigcap\{U | x \in U \in \mathscr{P}\}$.

Under the premise of keeping topology unchanged, the sub-base reduct of a family of sub-bases is defined according to the unique open neighborhood in [12, 13]. However, one may concern sub-base reducts related to several open sets. Hence, the concept of the sub-base local reduct is provided.

Definition 2.1. *Let $\mathscr{S}_i$ be a sub-base for finite topological space (X, τ_i) for $i = 1, 2, \ldots, n$ and $\Delta = \{\mathscr{S}_1, \mathscr{S}_2, \ldots, \mathscr{S}_n\}$. Suppose $\mathscr{F} = \{F | F \in \mathscr{S}_\Delta\}$. $\Delta_1 \subset \Delta$ is called a sub-base local consistent set with respect to $\mathscr{F}$ of Δ if $\mathscr{F} \subset \mathscr{S}_\Delta$. If Δ is a sub-base local consistent set with respect to $\mathscr{F}$ of Δ, and for any proper subset Δ_2 of Δ_1, $\mathscr{F} \subset \mathscr{S}_\Delta$, then Δ_1 is called a sub-base local reduct with respect to $\mathscr{F}$ of Δ.*

Remark 2.1. *Compared with the local reduct discussed in rough set theory, the sub-base local reduct in a family of sub-bases also focuses on a subset A of the given universal set X. Thus, the sub-base local reduct in a family of sub-bases is consistent with the local reduct discussed in rough set theory. When discussing the sub-base local reduct in a family of sub-bases, subset A is obtained via the union of those concerned open sets. But considering the local reduct in rough set theory, subset A is determined according to elements that are indispensable for certain decision classes.*

The sub-base local discernibility matrix is defined in the following.

Definition 2.2. *Let $\mathscr{S}_i$ be a sub-base for finite topological space (X, τ_i) for $i = 1, 2, \ldots, n$ and $\Delta = \{\mathscr{S}_1, \mathscr{S}_2, \ldots, \mathscr{S}_n\}$. Suppose $\mathscr{F} = \{F | F \in \mathscr{S}_\Delta\}$. The sub-base local discernibility matrix with respect to $\mathscr{F}$ of Δ is denoted by $\mathcal{D}_{\mathscr{F}}(\Delta) = \{\mathcal{D}(x, y) | x, y \in X\}$, where*

(1) if there exists $F \in \mathscr{F}$ such that $x \in F$, but $y \notin F$, then $\mathcal{D}(x, y) = \{\mathscr{S} \in \Delta | y \notin N_{\mathscr{S}}(x)\}$.

(2) Otherwise, $\mathcal{D}(x, y) = \varnothing$.

Using the sub-base local discernibility matrix, a result of the sub-base local consistent set is presented.

Theorem 2.1. *Let $\mathscr{S}_i$ be a sub-base for finite topological space (X, τ_i) for $i = 1, 2, \ldots, n$, and $\Delta = \{\mathscr{S}_1, \mathscr{S}, \ldots, \mathscr{S}_n\}$. Suppose $\mathscr{S} = \{F | F \in \mathscr{S}_\Delta\}$. $\Delta_1 \subset \Delta$ is a sub-base local consistent set with respect to $\mathscr{F}$ of Δ if and only if $\Delta_1 \cap \mathcal{D}(x, y) \neq \emptyset$ for $\mathcal{D}(x, y) \neq \emptyset$.*

Proof. Necessity. Assume there exist two points $x, y \in X$ such that $\mathcal{D}(x, y) \notin \emptyset$, but $\Delta_1 \cap \mathcal{D}(x, y) = \emptyset$. Then $\mathscr{S} \notin \mathcal{D}(x, y)$ for each $\mathscr{S} \in \Delta_1$, which means $y \in N_{\mathscr{S}}(x)$ for each $\mathscr{S} \in \Delta_1$. That is, $y \in N_{\mathscr{S}_{\Delta_1}}(x)$. Since $\mathcal{D}(x, y) \notin \emptyset$, for each $F \in \mathscr{F}$ satisfying $x \in F, y \notin F$, there exists $\mathscr{S} \in \Delta$ such that $y \notin N_{\mathscr{S}}(x)$. Because $\Delta_1 \subset \Delta$ is a sub-base local consistent set with respect to $\mathscr{F}$ of Δ, we have $\mathscr{F} \subset \mathscr{S}_{\Delta_1}$. Thus, $N_{\mathscr{S}_{\Delta_1}}(x) = (\bigcap\limits_{x \in A, A \in \mathscr{S}_{\Delta_1}, A \neq F} A) \cap F$, which contradicts with $y \in N_{\mathscr{S}_{\Delta_1}}$. Hence, $\Delta_1 \cap \mathcal{D}(x, y) \neq \emptyset$ for $\mathcal{D}(x, y) \neq \emptyset$.

Sufficiency. Assume Δ_1 is not a sub-base local consistent set with respect to $\mathscr{F}$ of Δ. Then there exists $F \in \mathscr{F}$ such that $F \notin \mathscr{S}_{\Delta_1}$. Thus, there exists a point $x \in X$ such that $N_{\mathscr{S}_{\Delta_1} \cup \{F\}}(x) \neq N_{\mathscr{S}_{\Delta_1}}(x)$. Since $N_{\mathscr{S}_{\Delta_1} \cup \{F\}}(x) \subset N_{\mathscr{S}_{\Delta_1}}(x)$, there exists a point $y \in X$ such that $y \in N_{\mathscr{S}_{\Delta_1}}(x)$, but $y \notin N_{\mathscr{S}_{\Delta_1} \cup \{F\}}(x)$. That is, $y \notin N_{\mathscr{S}_\Delta}(x)$ and $y \in N_{\mathscr{S}}(x)$ for each $\mathscr{S} \in \Delta_1$, which implies $\mathcal{D}(x, y) = \emptyset$, but $\Delta_1 \cap \mathcal{D}(x, y) = \emptyset$, which is a contradiction. Hence, Δ_1 is a sub-base local consistent set with respect to $\mathscr{F}$ of Δ. □

The definition of the sub-base local core is proposed.

Definition 2.3. *Let $\mathscr{S}_i$ be a sub-base for finite topological space (X, τ_i) for $i = 1, 2, \ldots, n$, and $\Delta = \{\mathscr{S}_2, \mathscr{S}_2, \ldots, \mathscr{S}_n\}$. Suppose $\mathscr{F} = \{F | F \in \Delta\}$. If $C_{\mathscr{F}}(\Delta) = \cap\{\Delta_1 | \mathscr{F} \subset \mathscr{S}_{\Delta_1}\}$, then $C_{\mathscr{F}}(\Delta)$ is called a sub-base local core with respect to $\mathscr{F}$ of Δ.*

Some equivalent conditions about the sub-base local core are provided.

Theorem 2.2. *Let $\mathscr{S}_i$ be a sub-base for finite topological space (X, τ_i) for $i = 1, 2, \ldots, n$, and $\Delta = \{\mathscr{S}_1, \mathscr{S}_2, \ldots, \mathscr{S}_n\}$, Suppose $\mathscr{F} = \{F | F \in \mathscr{S}_\Delta\}$. Then the following conclusions are equivalent.*

(1) $\mathscr{S} \in C_{\mathscr{F}}(\Delta)$.

(2) There exist $x, y \in X$ such that $\mathcal{D}(x, y) = \{\mathscr{S}\}$.

(3) There exists $F \in \mathscr{F}$ such that $F \notin \mathscr{S}_{\Delta \setminus \{\mathscr{S}\}}$.

Proof. (1)⇒(2). Assume $|\mathcal{D}(x, y)| \geq 2$ for any points $x, y \in X$. Denote $\Delta' = \bigcap\{\mathcal{D}(x, y) \setminus \{S\} | x, y \in X\}$. It is easy to see that $\Delta' \cap \mathcal{D}(x, y) \neq \emptyset$ for $\mathcal{D}(x, y) \neq \emptyset$. According to Theorem 2.1, Δ' is a sub-base local consistent set with respect to $\mathscr{F}$ of Δ. Thus, there exists $\Delta_1 \subset \Delta'$ such that Δ_1 is a sub-base local reduct with respect to $\mathscr{F}$ of Δ, which contradicts with $\mathscr{S} \in C_{\mathscr{F}}(\Delta)$. Hence, there exist $x, y \in X$ such that $\mathcal{D}(x, y) = \{\mathscr{S}\}$.

(2)⇒(3). Assume $\mathscr{F} \subset \mathscr{S}_{\Delta \setminus \{\mathscr{S}\}}$. Then, $\Delta \setminus \{\mathscr{S}\}$ is a sub-base local consistent set with respect to $\mathscr{F}$ of Δ. Based on Theorem 2.1, $(\Delta \setminus \{\mathscr{S}\}) \cap \mathcal{D}(x, y) \neq \emptyset$ for $\mathcal{D}(x, y) \neq \emptyset$, which contradicts with (2). Hence, there exists $F \in \mathscr{F}$ such that $F \notin \mathscr{S}_{\Delta \setminus \{\mathscr{S}\}}$.

(3)⇒(1). Since there exists $F \in \mathscr{F}$ such that $F \notin \mathscr{S}_{\Delta \setminus \{\mathscr{S}\}}$, one concludes that $\Delta \setminus \{\mathscr{S}\}$ is not a sub-base local consistent set with respect to $\mathscr{F}$ of Δ. Thus, for any $\Delta' \subset \Delta \setminus \{\mathscr{S}\}$, Δ' is not a sub-base local reduct with respect to $\mathscr{F}$ of Δ, which contradicts with (1). Hence, $\mathscr{S} \in C_{\mathscr{F}}(\Delta)$ is proved. □

3. Sub-base local reducts based on Boolean matrices

To provided a simple method to find a sub-base local reduct of a given Δ, the following definitions are used to construct a Boolean matrix.

Definition 3.1. *[5] Let $X = \{x_1, x_2, \dots, x_m\}$ and $A \subset X$. The characteristic function is defined as $f(A) = (f_1, f_2, \dots, f_m)'$ ($'$ denotes the transpose throughout this paper), where*

$$f_i = \begin{cases} 1, & x_i \in A, \\ 0, & x_i \notin A. \end{cases}$$

Definition 3.2. *[13] Let $\mathscr{P}$ be a family of subsets of X with $X = \{x_1, x_2, \dots, x_m\}$ and $\mathscr{P} = \{P_1, P_2, \dots, P_k\}$. The characteristic matrix of $\mathscr{P}$ is defined as $M_{\mathscr{P}} = (f(P_1), f(P_2), \dots, f(P_k))$.*

Definition 3.3. *[4] Let $M = (m_{ij})_{n\times m}$ be a matrix. Define two matrix operators $\sim$ and $\approx$ as follows:*
(1) $\sim M = (\sim m_{ij})_{n\times m}$, where

$$\sim m_{ij} = \begin{cases} 1, & m_{ij} = 0, \\ 0, & m_{ij} \neq 0. \end{cases}$$

(2) $\approx M = (\approx m_{ij})_{n\times m}$, where

$$\approx m_{ij} = \begin{cases} 0, & m_{ij} = 0, \\ 1, & m_{ij} \neq 0. \end{cases}$$

Definition 3.4. *[16] Let $A = (a_{ij})_{n\times m}$ and $B = (b_{ij})_{n\times m}$ be two matrices. The Hadamard product of A and B is defined as $A \circ B = (a_{ij}b_{ij})_{n\times m}$.*

The sub-base local discernibility Boolean matrix is defined.

Definition 3.5. *Let $\mathscr{S}_i$ be a sub-base for finite topological space (X, τ_i) for $i = 1, 2, \dots, n$ with $X = \{x_1, x_2, \dots, x_m\}$, and $\Delta = \{\mathscr{S}_1, \mathscr{S}_2, \dots, \mathscr{S}_n\}$. Suppose $\mathscr{F} = \{F | F \in \mathscr{S}_\Delta\}$. For any $\Delta_1 \subset \Delta$, define a sub-base local discernibility Boolean matrix $D_{\mathscr{F}}(\Delta_1) = (d_{ij})_{m\times m}$ satisfying:*
(1) If there exists $F \in \mathscr{F}$ such that $x_i \in F, x_j \notin F$ and $x_j \notin N_{\mathscr{S}_{\Delta_1}}(x_i)$, then $d_{ij} = 1$.
(2) Otherwise, $d_{ij} = 0$.

From the following theorem, the sub-base local discernibility Boolean matrix is computed.

Theorem 3.1. *Let $\mathscr{S}_i$ be a sub-base for finite topological space (X, τ_i) for $i = 1, 2, \dots, n$ with $X = \{x_1, x_2, \dots, x_m\}$. Suppose $\mathscr{F} = \{F | F \in \mathscr{S}_\Delta\}$. Then the following results hold.*
(1) $D_{\mathscr{F}}(\mathscr{S}) = \approx (M_{\mathscr{S}}(\sim M'_{\mathscr{S}}) \circ (M_{\mathscr{F}}(\sim M'_{\mathscr{F}})))$ for each $\mathscr{S} \in \Delta$.
(2) $D_{\mathscr{F}}(\mathscr{S}_{\Delta_1}) = \approx (\sum_{\mathscr{S}\in\Delta_1} D_{\mathscr{F}}(\mathscr{S}))$ for any $\Delta_1 \subset \Delta$.

Proof. Given a matrix M, denote its i-th row by $Row_i(M)$ and its element in the i-th row and j-th column by M_{ij}.

(1) Denote $\mathscr{S} = \{S_1, S_2, \dots, S_k\}$ and $\mathscr{F} = \{F_1, F_2, \dots, F_q\}$. It is easy to find that $Row_i(M_{\mathscr{S}}(\sim M'_{\mathscr{S}})) = \sum_{1\le j\le k} (M_{\mathscr{S}})_{ij} Row_j(\sim M'_{\mathscr{S}})$. According to Definitions 3.1 and 3.2, $(M_{\mathscr{S}})_{ij} = 1$ means $x_i \in S_j$, and $(M_{\mathscr{S}})_{ij} = 0$ means $x_i \notin S_j$. Thus, we get $Row_i(M_{\mathscr{S}}(\sim M'_{\mathscr{S}})) = \sum_{1\le j\le k} (M_{\mathscr{S}})_{ij} Row_j(\sim M'_{\mathscr{S}}) = \sum_{x_i\in S_j} Row_j(\sim M'_{\mathscr{S}})$. That is, $(M_{\mathscr{S}}(\sim M'_{\mathscr{S}}))_{il} = \sum_{x_i\in S_j} (\sim M'_{\mathscr{S}})_{il}$. Similarly, $(M_{\mathscr{F}}(\sim M'_{\mathscr{F}}))_{il} = \sum_{x_i\in S_j} (\sim M'_{\mathscr{F}})_{il}$. If $(D_{\mathscr{F}}(\mathscr{S}))_{il} = 1$, then there exists $F_p \in \mathscr{F}$ such that $x_i \in F_p, x_l \notin F_p$ and $x_l \notin N_{\mathscr{S}_\Delta}(x_i)$. Hence, we obtain $(M_{\mathscr{F}})_{ip} = 1$ and $(\sim M'_{\mathscr{F}})_{pl} = 1$. That is, $(M_{\mathscr{F}}(\sim M'_{\mathscr{F}}))_{il} \ge 1$. Since $x_l \notin N_{\mathscr{S}_\Delta}(x_i)$, there exists $S_{j_0} \in \mathscr{S}$ such that $x_i \in S_{j_0}$ but $x_l \notin S_{j_0}$. So we have $(M_{\mathscr{S}}(\sim M'_{\mathscr{S}}))_{il} \ge 1$. Therefore, we prove $(\approx (M_{\mathscr{S}}(\sim M'_{\mathscr{S}}) \circ (M_{\mathscr{F}}(\sim M'_{\mathscr{F}}))))_{il} = 1$. Similarly, if $(D_{\mathscr{F}}(\mathscr{S}))_{il} = 0$, then $(\approx (M_{\mathscr{S}}(\sim M'_{\mathscr{S}}) \circ (M_{\mathscr{F}}(\sim M'_{\mathscr{F}}))))_{il} = 0$. Consequently, $D_{\mathscr{F}}(\mathscr{S}) = \approx (M_{\mathscr{S}}(\sim M'_{\mathscr{S}}) \circ (M_{\mathscr{F}}(\sim M'_{\mathscr{F}})))$ for each $\mathscr{S} \in \Delta$.

(2) If $(D_{\mathscr{F}}(\mathscr{S}_{\Delta_1}))_{ij} = 1$, then there exists $F \in \mathscr{F}$ such that $x_i \in F, x_j \notin F$ and $x_j \notin N_{\mathscr{S}_{\Delta_1}}(x_i)$. Thus, there exists $\mathscr{S} \in \Delta_1$ such that $x_j \notin N_{\mathscr{S}}(x_i)$. From (1), we conclude that $(D_{\mathscr{F}}(\mathscr{S}))_{ij} = 1$, i.e., $(\approx (\sum_{\mathscr{S}\in\Delta_1} D_{\mathscr{F}}(\mathscr{S})))_{ij} = 1$. If $(D_{\mathscr{F}}(\mathscr{S}_{\Delta_1}))_{ij} = 0$, then it is similar to proving $(\approx (\sum_{\mathscr{S}\in\Delta_1} D_{\mathscr{F}}(\mathscr{S})))_{ij} = 0$. Consequently, $D_{\mathscr{F}}(\mathscr{S}_{\Delta_1}) =\approx (\sum_{\mathscr{S}\in\Delta_1} D_{\mathscr{F}}(\mathscr{S}))$ for any $\Delta_1 \subset \Delta$. □

Based on the results above, Theorem 3.2 is proved.

Theorem 3.2. *Let $\mathscr{S}_i$ be a sub-base for finite topological space (X, τ_i) for $i = 1, 2, \ldots, n$ with $X = \{x_1, x_2, \ldots, x_m\}$. Suppose $\mathscr{F} = \{F|F \in \mathscr{S}_\Delta\}$. Then the following results hold.*

(1) For each $\Delta_1 \subset \Delta$, Δ_1 is a sub-base local consistent set with respect to $\mathscr{F}$ of $\mathscr{S}$ if and only if $D_{\mathscr{F}}(\Delta_1) = D_{\mathscr{F}}(\Delta)$.

(2) For each $\mathscr{S} \in \Delta$, $\mathscr{S} \in C_{\mathscr{F}}(\Delta)$ if and only if $D_{\mathscr{F}}(\Delta \setminus \{\mathscr{S}\}) \neq D_{\mathscr{F}}(\Delta)$.

Proof. (1) According to Theorem 2.1, $\mathscr{S}$ is a sub-base local core with respect to $\mathscr{F}$ if and only if $\Delta_1 \cap \mathcal{D}(x_{i,j}) \neq \varnothing$ for $\mathcal{D}(x_i, x_j) \neq \varnothing$. From Definitions 2.2 and 3.5, $\mathcal{D}(x_i, x_j) \neq \varnothing$ is equivalent to $(D_{\mathscr{F}}(\Delta))_{ij} = 1$. $\Delta_1 \cap \mathcal{D}(x_i, x_j) \neq \varnothing$ is equivalent to $x_j \notin N_{\mathscr{S}_{\Delta_1}}(x_i)$, i.e., $(D_{\mathscr{F}}(\Delta_1))_{ij} = 1$. Hence, we conclude that $D_{\mathscr{F}}(\Delta_1) = D_{\mathscr{F}}(\Delta)$.

(2) From Theorem 2.2, $\mathscr{S}$ is a sub-base local core with respect to $\mathscr{F}$ if and only if there exists $x_i, x_j \in X$ such that $\mathcal{D}(x_i, x_j) = \{\mathscr{S}\}$. It is equivalent to $(D_{\mathscr{F}}(\Delta))_{ij} = 1$, but $(D_{\mathscr{F}}(\Delta \setminus \{\mathscr{S}\}))_{ij} = 0$. Hence, $D_{\mathscr{F}}(\Delta \setminus \{\mathscr{S}\}) \neq D_{\mathscr{F}}(\Delta)$. □

Moreover, a necessary and sufficient condition for the sub-base local reduct is presented.

Corollary 3.1. *Let $\mathscr{S}_i$ be a sub-base for finite topological space (X, τ_i) for $i = 1, 2, \ldots, n$ with $X = \{x_1, x_2, \ldots, x_m\}$. Suppose $\mathscr{F} = \{F|F \in \mathscr{S}_\Delta\}$. Then $\Delta_1 \subset \Delta$ is a sub-base local reduct with respect to $\mathscr{F}$ of Δ if and only if Δ_1 is a minimal subfamily of Δ satisfying $D_{\mathscr{F}}(\Delta_1) = D_{\mathscr{F}}(\Delta)$.*

On the basis of the analysis above, an algorithm is devised to find sub-base local reducts.

Algorithm 1 Sub-base local reducts based on Boolean matrices

Input: A family Δ of sub-bases and $\mathscr{F} = \{F|F \in \mathscr{S}_\Delta\}$.
Output: A minimal family Δ' of sub-bases.

Let $\Delta' = \varnothing$;
for each $\mathscr{S} \in \Delta$ **do**
    Compute $D_{\mathscr{F}}(\Delta \setminus \{\mathscr{S}\})$ according to Theorem 3.1;
    if $D_{\mathscr{F}}(\Delta \setminus \{\mathscr{S}\}) \neq D_{\mathscr{F}}(\Delta)$; **then**
        Let $\Delta' = \Delta' \cup \{\mathscr{S}\}$.//find all sub-base local cores;
    end if
end for
while $D_{\mathscr{F}}(\Delta') \neq D_{\mathscr{F}}(\Delta)$ **do**
    Let $\Delta' = \Delta' \cup \{\mathscr{S}_0\}$,
where $\mathscr{S}_0$ satisfies $|D_{\mathscr{F}}(\Delta' \cup \{\mathscr{S}_0\})| = max\{|D_{\mathscr{F}}(\Delta' \cup \{\mathscr{S}_0\})| \mid \mathscr{S} \in \Delta \setminus \Delta'\}$, and $|\cdot|$ is the total number of 1 in a matrix;
end while
Return Δ'.

Remark 3.1. *The time complexities of Steps 3-6 and Steps 8-10 are* $O(\sum_{\mathscr{S}\in\Delta}\alpha^2|\mathscr{S}|)$ *and* $O(\sum_{i=1}^{|\Delta|-1}\alpha^2(|\Delta|-i))$*, respectively, where* $\alpha = |\bigcup_{F\in\mathscr{F}} F|$*. Thus, the time complexity of Algorithm 1 is* $O(\sum_{\mathscr{S}\in\Delta}\alpha^2|\mathscr{S}| + \sum_{i=1}^{|\Delta|-1}\alpha^2(|\Delta|-i))$.

4. Conclusions

Sub-base local reducts in a family of sub-bases have been investigated in this paper. Firstly, using the defined sub-base local discernibility matrix, a necessary and sufficient condition for the sub-base local consistent set has been provided. Then the sub-base local discernibility matrix has been employed to study properties of the sub-base local core. Finally, an algorithm has been devised to obtain sub-base local reducts via the sub-base local discernibility matrix.

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Conflict of interest

The authors declare that they have no conflict of interest.

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