AIMS Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 7(7): 12650–12670.
DOI: 10.3934/math.2022700
Received: 30 November 2021
Revised: 05 April 2022
Accepted: 12 April 2022
Published: 28 April 2022

Research article

Generalized hesitant intuitionistic fuzzy N-soft sets-first result

Admi Nazra[1,*], Jenizon[1], Yudiantri Asdi[1] and Zulvera[2]

[1] Department of Mathematics, Andalas University, Kampus UNAND Limau Manis, Padang, Indonesia

[2] Department of Economic Social of Agriculture, Andalas University, Kampus UNAND Limau Manis, Padang, Indonesia

* **Correspondence:** Email: nazra@sci.unand.ac.id; Tel: +6282125306590.

Abstract: The study on N-soft sets (NSSs) has been significantly developed recently. Hybrid models such as fuzzy N-soft sets, Intuitionistic fuzzy N-soft sets, and hesitant fuzzy N-soft sets were introduced to combine fuzzy sets, intuitionistic fuzzy sets and hesitant fuzzy sets with NSSs. Related to the hybrid models, it was also constructed some complements, operations and related properties. This article aims to construct a new hybrid model called hesitant intuitionistic fuzzy N-soft sets (HIFNSSs) to combine intuitionistic fuzzy N-soft sets and hesitant fuzzy N-soft sets. Moreover, we generalise HIFNSSs to generalized hesitant intuitionistic fuzzy N-soft sets (GHIFNSSs) as a hybrid model between generalized hesitant intuitionistic fuzzy sets and N-soft sets. It was also defined some complements of GHIFNSSs, intersection and union operations between GHIFNSSs, and proved that the operations between some particular complements hold De Morgan Law. In applying a GHIFNSS, we provide an algorithm for decision-making problems and its numerical illustration.

Keywords: N-soft sets; intuitionistic fuzzy N-soft sets; hesitant fuzzy N-soft sets; generalized hesitant intuitionistic fuzzy sets; hesitant intuitionistic fuzzy N-soft sets
Mathematics Subject Classification: 03E72, 94D05

1. Introduction

N-soft sets (NSSs) theory and their applications were first introduced by Fatimah et al. [8] as a generalization to the concept of soft sets (SSs) defined by Molodtsov [11]. In the last three years, the NSS theory and its use in decision-making problems for various issues in daily life have been growing steadily. By combining the NSS theory with previous theories such as fuzzy sets (FSs) [16], fuzzy soft sets (FSSs) [10], hesitant fuzzy soft sets (HFSSs) [5], intuitionistic fuzzy sets (IFSs) [4], thus new concepts were constructed, among which were the theories of fuzzy N-soft sets (FNSSs) [3], hesitant fuzzy N-soft sets (HFNSSs) [2] and intuitionistic fuzzy N-soft sets (IFNSSs) [1]. Therefore, these results can be applied to a wider variety of problem models in everyday life.

On the other hand, Nazra et al. [12] have developed a new concept of hesitant intuitionistic fuzzy soft sets (HIFSSs) as a combination of HFSS and IFS concepts. However, the models constructed by Akram et al. [1–3] could not cover the decision-making problems that contain elements of hesitation and, at the same time, it also intuitionistic, because each model only suitable for elements of hesitation or intuitionistic separately. Another shortcoming of Akram's models is that the models do not consider the degree of importance (preference) of parameters. Therefore, the concept of HIFSSs and generalized hesitant intuitionistic fuzzy soft sets (GHIFSSs), considering the degree of preference of parameters that Nazra et al. [12, 13] have introduced, needs to be developed further in the context of N-soft sets (NSSs).

The problem in this research is how the generalization of the research results of Nazra et al. [13] on GHIFSS is related to the research results of Fatimah et al. [8], as well as the generalization of the research of Akram et al. [1–3]. Hence this new concept is called the generalized hesitant intuitionistic fuzzy N-soft set (GHIFNSS) concept. This research aims to formulate the definition of GHIFNSS, their complements, and some related operations. Then, we analytically prove some properties concerning the operations and complements. As an application of our new model, we construct a novel algorithm for decision-making problems. The algorithm is a generalization of that constructed by Cağman and Karatas [7], and Khan and Zhu [9] to solve decision-making problems based on intuitionistic fuzzy soft sets.

We organize this paper as follows. Section 2 recalls definitions of FSs, SSs and their combinations, and NSSs. Sections 3 and 4 are the main results. Section 3 introduces our new hybrid model GHIFNSS, its complements, operations and properties. In order to be easily understood, we provide some examples. In Section 4, we construct an algorithm as an application of a GHIFNSS and give a numerical example illustrating a decision-making problem in a GHIFNSS information using the algorithm. Section 5 concludes the paper.

2. Preliminaries

In this section, we review some definitions, such as, fuzzy set (FS), soft set (SS), fuzzy soft set (FSS), intuitionistic fuzzy set (IFS), hesitant intuitionistic fuzzy set (HIFS) and N-soft set (NSS).

The concept of fuzzy set is introduced by Zadeh [16]. A fuzzy set (FS) over a set of objects O is a set $F_s = \{(u, f(u))|u \in O\}$ where $f\colon O \to [0, 1]$. Here f and $f(u)$ are called the membership function of F_s and the membership value of u in F_s, respectively. Molodtsov, in [11], defined a kind of set called the soft set (SS).

Definition 2.1. [11] Let U be a universal set, P(U) be a power set of U, and E be a set of parameters. A pair $\langle F, E\rangle$ is called a soft set (SS) over U if and only if F is a function $F : E \to P(U)$, such that

$$\langle F, E\rangle = \{\langle \varepsilon, F(\varepsilon)\rangle|\varepsilon \in E, F(\varepsilon) \in P(U)\}. \tag{2.1}$$

As a generalization of a FS, it is introduced the concept of an intuitionistic fuzzy set (IFS).

Definition 2.2. [4] Let X be a universal set. An intuitionistic fuzzy set (IFS) I over X is

$$I = \{\langle x, \mu_I(x), \gamma_I(x)\rangle|x \in X\}, \tag{2.2}$$

where $\mu_I, \gamma_I : X \to [0,\ 1]$ are membership and non-membership functions on I. Moreover, for any $x \in X$, $0 \le \mu_I(x) + \gamma_I(x) \le 1$.

Beg and Rashid [6] generalized the concept of IFS to hesitant intuitionistic fuzzy set (HIFS). However, in this article, we revise the definition to make it simpler and more general.

Definition 2.3. Let X be a universal set. An hesitant intuitionistic fuzzy set (HIFS) H over X is

$$H = \{\langle x, \alpha(x), \beta(x)\rangle | x \in X\}, \tag{2.3}$$

where $\beta, \alpha : X \to \mathbb{P}([0,1])$ are membership and non-membership functions on H. The set $\mathbb{P}([0,\ 1])$ denotes the collection of non-empty subsets of real numbers in $[0,1]$, Moreover, for any $x \in X$, $0 \le max\{a|a \in \alpha(x)\} + max\{b|b \in \beta(x)\} \le 1$.

Fatimah et al. [8] expand the concept SS to N-soft set.

Definition 2.4. [8] Suppose that U is a set of objects, E is a set of parameters or attributes, $A \subseteq E$. $R = \{0, 1, 2, ..., N-1\}$ is a set of grades where $N \in \{2, 3, ...\}$. An N-soft set (NSS) (F, A, N) over U is defined as

$$(F, A, N) = \{(a, F(a)) \mid a \in A\}$$

where $F : A \to 2^{U \times R}$ such that $F(a) = \{(u, r_{au}) | u \in U, r_{au} \in R\}$. Here, for some $a \in A$, for any $u \in U$ there exist a unique $r_{au} \in R$ so that we may write $r_{au} = F(u)(a)$ as the grade of the object u related to the parameter a.

3. Main results

First of all, we define a new hybrid model called a hesitant intuitionistic fuzzy N-soft set as a combination of HIFS and NSS, and IFNSS and HFNSS.

Definition 3.1. Let X be a set of objects, E be a set of parameters, and $A \subseteq E$. A pair (H_A, NF) is called a hesitant intuitionistic fuzzy N-soft set (HIFNSS) over X where $NF = (F, A, N)$ is an NSS over U, if

$$H_A : A \to \bigcup_{a \in A} \hat{\mathcal{F}}(F(a)),$$

where $\hat{\mathcal{F}}(F(a))$ is a collection of all HIFSs over $F(a)$.

An HIFNSS may restate as

$$(H_A, NF) = \{(a, H_A(a)) | a \in A\},$$

where $H_A(a) = \{\langle (u, r_{au}), \mu_a(u, r_{au}), \gamma_a(u, r_{au})\rangle | (u, r_{au}) \in F(a)\}$, with $\mu_a, \gamma_a : F(a) \to \mathbb{P}([0,1])$. Here, $\mathbb{P}([0,\ 1])$ denotes the collection of non-empty subsets of real numbers in $[0,1]$, r_{au} is a grade of an object u corresponding to a parameter a, and μ_a and γ_a are called membership and non-membership functions respectively. For simplify, we denote $m_{au} := \mu_a(u, r_{au})$ and $w_{au} := \gamma_a(u, r_{au})$ as a possible membership degrees and a possible non-membership degrees of an object u related to a parameter a, respectively, so that

$$(H_A, NF) = \{(a, H_A(a)) | a \in A\}, \tag{3.1}$$

$$\text{with } H_A(a) = \left\{\langle\frac{(u, r_{au})}{(m_{au}, w_{au})}\rangle \middle| (u, r_{au}) \in F(a)\right\}$$
$$0 \leq max\{\gamma|\gamma \in m_{au}\} + max\{\gamma|\gamma \in w_{au}\} \leq 1.$$

Furthermore, the set $\{\langle(u, r_{au}), \mu_a(u, r_{au}), \gamma_a(u, r_{au})\rangle|(u, r_{au}) \in F(a)\}$ may be written as

$$\{H_A(a)(u, r_{au})|(u, r_{au}) \in F(a)\},$$

with $H_A(a)(u, r_{au}) = \langle\frac{(u,r_{au})}{(m_{au},w_{au})}\rangle$. An HIFNSS over a set U may be represented in a table called Representation Table of an HIFNSS as in Table 1.

Table 1. Representation table of an HIFNSS.

(H_A, NF)	a_1	a_2	$\dots$	a_n
u_1	(r_{11}, m_{11}, w_{11})	(r_{12}, m_{12}, w_{12})	$\dots$	(r_{1n}, m_{1n}, w_{1n})
u_2	(r_{21}, m_{21}, w_{21})	(r_{22}, m_{22}, w_{22})	$\dots$	(r_{2n}, m_{2n}, w_{2n})
$\vdots$				
$\vdots$				
u_m	(r_{m1}, m_{m1}, w_{m1})	(r_{m2}, m_{m2}, w_{m2})	$\dots$	(r_{mn}, m_{mn}, w_{mn})

In Table 1, $u_i \in U, i = 1, ..., m, a_j \in A, j = 1, ..., n$, and (r_{ij}, m_{ij}, w_{ij}) at the cell (i, j) represents that $\langle(u_i, r_{ij}), m_{ij}, w_{ij}\rangle \in H_A(a_j)$ where $r_{ij} = r_{a_j u_i}$, $m_{ij} = \mu_{a_j}(u_i, r_{ij})$, and $w_{ij} = \gamma_{a_j}(u_i, r_{ij})$.

Example 1. The Indonesian Ministry of Agriculture conducts a selection of candidates for agricultural extension workers. The candidates taking the test are $u_1.u_2, u_3, u_4$ which is expressed in the set of objects $U = \{u_1.u_2, u_3, u_4\}$. Competencies (parameters) tested are e_1 =Development of Extension Programs, e_2 =Development of Farmer Participation and e_3 =Farmers' Education. Suppose $A = \{e_1, e_2, e_3\}$. The selection process is carried out in two stages: the written test stage and the interview stage of testing all types of competencies. At the written test stage, the test score s of each candidate is stated in grades as follows:
a) grade 4, if $8 < s \leq 10$.
b) grade 3, if $6 < s \leq 8$.
c) grade 2, if $4 < s \leq 6$.
d) grade 1, if $2 < s \leq 4$.
e) grade 0, if $0 \leq s \leq 2$.

Furthermore, the candidate's ability and inability to explain all the competencies tested will be assessed, from the interview test. The results of this assessment are expressed as real numbers in [0,1], which are the membership and non-membership values of each candidate for each parameter.

Following are the results of the assessment of all candidates, which can be stated in the table of representation of an HIFNSS (see Table 2).

Table 2. Representation table of an HIFNSS.

(H_A, NF)	e_1	e_2	a_3
u_1	$(4, \{0.60, 0.70\}, \{0.30, 0.25\})$	$(3, \{0.65, 0.75\}, \{0.20, 0.25\})$	$(2, \{0.60, 0.55\}, \{0.30, 0.35\})$
u_2	$(3, \{0.50, 0.55\}, \{0.30, 0.35\})$	$(2, \{0.50, 0.55\}, \{0.30, 0.35\})$	$(1, \{0.45, 0.30\}, \{0.55, 0.50\})$
u_2	$(2, \{0.40, 0.35\}, \{0.55, 0.50\})$	$(1, \{0.40, 0.35\}, \{0.55, 0.50\})$	$(3, \{0.65, 0.75\}, \{0.20, 0.25\})$
u_4	$(4, \{0.75, 0.80\}, \{0.20, 0.10\})$	$(4, \{0.75, 0.70\}, \{0.20, 0.15\})$	$(4, \{0.70, 0.80\}, \{0.20, 0.10\})$

Now, we define some complements of an HIFNSS.

Definition 3.2. The top grade complement of an HIFNSS (H_A, NF), as in (3.1), is defined as

$$(H_A^{tg}, NF) = \{(a, H_A^{tg}(a)) | a \in A\}, \tag{3.2}$$

where $H_A^{tg}(a) = \left\{ H_A^{tg}(a)(u, r_{au}^{tg}) \middle| (u, r_{au}) \in F(a) \right\}$ with

$$H_A^{tg}(a)(u, r_{au}^{tg}) := \begin{cases} \left\langle \frac{(u, N-1)}{(m_{au}, w_{au})} \right\rangle, & \text{if } r_{au} < N-1, \\ \left\langle \frac{(u, 0)}{(m_{au}, w_{au})} \right\rangle, & \text{if } r_{au} = N-1. \end{cases}$$

Definition 3.3. The bottom grade complement of an HIFNSS (H_A, NF), as in (3.1), is defined as

$$(I_A^{bg}, NF) = \{(a, H_A^{bg}(a)) | a \in A\} \tag{3.3}$$

where $H_A^{bg}(a) = \left\{ H_A^{bg}(a)(u, r_{au}^{bg}) \middle| (u, r_{au}) \in F(a) \right\}$ with

$$H_A^{bg}(a)(u, r_{au}^{bg}) := \begin{cases} \left\langle \frac{(u, 0)}{(m_{au}, w_{au})} \right\rangle, & \text{if } r_{au} > 0, \\ \left\langle \frac{(u, N-1)}{(m_{au}, w_{au})} \right\rangle, & \text{if } r_{au} = 0. \end{cases}$$

Definition 3.4. The top grade hesitant intuitionistic complement of an HIFNSS (H_A, NF), as in (3.1), is defined as

$$(H_A^{th}, NF) = \{(a, H_A^{th}(a)) | a \in A\}, \tag{3.4}$$

where $H_A^{th}(a) = \left\{ H_A^{th}(a)(u, r_{au}^{th}) \middle| (u, r_{au}) \in F(a) \right\}$ with

$$H_A^{th}(a)(u, r_{au}^{th}) := \begin{cases} \left\langle \frac{(u, N-1)}{(w_{au}, m_{au})} \right\rangle, & \text{if } r_{au} < N-1, \\ \left\langle \frac{(u, 0)}{(w_{au}, m_{au})} \right\rangle, & \text{if } r_{au} = N-1. \end{cases}$$

Definition 3.5. The bottom grade hesitant intuitionistic complement of an HIFNSS (H_A, NF), as in (3.1), is defined as

$$(H_A^{bh}, NF) = \{(a, H_A^{bh}(a)) | a \in A\}, \tag{3.5}$$

where $H_A^{bh}(a) = \left\{ H_A^{bh}(a)(u, r_{au}^{bh}) \middle| (u, r_{au}) \in F(a) \right\}$ with

$$H_A^{bh}(a)(u, r_{au}^{bh}) := \begin{cases} \left\langle \frac{(u,0)}{(w_{au},m_{au})} \right\rangle, & \text{if } r_{au} > 0, \\ \left\langle \frac{(u,N-1)}{(w_{au},m_{au})} \right\rangle, & \text{if } r_{au} = 0. \end{cases}$$

As a generalization of HIFNSS, and a study of HIFSS in context NSS, we propose the following concept called a generalized hesitant intuitionistic fuzzy N-soft set.

Definition 3.6. Let U be a set of objects and P be a set of parameters, $A \subseteq P$. Suppose (H_A, NF) is an HIFNSS over U and α is a FS over A, with $\alpha : A \to [0, 1]$. The triple (G_α, G, α), where $G = (F, A, N)$ is an NSS, is called a generalized hesitant intuitionistic fuzzy N-soft set (GHIFNSS) over U, if

$$G_\alpha : A \to \bigcup_{a \in A} \hat{\mathcal{F}}(F(a)) \times [0, 1],$$

which is defined as $G_\alpha(a) = (H_A(a), \alpha(a))$, with $\hat{\mathcal{F}}(F(a))$ is the collection of all hesitant intuitionistic fuzzy sets over $F(a)$. In more detail, a GHIFNSS can be written in the form

$$\begin{aligned} (G_\alpha, G, \alpha) &= \{(a, G_\alpha(a)) | a \in A\} \\ &= \{(a, H_A(a), \alpha(a)) | a \in A\} \end{aligned} \tag{3.6}$$

where $G_\alpha(a) = (H_A(a), \alpha(a)) = (\{\langle (u, r_{au}), \mu_a(u, r_{au}), \gamma_a(u, r_{au}) \rangle | (u, r_{au}) \in F(a)\}, \alpha(a))$.

Next, we may write

$$\begin{aligned} G_\alpha(a) &= (\{H_A(a)(u, r_{au}) | (u, r_{au}) \in F(a)\}, \alpha(a)) \\ &= \left(\left\{ \left\langle \frac{(u, r_{au})}{(m_{au}, w_{au})} \right\rangle \middle| (u, r_{au}) \in F(a) \right\}, \alpha(a) \right), \end{aligned} \tag{3.7}$$

with $H_A(a)(u, r_{au}) = \langle \frac{(u,r_{au})}{(m_{au},w_{au})} \rangle$, $m_{au} = \mu_a(u, r_{au})$ and $w_{au} = \gamma_a(u, r_{au})$. Since the definition is closely related to a hesitant intuitionistic fuzzy set, then it must be $0 \leq max\{\gamma | \gamma \in m_{au}\} + max\{\gamma | \gamma \in w_{au}\} \leq 1$.

A GHIFNSS contains not only a grade, degrees of membership, and degrees of non-membership for each object based on specific parameter, but also a degree of importance for each of these parameters, which is expressed by $\alpha(a)$. A GHIFNSS can also be represented in a table. The following table illustrates the representation of a GHIFNSS (Table 3), called representation table of a GHIFNSS.

Table 3. Representation table of a GHIFNSS.

(G_α, G, α)	$a_1; \alpha(a_1)$	$a_2; \alpha(a_2)$	...	$a_n; \alpha(a_n)$
u_1	(r_{11}, m_{11}, w_{11})	(r_{12}, m_{12}, w_{12})	...	(r_{1n}, m_{1n}, w_{1n})
u_2	(r_{21}, m_{21}, w_{21})	(r_{22}, m_{22}, w_{22})	...	(r_{2n}, m_{2n}, w_{2n})
⋮				
⋮				
u_m	(r_{m1}, m_{m1}, w_{m1})	(r_{m2}, m_{m2}, w_{m2})	...	(r_{mn}, m_{mn}, w_{mn})

In Table 3, $u_i \in U, i = 1, ..., m$, $a_j \in A, j = 1, ..., n$, and (r_{ij}, m_{ij}, w_{ij}) in each cell (i, j) represents that $\langle (u_i, r_{ij}), m_{ij}, w_{ij} \rangle \in H_A(a_j)(u_i, r_{a_j u_i})$ where $r_{ij} = r_{a_j u_i}$, $m_{ij} = \mu_{a_j}(u_i, r_{a_j u_i})$, $w_{ij} = \gamma_{a_j}(u_i, r_{a_j u_i})$ and $\alpha(a_j) \in [0, 1]$.

Example 2. Given a case as in Example 1. Assume that a decision maker defines that degrees of importance for each parameter as follows.

$$\alpha(e_1) = 0.5, \alpha(e_2) = 0.3, \alpha(e_3) = 0.2.$$

By this assumption we obtain a GHIFNSS as represented in Table 4.

Table 4. Representation table of a GHIFNSS.

(G_α, G, α)	$e_1; \alpha(e_1) = 0.5$	$e_2; \alpha(e_2) = 0.3$	$a_3; \alpha(e_3) = 0.2$
u_1	$(4, \{0.60, 0.70\}, \{0.30, 0.25\})$	$(3, \{0.65, 0.75\}, \{0.20, 0.25\})$	$(2, \{0.60, 0.55\}, \{0.30, 0.35\})$
u_2	$(3, \{0.50, 0.55\}, \{0.30, 0.35\})$	$(2, \{0.50, 0.55\}, \{0.30, 0.35\})$	$(1, \{0.45, 0.30\}, \{0.55, 0.50\})$
u_2	$(2, \{0.40, 0.35\}, \{0.55, 0.50\})$	$(1, \{0.40, 0.35\}, \{0.55, 0.50\})$	$(3, \{0.65, 0.75\}, \{0.20, 0.25\})$
u_4	$(4, \{0.75, 0.80\}, \{0.20, 0.10\})$	$(4, \{0.75, 0.70\}, \{0.20, 0.15\})$	$(4, \{0.70, 0.80\}, \{0.20, 0.10\})$

The following are developed forms of complements and operations on generalized hesitant intuitionistic fuzzy N-soft sets.

Definition 3.7. A weak complement of a GHIFNSS (G_α, G, α) as in (3.7), is defined by

$$(G^w_\alpha, G, \alpha) = \{(a, G^w_\alpha(a)) | a \in A\} \tag{3.8}$$

where

$$\begin{aligned} G^w_\alpha(a) &= (H'_A(a), \alpha(a)) \\ &= \left(\left\{ \left\langle \frac{(u, r'_{au})}{(m_{au}, w_{au})} \right\rangle \middle| (u, r_{au}) \in F(a) \right\}, \alpha(a) \right), \end{aligned}$$

with $H'_A(a) = \left\{ \left\langle \frac{(u, r'_{au})}{(m_{au}, w_{au})} \right\rangle \middle| (u, r_{au}) \in F(a) \right\}$, $r'_{au} \in R$, $r'_{au} \neq r_{au}$.

Grades in a weak complement of a GHIFNSS are different with those in the GHIFNSS. In contrast, the degrees of membership and non-membership and the degree of importance remain.

Definition 3.8. The hesitant intuitionistic fuzzy complement of a GHIFNSS (G_α, G, α) as in (3.7), is defined by

$$(G^h_\alpha, G, \alpha) = \{(a, G^h_\alpha(a)) | a \in A\}, \tag{3.9}$$

where

$$\begin{aligned} G^h_\alpha(a) &= (H^c_A(a), \alpha(a)), \\ &= \left(\left\{ \left\langle \frac{(u, r_{au})}{(w_{au}, m_{au})} \right\rangle \middle| (u, r_{au}) \in F(a) \right\}, \alpha(a) \right), \end{aligned}$$

with $H^c_A(a) = \left\{ \left\langle \frac{(u, r_{au})}{(w_{au}, m_{au})} \right\rangle \middle| (u, r_{au}) \in F(a) \right\}$.

In this complement, the degrees of membership of each object in (G_α, G, α), will be the degrees of non-membership in (G^h_α, G, α) and vice versa. At the same time, the grade and the degree of importance have not changed. Hence, the hesitant intuitionistic fuzzy complement of a GHIFNSS is unique.

Definition 3.9. The preference complement of a GHIFNSS (G_α, G, α) as in (3.7), is defined by

$$(G^p_\alpha, G, \alpha) = \{(a, G^p_\alpha(a))|a \in A\}, \tag{3.10}$$

where

$$\begin{aligned} G^p_\alpha(a) &= (H_A(a), \alpha^p(a)) \\ &= \left(\left\{\left\langle \frac{(u, r_{au})}{(m_{au}, w_{au})}\right\rangle \middle| (u, r_{au}) \in F(a)\right\}, 1 - \alpha(a)\right), \end{aligned}$$

with $\alpha^p(a) = 1 - \alpha(a)$.

In this complement, the degree of importance in the preference complement of a GHIFNSS, is one minus the corresponding degree of importance in the GHIFNSS. Meanwhile, grades, membership and non-membership degrees do not change. Therefore, the preference complement of a GHIFNSS is unique.

Definition 3.10. A weak hesitant intuitionistic fuzzy complement of a GHIFNSS (G_α, G, α) as in (3.7), is defined by

$$(G^{wh}_\alpha, G, \alpha) = \{(a, G^{wh}_\alpha(a))|a \in A\}, \tag{3.11}$$

where

$$\begin{aligned} G^{wh}_\alpha(a) &= (H^{c'}_A(a), \alpha(a)), \\ &= \left(\left\{\left\langle \frac{(u, r^{c'}_{au})}{(w_{au}, m_{au})}\right\rangle \middle| (u, r_{au}) \in F(a)\right\}, \alpha(a)\right), \end{aligned}$$

with $H^{c'}_A(a) = \left\{\left\langle \frac{(u, r^{c'}_{au})}{(w_{au}, m_{au})}\right\rangle \middle| (u, r_{au}) \in F(a)\right\}$, $r^{c'}_{au} \neq r_{au}$.

In this complement, the grades in a weak hesitant intuitionistic fuzzy complement of a GHIFNSS, are different from the corresponding grades in the GHIFNSS. In addition, the degrees of membership in a weak hesitant intuitionistic fuzzy complement of a GHIFNSS, will be the degrees of non-membership of the GHIFNSS, and vice versa. At the same time, the degrees of importance have not changed.

Definition 3.11. A weak preference complement of a GHIFNSS (G_α, G, α) as in (3.7), is defined by

$$(G^{wp}_\alpha, G, \alpha) = \{(a, G^{wp}_\alpha(a))|a \in A\}, \tag{3.12}$$

where

$$\begin{aligned} G^{wp}_\alpha(a) &= (H'_A(a), \alpha^p(a)), \\ &= \left(\left\{\left\langle \frac{(u, r'_{au})}{(m_{au}, w_{au})}\right\rangle \middle| (u, r_{au}) \in F(a)\right\}, 1 - \alpha(a)\right). \end{aligned}$$

In this complement, membership and non-membership degrees are the same as those in the GHIFNSS.

Definition 3.12. The hesitant intuitionistic fuzzy preference complement of a GHIFNSS (G_α, G, α) as in (3.7), is defined by

$$(G_\alpha^{hp}, G, \alpha) = \{(a, G_\alpha^{hp}(a))|a \in A\}, \tag{3.13}$$

where

$$\begin{aligned} G_\alpha^{hp}(a) &= (H_A^c(a), \alpha^p(a)), \\ &= \left(\left\{\left\langle \frac{(u, r_{au})}{(w_{au}, m_{au})} \right\rangle \middle| (u, r_{au}) \in F(a)\right\}, 1 - \alpha(a)\right). \end{aligned}$$

This complement is unique for a GHIFNSS, and the grades are remain.

Definition 3.13. A weak hesitant intuitionistic fuzzy preference complement of a GHIFNSS (G_α, G, α) as in (3.7), is defined by

$$(G_\alpha^{whp}, G, \alpha) = \{(a, G_\alpha^{whp}(a))|a \in A\}, \tag{3.14}$$

where

$$\begin{aligned} G_\alpha^{whp}(a) &= (H_A^{c'}(a), \alpha^p(a)), \\ &= \left(\left\{\left\langle \frac{(u, r_{au}^{c'})}{(w_{au}, m_{au})} \right\rangle \middle| (u, r_{au}) \in F(a)\right\}, 1 - \alpha(a)\right). \end{aligned}$$

The following are some interesting special complements of a weak complement, a weak hesitant intuitionistic fuzzy complement and a weak hesitant intuitionistic fuzzy preference complement of a GHIFNSS.

Definition 3.14. The top grade complement of a GHIFNSS (G_α, G, α) as in (3.7) is defined by

$$(G_\alpha^{tg}, G, \alpha) = \{(a, G_\alpha^{tg}(a))|a \in A\}, \tag{3.15}$$

where $G_\alpha^{tg}(a) = (H_A^{tg}(a), \alpha(a))$.

Compared to the GHIFNSS, the changes part from its complement is the grade, where the grade becomes $N - 1$ if the corresponding grade in the GHIFNSS is less than $N - 1$ and becomes 0 if that is equal to $N - 1$. By this definition, it is clear that the top grade complement of a GHIFNSS is unique.

Definition 3.15. The top grade hesitant intuitionistic fuzzy complement of a GHIFNSS (G_α, G, α) as in (3.7) is defined by

$$(G_\alpha^{th}, G, \alpha) = \{(a, G_\alpha^{th}(a))|a \in A\}, \tag{3.16}$$

where $G_\alpha^{th}(a) = (H_A^{th}(a), \alpha(a))$.

In this complement, the determination of the grades is the same as that of the top grade complement. At the same time, the membership and non-membership degrees in the GHIFNSS and the top grade hesitant intuitionistic fuzzy complement are interchanging each other. The top grade hesitant intuitionistic fuzzy complement of a GHIFNSS is also unique.

Definition 3.16. The top grade preference complement of a GHIFNSS (G_α, G, α) as in (3.7) is defined by

$$(G_\alpha^{tp}, G, \alpha) = \{(a, G_\alpha^{tp}(a))|a \in A\}, \tag{3.17}$$

where $G_\alpha^{tp}(a) = (H_A^{tg}(a), \alpha^p(a))$.

The difference between Definitions 3.16 and 3.14 is in the degree of importance. It is also clear that $(G_\alpha^{tp}, G, \alpha)$ is unique.

Definition 3.17. The top grade hesitant intuitionistic fuzzy preference complement of a GHIFNSS (G_α, G, α) as in (3.7) is defined as

$$(G_\alpha^{thp}, G, \alpha) = \{(a, G_\alpha^{thp}(a))|a \in A\}, \tag{3.18}$$

where $G_\alpha^{thp}(a) = (H_A^{th}(a), \alpha^p(a))$.

The difference between Definitions 3.15 and 3.17 is in the degree of importance. This kind of complement is also unique.

Definition 3.18. The bottom grade complement of a GHIFNSS (G_α, G, α) as in (3.7) is defined as the following

$$(G_\alpha^{bg}, G, \alpha) = \{(a, G_\alpha^{bg}(a))|a \in A\}, \tag{3.19}$$

where $G_\alpha^{bg}(a) = (H_A^{bg}(a), \alpha(a))$.

Compared to the GHIFNSS, the changes part from its complement is the grade, where the grade becomes 0 if the corresponding grade in the GHIFNSS is greater than 0 and becomes $N - 1$ if that is equal to 0. By this definition, it is clear that the top bottom grade complement of a GHIFNSS is unique.

Definition 3.19. The bottom grade hesitant intuitionistic fuzzy complement of a GHIFNSS (G_α, G, α) as in (3.7) is defined as

$$(G_\alpha^{bh}, G, \alpha) = \{(a, G_\alpha^{bh}(a))|a \in A\}, \tag{3.20}$$

where $G_\alpha^{bh}(a) = (H_A^{bh}(a), \alpha(a))$.

In this complement, the determination of the grades is the same as that of the bottom grade complement. However, the membership and non-membership degrees in the GHIFNSS and the bottom hesitant intuitionistic fuzzy complement are interchanging each other. This complement is also unique.

Definition 3.20. The bottom grade preference complement of a GHIFNSS (G_α, G, α) as in (3.7) is defined as

$$(G_\alpha^{bp}, G, \alpha) = \{(a, G_\alpha^{bp}(a))|a \in A\}, \tag{3.21}$$

where $G_\alpha^{bp}(a) = (H_A^{bg}(a), \alpha^p(a))$.

The difference between Definitions 3.20 and 3.18 is in the degree of importance. It is also clear that $(G_\alpha^{bp}, G, \alpha)$ is unique.

Definition 3.21. The bottom grade hesitant intuitionistic fuzzy preference complement of a GHIFNSS (G_α, G, α) as in (3.7) is defined as

$$(G_\alpha^{bhp}, G, \alpha) = \{(a, G_\alpha^{bhp}(a))|a \in A\}, \tag{3.22}$$

where $G_\alpha^{bhp}(a) = (H_A^{bh}(a), \alpha^p(a))$.

The difference between Definitions 3.19 and 3.21 is in the degree of importance. This kind of complement is also unique.

In the following, we define some operations on generalized hesitant intuitionistic fuzzy N-soft sets.

Definition 3.22. Suppose that $(G_\alpha, G_1, \alpha) = \{(a, H_A(a), \alpha(a)) \mid a \in A\}$ and $(G_\beta, G_2, \beta) = \{(a, H_B(a), \beta(a)) \mid b \in B\}$ are two GHIFNSSs over U, where $G_1 = (F_1, A, N_1)$ and $G_2 = (F_2, B, N_2)$ are two NSSs over U,

$$\begin{aligned} G_\alpha(a) &= (H_A(a), \alpha(a)) \\ &= \left(\left\{\left\langle \frac{(u, r_{au}^{(1)})}{(m_{au}^{(1)}, w_{au}^{(1)})} \right\rangle \middle| (u, r_{au}^{(1)}) \in F_1(a)\right\}, \alpha(a)\right), \end{aligned}$$

$$\begin{aligned} G_\beta(a) &= (H_B(a), \beta(a)) \\ &= \left(\left\{\left\langle \frac{(u, r_{au}^{(2)})}{(m_{au}^{(2)}, w_{au}^{(2)})} \right\rangle \middle| (u, r_{au}^{(2)}) \in F_2(a)\right\}, \beta(a)\right). \end{aligned}$$

A restricted intersection between (G_α, G_1, α) and (G_β, G_2, β), denoted by $(G_\alpha, G_1, \alpha) \cap_{\Re} (G_\beta, G_2, \beta)$, is defined as $(G_{\delta_{\Re\Im}}, G_3, \delta)$ where $G_3 = (F_3, A \cap B, min(N_1, N_2))$, and $\forall c \in C = A \cap B,\ u \in U$,

$$(G_{\delta_{\Re\Im}}, G_3, \delta) = \{(c, G_{\delta_{\Re\Im}}(c)) | c \in C\}. \tag{3.23}$$

Here,

$$\begin{aligned} G_{\delta_{\Re\Im}}(c) &= (H_C(c), \delta(c)) \\ &= \left(\left\{\left\langle \frac{(u, r_{cu}^{(3)})}{(m_{cu}^{(3)}, w_{cu}^{(3)})} \right\rangle \middle| (u, r_{cu}^{(3)}) \in F_3(c)\right\}, \delta(c)\right) \end{aligned}$$

with

$$\begin{aligned} r_{cu}^{(3)} &= min(r_{cu}^{(1)}, r_{cu}^{(2)}), \\ m_{cu}^{(3)} &= \{\gamma | \gamma = min\{p, q\}, p \in m_{cu}^{(1)}, q \in m_{cu}^{(2)}\}, \\ w_{cu}^{(3)} &= \{\gamma | \gamma = max\{p, q\}, p \in w_{cu}^{(1)}, q \in w_{cu}^{(2)}\}, \\ \delta(c) &= min(\alpha(c), \beta(c)). \end{aligned}$$

Definition 3.23. Suppose that $(G_\alpha, G_1, \alpha) = \{(a, H_A(a), \alpha(a)) \mid a \in A\}$ and $(G_\beta, G_2, \beta) = \{(a, H_B(a), \beta(a)) \mid b \in B\}$ are two GHIFNSSs over U, where $G_1 = (F_1, A, N_1)$ and $G_2 = (F_2, B, N_2)$ are two NSSs over U,

$$\begin{aligned} G_\alpha(a) &= (H_A(a), \alpha(a)) \\ &= \left(\left\{\left\langle \frac{(u, r_{au}^{(1)})}{(m_{au}^{(1)}, w_{au}^{(1)})} \right\rangle \middle| (u, r_{au}^{(1)}) \in F_1(a)\right\}, \alpha(a)\right), \end{aligned}$$

$$G_\beta(a) = (H_B(a), \beta(a))$$

$$= \left(\left\{\left\langle\frac{(u, r_{au}^{(2)})}{(m_{au}^{(2)}, w_{au}^{(2)})}\right\rangle\middle|(u, r_{au}^{(2)}) \in F_2(a)\right\}, \beta(a)\right).$$

An extended intersection between (G_α, G_1, α) and (G_β, G_2, β) denoted by $(G_\alpha, G_1, \alpha) \cap_{\mathfrak{E}} (G_\beta, G_2, \beta)$ is defined as $(G_{\delta_{\mathfrak{E}\mathfrak{I}}}, G_3, \delta)$ where $G_3 = (F_3, A \cap B, max(N_1, N_2))$, and $\forall c \in C = A \cup B,\ u \in U$,

$$(G_{\delta_{\mathfrak{E}\mathfrak{I}}}, G_3, \delta) = \{(c, G_{\delta_{\mathfrak{E}\mathfrak{I}}}(c)) | c \in C\}, \tag{3.24}$$

with $G_{\delta_{\mathfrak{E}\mathfrak{I}}}(c) = \begin{cases} G_\alpha(c), & \text{if } c \in A - B, \\ G_\beta(c), & \text{if } c \in B - A, \\ G_{\delta_{\mathfrak{R}\mathfrak{I}}}(c), & \text{if } c \in A \cap B. \end{cases}$

Definition 3.24. Suppose that $(G_\alpha, G_1, \alpha) = \{(a, H_A(a), \alpha(a)) \mid a \in A\}$ and $(G_\beta, G_2, \beta) = \{(a, H_B(a), \beta(a)) \mid b \in B\}$ are two GHIFNSSs over U, where $G_1 = (F_1, A, N_1)$ and $G_2 = (F_2, B, N_2)$ are two NSSs over U,

$$\begin{aligned} G_\alpha(a) &= (H_A(a), \alpha(a)) \\ &= \left(\left\{\left\langle\frac{(u, r_{au}^{(1)})}{(m_{au}^{(1)}, w_{au}^{(1)})}\right\rangle\middle|(u, r_{au}^{(1)}) \in F_1(a)\right\}, \alpha(a)\right), \end{aligned}$$

$$\begin{aligned} G_\beta(a) &= (H_B(a), \beta(a)) \\ &= \left(\left\{\left\langle\frac{(u, r_{au}^{(2)})}{(m_{au}^{(2)}, w_{au}^{(2)})}\right\rangle\middle|(u, r_{au}^{(2)}) \in F_2(a)\right\}, \beta(a)\right). \end{aligned}$$

A restricted union between (G_α, G_1, α) and (G_β, G_2, β) denoted by $(G_\alpha, G_1, \alpha) \cup_{\mathfrak{R}} (G_\beta, G_2, \beta)$ is defined as $(G_{\delta_{\mathfrak{R}\mathfrak{U}}}, G_3, \delta)$ where $G_3 = (F_3, A \cap B, max(N_1, N_2))$, and $\forall c \in C = A \cap B,\ u \in U$,

$$(G_{\delta_{\mathfrak{R}\mathfrak{U}}}, G_3, \delta) = \{(c, G_{\delta_{\mathfrak{R}\mathfrak{U}}}(c)) | c \in C\}. \tag{3.25}$$

Here,

$$\begin{aligned} G_{\delta_{\mathfrak{R}\mathfrak{U}}}(c) &= (H_C(c), \delta(c)) \\ &= \left(\left\{\left\langle\frac{(u, r_{cu}^{(3)})}{(m_{cu}^{(3)}, w_{cu}^{(3)})}\right\rangle\middle|(u, r_{cu}^{(3)}) \in F_3(c)\right\}, \delta(c)\right) \end{aligned}$$

with

$$\begin{aligned} r_{cu}^{(3)} &= max(r_{cu}^{(1)}, r_{cu}^{(2)}), \\ m_{cu}^{(3)} &= \{\gamma | \gamma = max\{p, q\}, p \in m_{cu}^{(1)}, q \in m_{cu}^{(2)}\}, \\ w_{cu}^{(3)} &= \{\gamma | \gamma = min\{p, q\}, p \in w_{cu}^{(1)}, q \in w_{cu}^{(2)}\}, \\ \delta(c) &= max(\alpha(c), \beta(c)). \end{aligned}$$

Definition 3.25. Suppose that $(G_\alpha, G_1, \alpha) = \{(a, H_A(a), \alpha(a)) \mid a \in A\}$ and $(G_\beta, G_2, \beta) = \{(a, H_B(a), \beta(a)) \mid b \in B\}$ are two GHIFNSSs over U, where $G_1 = (F_1, A, N_1)$ and $G_2 = (F_2, B, N_2)$ are two NSSs over U,

$$G_\alpha(a) = (H_A(a), \alpha(a))$$

$$
\begin{aligned}
&= \left(\left\{\left\langle \frac{(u, r_{au}^{(1)})}{(m_{au}^{(1)}, w_{au}^{(1)})} \right\rangle \middle| (u, r_{au}^{(1)}) \in F_1(a)\right\}, \alpha(a)\right), \\
G_\beta(a) &= (H_B(a), \beta(a)) \\
&= \left(\left\{\left\langle \frac{(u, r_{au}^{(2)})}{(m_{au}^{(2)}, w_{au}^{(2)})} \right\rangle \middle| (u, r_{au}^{(2)}) \in F_2(a)\right\}, \beta(a)\right).
\end{aligned}
$$

An extended union between (G_α, G_1, α) and (G_β, G_2, β), denoted by $(G_\alpha, G_1, \alpha) \cup_{\mathfrak{E}} (G_\beta, G_2, \beta)$, is defined as $(G_{\delta_{\mathfrak{EU}}}, G_3, \delta)$ where $G_3 = (F_3, A \cup B, max(N_1, N_2))$, and $\forall c \in C = A \cup B,\ u \in U$,

$$
(G_{\delta_{\mathfrak{EU}}}, G_3, \delta) = \{(c, G_{\delta_{\mathfrak{EU}}}(c)) | c \in C\}, \tag{3.26}
$$

with $G_{\delta_{\mathfrak{EU}}}(c) = \begin{cases} G_\alpha(c), & \text{if } c \in A - B, \\ G_\beta(c), & \text{if } c \in B - A, \\ G_{\delta_{\mathfrak{RU}}}(c), & \text{if } c \in A \cap B. \end{cases}$

Now, we prove some properties of GHIFNSS corresponding to the operations and the top or bottom grade hesitant intuitionistic fuzzy preference complements.

Theorem 3.26. *Given two top grade hesitant intuitionistic fuzzy preference complements $(G_\alpha^{thp}, G_1, \alpha)$ and $(G_\beta^{thp}, G_2, \beta)$ of (G_α, G_1, α) and (G_β, G_2, β) over U respectively, where $G_1 = (F_1, A, N_1)$ and $G_2 = (F_2, B, N_2)$ are two NSSs over U. Let $G_\alpha^{thp}(a) = (H_A^{th}(a), \alpha^p(a))$, where*

$$
H_A^{th}(a)(u, r_{au}^{th}) := \begin{cases} \left\langle \frac{(u, N-1)}{(w_{au}^{(1)}, m_{au}^{(1)})} \right\rangle, & if\ r_{au}^{(1)} < N-1, \\ \left\langle \frac{(u, 0)}{(w_{au}^{(1)}, m_{au}^{(1)})} \right\rangle, & if\ r_{au}^{(1)} = N-1, \end{cases}
$$

with $(u, r_{au}^{(1)}) \in F_1(a)$, and $G_\beta^{thp}(b) = (H_B^{th}(b), \beta^p(b))$, where

$$
H_B^{th}(b)(u, r_{bu}^{th}) := \begin{cases} \left\langle \frac{(u, N-1)}{(w_{bu}^{(2)}, m_{bu}^{(2)})} \right\rangle, & if\ r_{bu}^{(2)} < N-1, \\ \left\langle \frac{(u, 0)}{(w_{bu}^{(2)}, m_{bu}^{(2)})} \right\rangle, & if\ r_{bu}^{(2)} = N-1, \end{cases}
$$

with $(u, r_{bu}^{(2)}) \in F_2(b)$.

Then the following holds.

(1) Let $(G_\alpha, G_1, \alpha) \cap_{\mathfrak{R}} (G_\beta, G_2, \beta) = (G_{\delta_{\mathfrak{RI}}}, G_3, \delta)$ with $G_3 = (F_3, A \cap B, N = min(N_1, N_2))$. Then $(G_{\delta_{\mathfrak{RI}}}^{thp}, G_3, \delta) = (G_\alpha^{thp}, G_1, \alpha) \cup_{\mathfrak{R}} (G_\beta^{thp}, G_2, \beta)$.

(2) Let $(G_\alpha, G_1, \alpha) \cup_{\mathfrak{R}} (G_\beta, G_2, \beta) = (G_{\delta_{\mathfrak{RI}}}, G_3, \delta)$ with $G_3 = (F_3, A \cap B, N = max(N_1, N_2))$. Then $(G_{\delta_{\mathfrak{RI}}}^{thp}, G_3, \delta) = (G_\alpha^{thp}, G_1, \alpha) \cap_{\mathfrak{R}} (G_\beta^{thp}, G_2, \beta)$.

Proof. Let $(G_\alpha, G_1, \alpha) = \{(a, \{H_A(a)(u, r_{au}^{(1)}) | (u, r_{au}^{(1)}) \in F_1(a)\}, \alpha(a)) | a \in A\}$, with

$$
H_A(a)(u, r_{au}^{(1)}) = \left\langle \frac{(u, r_{au}^{(1)})}{(m_{au}^{(1)}, w_{au}^{(1)})} \right\rangle,
$$

and $(G_\beta, G_2, \beta) = \{(b, \{H_B(b)(u, r_{bu}^{(2)}) | (u, r_{bu}^{(2)}) \in F_2(b)\}, \beta(b)) | b \in B\}$, with

$$
H_B(b)(u, r_{bu}^{(2)}) = \left\langle \frac{(u, r_{bu}^{(2)})}{(m_{bu}^{(2)}, w_{bu}^{(2)})} \right\rangle.
$$

Using Definition 3.22 for $c \in C = A \cap B$

$$\begin{aligned} G_{\delta_{\Re\Im}}(c) &= (H_C(c), \delta(c)) \\ &= \left(\left\{\left\langle \frac{(u, r_{cu}^{(3)})}{(m_{cu}^{(3)}, w_{cu}^{(3)})} \right\rangle \middle| (u, r_{cu}^{(3)}) \in F_3(c)\right\}, \delta(c)\right) \end{aligned}$$

with

$$\begin{aligned} r_{cu}^{(3)} &= min(r_{cu}^{(1)}, r_{cu}^{(2)}), \\ m_{cu}^{(3)} &= \{\gamma | \gamma = min\{p, q\}, p \in m_{cu}^{(1)}, q \in m_{cu}^{(2)}\}, \\ w_{cu}^{(3)} &= \{\gamma | \gamma = max\{p, q\}, p \in w_{cu}^{(1)}, q \in w_{cu}^{(2)}\}, \\ \delta(c) &= min(\alpha(c), \beta(c)). \end{aligned}$$

Then, by Definition 3.17, $G^{thp}_{\delta_{\Re\Im}}(c) = (H_C^{th}(c), 1 - \delta(c))$ with

$$H_C^{th}(c)(u, r_{cu}^{th}) = \begin{cases} \left\langle \frac{(u,N-1)}{(w_{cu}^{(3)}, m_{cu}^{(3)})} \right\rangle, & \text{if } r_{cu}^{(3)} < N-1, (in\,case,\ r_{cu}^{(1)}, r_{cu}^{(2)} < N-1) \\ \left\langle \frac{(u,N-1)}{(w_{cu}^{(3)}, m_{cu}^{(3)})} \right\rangle, & \text{if } r_{cu}^{(3)} < N-1, (in\,case,\ r_{cu}^{(1)} < N-1, r_{cu}^{(2)} = N-1) \\ \left\langle \frac{(u,N-1)}{(w_{cu}^{(3)}, m_{cu}^{(3)})} \right\rangle, & \text{if } r_{cu}^{(3)} < N-1, (in\,case,\ r_{cu}^{(1)} = N-1, r_{cu}^{(2)} < N-1) \\ \left\langle \frac{(u,0)}{(w_{cu}^{(3)}, m_{cu}^{(3)})} \right\rangle, & \text{if } r_{bu}^{(3)} = N-1, (in\,case,\ r_{cu}^{(1)} = N-1 = r_{cu}^{(2)}). \end{cases}$$

On the other hand, by Definition 3.24,
$(G_\alpha^{thp}, G_1, \alpha) \cup_{\Re} (G_\beta^{thp}, G_2, \beta) = (G^{thp}_{\gamma_{\Re\mathfrak{U}}}, G_4, \gamma)$ with $G_4 = (F_4, C = A \cap B, max(N_1, N_2))$. Let, for $c \in C = A \cap B$,

$$\begin{aligned} G^{thp}_{\gamma_{\Re\mathfrak{U}}}(c) &= (H_C(c), \gamma(c)) \\ &= \left(\left\{\left\langle \frac{(u, r_{cu}^{(4)})}{(m_{cu}^{(4)}, w_{cu}^{(4)})} \right\rangle \middle| (u, r_{cu}^{(4)}) \in F_4(c)\right\}, \gamma(c)\right). \end{aligned}$$

Suppose that $H_C(c)(u, r_{cu}^{(4)}) = \left\langle \frac{(u, r_{cu}^{(4)})}{(m_{cu}^{(4)}, w_{cu}^{(4)})} \right\rangle$. Then
$m_{cu}^{(4)} = \{\gamma | \gamma = max\{p, q\}, p \in w_{cu}^{(1)}, q \in w_{cu}^{(2)}\} = w_{cu}^{(3)}$ and $w_{cu}^{(4)} = \{\gamma | \gamma = min\{p, q\}, p \in m_{cu}^{(1)}, q \in m_{cu}^{(2)}\} = m_{cu}^{(3)}$,

$$\begin{aligned} H_C(c)(u, r_{cu}^4) &= \begin{cases} \left\langle \frac{(u,N-1)}{(m_{cu}^{(4)}, w_{cu}^{(4)})} \right\rangle, & \text{if } r_{cu}^{(1)}, r_{cu}^{(2)} < N-1 \\ \left\langle \frac{(u,N-1)}{(m_{cu}^{(4)}, w_{cu}^{(4)})} \right\rangle, & \text{if } r_{cu}^{(1)} < N-1, r_{cu}^{(2)} = N-1 \\ \left\langle \frac{(u,N-1)}{(m_{cu}^{(4)}, w_{cu}^{(4)})} \right\rangle, & \text{if } r_{cu}^{(1)} = N-1, r_{cu}^{(2)} < N-1 \\ \left\langle \frac{(u,0)}{(m_{cu}^{(4)}, w_{cu}^{(4)})} \right\rangle, & \text{if } r_{cu}^{(1)} = N-1 = r_{cu}^{(2)}. \end{cases} \\ &= H_C^{th}(c)(u, r_{cu}^{th}) \end{aligned}$$

and $\gamma(c) = max(\alpha^p(c), \beta^p(c)) = max(1 - \alpha(c), 1 - \beta(c)) = 1 - min(\alpha(c), \beta(c)) = 1 - \delta(c)$. Therefore, Theorem 3.26 (1) is proved.
The proof of Theorem 3.26 (2) is similar.

□

Theorem 3.27. *Given two bottom grade hesitant intuitionistic fuzzy preference complements $(G^{bhp}_{\alpha}, G_1, \alpha)$ and $(G^{bhp}_{\beta}, G_2, \beta)$ of (G_α, G_1, α) and (G_β, G_2, β) over U respectively, where $G_1 = (F_1, A, N_1)$ and $G_2 = (F_2, B, N_2)$ are two NSSs over U. Let $G^{bhp}_{\alpha}(a) = (H^{bh}_A(a), \alpha^p(a))$, where*

$$H^{bh}_A(a)(u, r^{bh}_{au}) := \begin{cases} \left\langle \frac{(u,0)}{(w^{(1)}_{au}, m^{(1)}_{au})} \right\rangle, & if\ r^{(1)}_{au} > 0, \\ \left\langle \frac{(u,N-1)}{(w^{(1)}_{au}, m^{(1)}_{au})} \right\rangle, & if\ r^{(1)}_{au} = 0, \end{cases}$$

with $(u, r^{(1)}_{au}) \in F_1(a)$, and $G^{bhp}_{\beta}(b) = (H^{bh}_B(b), \beta^p(b))$, where

$$H^{bh}_B(b)(u, r^{bh}_{bu}) := \begin{cases} \left\langle \frac{(u,0)}{(w^{(2)}_{bu}, m^{(2)}_{bu})} \right\rangle, & if\ r^{(2)}_{bu} > 0, \\ \left\langle \frac{(u,N-1)}{(w^{(2)}_{bu}, m^{(2)}_{bu})} \right\rangle, & if\ r^{(2)}_{bu} = 0, \end{cases}$$

with $(u, r^{(2)}_{bu}) \in F_2(b)$.

Then the following holds.

(1) Let $(G_\alpha, G_1, \alpha) \cap_{\Re} (G_\beta, G_2, \beta) = (G_{\delta_{\Re\Im}}, G_3, \delta)$ with $G_3 = (F_3, A \cap B, N = min(N_1, N_2))$. Then $(G^{bhp}_{\delta_{\Re\Im}}, G_3, \delta) = (G^{bhp}_{\alpha}, G_1, \alpha) \cup_{\Re} (G^{bhp}_{\beta}, G_2, \beta)$.

(2) Let $(G_\alpha, G_1, \alpha) \cup_{\Re} (G_\beta, G_2, \beta) = (G_{\delta_{\Re\Im}}, G_3, \delta)$ with $G_3 = (F_3, A \cap B, N = max(N_1, N_2))$. Then $(G^{bhp}_{\delta_{\Re\Im}}, G_3, \delta) = (G^{bhp}_{\alpha}, G_1, \alpha) \cap_{\Re} (G^{bhp}_{\beta}, G_2, \beta)$.

Proof. Let $(G_\alpha, G_1, \alpha) = \{(a, \{H_A(a)(u, r^{(1)}_{au}) | (u, r^{(1)}_{au}) \in F_1(a)\}, \alpha(a)) | a \in A\}$, with

$$H_A(a)(u, r^{(1)}_{au}) = \left\langle \frac{(u, r^{(1)}_{au})}{(m^{(1)}_{au}, w^{(1)}_{au})} \right\rangle,$$

and $(G_\beta, G_2, \beta) = \{(b, \{H_B(b)(u, r^{(2)}_{bu}) | (u, r^{(2)}_{bu}) \in F_2(b)\}, \beta(b)) | b \in B\}$, with

$$H_B(b)(u, r^{(2)}_{bu}) = \left\langle \frac{(u, r^{(2)}_{bu})}{(m^{(2)}_{bu}, w^{(2)}_{bu})} \right\rangle.$$

Using Definition 3.22 for $c \in C = A \cap B$

$$\begin{aligned} G_{\delta_{\Re\Im}}(c) &= (H_C(c), \delta(c)) \\ &= \left(\left\{ \left\langle \frac{(u, r^{(3)}_{cu})}{(m^{(3)}_{cu}, w^{(3)}_{cu})} \right\rangle \middle| (u, r^{(3)}_{cu}) \in F_3(c) \right\}, \delta(c) \right) \end{aligned}$$

with

$$\begin{aligned} r^{(3)}_{cu} &= min(r^{(1)}_{cu}, r^{(2)}_{cu}), \\ m^{(3)}_{cu} &= \{\gamma | \gamma = min\{p, q\}, p \in m^{(1)}_{cu}, q \in m^{(2)}_{cu}\}, \\ w^{(3)}_{cu} &= \{\gamma | \gamma = max\{p, q\}, p \in w^{(1)}_{cu}, q \in w^{(2)}_{cu}\}, \\ \delta(c) &= min(\alpha(c), \beta(c)). \end{aligned}$$

Then, by Definition 3.21, $G^{bhp}_{\delta_{\Re\Im}}(c) = (H^{bh}_C(c), 1 - \delta(c))$ with

$$H^{bh}_C(c)(u, r^{bh}_{cu}) := \begin{cases} \left\langle \frac{(u,0)}{(w^{(3)}_{cu}, m^{(3)}_{cu})} \right\rangle, & \text{if } r^{(3)}_{cu} > 0, (in\,case,\ r^{(1)}_{cu}, r^{(2)}_{cu} > 0) \\ \left\langle \frac{(u,N-1)}{(w^{(3)}_{cu}, m^{(3)}_{cu})} \right\rangle, & \text{if } r^{(3)}_{cu} = 0, (in\,case,\ r^{(1)}_{cu} > 0, r^{(2)}_{cu} = 0) \\ \left\langle \frac{(u,N-1)}{(w^{(3)}_{cu}, m^{(3)}_{cu})} \right\rangle, & \text{if } r^{(3)}_{cu} = 0, (in\,case,\ r^{(1)}_{cu} = 0, r^{(2)}_{cu} > 0) \\ \left\langle \frac{(u,N-1)}{(w^{(3)}_{cu}, m^{(3)}_{cu})} \right\rangle, & \text{if } r^{(3)}_{bu} = 0, (in\,case,\ r^{(1)}_{cu} = 0 = r^{(2)}_{cu}). \end{cases}$$

On the other hand, by Definition 3.24,
$(G^{bhp}_\alpha, G_1, \alpha) \cup_{\Re} (G^{bhp}_\beta, G_2, \beta) = (G^{bhp}_{\gamma_{\Re\mathfrak{U}}}, G_4, \gamma)$ with $G_4 = (F_4, C = A \cap B, max(N_1, N_2))$. Let, for $c \in C = A \cap B$,

$$\begin{aligned} G^{bhp}_{\gamma_{\Re\mathfrak{U}}}(c) &= (H_C(c), \gamma(c)) \\ &= \left(\left\{ \left\langle \frac{(u, r^{(4)}_{cu})}{(m^{(4)}_{cu}, w^{(4)}_{cu})} \right\rangle \middle| (u, r^{(4)}_{cu}) \in F_4(c) \right\}, \gamma(c) \right). \end{aligned}$$

Suppose that $H_C(c)(u, r^{(4)}_{cu}) = \left\langle \frac{(u, r^{(4)}_{cu})}{(m^{(4)}_{cu}, w^{(4)}_{cu})} \right\rangle$. Then
$m^{(4)}_{cu} = \{\gamma | \gamma = max\{p, q\}, p \in w^{(1)}_{cu}, q \in w^{(2)}_{cu}\} = w^{(3)}_{cu}$ and $w^{(4)}_{cu} = \{\gamma | \gamma = min\{p, q\}, p \in m^{(1)}_{cu}, q \in m^{(2)}_{cu}\} = m^{(3)}_{cu}$,

$$\begin{aligned} H_C(c)(u, r^4_{cu}) &= \begin{cases} \left\langle \frac{(u,0)}{(m^{(4)}_{cu}, w^{(4)}_{cu})} \right\rangle, & \text{if } r^{(1)}_{cu}, r^{(2)}_{cu} > 0 \\ \left\langle \frac{(u,N-1)}{(m^{(4)}_{cu}, w^{(4)}_{cu})} \right\rangle, & \text{if } r^{(1)}_{cu} > 0, r^{(2)}_{cu} = 0 \\ \left\langle \frac{(u,N-1)}{(m^{(4)}_{cu}, w^{(4)}_{cu})} \right\rangle, & \text{if } r^{(1)}_{cu} = 0, r^{(2)}_{cu} > 0 \\ \left\langle \frac{(u,N-1)}{(m^{(4)}_{cu}, w^{(4)}_{cu})} \right\rangle, & \text{if } r^{(1)}_{cu} = 0 = r^{(2)}_{cu}. \end{cases} \\ &= H^{th}_C(c)(u, r^{th}_{cu}) \end{aligned}$$

and $\gamma(c) = max(\alpha^p(c), \beta^p(c)) = max(1 - \alpha(c), 1 - \beta(c)) = 1 - min(\alpha(c), \beta(c)) = 1 - \delta(c)$. Therefore, Theorem 3.27 (1) is proved.
The proof of Theorem 3.27 (2) is similar. □

Using Definitions 3.17, 3.21, 3.23 and 3.25, we obtain the following two theorems, in which proving is similar to Theorems 3.26 and 3.27 respectively.

Theorem 3.28. *Given two top grade hesitant intuitionistic fuzzy preference complements* $(G^{thp}_\alpha, G_1, \alpha)$ *and* $(G^{thp}_\beta, G_2, \beta)$ *of* (G_α, G_1, α) *and* (G_β, G_2, β) *over U respectively.*
Then the following holds.

(1) Let $(G_\alpha, G_1, \alpha) \cap_{\mathfrak{E}} (G_\beta, G_2, \beta) = (G_{\delta_{\Re\Im}}, G_3, \delta)$ *with* $G_3 = (F_3, A \cap B, N = min(N_1, N_2))$*. Then* $(G^{thp}_{\delta_{\Re\Im}}, G_3, \delta) = (G^{thp}_\alpha, G_1, \alpha) \cup_{\mathfrak{E}} (G^{thp}_\beta, G_2, \beta)$*.*
(2) Let $(G_\alpha, G_1, \alpha) \cup_{\mathfrak{E}} (G_\beta, G_2, \beta) = (G_{\delta_{\Re\Im}}, G_3, \delta)$ *with* $G_3 = (F_3, A \cap B, N = max(N_1, N_2))$*. Then* $(G^{thp}_{\delta_{\Re\Im}}, G_3, \delta) = (G^{thp}_\alpha, G_1, \alpha) \cap_{\mathfrak{E}} (G^{thp}_\beta, G_2, \beta)$*.*

Theorem 3.29. *Given two bottom grade hesitant intuitionistic fuzzy preference complements* $(G_\alpha^{bhp}, G_1, \alpha)$ *and* $(G_\beta^{bhp}, G_2, \beta)$ *of* (G_α, G_1, α) *and* (G_β, G_2, β) *over U respectively.*
Then the following holds.

(1) Let $(G_\alpha, G_1, \alpha) \cap_{\mathfrak{E}} (G_\beta, G_2, \beta) = (G_{\delta_{\mathfrak{R}\mathfrak{Z}}}, G_3, \delta)$ *with* $G_3 = (F_3, A \cap B, N = min(N_1, N_2))$*. Then* $(G_{\delta_{\mathfrak{R}\mathfrak{Z}}}^{bhp}, G_3, \delta) = (G_\alpha^{bhp}, G_1, \alpha) \cup_{\mathfrak{E}} (G_\beta^{bhp}, G_2, \beta)$*.*

(2) Let $(G_\alpha, G_1, \alpha) \cup_{\mathfrak{E}} (G_\beta, G_2, \beta) = (G_{\delta_{\mathfrak{R}\mathfrak{Z}}}, G_3, \delta)$ *with* $G_3 = (F_3, A \cap B, N = max(N_1, N_2))$*. Then* $(G_{\delta_{\mathfrak{R}\mathfrak{Z}}}^{bhp}, G_3, \delta) = (G_\alpha^{bhp}, G_1, \alpha) \cap_{\mathfrak{E}} (G_\beta^{bhp}, G_2, \beta)$*.*

4. Applications of generalized hesitant intuitionistic fuzzy N-soft sets

In this section, we propose a decision-making algorithm for a decision-making problem represented as a GHIFNSS. Before that, several definitions used in the decision-making process will be introduced.

Definition 4.1. Given a GHIFNSS (G_α, G, α) over U as in Eq (3.6). An induced generalized intuitionistic fuzzy N-soft set (IGIFNSS) (IG_α, G, α) over U is defined as follows.

$$\begin{aligned}(IG_\alpha, G, \alpha) &= \{(a, IG_\alpha(a))|a \in A\} \\ &= \{(a, IF_A(a), \alpha(a))|a \in A\}\end{aligned} \tag{4.1}$$

where $IG_\alpha(a) = (IF_A(a), \alpha(a))$ and $IF_A(a) = \{\langle (u, r_{au}), \bar{\mu}_a(u, r_{au}), \bar{\gamma}_a(u, r_{au})\rangle | (u, r_{au}) \in F(a)\}$, with $\bar{\mu}_a(u, r_{au}) := \frac{\sum_{\mu \in m_{au}} \mu}{|m_{au}|}$ and $\bar{\gamma}_a(u, r_{au}) := \frac{\sum_{\gamma \in w_{au}} \gamma}{|w_{au}|}$.

An IGIFNSS over U may be represented in a table called representation table of an IGIFNSS as in Table 5.

Table 5. Representation table of a IGIFNSS.

(IG_α, G, α)	$a_1; \alpha(a_1)$	$a_2; \alpha(a_2)$	...	$a_n; \alpha(a_n)$
u_1	$(r_{11}, \bar{m}_{11}, \bar{w}_{11})$	$(r_{12}, \bar{m}_{12}, \bar{w}_{12})$	...	$(r_{1n}, \bar{m}_{1n}, \bar{w}_{1n})$
u_2	$(r_{21}, \bar{m}_{21}, \bar{w}_{21})$	$(r_{22}, \bar{m}_{22}, \bar{w}_{22})$	...	$(r_{2n}, \bar{m}_{2n}, \bar{w}_{2n})$
⋮				
⋮				
u_m	$(r_{m1}, \bar{m}_{m1}, \bar{w}_{m1})$	$(r_{m2}, \bar{m}_{m2}, \bar{w}_{m2})$	...	$(r_{mn}, \bar{m}_{mn}, \bar{w}_{mn})$

In Table 5, $u_i \in U, i = 1, ..., m$, $a_j \in A, j = 1, ..., n$, and $(r_{ij}, \bar{m}_{ij}, \bar{w}_{ij})$ in each cell (i, j) represents that $\langle (u_i, r_{ij}), \bar{m}_{ij}, \bar{w}_{ij}\rangle \in IF_A(a_j)(u_i, r_{a_j u_i})$ where $r_{ij} = r_{a_j u_i}$, $\bar{m}_{ij} = \bar{\mu}_{a_j}(u_i, r_{a_j u_i})$, $\bar{w}_{ij} = \bar{\gamma}_{a_j}(u_i, r_{a_j u_i})$ and $\alpha(a_j) \in [0, 1]$.

Cağman and Karatas [7] (see also Khan and Zhu [9]) developed a novel algorithm to solve decision-making problems based on Intuitionistic Fuzzy Soft sets (IFSSs). Referring to these algorithms, we propose a similar algorithm but in the field of IGIFNSS information. For this, we define the following definitions. For an object u_i at e_j, it is defined the left and right values of an IGIFNSS, as

$$\bar{m}_{ij}^l := \bar{m}_{ij} \text{ and } \bar{m}_{ij}^r := 1 - \bar{w}_{ij} \tag{4.2}$$

respectively.

Definition 4.2. Given an IGIFNSS (IG_α, G, α) over U where $G = \{F, A, N\}$, $U = \{u_1, u_2, \ldots, u_m\}$ and $A = \{e_1, e_2, \ldots, e_n\}$. The intuitionistic value of an object u_i at e_j, of the IGIFNSS, is defined by

$$\psi_{e_j}(u_i) = \bar{m}^l_{ij} + \bar{m}^r_{ij}, \tag{4.3}$$

with $\bar{m}^l_{ij}$ and $\bar{m}^r_{ij}$ are the left and right values of u_i at e_j respectively.

Definition 4.3. Given an IGIFNSS (IG_α, G, α) over U where $G = \{F, A, N\}$, $U = \{u_1, u_2, \ldots, u_m\}$ and $A = \{e_1, e_2, \ldots, e_n\}$. The grade score and the membership score of an object u_i at e_j, of the IGIFNSS, is defined by

$$\tilde{g}_{e_j}(u_i) = \sum_{k=1}^{m} (r_{ij} - r_{kj}), \tag{4.4}$$

and

$$\tilde{s}_{e_j}(u_i) = \sum_{k=1}^{m} (\psi_{e_j}(u_i) - \psi_{e_j}(u_k)), \tag{4.5}$$

respectively.

Definition 4.4. Given an IGIFNSS (IG_α, G, α) over U where $G = \{F, A, N\}$, $U = \{u_1, u_2, \ldots, u_n\}$ and $A = \{e_1, e_2, \ldots, e_m\}$. The total score of an object u_i, of the IGIFNSS, is defined by

$$T_i = \sum_{k=1}^{n} \alpha(e_k)(\tilde{g}_{e_k}(u_i) + \tilde{s}_{e_k}(u_i)). \tag{4.6}$$

The optimal score to determine the best u_b is computed by

$$T_b = \max_{1 \le i \le n}(T_i). \tag{4.7}$$

Note that the evaluation score of an object u_i at e_j as in Theorem 1 [9] is a special case of the total score (4.6) with $\alpha(e_k) = 1$ and $\tilde{g}_{e_k}(u_i) = 0$ for any $e_k \in A$ and $u_i \in U$. This means that the GHIFNSS as a generalization of the IFSS gives the total score formula as a generalization of the evaluation score formula.

Now, we present an algorithm for decision-making problems as an application of GHIFNSSs.

Algorithm

(1) Define a representation table of a GHIFNSS (G_α, G, α) over U.
(2) Using (G_α, G, α) over U, set the IGIFNSS (IG_α, G, α) over U.
(3) Using (IG_α, G, α) over U, compute the left and the right values for any object u_i at any parameter e_j.
(4) Compute the intuitionistic value $\psi_{e_j}(u_i)$ for any object u_i at any parameter e_j.
(5) Calculate the grade and membership scores $\tilde{g}_{e_j}(u_i)$ and $\tilde{s}_{e_j}(u_i)$ respectively, for any object u_i at any parameter e_j.
(6) Calculate the total score T_i for any object u_i
(7) If $T_b = \max_{1 \le i \le n}(T_i)$ is the maximum score, then the object u_b is the best choice.

Example 3. Given the decision-making problem as in Example 2. To determine the best candidate for agricultural extension worker, we apply the Algorithm above.

(1) Defined the representation table of a GHIFNSS (G_α, G, α) over U as in Table 4.
(2) Using Definition 4.1, we obtain the IGIFNSS (IG_α, G, α) over U as in Table 6.
(3) Using (IG_α, G, α) over U, we compute the left and the right values for any object u_i at any parameter e_j, of the IGIFNSS (IG_α, G, α) over U, by using Eq (4.2) and we present in Table 7.
(4) We compute the intuitionistic value $\psi_{e_j}(u_i)$, using Definition 4.2 for any object u_i at any parameter e_j and represented in Table 8.
(5) Calculate the grade and the membership scores $\tilde{g}_{e_j}(u_i)$ and $\tilde{s}_{e_j}(u_i)$ respectively, for any object u_i at any parameter e_j, using Definition 4.3 and we obtain Table 9.
(6) We get the total score T_i for any object u_i as in Table 10, using Definition 4.4.
(7) Since $T_b = \max_{a \leq i \leq n}(T_i) =$ is the maximum score, then the object u_b is the best choice.

Table 6. Representation table of an IGIFNSS.

(IG_α, G, α)	$e_1; \alpha(e_1) = 0.5$	$e_2; \alpha(e_2) = 0.3$	$a_3; \alpha(e_3) = 0.2$
u_1	$(4, 0.650, 0.275)$	$(3, 0.700, 0.225)$	$(2, 0.575, 0.325)$
u_2	$(3, 0.525, 0.325)$	$(2, 0.525, 0.325)$	$(1, 0.375, 0.525)$
u_2	$(2, 0.375, 0.525)$	$(1, 0.375, 0.525)$	$(3, 0.700, 0.225)$
u_4	$(4, 0.775, 0.150)$	$(4, 0.725, 0.175)$	$(4, 0.750, 0.150)$

Table 7. Table of the left and the right values of the IGIFNSS.

$(r_{ij}, \bar{m}^l_{ij}, \bar{m}^r_{ij})$	$e_1; \alpha(e_1) = 0.5$	$e_2; \alpha(e_2) = 0.3$	$a_3; \alpha(e_3) = 0.2$
u_1	$(4, 0.650, 0.725)$	$(3, 0.700, 0.775)$	$(2, 0.575, 0.675)$
u_2	$(3, 0.525, 0.675)$	$(2, 0.525, 0.675)$	$(1, 0.375, 0.475)$
u_2	$(2, 0.375, 0.475)$	$(1, 0.375, 0.475)$	$(3, 0.700, 0.775)$
u_4	$(4, 0.775, 0.850)$	$(4, 0.725, 0.825)$	$(4, 0.750, 0.850)$

Table 8. Table of the intuitionistic values of the IGIFNSS.

$(r_{ij}, \psi_{e_j}(u_i))$	$e_1; \alpha(e_1) = 0.5$	$e_2; \alpha(e_2) = 0.3$	$a_3; \alpha(e_3) = 0.2$
u_1	$(4, 1.375)$	$(3, 1.475)$	$(2, 1.250)$
u_2	$(3, 1.200)$	$(2, 1.200)$	$(1, 0.850)$
u_2	$(2, 0.875)$	$(1, 0.875)$	$(3, 1.475)$
u_4	$(4, 1.625)$	$(4, 1.550)$	$(4, 1.600)$

Table 9. Table of the grade and the membership scores of the IGIFNSS.

$(\tilde{g}_{e_j}(u_i), \tilde{s}_{e_j}(u_i))$	$e_1; \alpha(e_1) = 0.5$	$e_2; \alpha(e_2) = 0.3$	$a_3; \alpha(e_3) = 0.2$
u_1	$(3, 0.45)$	$(2, 0.83)$	$(-2, -0.18)$
u_2	$(-1, -0.25)$	$(-2, -0.27)$	$(-6, -1.78)$
u_2	$(-5, -1.65)$	$(-6, -1.68)$	$(2, 0.73)$
u_4	$(3, 1.45)$	$(6, 1.13)$	$(6, 1.23)$

Table 10. Table of total scores of the IGIFNSS.

u_i	T_i
u_1	2.14
u_2	-2.86
u_2	-5.08
u_4	5.81

Since T_4 is the maximum, then the best candidate is u_4.

5. Conclusions

Previous scholars have proposed fuzzy N-soft sets and hesitant fuzzy N-soft sets. Furthermore, we generalize HIFNSSs to generalized hesitant intuitionistic fuzzy N-soft sets (GHIFNSSs) as a hybrid model between generalized hesitant intuitionistic fuzzy sets and N-soft sets. Then, it was introduced some complements of the GHIFNSSs, intersection and union operations between GHIFNSSs, and proved that the operations between some particular complements hold De Morgan Law. An algorithm for decision-making problems in GHIFNSSs information was constructed, and a numerical example was given.

This research can be extended by combining the concept of generalized interval-valued hesitant intuitionistic fuzzy soft sets (see Nazra et al. [14]) and NSS. Therefore, the study on NSS is more complete and more general. On the other hand, the study on the distance measure introduced by Xiao [15], may be constructed in the field of GHIFNSS.

Acknowledgments

This research is supported by research fund from Universitas Andalas in accordance with contract of Professor's acceleration research cluster scheme (Batch II), T/25/UN.16.17/PP.Energi-PDU-KRP2GB-Unand/2021.

Conflict of interest

The authors declare that there are no conflicts of interest.

References

1. M. Akram, G. Ali, J. C. R. Alcantud, New decision-making hybridmodel: Intuitionistic fuzzy N-soft rough sets, *Soft Comput.*, **23** (2019), 9853–9868.
2. M. Akram, A. Adeel, J. C. R. Alcantud, Hesitant fuzzy N-soft sets: A novel model with applications in decision making, *J. Inte. Fuzzy Syst.*, **36** (2019), 6113–6127. https://doi.org/10.3233/JIFS-181972
3. M. Akram, A. Adeel, J. C. R. Alcantud, Fuzzy N-soft sets: A novel model with applications in decision making, *J. Intell. Fuzzy Syst.*, **35** (2018), 4757–4771.
4. K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Set. Syst.*, **20** (1986), 87–96. https://doi.org/10.1007/978-3-7908-1870-3
5. K. V. Babitha, S. J. John, Hesitant fuzzy soft sets, *J. New Result. Sci.,* **3** (2013), 98–107.
6. I. Beg, T. Rashid, Group decision making using intuitionistic hesitant fuzzy sets, *Int. J. Fuzzy Log. Intell.*, **14** (2014), 181–187. https://doi.org/10.5391/IJFIS.2014.14.3.181
7. N. Cağman, S. Karatas, Intuitionistic fuzzy soft set theory and its decision making, *J. Intell. Fuzzy Syst.*, **24** (2013), 829–836.
8. F. Fatimah, D. Rosadi, R. B. F. Hakim, J. C. R. Alcantud, N-soft sets and their decision making algorithms, *Soft Comput.*, **22** (2018), 3829–3842. https://doi.org/10.1007/s00500-017-2838-6
9. A. Khan, Y. Zhu, New algorithms for parameter reduction of intuitionistic fuzzy soft sets, *Comput. Appl. Math.*, **39** (2020), 232. https://doi.org/10.1007/s40314-020-01279-4
10. P. K. Maji, R. Biswas, A. R. Roy, Fuzzy soft sets, *J. Fuzzy Math.*, **9** (2001), 589–602.
11. D. Molodtsov, Soft set theory-first result, *Comput. Math. Appl.*, **37** (1999), 19–31. https://doi.org/10.1016/S0898-1221(99)00056-5
12. A. Nazra, Syafruddin, R. Lestari, G. C. Wicaksono, Hesitant intuitionistic fuzzy soft sets, *J. Phys.- Conf. Ser.*, **890** (2017), 012118. https://doi.org/10.1088/1742-6596/890/1/012118
13. A. Nazra, Syafruddin, G. C. Wicaksono, M. Syafwan, A study on generalized hesitant intuitionistic fuzzy soft sets, *J. Phys.- Conf. Ser.*, **983** (2018), 012127. https://doi.org/10.1088/1742-6596/983/1/012127
14. A. Nazra, Y. Asdi, S. Wahyuni, H. Ramadhani, Zulvera, Generalized interval-valued hesitant intuitionistic fuzzy soft sets, *J. Intell. Fuzzy Syst.*, **40** (2021), 11039–11050. https://doi.org/10.3233/JIFS-202185
15. F. Xiao, A distance measure for intuitionistic fuzzy sets and its application to pattern classification problems, *IEEE T. Syst. Man Cy-S.*, **51** (2021), 3980–3992. https://doi.org/10.1109/TSMC.2019.2958635
16. L. A. Zadeh, Fuzzy set, *Inform. Control*, **8** (1965), 338–353.