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Research article

Maximum likelihood DOA estimation based on improved invasive weed optimization algorithm and application of MEMS vector hydrophone array

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Abstract: Direction of arrival (DOA) estimation based on Maximum Likelihood is a common method in array signal processing, with many practical applications, but the huge amount of calculation limits the practical application. To deal with such an Maximum Likelihood (ML) DOA estimation problem, firstly, the DOA estimation model with ML for acoustic vector sensor array is developed, where the optimization standard in various cases can be unified by converting the maximum of objective function to the minimum. Secondly, based on the Invasive Weed Optimization (IWO) method which is a novel biological evolutionary algorithm, a new Improved IWO (IIWO) algorithm for DOA estimation of the acoustic vector sensor array is proposed by using ML estimation. This algorithm simulates weed invasion process for DOA estimation by adjusting the non-linear harmonic exponent of IWO algorithm adaptively. The DOA estimation accuracy has been improved, and the computation of multidimensional nonlinear optimization for the ML method has been greatly reduced in the IIWO algorithm. Finally, compared with Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Differential Evolution (DE) method and Tuna Swarm Optimization(TSO) algorithm, numerical simulations show that the proposed algorithm has faster convergence rate, improved accuracy in terms of Root Mean Square Error (RMSE), lower computational complexity and more robust estimation performance for ML DOA estimation. The experiment with tracking the orientation of the motorboat by Microelectronic mechanical systems (MEMS) vector hydrophone array shows the superior performance of proposed IIWO algorithm in engineering application. Therefore, the proposed ML-DOA estimation with IIWO algorithm can take into account both resolution and computation. which can meet the requirements of real-time calculation and estimation accuracy in the actual environment.

Keywords: Direction of arrival (DOA) estimation; Invasive Weed Optimization (IWO); Maximum Likelihood (ML); acoustic vector sensor array; MEMS vector hydrophone
Mathematics Subject Classification: 65K05, 65K10

1. Introduction

Signal processing of acoustic vector sensor array has been an active research area for decades. Compared with traditional acoustic pressure hydrophone, a vector hydrophone can measure the acoustic pressure and particle velocity in the acoustic field simultaneously. The main advantage of these vector sensors is that they make better use of available acoustic information, which could improve the performance of Direction of arrival (DOA) estimation without increasing array aperture size [1–4]. For instance, Micro Electronic Mechanical Systems (MEMS) vector hydrophone is one of the excellent hydrophones, which has been used in some practical applications [5,6]. Since the measurement model of the acoustic vector sensor array for dealing with narrowband signals [7] has been proposed by Nehorai and Paldi in 1994, many useful estimation methods have been proposed including Maximum Likelihood (ML) methods [8], Multiple Signal Classification (MUSIC) methods [9], Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) [10], etc. [5,11–13].

The ML method is an excellent and robust estimation technique with better statistical performance, whose performance is better than subspace decomposition methods such as MUSIC and ESPRIT, especially under the conditions of lower Signal to Noise Ratio (SNR) or smaller snapshot number. However, implementing ML estimation requires maximizing a nonlinear multidimensional cost function under the critical condition of SNR [14, 15]. Therefore, many multi-dimensional search methods which can reduce the computation complexity have been proposed for decades, such as Method of Direction Estimation with EXtra-roots (MODEX) [16], Alternating Projection (AP) [17], Expectation Maximization (EM) [18, 19], Space Alternating Generalized Expectation (SAGE) maximization method [20], etc.

However, these methods also have some weaknesses, which limit their performance in practical applications. The general MODEX method needs to decompose the covariance matrix of data, and there is a threshold at the same time. When the number of SNR or snapshots is lower than the threshold, the estimation performance will be greatly reduced. AP search method can reduce the computation by transforming multi-dimensional search into one-dimensional search, but its convergence speed will tend to be slow with the increase of the number of signal sources. The results of the EM method often fall into the local optimum solution, but can not get the global optimum solution. The SAGE method requires a large number of iterative operations, which leads to high computational complexity.

Considering the estimation accuracy and computational complexity, an improved Maximum Likelihood (IML) method is proposed [21], which uses moment estimation for search optimization processing. In [22], a two-dimensional ML DOA estimation algorithm based on the uniform rectangular array is proposed, which utilizes Pincus theorem and Monte Carlo method to achieve global optimum, and can achieve Cramer-Rao lower bound (CRLB) even in low signal-to-noise ratio (SNR) scenarios.

In recent years, many algorithms have emerged to deal with practical problems. Liu et al. designed a new mixed variable differential evolution (MVDE) and differentiated it with some specific operators to solve the EVCS (vehicle charging scheduling) problem of hierarchical mixed variables [23]. Zhao et al. proposed a two-stage co-evolutionary algorithm with problem-specific knowledge to solve the wait-free flow shop problem with maximum power consumption and total power consumption minimization energy-saving scheduling problem [24]. Zhou et al. proposed an adaptive adaptive differential evolution (SDE) algorithm for a single Batch-Processing Machine (BPM) scheduling problem with different job sizes and release times [25]. Zhao et al. proposed an ensemble discrete differential evolution (EDE) algorithm to solve the minimized jam-flow shop scheduling problem in a distributed manufacturing environment [26]. Zhou et al. proposed a Self-learning Discrete Jaya (SD-Jaya) algorithm to address the energy-efficient distributed no-idle flow-shop scheduling problem in a heterogeneous factory system with the criteria of minimizing the total tardiness, total energy consumption, and factory load balancing [27].

With the development of bionic evolutionary algorithms, some methods are used in ML DOA estimation, such as Genetic Algorithm (GA) [28], Particle Swarm Optimization (PSO) [29], Differential Evolution (DE) method [30] and Tuna Swarm Optimization (TSO) algorithm [31], etc. In [28], a GA-ML estimation method with almost guarantees global convergence is proposed, and simulation experiments show that the estimation performance of this method is better than that of the traditional MUSIC algorithm in SNR, number of snapshots and computation complexity. In [29], a method to optimize complex nonlinear multimodal functions in high-dimensional space based on PSO-ML estimation is proposed. Numerical simulation shows that it has good statistical performance in correlation and coherent signals environment compared with traditional DOA estimation methods. In [30], an evolutionary algorithm for jointing amplitude and DOA estimation of signals is proposed and Differential Evolution algorithm with Mean Square Error is used as a fitness evaluation function. The algorithm demands only one snapshot to converge and produce fairly good results even in the presence of low SNR. In [32], the study of some populational meta-heuristics for natural computation in DOA estimation is presented. Simulation results show that, regardless of the number and source of signals, these methods can achieve the balance between global search and local improvement of ML function, to achieve global optimization. In [33], a new ML DOA estimation with Ant Colony Optimization (ACO) is proposed, which has the lower computational complexity and can achieve the global optimum solution of the ML function by extending the pheromone residual process to Gaussian kernel probability distribution function in continuous space. In [34], a new ML DOA estimation with Artificial Bee Colony (ABC) algorithm is presented. ABC algorithm optimizeS the multivariable function by imitating the behavior of the bee colony looking for good nectar sources in the natural environment. The simulation results show that the ML DOA estimation with ABC algorithm has high computational efficiency and statistical performance. In [35], a spatial aliasing approach produced by a nested array structure with double-magnified apertures is studied, which reduces the computational complexity of ML DOA estimation significantly. In [36], an algorithm based on Alternating Minimization (AM) is given to solve the problem of reducing the computational complexity of Stochastic ML estimation. In [37], an improved squirrel search algorithm for ML DOA estimation is presented, which achieve better estimation accuracy.

The Invasive Weed optimization (IWO) algorithm proposed in recent years is a novel numerical optimization algorithm that has received increasing attention in academic and engineering optimization fields. The IWO algorithm is inspired by the aggressiveness and community of weeds, and shows strong robustness, adaptability and randomness in the process of colonization [38]. The greatest advantage of IWO is that it allows global and local optimization to be searched in each iteration, which increases the probability of finding the global optimum solution and avoids local optimum solutions. These features are very useful to improve the convergence speed of IWO algorithm and accuracy of DOA estimate. However, the fixed value of the nonlinear harmonic index affects the performance of the IWO algorithm.

In this paper, a new ML DOA estimation with Improved IWO (IIWO) is proposed, which has advantages over other biomimetic evolution methods in terms of lower SNR, computational complexity, convergence speed and number of iterations. The paper is organized as follows: The signal model and ML DOA estimation for acoustic vector sensor array are presented in Section 2, the improved IWO algorithm for ML DOA estimation is proposed in Section 3, the simulation results to demonstrate convergence property and statistical performance about IIWO-ML, GA-ML, PSO-ML, DE-ML and TSO-ML estimation are given in Section 4.1, the test results in lake trials from MEMS vector hydrophone array are shown in Section 4.2, the paper is summarized in Section 5.

2. Signal model and ML DOA estimation for acoustic vector hydrophone array

2.1. Signal model

Suppose that N far-field narrowband signals from directions $\theta = [\theta_1, \theta_2, \cdots, \theta_N]^T$, are incident on an uniform line array of M acoustic vector sensors along the x-axis in space, the signal vector received by the array can be expressed as

$$Z(t) = A(\theta)S(t) + N(t), \tag{2.1}$$

in which $Z(t) \in \mathbb{C}^{3M\times 1}$ is the snapshot vector of the acoustic vector hydrophone array, $A(\theta)$ is steering vector matrix of the array, $S(t) \in \mathbb{C}^{N\times 1}$ is the signal vector, and $N(t) \in \mathbb{C}^{3M\times 1}$ is Gaussian noise vector, and the signal and noise are statistically independent.

$$\begin{aligned} A(\theta) &= [a(\theta_1), a(\theta_2), \cdots, a(\theta_N)] \\ &= [a_1(\theta_1) \otimes u_1, a_2(\theta_2) \otimes u_2, \cdots, a_N(\theta_N) \otimes u_N], \end{aligned} \tag{2.2}$$

where $a_k(\theta_k) = [1, e^{-j\beta_k}, e^{-j2\beta_k}, \cdots, e^{-j(M-1)\beta_k}]^T$ is acoustic pressure vector corresponding to the kth signal. $\beta_k = \frac{2\pi}{\lambda} d \sin\theta_k$, and d is inter-element spacing. λ is wavelength corresponding to the maximum frequency of signals. $u_k = [1, \cos\theta_k, \sin\theta_k]^T$ is direction vector of the kth signal, and the notation $\otimes$ denotes the Kronecker product. Then the covariance matrix for the array of received signal is given by

$$\begin{aligned} R &= E[Z(t)Z^H(t)] \\ &= AE[S(t)S^H(t)]A^H + E[N(t)N^H(t)] \\ &= AR_S A^H + \sigma^2 I, \end{aligned} \tag{2.3}$$

in which R_S is the covariance matrix of signal, σ^2 is the energy of Gaussian white noise, I is normalized covariance matrix of noise, and $(\cdot)^H$ stands for complex conjugate transpose operation. In practical calculation, since the received data is limited, the covariance matrix R can be replaced by the estimated value in Eq (2.4),

$$\hat{R} = \frac{1}{K} \sum_{k=1}^{K} Z(k)Z^H(k), \tag{2.4}$$

in which K is the number of snapshots.

2.2. ML estimation

In view of the hypothesis proposed in the previous section and the principle of deterministic maximum likelihood estimation, the received signals of the array are sampled independently and uniformly (i.e., the number of snapshots), the joint probability density function of the sample data is as

$$f(Z_1, Z_2, \cdots, Z_K) = \prod_{i=1}^{K} \frac{\exp\left(-\frac{1}{\sigma^2}|Z_i - A(\tilde{\theta})s(i)|\right)}{\det(\pi\sigma^2 I)}, \tag{2.5}$$

in which $\det(\cdot)$ stands for the determinant of matrix $(\cdot)$, and $\tilde{\theta}$ is unknown signal azimuth to be estimated.

Take the negative logarithm of Eq (2.5),

$$-\ln f = K \ln \pi + 3MK \ln \sigma^2 + \frac{1}{\sigma^2} \sum_{k=1}^{K} ||Z_i - A(\tilde{\theta})S(i)||^2, \tag{2.6}$$

the deterministic ML DOA estimation of the unknown parameters σ^2 and S from Eq (2.6) is

$$\sigma^2 = \frac{1}{3M} \mathrm{tr}\left\{P_A^{\perp}\hat{R}\right\}, \tag{2.7}$$

$$\hat{S} = A^{+}Z, \tag{2.8}$$

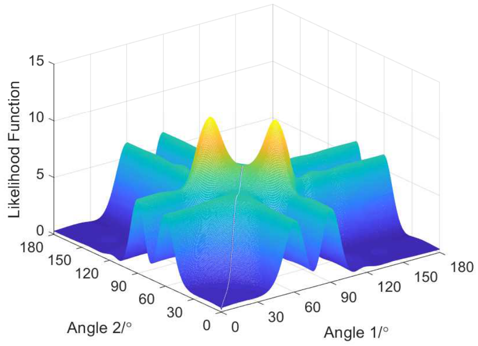
in which $\mathrm{tr}\{\cdot\}$ is the trace of a matrix, $P_A^{\perp}$ is the orthogonal projection matrix of the matrix A, and $A^{+} = (A^H A)^{-1} A^H$ is the pseudoinverse of the matrix A.

Substituting Eqs (2.7) and (2.8) into Eq (2.6), then the ML estimation of the parameter $\tilde{\theta}$ can be written as Eq (2.9),

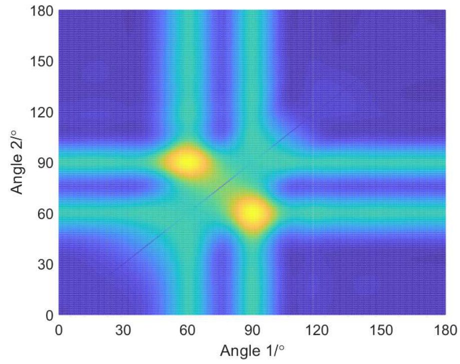
$$\hat{\theta} = \arg\max_{\tilde{\theta}} g(\tilde{\theta}), \tag{2.9}$$

where $g(\tilde{\theta}) = \mathrm{tr}\left\{\left[A(\tilde{\theta})(A^H(\tilde{\theta})A(\tilde{\theta}))^{-1}A^H(\tilde{\theta})\right]\hat{R}\right\}$, and $\mathrm{tr}(\cdot)$ denotes the trace of matrix $(\cdot)$.

The spectrum and project of likelihood function from an example can be seen in Figure 1, where two signals from 60 and 90 degrees impinge on the uniform linear array composed of 6 acoustic vector sensors, the SNR is 0 dB and the number of snapshots is 300. The location of the maximum for the likelihood function is the DOA estimation of signals.



(a) Spectrum



(b) Project

Figure 1. The spectrum and project of likelihood function with SNR=0dB.

The basic idea of ML DOA estimation is to find the maximum value of likelihood function $g(\theta)$ in Eq (2.9), but the maximum value of $g(\theta)$ are not same in different conditions such as SNR, Snapshots, the number of sensors and signal etc. In order to maintain consistency under different conditions and compare the convergence performance of different methods, a new function with the same effect as Eq (2.9) is expressed by

$$F(\tilde{\theta}) = |g(\tilde{\theta}) - g(\theta)|, \tag{2.10}$$

where

$$g(\theta) = \mathrm{tr}\left\{\left[A(\theta)(\mathbf{A}^H(\theta)\mathbf{A}(\theta))^{-1}\mathbf{A}^H(\theta)\right]\hat{R}\right\}, \tag{2.11}$$

then the optimization problem of Eq (2.9) is further expressed as

$$\hat{\theta} = \arg\min_{\tilde{\theta}} F(\tilde{\theta}), \tag{2.12}$$

in which the minimum value of $F(\tilde{\theta})$ is close to 0 infinitely.

Grid search is one of the methods to find the global optimal solution of the likelihood function. Its computational load C_{gs} depends on the searching range, grid size, and the number of signals, which can be expressed as Eq (2.13)

$$C_{gs} = \left(\frac{\theta_{\max} - \theta_{\min}}{r}\right)^N \cdot \triangle, \tag{2.13}$$

where $(\theta_{\min}, \theta_{\max})$ is the maximum searching range, $\triangle$ is the computational load for single DOA , r is the distance between each grid point, and N is the number of signals. Obviously, with the increase of the number of signals, the computational complexity will grow exponentially, which limits the application of grid search in engineering.

3. ML DOA estimation based on improved IWO algorithm

Invasive Weed Optimization, proposed by A.R. Mehrabian and C. Lucas firstly [38], is a weed-inspired, population-based numerical optimization calculation method by simulating the weed invasion process. IWO algorithm is an excellent swarm intelligence algorithm based on the meta-heuristic global optimization algorithm. It has many advantages, such as low computational complexity, fast convergence, and few adjustment items, and has been widely used to solve various kinds of optimization problems. The general process of weed invasion is, adapt to the environment, take the opportunity to occupy land and site, seed breeding, support the population, adapt to the situation, gradually intensive, the survival of the fittest, and die out of the competition. The adaptable individuals gain more chances of survival. The general characteristics of weed invasion behavior in a certain area are as follows, resources that are underutilized by crops create space for weeds to seize the opportunity to enter the field through the spread of seeds, and then continue to colonize through reproduction and ultimately control the entire land. Biodiversity makes weeds have a good ability to capture favorable living space, survive through natural selection and competition for a long time, and become localized improved weeds. Weeds always maximize their adaptability in plant communities at the best time of the start of the season.

In the process of weed invasion, adjacent plants will interact with each other due to factors such as growth years, plant size, and relative distance. Thus, the birth, growth, and reproduction of plants are influenced by plant density, population fitness and flora. In plant communities, there is a conflict among three main factors of adaptation (reproduction, struggle with competitors, and avoid predators). The improvement of adaptability can make plants live longer, and the above three factors must be taken into account in the estimation of plant adaptability. There are many ways to choose the natural evolution of plants, two of which are more important, i.e., R-selection and K-selection. R-selection is to choose plants from fast-growing, fast-reproducing and die young, and let them occupy an unstable and unpredictable environment. K-selection is to choose plants with strong competitiveness from slow-growing, slow-reproducing, and die old, and let them occupy the environment with highly competitive pressure, limited resources, stable and predictable. That is, R-selection corresponds to the global search mode, and K-selection corresponds to the local search mode of the IWO algorithm.

In this section, the IIWO algorithm to the optimization of ML for DOA estimation is proposed. The process is addressed in details as follows.

Step 1. Initialize a population

Suppose P_{size} is the initial population size, Q_{size} is the maximum population size, $\text{iter}_{\max}$ is the maximum number of iterations, and $s_{\max}$, $s_{\min}$ are the maximum and minimum number of produced seeds, respectively. The search dimension is equal to the number of signals N and the search range is $[\theta_{\min}, \theta_{\max}]$. A set of initial solutions θ is distributed in the N dimensional space with random position.

Step 2. Reproduction

Each weed seed blooms and then produces seeds based on its adaptability (reproductive ability). A member of the population of plants is allowed to produce seeds which depend on its own and the colony's lowest and highest fitness. The number of seeds produced by each plant increases linearly from minimum possible seed production to its maximum. In other words, a plant will produce seeds based on its fitness, the colony's lowest fitness and highest fitness to make sure that the increase is linear. The procedure is illustrated in Figure 2. The number of seeds produced by the parent weeds is linear with the maternal fitness

$$N_s = \frac{F - F_{\min}}{F_{\max} - F_{\min}}(s_{\max} - s_{\min}) + s_{\min}, \tag{3.1}$$

where $F(\cdot)$ is the fitness function, which equals to the objective function as Eq (2.10), and $F_{\max}, f_{\min}$ are the maximum and minimum value of $F(\theta)$, respectively.

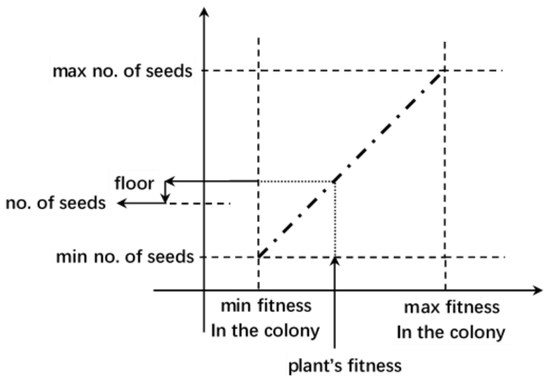


Figure 2. Seed production procedure in a colony of weeds.

Step 3. Spatial dispersal

Let the parent be the axis (mean) and the offsprings are diffused in the N dimensional space in a normal distribution. In the iterative process, the standard deviation of each generation changes according to the rules in Eq (3.2)

$$\sigma_{\text{iter}} = \frac{(\text{iter}_{\max} - \text{iter})^n}{(\text{iter}_{\max})^n}(\sigma_{\text{initial}} - \sigma_{\text{final}}) + \sigma_{\text{final}}, \tag{3.2}$$

where σ_{iter} is standard deviation at the present time step, σ_{initial} is initial value of standard deviation, σ_{final} is final value of standard deviation, $\text{iter}_{\max}$ is maximum iteration number and n is nonlinear harmonic index. Eq (3.2) ensures that the probability of seed production in a far field is gradually reduced in a nonlinear way, so that the individuals with good adaptability will gather together and some of the individuals with no adaptability will be removed.

The parameter n in Eq (3.2) is a non-linear harmonic index which is 3 in most literatures [38]. The fixed parameter limits the performance of IWO to a certain extent. Therefore, An IIWO algorithm can be obtained by adjusting the non-linear harmonic index self-adaptively, where n can be expressed as Eq (3.3),

$$n = \frac{F_{\max}}{\overline{F}}, \tag{3.3}$$

in which $\overline{F}$ is the average of fitness funtion value $F(\theta)$ in certain population.

This self-adaptive non-linear harmonic index is related to the fitness function of each generation. As the number of iterations increases, the fitness function value of each generation becomes more and more concentrated. So n decreases with iteration, and it will tend to 1 with the convergence process of the algorithm. Simulation experiments verify the better performance of IIWO algorithm than IWO algorithm in section 4.1.

Step 4. Competitive exclusion

After several generations of reproduction, the number of progeny produced by the cloning will exceed the acceptable number of environmental resources, and the maximum population size will be determined by the maximum number of populations set in advance. When the maximum population number is reached, it is freely propagated according to the previous rules. After the diffusion is completed, the parents and the children are arranged to be eliminated according to the size of the adaptation value to reach the upper limit of the population.

Therefore, the minimum value of $F(\theta)$ and corresponding DOAs can be found through multiple iterations. The pseudocode of IIWO is present in Algorithm 1, and the flow chart of the IIWO algorithm can be seen in Figure 3.

Algorithm 1. Pseudocode of IIWO

Begin
Define input parameters.
Generate random locations θ (parent weed seeds) for DOAs of signals.
While the stop criterion is not satisfied do
    For each weed seed θ_i do
        Determine the number of weed seeds for growth and reproduction by Eq (3.1)).
        Generate offspring weed seeds by normal distribution with a mean of parent weed seeds and a standard deviation σ_{iter} by Eqs (3.2) and (3.3).
        Calculate the fitness function $F(\theta)$ values of parent and child populations, sort and eliminate the weed seeds with large fitness function value.
    End For.
End while.
Find the locatin of optimal solution as the DOAs of signals.

To sum up, the advantages of IIWO algorithm can be summarized as follows.

(1) IIWO algorithm reproduces based on fitness. The reproduction process gives infeasible individuals the opportunity to survive and reproduce according to the reproduction rules in nature.

(2) IIWO algorithm propagates offspring in a normal distribution around the parent individual, so that it has a certain depth (local search) and breadth (global search). At the initial stage of iteration, the seed individuals can spread far away from the parent weeds in a normal distribution through large standard deviation. At this time, the population exploration ability is strong (R-selection). When the iteration goes on to the later stage, the standard deviation gradually decreases, so as to narrow the diffusion range of seeds, and the original dominant groups are easier to prosper and develop. At this time, the mining capacity is strong (K-selection). Weed algorithm takes into account both global search and local search, and can adjust their intensity according to the number of iterations.

(3) IIWO algorithm adopts the competitive exclusion mechanism of offspring and parents, which gives individuals with low fitness the opportunity to reproduce. If their offspring have better fitness, these offspring can survive and retain useful information to the greatest extent, while avoiding premature and falling into local optimization.

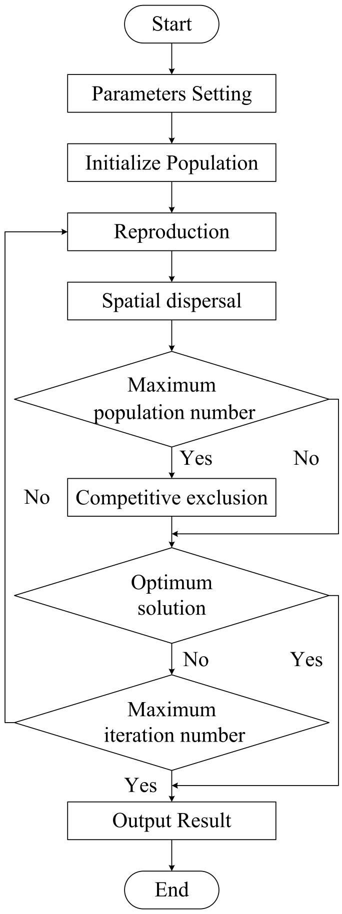


Figure 3. The flow chart of the IIWO algorithm.

4. Results and discussion

4.1. Simulation experiments and discussion

In this section, some simulation results about the estimation performance of ML DOA estimation with the IIWO method are demonstrated, which include the statistical performance comparison with other methods, such as IWO, GA, PSO, DE and TSO algorithm. In the experiments, the received array is supposed as the uniform linear array which is composed of 6 acoustic vector hydrophones, the noise is Gaussian white noise, and the number of snapshots is 300. In the experiment of different number of signal, two signals from the directions of $\theta = [\theta_1, \theta_2] = [30°, 60°]$, three signals from the directions of $\theta = [\theta_1, \theta_2, \theta_3] = [30°, 60°, 80°]$, and four signals from the directions of $\theta = [\theta_1, \theta_2, \theta_3, \theta_4] = [30°, 60°, 80°, 150°]$ are taken, respectively.

4.1.1. Camparisons between IWO and IIWO algorithm

Assuming that the SNR is 10dB, the maximum number of iterations is 30, the initial population size is 20 when the number of signal sources is 2, 3, 4, respectively, the estimation error for each signal and RMSE of IWO and IIWO algorithm is obtained by 100 independent Monto Carlo trials, which are shown in Table 1. From Table 1, we can find out that the estimation error and RMSE of IIWO are lower than one of IWO whether the number of signals is 2, 3, or 4. For example, when the number of signals is 4, the average estimation error of IIWO is only 11.1% of IWO and the average RMSE is only 5.7%. Moreover, the maximum number of iterations in this experiment is only 30, which shows that the convergence speed of the IIWO algorithm is more faster.

From the above analysis, the experimental results show that the IIWO algorithm can not only improve the estimation accuracy effectively but also accelerate the convergence speed when the population size is small. Therefore, we only compare the performance between IIWO and other algorithms in the following comparison experiments.

Table 1. Estimation error and RMSE values under IWO and IIWO algorithm with two, three and four signals when SNR=10dB.

	No. of signals	DOAs	Estimation error/°	Average estimation error/°	RMSE	Average RMSE
IWO	2	θ_1	0.3457	0.5140	1.6250	2.8579
		θ_2	0.6822		4.0908	
IIWO	2	θ_1	0.0813	0.0655	0.1026	0.0833
		θ_2	0.0498		0.0639	
IWO	3	θ_1	1.1592	0.7931	4.4050	2.8930
		θ_2	0.6328		2.3231	
		θ_3	0.5872		1.9510	
IIWO	3	θ_1	0.1097	0.0864	0.1366	0.1070
		θ_2	0.0814		0.0977	
		θ_3	0.0680		0.0868	
IWO	4	θ_1	1.3882	0.9587	3.5723	2.3007
		θ_2	0.8880		2.2423	
		θ_3	0.7205		1.8307	
		θ_4	0.8382		1.5575	
IIWO	4	θ_1	0.1332	0.1066	0.1640	0.1313
		θ_2	0.0927		0.1149	
		θ_3	0.0839		0.1015	
		θ_4	0.1165		0.1447	

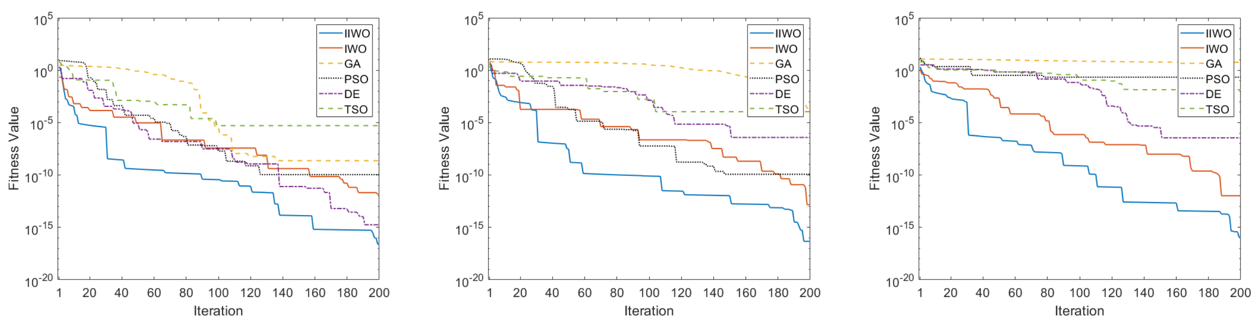
4.1.2. Convergence performance

Convergence rate are one of the important factors to evaluate the efficiency of an intelligent algorithm. To make the simulation results accurate and reliable, in each iteration, the maximum value of likelihood function is the average value of 100 independent Monte Carlo tests, thus the curve of the maximum value change process of fitness value (likelihood function) is obtained. For the sake of fairness, the maximum number of iterations of all algorithms is 200, the population sizes are 30, and all parameters of these algorithms are provided in Table 2.

Figure 4 shows the convergence properties of IIWO, IWO, GA, PSO, DE and TSO algorithm with SNR = 0dB, respectively. It is observed that the speed of finding a global optimum solution for the IIWO algorithm is faster than the others whatever two signals, three signals, or four signals. It means that the IIWO algorithm can always find the maximum of the ML DOA estimation with fewer iterations, which influence the computational complexity of the algorithms. From Figure 4(c), it is particularly important to note that the GA, PSO and TSO can only converge to the local maximum of ML DOA estimation, which signifies the performance of GA, PSO and TSO algorithm decreased severely with the increase of dimension or the number of signals.

Table 2. Parameter values of different algorithm for ML DOA estimation.

Name of Parameter	GA	PSO [39]	DE [40]	IIWO
Problem dimension	2(3,4)	2(3,4)	2(3,4)	2(3,4)
Population Size	30	30	30	30
Maximum number of iterations	200	200	200	200
Initial search area	[0, 180]	[0, 180]	[0, 180]	[0, 180]
Crossover Fraction	0.8	-	-	-
Migration Fraction	0.2	-	-	-
Cognitive Constants	-	1.25	-	-
Social Constants	-	0.5	-	-
Inertial Weight	-	0.9	-	-
Mutation operator	-	-	0.5	-
Selection operator	-	-	0.9	-
Maximum population size	-	-	-	50
Maximum number of seeds	-	-	-	5
Minimum number of seeds	-	-	-	2
Initial value of standard deviation	-	-	-	3
Final value of standard deviation	-	-	-	0.001



(a) Two signals (b) Three signals (c) Four signals

Figure 4. The convergence curves of likelihood function with SNR=0dB

Based on the simulation results, it could be concluded that the IIWO algorithm has faster convergence speed for the ML-DOA estimation problem than IWO, GA, PSO, DE and TSO algorithms, which estimates DOAs with smaller computational complexity than the other three bio-inspired computing ones.

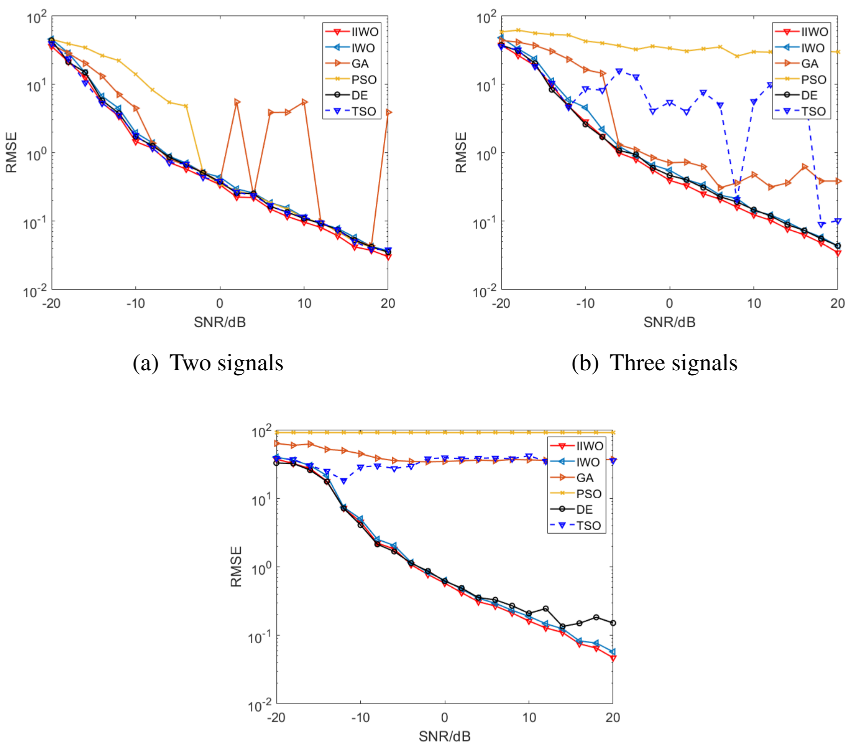
4.1.3. Statistical performance

To evaluate the statistical performance of the algorithm, we compare the RMSE of 100 independent Monte Carlo experiments. The formula of RMSE is Eq (4.1)

$$\mathrm{RMSE}=\sqrt{\frac{1}{N\cdot L}\sum_{j=1}^{L}\sum_{i=1}^{N}\left[\hat{\theta}_i(j)-\theta_i\right]^2}, \tag{4.1}$$

in which L is the number of experiments, N is the number of signals, θ_i is the DOA of the ith signal, $\hat{\theta}_i(j)$ denotes the estimate of the ith DOA achieved in the jth experiment.

Figure 5 shows the simulation results of RMSE and SNR of DOA estimation with two signals, three signals, and four signals, respectively. It can be found that the RMSE of the IIWO algorithm is smaller at low SNR, while the performance of IWO and DE algorithms is very close at higher SNR. In the case of four signals, GA, PSO and TSO algorithms can not converge to the global optimal solution, which leads to a large RMSE, but IIWO algorithm can always maintain stable performance and faster convergence speed even if the population size is increased, GA, PSO and TSO algorithm still can not converge to the globally optimal solution (the simulation experiments in next section illustrate this point).



(a) Two signals (b) Three signals

(c) Four signals

Figure 5. DOA estimation RMSEs versus SNR.

4.1.4. Computational complexity

The computation load of ML DOA estimation of the grid search method in Eq (2.13) increases exponentially with the number of signals. The maximum computational load of the ML DOA estimation with the IIWO algorithm can be expressed as Eq (4.2),

$$C_{\mathrm{IIWO}} = \mathrm{iter}_{\max} \cdot Q_{\mathrm{size}} \cdot \triangle. \tag{4.2}$$

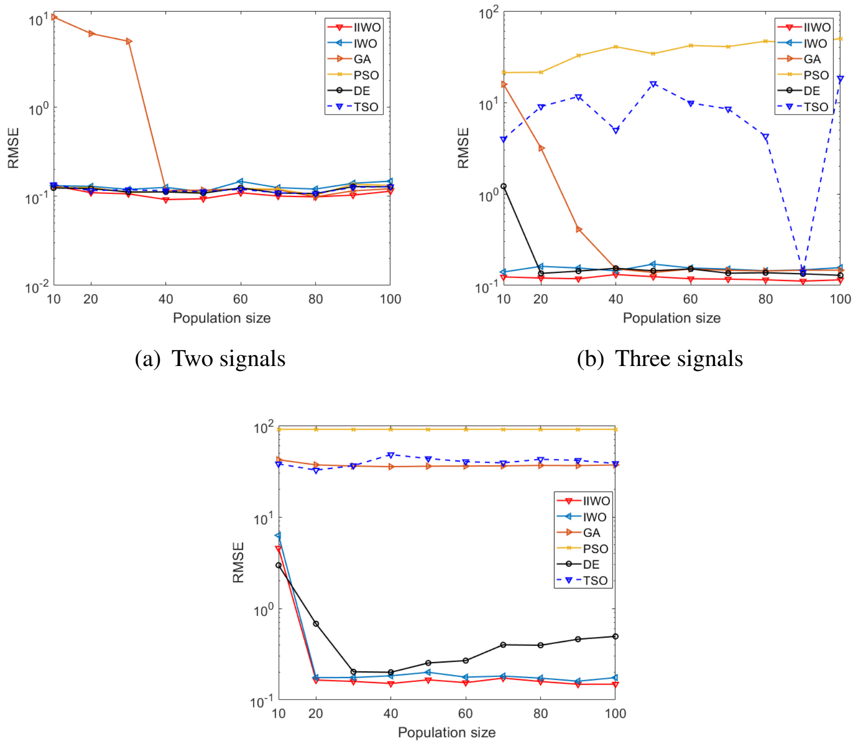
From (4.2), we can find that the computation load of ML DOA estimation with the IIWO algorithm only depends on the population number and the maximum number of iterations, but not the number of signals to be estimated. Through the above simulation experiments, we find that the IIWO algorithm needs fewer iterations than other algorithms in all cases. Therefore, we next analyze the impact of population size.

Population size is the most important parameter in the biological evolutionary algorithm, which directly affects the computational complexity of the algorithm. For the ML DOA estimation, the population size determines the number of times to calculate the likelihood function in each iteration, which has a direct impact on the convergence speed. Therefore, the algorithm with higher precision and lower population size is needed in practice.

Suppose that two signals, three signals, and four signals are incidence on the array with SNR = 10dB, respectively, Figure 6 shows the RMSE curves of DOA estimation versus the population size of IIWO, IWO, GA, PSO, DE and TSO algorithm. It can be seen from Figure 6(a) that in the case of two signals, when the population size is only 10, the estimation performance of ML DOA estimation using these methods except GA is relatively close, while the ML DOA estimation using GA needs more than 40 populations to be close to other algorithms. When the population number is greater than 10, the RMSE of ML DOA estimation with IIWO algorithm is the smallest. Especially when the population number of IIWO is 40, its estimation performance is the best. Even if the population size is more, its estimation performance is hardly improved, which fully shows that IIWO algorithm can achieve higher estimation accuracy in a small population size. From Figure 6(b) and 6(c), in the case of three signals, the ML DOA estimation with PSO and TSO is almost invalid, and the estimated RMSE is very large. In the case of four signals, the estimation performance of ML estimation with GA, PSO and TSO algorithms are also very poor. Regardless of the number of signal sources, the ML DOA estimation performance based on IIWO algorithm is the most stable and excellent.

Additionally, in the case of two signals from the directions of $\theta = [30°, 60°]$, Figure 7 shows the average iteration number of four algorithms in 100 independent Monte Carlo trials when SNR changes from −20dB to 20dB. The iteration number of the IIWO algorithm is much lower than the one of IWO, DE, PSO, GA and TSO methods.

Compared with the other algorithms, the IIWO algorithm can maintain stable calculation accuracy when the population size is small, i.e., the convergence rate of ML-DOA estimation based on IIWO is the fastest and computation load is smallest.



(a) Two signals

(b) Three signals

(c) Four signals

Figure 6. DOA estimation RMSEs versus population size.

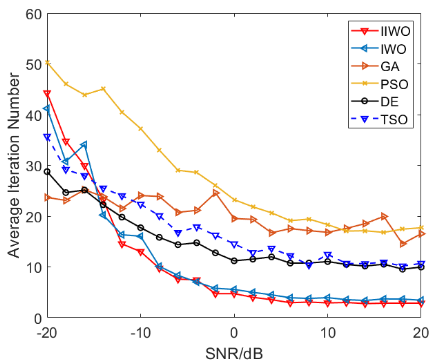


Figure 7. The average iteration number of four algorithms versus SNR with two signals.

Finally, the computation time of the IIWO, IWO, GA, PSO, DE and TSO methods are compared. Table 3 presents the simulation results. The experiment was carried out on a computer with Intel Core i7-8700 and memory using MATLAB 2018b. The RMSE value and calculation time are the average of 100 independent experiments. The population size of the four algorithms is 30, and the maximum number of iterations is 200. From Table 3, It can be found that IIWO algorithm has shorter computation time and lower RMSE no matter the number of signals is 2, 3 or 4. Among them, when the number of signals is 2, the computation time of IIWO is only 90.57%, 61.64%, 79.65%, 20.83% and 31.50% of that of IIWO, GA, PSO, DE and TSO methods respectively, while the RMSE of IIWO is only 77.38%, 91.04%, 98.07%, 90.53% and 88.94% respectively. When the number of signal sources is more, the advantages of IIWO algorithm are more obvious. This means that compared with other algorithms, the IIWO algorithm can get the same estimation accuracy with a lower computation load and shorter time.

Table 3. The computing time of different algorithms with two, three and four signals when SNR=0dB.

Algorithms	No. of signals	IIWO	IWO	GA	PSO	DE	TSO
Computation times/s	2	0.1816	0.2005	0.2946	0.2280	0.8720	0.5765
	3	0.2134	0.2323	0.4659	0.2941	0.9918	0.6304
	4	0.3869	0.4108	0.7163	0.6332	1.6810	1.0145
RMSE/°	2	0.3403	0.4398	0.3738	0.3470	0.3759	0.3826
	3	0.3949	0.5532	0.7105	33.0676	0.4656	5.4423
	4	0.5775	0.6373	34.4628	90.8568	0.6190	39.0268

4.2. Lake test

MEMS vector hydrophone in Figure 8 is a new type of bionic vector hydrophone combined with MEMS technology [41–44], Its microstructure can achieve batch manufacturing and one-time integration, so it has the advantages of small size, good consistency, and is more suitable for group array.

Figure 8. MEMS vector hydrophone.

The test has been conducted in Fenhe River (Figure 9). A uniform linear array is composed of three MEMS vector hydrophones, with the array element spacing of 0.5 meter and horizontal fixation of 10 meters. In the experiment, a motorboat is used as the moving target, whose initial position is 100 meters away from the array, and the real-time azimuth data is recorded by GPS device. The motorboat starts from about 45 degrees and moves to 150 degrees after 39s. The acoustic pressure signal p and vibration velocity signal v_x, v_y received by No.1 hydrophone are shown in Figure 10, in which the received signals are filtered by 800Hz center frequency.

Figure 9. Lake test field.

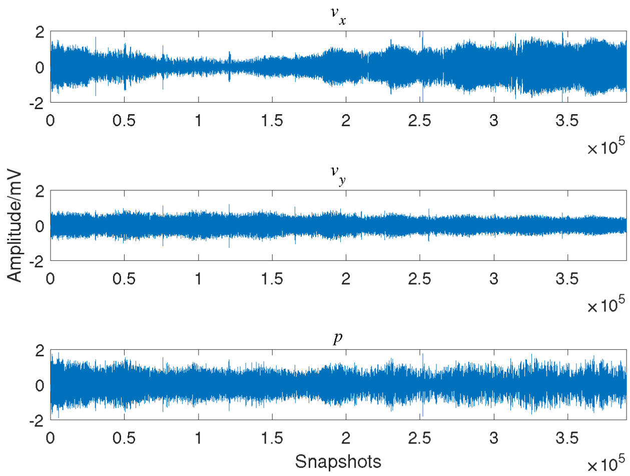


Figure 10. Signal of vibration velocity v_x, v_y and acoustic pressure p received by No.1 MEMS vector hydrophone.

The direction of the motorboat is estimated by using the ML method with six intelligent algorithms, once per second. Among them, the initial population size of IIWO, IWO, GA, PSO, DE and TSO algorithms are all set to 10, and the maximum iteration number is 30. Figure 11 shows the real-time GPS records and time-bearing display of different methods. Table 4 shows the average estimation error and RMSE of different methods compared with GPS data. We can find that the IIWO method can maintain accurate estimation performance, followed by the TSO, DE and IWO method, while GA and PSO methods have a poor effect. The main reason is that the maximum iteration times are only 30 due to the actual real-time calculation needs, which makes the ML function possibly unable to converge in some direction estimation. Table 4 also gives the average error and RMSE data of the six methods when the maximum number of iterations is 50, 100 and 200, respectively. At this time, the estimation results of the ML estimation with six methods are close. Therefore, from this point of view, the IIWO algorithm can still maintain faster convergence speed and is more suitable for engineering applications.

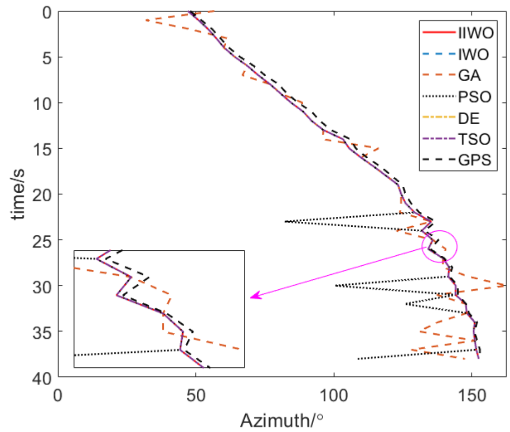


Figure 11. Time-bearing display of motor boat using ML estimation with four methods when the maxinum iteration number is 30.

Table 4. The average estimation error and RMSE for motor boat under different methods.

	Maximum iteration number	IIWO	IWO	GA	PSO	DE	TSO
Average estimation error/°	30	1.5929	1.6396	4.4623	52.9495	1.5988	1.5982
	50	1.5928	1.7230	6.0691	6.5472	1.5927	1.5978
	100	1.5927	1.5927	3.3691	1.5927	1.5927	1.5927
	200	1.5927	1.5927	2.3737	1.5932	1.5927	1.5927
Average RMSE	30	1.7945	1.8882	6.8583	63.2752	1.7945	1.7998
	50	1.7945	2.0557	10.0797	16.3230	1.7945	1.7945
	100	1.7945	1.7945	5.6411	1.7945	1.7945	1.7945
	200	1.7945	1.7945	3.6862	1.7948	1.7945	1.7945

5. Conclusions

DOA estimation is one of the core problems in underwater acoustic signal processing. Especially with the development of acoustic vector hydrophone technology, there is an urgent need for some algorithms with higher estimation accuracy and lower computation to meet the needs of engineering practice. Among them, traditional DOA estimation methods such as MUSIC and ESPRIT algorithm have the advantages of high resolution, but their huge amount of calculation hinders their practical application. However, the DOA estimation model based on ML needs to find the maximum value of multi-dimensional nonlinear function to obtain the azimuth of the signal. Its resolution is directly related to the interval of grid division. The smaller the interval of grid division, the higher the resolution, and the greater the amount of calculation, which leads to contradiction, Therefore, the ML-DOA estimation model based on intelligent optimization algorithm can take into account the problems of resolution and computation. The main work of this paper is as follows:

(1) The ML-DOA estimation model based on acoustic vector sensor array is developed, and the bridge of applying intelligent optimization algorithm to signal DOA estimation is established. By converting the maximum of objective function to the minimum, the optimization standard in various

cases can be unified, which is convenient to compare the performance of different algorithms

(2) Based on the IWO algorithm, an improved Iwo algorithm is proposed by adaptively adjusting the non linear harmonic index. Compared with the IWO algorithm, the calculation efficiency and estimation accuracy are improved.

(3) Compared with the simulation experiments of several typical optimization algorithms GA, PSO, DE and TSO, IIWO algorithm has obvious advantages over other algorithms in terms of RMSE, convergence performance and calculation time under the conditions of different SNR, different number of signals and different initial population size.

(4) The experimental results of MEMS vector hydrophone tracking motorboat show that the proposed model and method can be successfully applied, and the performance of IIWO algorithm is the best even when the number of iterations is fewer, which can meet the requirements of real-time calculation and estimation accuracy in the actual environment.

Therefore, ML DOA estimation with the IIWO algorithm is a more suitable method for engineering application, which is worthy of further research.

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Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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