



Research article

New solutions for perturbed chiral nonlinear Schrödinger equation

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Abstract: In this article, we extract stochastic solutions for the perturbed chiral nonlinear Schrödinger equation (PCNLSE) forced by multiplicative noise in Itô sense with the aid of $\exp[-\varphi(\xi)]$ -expansion and unified solver methods. The PCNLSE meditate on the quantum behaviour, like quantum features are closely related to its particular features. The proposed techniques introduce the closed form structure of waves in explicit form. The behaviour of the gained solutions are of qualitatively different nature, based on the physical parameters. The acquired solutions are extremely viable in nonlinear optics, superfluid, plasma physics, electromagnetism, nuclear physics, industrial studies and in many other applied sciences. We also illustrate the profile pictures of some acquired solutions to show the physical dynamical representation of them, utilizing Matlab release. The proposed techniques in this article can be implemented to other complex equations arising in applied sciences.

Keywords: PCNLSE; $\exp[-\varphi(\xi)]$ -expansion technique; unified solver technique; physical applications

Mathematics Subject Classification: 35C07, 35Q40, 35Q55, 60H15, 81Q15

1. Introduction

The theory of solitons plays a crucial role in different fields of natural sciences, such as superfluid, biology, nuclear physics, solid state physics, quantum mechanics, engineering, plasma physics, optical physics, super conductivity, industrial studies, and in various other physical and natural sciences [1–5]. Solitons are of great significant in biological sciences in the filed of neurosciences [6, 7]. Solitons have gained much attention on account of their sturdy nature and vital applications in optical communications and all long-distance [8–14]. The nonlinear Schrödinger's equations with their huge

number of vital applications are the most commonly use nonlinear models in field of nonlinear wave propagation in natural sciences [15–24]. There are many scientists introduced and developed some standing wave stabilities for nonlinear Schrödinger's equations and related topics; such as Hartree, spatio-temporal dispersion, inter-modal dispersion, Choquard, perturbed and their fractional forms [25–28].

Recently, more and more attentiveness has been gone recently to the influence of a noise on the propagation of the travelling wave solutions. This influence is so important in order to describe various vital phenomena in many fields in natural sciences, such as quantum mechanics, fluid mechanics, molecular biology, chemical engineering, plasma physics, electromagnetic theory, bio science, nano-fibers, optoelectronics and photonics [29–33]. The dynamics of optical pulses in fibers communications with birefringence, randomly varied, may be reflected to coupled of nonlinear Schrödinger's equations with random coefficients [34]. The nonlinear Schrödinger's equations with multiplicative noise proposed as a model of energy transfer in a monolayer molecular aggregate in the existence of thermal fluctuations [35, 36]. Abdelrahman and Sohaly in [37] implemented Riccati-Bernoulli sub-ODE approach to extract stochastic solutions via probability distribution function, where the stochastic terms are the random variables. With fast evolution of symbolic computation approaches, the seeking of stochastic solutions for the stochastic nonlinear partial differential equations have attracted a lot of attention.

Obviously, the chiral solitons are of great important in the evolution of practical quantum Hall effect, where chiral excitations are known to become visible. The chiral nonlinear Schrödinger equation, was considered earlier where bright and dark soliton waves were given [38]. Thereafter in 2004, Bohm potential was introduced via many scientific researches were done with PNLSE where Bohm potential was taken as perturbation term [39, 40]. This paper is concerned with the PNLSE forced by multiplicative noise in Itô sense given by:

$$i u_t + a u_{xx} + i b (u u_x^* - u^* u_x) u + \sigma u W_t = i \alpha u \frac{|u|_{xx}}{|u|}, \quad i = \sqrt{-1} \quad (1.1)$$

u is a complex function, $*$ denotes complex conjugate. The 1st term denotes the evolution term and a represents the coefficient of dispersion and b represents a nonlinear coupling constant. The noise in time $W_t = \frac{dW}{dt}$ is the distributional derivative of the Brownian motion. Whereas, the right side, represents the perturbation term; α is named Bohm potential. The solutions of PCNLSE have valuable applications in the development of quantum mechanics. Hence it is so significant to gain solutions of PCNLSE.

Our work is motivated to extract new solutions for the PCNLSE in the presence of noise term, utilizing $\exp[-\varphi(\xi)]$ -expansion technique [41, 42] and unified solver technique [43–45]. These approaches present various classes of solutions via free physical parameters. These solutions are very profitable in various fields of natural sciences, like, superfluid, optical fibers, plasma physics, nano-technology, nuclear physics, quantum mechanics. The proposed techniques are direct, robust, adequate and efficacious.

This paper is organized as follows. Section 3 presents the solutions for the PCNLSE, utilizing $\exp[-\varphi(\xi)]$ -expansion and unified solver techniques. Section 4 presents physical interpretation for the acquired solutions. Conclusions are reported in section 5.

2. Description of the methods

Here, we briefly introduce the $\exp[-\varphi(\xi)]$ -expansion method [41,42,46] and unified solver method [43].

Consider the following nonlinear partial differential equations (NPDEs):

$$\mathcal{G}(\Xi, \Xi_x, \Xi_t, \Xi_{xx}, \Xi_{xt}, \Xi_{tt}, \dots) = 0. \quad (2.1)$$

Using the wave transformation:

$$\Xi(x, t) = \Xi(\zeta), \quad \zeta = x - wt, \quad (2.2)$$

where w is the speed of the travelling wave, Eq (2.1) reduced to the following ODE:

$$\mathcal{H}(\Xi, \Xi', \Xi'', \Xi''', \dots) = 0. \quad (2.3)$$

Various NPDEs in applied science reduced to the following ODE:

$$\Gamma_1 \Xi'' + \Gamma_2 \Xi^3 + \Gamma_3 \Xi = 0, \quad (2.4)$$

where Γ_1, Γ_2 and Γ_3 are constants depend on the constants of the proposed equation and the velocity speed of the wave transformations.

2.1. The $\exp[-\varphi(\xi)]$ -expansion method

According to the $\exp[-\varphi(\xi)]$ -expansion technique [41,42,46], the solution of Eq (2.4) can be written in a polynomial form of $\exp[-\varphi(\xi)]$ as follows

$$\Xi(\xi) = B_0 + B_1 \exp[-\varphi(\xi)] \quad (2.5)$$

B_0 and $B_1 \neq 0$ are constants and $\varphi(\xi)$ obeys the following ODE

$$\varphi'(\xi) = \exp[-\varphi(\xi)] + \nu \exp[\varphi(\xi)] + \lambda. \quad (2.6)$$

Equation (2.6) has the following solutions:

1) For $\nu \neq 0, \lambda^2 - 4\nu > 0$,

$$\varphi(\xi) = \ln \left(\frac{-\sqrt{\lambda^2 - 4\nu} \tanh \left(\frac{\sqrt{\lambda^2 - 4\nu}}{2} (\xi + K) \right) - \lambda}{2\nu} \right), \quad (2.7)$$

2) For $\nu \neq 0, \lambda^2 - 4\nu < 0$,

$$\varphi(\xi) = \ln \left(\frac{\sqrt{4\nu - \lambda^2} \tan \left(\frac{\sqrt{4\nu - \lambda^2}}{2} (\xi + K) \right) - \lambda}{2\nu} \right), \quad (2.8)$$

3) For $\nu = 0, \lambda \neq 0, \lambda^2 - 4\nu > 0$

$$\varphi(\xi) = -\ln\left(\frac{\lambda}{\exp[\lambda(\xi + K)] - 1}\right), \quad (2.9)$$

4) For $\nu \neq 0, \lambda \neq 0, \lambda^2 - 4\nu = 0$,

$$\varphi(\xi) = \ln\left(-\frac{2(\lambda(\xi + C) + 2)}{\lambda^2(\xi + K)}\right), \quad (2.10)$$

5) For $\nu = 0, \lambda = 0, \lambda^2 - 4\nu = 0$,

$$\varphi(\xi) = \ln(\xi + K). \quad (2.11)$$

Here K is an arbitrary constant.

Finally, putting Eq (2.5) and (2.6) into Eq (2.4) and aggregating all terms of the same power $\exp[-m\varphi(\xi)]$, $m = 0, 1, 2, 3$. After that equating them to zero, gives an algebraic equations. Solving these equations by Mathematica, gives the values of a_i . Then, we get the solutions (2.5), which give the exact solutions of Eq (2.3).

2.2. Unified solver method

In view of the unified solver approach [43], the solutions of Eq (2.4) are:

(i) **Rational function solutions:** (at $\Gamma_3 = 0$)

$$\Xi_{1,2}(x, t) = \left(\mp \sqrt{\frac{-\Gamma_2}{2\Gamma_1}} (\xi + \varsigma)\right)^{-1}. \quad (2.12)$$

(ii) **Trigonometric function solutions:** (at $\frac{\Gamma_3}{\Gamma_1} < 0$)

$$\Xi_{3,4}(x, t) = \pm \sqrt{\frac{\Gamma_3}{\Gamma_2}} \tan\left(\sqrt{\frac{-\Gamma_3}{2\Gamma_1}} (\eta + \varrho)\right) \quad (2.13)$$

and

$$\Xi_{5,6}(x, t) = \pm \sqrt{\frac{\Gamma_3}{\Gamma_2}} \cot\left(\sqrt{\frac{-\Gamma_3}{2\Gamma_1}} (\eta + \varrho)\right). \quad (2.14)$$

(iii) **Hyperbolic function solutions:** (at $\frac{\Gamma_3}{\Gamma_1} > 0$)

$$\Xi_{7,8}(x, t) = \pm \sqrt{\frac{-\Gamma_3}{\Gamma_2}} \tanh\left(\sqrt{\frac{\Gamma_3}{2\Gamma_1}} (\eta + \varrho)\right) \quad (2.15)$$

and

$$\Xi_{9,10}(x, t) = \pm \sqrt{\frac{-\Gamma_3}{\Gamma_2}} \coth\left(\sqrt{\frac{\Gamma_3}{2\Gamma_1}} (\eta + \varrho)\right). \quad (2.16)$$

Here ϱ is an arbitrary constant.

3. Mathematical analysis

Here, we implement the $\exp[-\varphi(\xi)]$ -expansion and unified solver techniques, to introduce the chiral solitons for the PCNLSE in the presence of noise term. Using the following wave transformation [47]:

$$u(x, t) = e^{i[-rx+wt+\vartheta+\sigma W(t)]} U(\xi), \quad \xi = x + ct, \quad (3.1)$$

r , w and ϑ are, respectively, wave number, frequency of solitons and phase constant.

Superseding Eq (3.1) into Eq (1.1), the real and imaginary parts, yields

$$a \frac{\partial^2 U(\xi)}{\partial \xi^2} - 2rbU^3(\xi) - (w + r^2 a)U(\xi) = 0, \quad (3.2)$$

$$(c - 2ra) \left(\frac{\partial U(\xi)}{\partial \xi} \right) - \alpha \left(\frac{\partial^2 U(\xi)}{\partial \xi^2} \right) = 0. \quad (3.3)$$

Hence, Eq (3.2) can be reformulated as follows

$$\Gamma_1 \frac{\partial^2 U(\xi)}{\partial \xi^2} + \Gamma_2 U^3(\xi) + \Gamma_3 U(\xi) = 0, \quad (3.4)$$

where $\Gamma_1 = a$, $\Gamma_2 = -2rb$, $\Gamma_3 = -(w + r^2 a)$.

3.1. Solutions of Eq (1.1) via $\exp[-\varphi(\xi)]$ -expansion method

In light of the above $\exp[-\varphi(\xi)]$ -expansion approach, the solutions are:

$$U = B_0 + B_1 \exp[-\varphi(\xi)], \quad \varphi' = e^{-\varphi} + \mu e^\varphi + \lambda, \quad (3.5)$$

B_0 and $B_1 \neq 0$ are constants. Superseding U , U'' , U^3 into Eq (3.4) and setting all coefficients of $\exp[-\varphi]$ equal to zero, yields a system of algebraic equations. Solving this system gives:

$$U(\xi) = \pm \frac{\sqrt{a}}{2\sqrt{kb}} (\lambda + 2\exp[-\varphi(\xi)]). \quad (3.6)$$

Then the solutions of Eq (3.4) [41, 42, 46] are

Family 1: When $\lambda^2 - 4\mu > 0$ & $\mu \neq 0$

$$U_{1,2}(x, t) = \pm \frac{\sqrt{a}\lambda}{2\sqrt{rb}} \left(\lambda - \frac{4\mu}{\sqrt{\lambda^2 - 4\mu} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(x + ct + K)\right) + \lambda} \right), \quad (3.7)$$

$$U_{3,4}(x, t) = \pm \frac{\sqrt{a}\lambda}{2\sqrt{rb}} \left(\lambda - \frac{4\mu}{\sqrt{\lambda^2 - 4\mu} \coth\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(x + ct + K)\right) + \lambda} \right). \quad (3.8)$$

As a result, Eq (1.1) has solutions

$$u_{1,2}(x, t) = \pm \frac{\sqrt{a}\lambda}{2\sqrt{rb}} e^{i[-rx+wt+\theta+\sigma W(t)]} \left(\lambda - \frac{4\mu}{\sqrt{\lambda^2 - 4\mu} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(x + ct + K)\right) + \lambda} \right). \quad (3.9)$$

$$u_{3,4}(x, t) = \pm \frac{\sqrt{a}\lambda}{2\sqrt{rb}} e^{i[-rx+wt+\theta+\sigma W(t)]} \left(\lambda - \frac{4\mu}{\sqrt{\lambda^2 - 4\mu} \coth\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(x + ct + K)\right) + \lambda} \right). \quad (3.10)$$

Family 2: When $\lambda^2 - 4\mu < 0$ & $\mu \neq 0$

$$U_{5,6}(x, t) = \pm \frac{\sqrt{a}}{2\sqrt{rb}} \left(\lambda + \frac{4\mu}{\sqrt{4\mu - \lambda^2} \tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2}(x + ct + K)\right) - \lambda} \right). \quad (3.11)$$

$$U_{7,8}(x, t) = \pm \frac{\sqrt{a}}{2\sqrt{rb}} \left(\lambda + \frac{4\mu}{\sqrt{4\mu - \lambda^2} \cot\left(\frac{\sqrt{4\mu - \lambda^2}}{2}(x + ct + K)\right) - \lambda} \right). \quad (3.12)$$

As a result, Eq (1.1) has solutions

$$u_{5,6}(x, t) = \pm \frac{\sqrt{a}}{2\sqrt{rb}} e^{i[-rx+wt+\theta+\sigma W(t)]} \left(\lambda + \frac{4\mu}{\sqrt{4\mu - \lambda^2} \tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2}(x + ct + K)\right) - \lambda} \right). \quad (3.13)$$

$$u_{7,8}(x, t) = \pm \frac{\sqrt{a}}{2\sqrt{rb}} e^{i[-rx+wt+\theta+\sigma W(t)]} \left(\lambda + \frac{4\mu}{\sqrt{4\mu - \lambda^2} \cot\left(\frac{\sqrt{4\mu - \lambda^2}}{2}(x + ct + K)\right) - \lambda} \right). \quad (3.14)$$

Family 3: When $\lambda^2 - 4\mu > 0$ & $\lambda \neq 0$ & $\mu = 0$

$$U_{9,10}(x, t) = \pm \frac{\sqrt{a}}{2\sqrt{rb}} \left(\lambda + \frac{2\lambda}{\exp[\lambda(x + ct + K)] - 1} \right). \quad (3.15)$$

As a result, Eq (1.1) has solutions

$$\tilde{\psi}_{9,10}(x, t) = \pm \frac{\sqrt{a}}{2\sqrt{rb}} e^{i[-rx+wt+\theta+\sigma W(t)]} \left(\lambda + \frac{2\lambda}{\exp[\lambda(x + ct + K)] - 1} \right). \quad (3.16)$$

Family 4: When $\lambda^2 - 4\mu = 0$ & $\lambda \neq 0$ & $\mu \neq 0$

$$U_{11,12}(x, t) = \pm \frac{\sqrt{a}}{2\sqrt{rb}} \left(\lambda - \frac{\lambda^2(\xi + C)}{\lambda(x + ct + K) + 2} \right). \quad (3.17)$$

As a result, Eq (1.1) has solutions

$$u_{11,12}(x, t) = \pm \frac{\sqrt{a}}{2\sqrt{rb}} e^{i[-rx+wt+\theta+\sigma W(t)]} \left(\lambda - \frac{\lambda^2(\xi + C)}{\lambda(x + ct + K) + 2} \right). \quad (3.18)$$

Family 5: When $\lambda^2 - 4\mu = 0$ & $\lambda = 0$ & $\mu = 0$

$$U_{13,14}(x, t) = \pm \frac{\sqrt{a}}{2\sqrt{rb}} \left(\frac{1}{x + ct + K} \right). \quad (3.19)$$

As a result, Eq (1.1) has solutions

$$u_{13,14}(x, t) = \pm \frac{\sqrt{a}}{2\sqrt{rb}} e^{i[-rx+wt+\theta+\sigma W(t)]} \left(\frac{1}{x + ct + K} \right). \quad (3.20)$$

3.2. Solutions of Eq (1.1) via unified solver method

In light of the the above unified solver technique, the solutions of Eq (1.1) are:

Rational function solutions:

$$\tilde{U}_{1,2}(x, t) = \left(\mp \sqrt{\frac{rb}{a}} (x + ct + \varrho) \right)^{-1}. \quad (3.21)$$

As a result, Eq (1.1) has solutions

$$\tilde{u}_{1,2}(x, t) = e^{i[-rx+wt+\theta+\sigma W(t)]} \left(\mp \sqrt{\frac{rb}{a}} (x + ct + \varrho) \right)^{-1}. \quad (3.22)$$

Trigonometric function solutions:

$$\tilde{U}_{3,4}(x, t) = \pm \sqrt{\frac{w + r^2 a}{2rb}} \tan \left(\sqrt{\frac{w + r^2 a}{2a}} (x + ct + \varsigma) \right) \quad (3.23)$$

and

$$\tilde{U}_{5,6}(x, t) = \pm \sqrt{\frac{w + r^2 a}{2rb}} \cot \left(\sqrt{\frac{w + r^2 a}{2a}} (x + ct + \varsigma) \right). \quad (3.24)$$

As a result, Eq (1.1) has solutions

$$\tilde{u}_{3,4}(x, t) = \pm \sqrt{\frac{w + r^2 a}{2rb}} e^{i[-rx+wt+\theta+\sigma W(t)]} \tan \left(\sqrt{\frac{w + r^2 a}{2a}} (x + ct + \varsigma) \right) \quad (3.25)$$

and

$$\tilde{u}_{5,6}(x, t) = \pm \sqrt{\frac{w + r^2 a}{2kb}} e^{i[-rx+wt+\theta+\sigma W(t)]} \cot \left(\sqrt{\frac{w + r^2 a}{2a}} (x + ct + \varsigma) \right). \quad (3.26)$$

Hyperbolic function solutions:

$$\tilde{U}_{7,8}(x, t) = \pm \sqrt{\frac{-(w + r^2 a)}{2rb}} \tanh \left(\sqrt{\frac{-(w + r^2 a)}{2a}} (x + ct + \varsigma) \right) \quad (3.27)$$

and

$$\tilde{U}_{9,10}(x, t) = \pm \sqrt{\frac{-(w + r^2 a)}{2rb}} \coth \left(\sqrt{\frac{-(w + r^2 a)}{2a}} (x + ct + \varsigma) \right). \quad (3.28)$$

As a result, Eq (1.1) has solutions

$$\tilde{u}_{7,8}(x, t) = \pm \sqrt{\frac{-(w + r^2 a)}{2rb}} e^{i[-kx + wt + \vartheta + \sigma W(t)]} \tanh \left(\sqrt{\frac{-(w + r^2 a)}{2a}} (x + ct + \varsigma) \right) \quad (3.29)$$

and

$$\tilde{u}_{9,10}(x, t) = \pm \sqrt{\frac{-(w + r^2 a)}{2rb}} e^{i[-kx + wt + \vartheta + \sigma W(t)]} \coth \left(\sqrt{\frac{-(w + r^2 a)}{2a}} (x + ct + \varsigma) \right). \quad (3.30)$$

4. Results and discussion

In this work we consider the PCNLSE with multiplicative noise term, where the noise has a rather complex spectrum. In crystals this kind of noise related to scattering of excitons by phonons fields, as a result of thermal vibrations of the molecules. We have constructed some new stochastic solutions to PCNLSE in the presence of noise term, utilizing the $\exp[-\varphi(\xi)]$ -expansion and unified solver techniques. These solutions are presented in explicit form. The acquired chiral solitons play an important role in the development of the practical quantum Hall effect, where chiral excitations are known to emerge [48, 49].

The behavior of these chiral solutions being solitons, shock, dissipative, rough, periodic, explosive, etc., is depend on the physical parameters given in the PCNLSE. For example the profile picture of wave changes from compressive to rarefactive at critical points and stability regions changed to unstable regions at some critical values [50, 51]. The obtained new solutions in this study are play a crucial role in nuclear physics, plasma physics, nano-technology, nonlinear optics, superfluid. Our study shows that the proposed techniques are direct, simple, sturdy and functional. Furthermore, these approaches are so vital to solve other complicated nonlinear partial differential equations in the presence of noise term, which arising in natural sciences. We give some profile pictures of some selected solutions for the PCNLSE in the presence of noise term. These pictures depict the effect of noise term in Itô sense on the presented solutions.

The amplitude (height) of the wave is the half the distance from trough to crest. Amplitude can be determined for sound wave traveling through water waves, air or for any other kind of wave along a gas or a liquid. Even waves traveling via a solid have an amplitude, as in waves fluctuating the earth because of an earthquake. One of the main feature of the multiplicative noise term in Itô sense, is the effect of this noise on the amplitude of the waves. By controlling the value of the noise term, the disastrous power of some massive natural disasters can be decreasing or turn them into beneficial energy sources.

Figures 1–4 illustrate the behaviour of solution u_1, \tilde{u}_7 via various values of noise term. These figure show the variation of the attitude of the waves.

Remark 1. The two presented techniques in this study can be implemented for solving the nonlinear fractional differential equations [52–54].

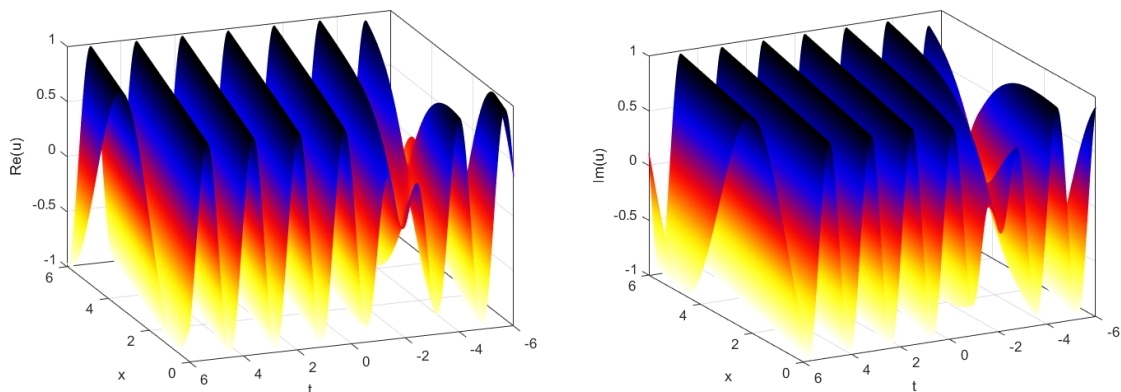


Figure 1. Profile picture of u_1 for $\sigma = 0$.

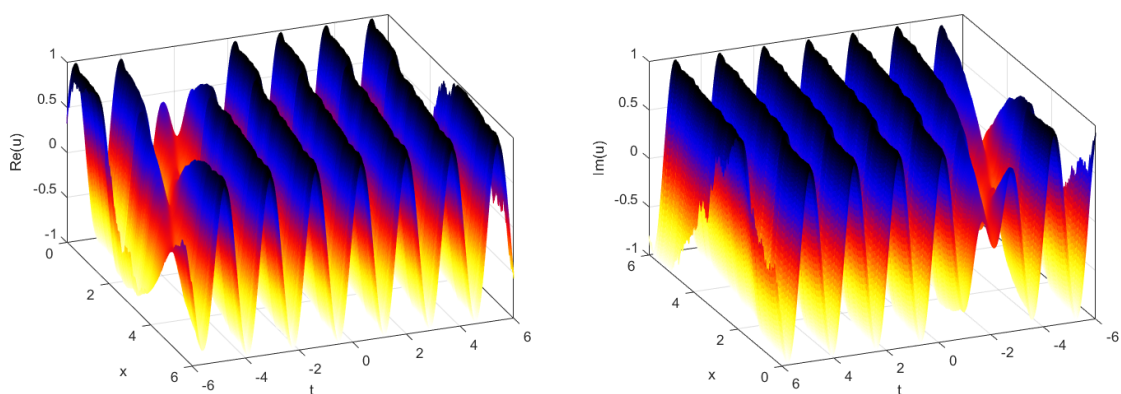


Figure 2. Profile picture of u_1 for $\sigma = 1$.

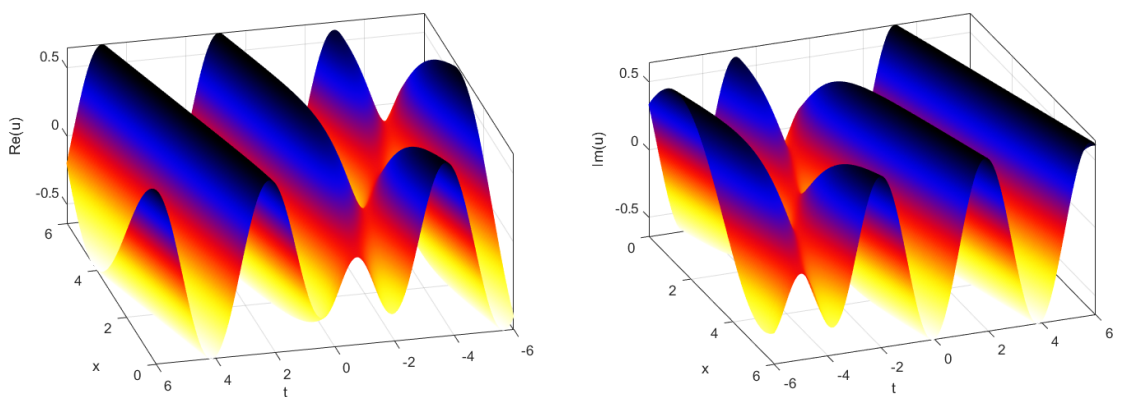


Figure 3. Profile picture of \tilde{u}_7 for $\sigma = 0$.

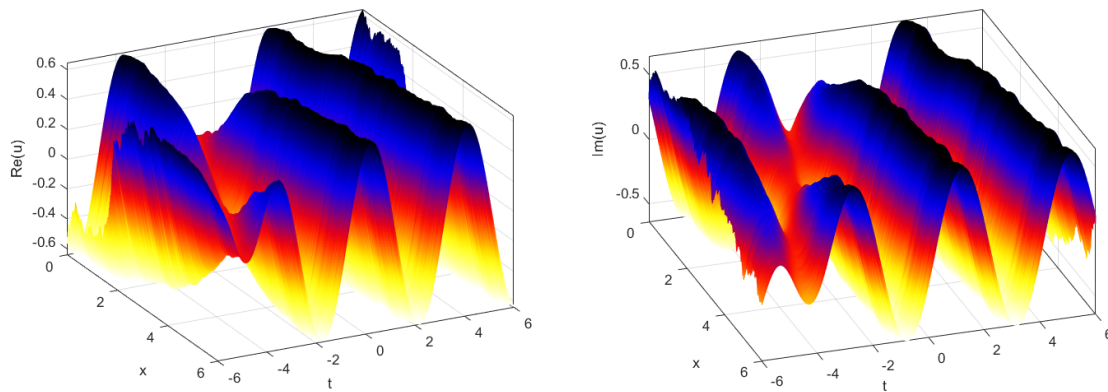


Figure 4. Profile picture of \tilde{u}_7 for $\sigma = 1$.

5. Conclusions

We have given a rich variety of classes of solutions with physical parameters to PCNLSE, using the unified solver and $\exp(-\varphi(\xi))$ -expansion methods. Indeed, the proposed methods are not only direct and simple but also succinct and give vital new results. The presented Chiral solitons are so important in evolution of quantum mechanics, specifically, in quantum hall effect complete computer industry, nano-science and nuclear medicine. We have illustrated graphical structures for some selected solutions utilizing Matlab 18, that are beneficial to understand more obviously about the dynamics of solutions. Finally, the proposed two techniques can be implemented for further physical equations arising in applied sciences.

Conflict of interest

The authors declare no conflict of interest.

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