Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 7(7): 12144–12153.
DOI: 10.3934/math.2022675
Received: 21 February 2022
Revised: 05 April 2022
Accepted: 10 April 2022
Published: 22 April 2022

Research article

Soft topological approaches via soft γ-open sets

Samirah Alzahrani[1], A. A. Nasef[2], N. Youns[3], A. I. EL-Maghrabi[3] and M. S. Badr[4,*]

[1] Department of Mathematics and Statistics, college of Science, Taif University, P.O. Box 11099, Taif 21944, Saudi Arabia
[2] Department of Physics and Engineering Mathematics, Faculty of Engineering, Kafrelsheikh University, Kafrelsheikh, Egypt
[3] Department of Mathematics, Faculty of Science, Kafrelsheikh University, Kafrelseikh, Egypt
[4] Department of Mathematics, Faculty of Science, New Valley University, El Kharga, Egypt

* **Correspondence:** Email: m.shaban@sci.nvu.edu.eg.

Abstract: The purpose of this research is to present, study, and prove numerous features of soft γ-open ($\mathcal{S}\gamma$o) and soft γ-closed ($\mathcal{S}\gamma$c) sets in soft topological structure ($\mathcal{S}\tau s$). Also, we show that the collection of $\mathcal{S}\gamma$o sets is a soft supra topology ($\mathcal{S}s\tau$) by stating and proving the conditions. Finally, we study soft γ-continuous functions and soft γ-irresolute functions. Some related properties of these new soft of discussed with help of some examples.

Keywords: soft topology; soft γ-open sets; soft γ-continuous
Mathematics Subject Classification: 54C99, 54D10, 54D15, 54D30, 54F05

1. Introduction

Uncertainty is a part of many real-world challenges in various sectors like economics, social science, medicine, etc. A variety of theoretical frameworks, such as probability theory and the theory of fuzzy sets [8], can be used to cope with these uncertainties because typical mathematical techniques are inadequate, theory of intuitionistic fuzzy sets [10], theory of vague sets [5], theory of interval mathematics [6], and theory of rough sets [4]. However, as pointed out in [2] that all these theories have their own difficulties.

Molodtsov [9] pioneered the theory of soft set ($\mathcal{S}$-set) which is a new way to deal with uncertainties that cannot be captured by more traditional mathematical techniques. He demonstrated numerous applications of this theory in the fields of economics, engineering, social science, and medicine, among others. In the recent years, a paper about $\mathcal{S}$-sets theory and their applications in many fields. $\mathcal{S}$-sets and its applications had seen significant advancements during the last few years with new developments occurring at fast rate. The concept of $\mathcal{S}\tau$ *spaces* was defined by Shabir and Naz (2011) who defined them as existing in an initial universe with a fixed set of parameters. Soft open ($\mathcal{S}$o) sets, soft closed ($\mathcal{S}$c) sets, soft interior and soft closure were defined by [10]. Chen [4] introduced the concept of soft semi open ($\mathcal{S}s$o) sets and their attributes. Soft continuous functions defined by [1]. Mahanta and Das [7] defined and investigated many forms of soft functions, Like soft semi-continuous and soft irresolute. Also, soft gb-closed sets and soft $gs\beta$-closed sets in $\mathcal{S}\tau$ structures defined by [2]. Al-shami [12] studied Soft α-sets. The main contribution of this paper introduces the class of $\mathcal{S}\gamma$o sets as generalization to each of soft semi open and soft preopen soft. Also, we construct a group of related topics like soft γ-continuity, $\mathcal{S}\gamma$-irresolute and soft γ-homeomorphism. Some related properties of these new soft of discussed with help of some examples. In our study, firstly, we have focused $\mathcal{S}\gamma$o and $\mathcal{S}\gamma$c sets over the $\mathcal{S}\tau s$ and have studied some of their properties. Secondly, we are introduced soft γ-continuity, soft γ-irresolute ($\mathcal{S}\gamma r$) and soft γ-homeomorphism ($\mathcal{S}\gamma$-h) on $\mathcal{S}\tau$ structures. Finally, we've gotten some properties of these functions.

2. Preliminaries

Unless otherwise indicated, the spaces U and W denoted $\mathcal{S}\tau$ *spaces* with (U, τ, E) and (W,ν, T). Additionally, a soft mapping f: U→W denotes a mapping, where f: (U, τ, E)→(W, ν, T), u: U→W and p: E→T denote assumed mappings.

Definition 2.1. [8] A $\mathcal{S}$-set (F_1, H) is called:

1) Null $\mathcal{S}$-set and represented by $\tilde{\emptyset}$ if F_1(e)=∅, $\forall e \in H$.
2) Absolute $\mathcal{S}$-set, written as $\tilde{U}$ if F_1(e)=U, $\forall e \in H$.

Definition 2.2. [5,8] If (F_1, H), (F_2, D) are two a $\mathcal{S}$-sets, then $(F_1, H)\tilde{\cup}(F_2$, D)=(V, J) is a $\mathcal{S}$-set, since J=H$\tilde{\cup}$D and for all s∈J.

$$V(s) = \begin{cases} F_1(\mathrm{s}) & \text{if s} \in \mathrm{H} - \mathrm{D}, \\ F_2(\mathrm{s}) & \text{if s} \in \mathrm{D} - \mathrm{H}, \\ F_1(\mathrm{s}) \cup F_2(\mathrm{s}) & \text{if s} \in \mathrm{H} \tilde{\cap} \mathrm{D}. \end{cases}$$

And $(F_1, H)\tilde{\cap}(F_2$, D)=(V, J) is a $\mathcal{S}$-sets defined as J=H$\tilde{\cap}$D, and V(s)=F_1(s)$\tilde{\cap}F_2$(s), ∀s∈J.

Definition 2.3. [10] The relative complement of $\mathcal{S}$-set (F_1, E) is written as $(F_1, \mathrm{E})^c$, $(F_1, \mathrm{E})^c=(F_1{}^c, E)$ and $F_1{}^c$: $E\to p(U)$, $F_1{}^c(e)=UF_1(e)$, ∀ $e\in E$.

Definition 2.4. Let (U, τ, E) be a $\mathcal{S}\tau$ space and (F_2, H) be a $\mathcal{S}$-set. Then

(1) A soft interior [6] of (F_2, H) defined as sint(F_2, H)=$\tilde{\cup}${(D, H): (D, H) is $\mathcal{S}$o set and (D, H)$\tilde{\subseteq}$(F_2, H)}.
(2) A soft closure [10] of (F_2, H) defined as $\tilde{\mathrm{s}}$ cl(F_2, H)=$\tilde{\cap}${(D, H): (D, H) is $\mathcal{S}$c set and (F_2, H)$\tilde{\subseteq}$(D, H)}.

Definition 2.5. A $\mathcal{S}$-set (F_1, H) in a $\mathcal{S}\tau$ structure (U, τ, E) is called:
(1) Soft semi-open ($\mathcal{S}so$) [4] if (F_1, H)⊆$c(int(F_1$, H)).
(2) Soft preopen ($\mathcal{S}po$) [2] if (F_1, H)⊆$in(cl(F_1$, H)).
(3) Soft α-open ($\mathcal{S}$αo) [2] if (F_1, H)⊆$in(cl(int(F_1$, H))).
(4) Soft β-open ($\mathcal{S}$βo) [2] if (F_1, H)⊆$c(int(cl(F_1$, H))).
(5) Soft regular closed ($\mathcal{S}rc$) [2] if (F_1, H)=$c(int(F_1$, H)).

Definition 2.6. [3] The $\mathcal{S}$-set (F_1, H) is called a soft point, write as (U_e, H), if $e \in H$, $F_1(e)=\{x\}$ and $F_1(e^c)=\varnothing$, $\forall e^c \in H-\{e\}$.

Definition 2.7. Let (U, τ, S) and (Y, σ, S) be two $\mathcal{S}\tau$ *spaces*. A function f: (U, τ, S)→(Y, σ, S) is called:
(1) Soft semi-continuous [7] if $f^{-1}((G,S))$ is $\mathcal{S}so$ in (U, τ, S), for each $\mathcal{S}o$ set (G, S) of (Y, σ, S).
(2) Soft precontinuous [11] if $f^{-1}((G,S))$ is $\mathcal{S}po$ in (U, τ, S), for each $\mathcal{S}o$ set (G, S) of (Y, σ, S).
(3) Soft α-continuous [11] if $f^{-1}((G,S))$ is $\mathcal{S}$αo in (U, τ, S), for each $\mathcal{S}o$ set (G, S) of (Y, σ, S).
(4) Soft β-continuous [11] if $f^{-1}((G,S))$ is $\mathcal{S}\beta$o in (U, τ, S), for each $\mathcal{S}o$ set (G, S) of (Y, σ, S).
(5) Soft β-irresolute [11] if $f^{-1}((G,S))$ is $\mathcal{S}\beta$o in (U, τ, S), for each $\mathcal{S}\beta$o set (G, S) of (Y, σ, S).

3. Soft γ-open sets and soft γ-closed sets

We define $\mathcal{S}$γo and $\mathcal{S}$γc sets in $\mathcal{S}\tau$ space and study some of their characteristics in this section.

Definition 3.1. A $\mathcal{S}$-set (F_1, H) in a $\mathcal{S}\tau$ (U, τ, S) is called:

(1) $\mathcal{S}\gamma$o if (F_1, H)$\tilde{\subseteq}$cl($int(F_1$, H))$\tilde{\cup}int(cl((F_1$, H)).

(2) $\mathcal{S}\gamma$c if its soft complement is $\mathcal{S}\gamma$o.

The family of all $\mathcal{S}\gamma o$ sets (resp. $\mathcal{S}\gamma c$ sets) in a $\mathcal{S}\tau$ structure (U, τ, S) will be written as $\mathcal{S}\gamma O(U)$ (resp. $\mathcal{S}\gamma C(U)$).

Remark 3.2. The family of $\mathcal{S}\gamma$O(U) contains each of $\mathcal{S}$O(U), $\mathcal{S}$αO(U), $\mathcal{S}$PO(U), $\mathcal{S}$SO(U) and contained in $\mathcal{S}$βO(U), as the following implication.

$\mathcal{S}o$ set⇒$\mathcal{S}$αo set⇒$\mathcal{S}p$o set

⇓ ⇓

$\mathcal{S}s$o set⇒$\mathcal{S}\gamma$o set⇒$\mathcal{S}\beta$o set

In the following example, we will prove that the converses may not always have to be true.

Example 3.3. Assume that $U=\{x_1, x_2, x_3, x_4\}$, $E=\{u_1, u_2\}$ and $\tau=\{\varnothing, \tilde{U}, (K_1, E), (K_2, E), (K_3, E)\}$ such that $(K_1, E), (K_2, E), (K_3, E)$ are $\mathcal{S}$-sets, which defined as:
(i) $K_1(u_1)=\{x_1\}$, $K_1(u_2)=\{x_1\}$, $K_2(u_1)=\{x_1, x_3\}$, $K_2(u_2)=\{x_2, x_3\}$, $K_3(u_1)=\{x_1, x_2, x_3\}$, $K_3(u_2)=\{x_1, x_2, x_3\}$. Then, the $\mathcal{S}$-set (S, E) defines as: $(u_1)=\{x_2\}$, $S(u_2)=\{x_2\}$ is a $\mathcal{S}\beta$o set but not $\mathcal{S}\gamma$o.
(ii) $K_1(u_1)=\{x_1\}$, $K_1(u_2)=x_1\}$, $K_2(u_1)=\{x_2\}$, $K_2(u_2)=\{x_2\}$, $K_3(u_1)=\{x_1, x_2\}$, $K_3(u_2)=\{x_1, x_2\}$. Hence, the $\mathcal{S}$-set (S, E) defined as: $S(u_1)=\{x_2, x_3\}$, $S(u_2)=\{x_2, x_3\}$ is a $\mathcal{S}\gamma$o set but not $\mathcal{S}p$o.
(iii) $K_1(u_1)=\{x_1\}$, $K_1(u_2)=\{x_2\}$, $K_2(u_1)=\{x_1\}$, $K_2(u_2)=U$, $K_3(u_1)=U$, $K_3(u_2)=\{x_2\}$. Then the $\mathcal{S}$-set (S, E) defined as: $S(u_1)=\{x_2\}$, $S(u_2)=U$ is a $\mathcal{S}\gamma$o set but not $\mathcal{S}s$o.

Proposition 3.4. A collection of $\mathcal{S}\gamma O(U)$ of all $\mathcal{S}$-sets of (U, τ, E) forms a $\mathcal{S}s\tau$.

Proof. Suppose that $(F_i, E) \in \mathcal{S}\gamma O(U), \forall i \in I=\{1,2,3\ldots.\}$. Then

$$\forall i \in I,\ (Fi, E) \tilde{\subseteq} int(cl(Fi, E)) \tilde{\cup} cl(int(Fi, E)),$$

so

$$\tilde{\cup}(Fi, E) \tilde{\subseteq} \tilde{\cup}[int(cl(Fi, E)) \tilde{\cup} cl(int(Fi, E))]$$

$$=\{\tilde{\cup}[\,int(cl(F_i, E))]\} \tilde{\cup} \{\,\tilde{\cup} cl(int(F_i, E))\}$$

$$\tilde{\subseteq} int(cl(\tilde{\cup}(Fi, E))) \tilde{\cup} cl(int(\tilde{\cup}(Fi, E))).$$

Then $\tilde{\cup}(F_i, E)$ is $\mathcal{S}\gamma O(U)$.

In the following example, we will prove that the intersection of two $\mathcal{S}\gamma$o sets is not $\mathcal{S}\gamma$o.

Example 3.5. Assume that (U, τ, E) is a $\mathcal{S}\tau s$ as shows in Example 3.3 and (G, E), (H, E) are $\mathcal{S}\gamma$o sets.

Then, $(G, E) \tilde{\cap} (H, E)=\{\{x_2\}, \varnothing\}=(K, E)$ *and* $cl(int(F, E)) \tilde{\cap} int(cl(F, E))=\varnothing$. Thus, (K, E) is not $\mathcal{S}\gamma$o.

Proposition 3.6. Any intersection of $\mathcal{S}\gamma$c sets is $\mathcal{S}\gamma$c.

Proof. Suppose that $(F_i, E) \in S\gamma O(U), \forall i \in I=\{1,2,3,\ldots\}$. Then,

$$\forall i \in I, (F_i, E) \tilde{\subseteq} nt(cl((F_i, E))) \tilde{\cup} cl(int((F_i, E)))$$

and

$$\tilde{\cap}(F_i, E) \tilde{\subseteq} \tilde{\cap}[int(cl((F_i, E))) \tilde{\cup} cl(int((F_i, E)))]$$

$$=\{\tilde{\cap}[int\ (cl((F_i, E)))]\} \tilde{\cup} \{\tilde{\cap}[cl(int((F_i, E))]\})$$

$$\tilde{\subseteq} int(cl(\tilde{\cap}(F_i, E))) \tilde{\cup} cl(int(\tilde{\cap}(F_i, E))).$$

Then $\tilde{\cap}(F_i, E)$ is $\mathcal{S}\gamma$c.

In this example, we will prove that the union of two $\mathcal{S}\gamma$c sets need not be $\mathcal{S}\gamma$c set.

Example 3.7. Assume that (U, τ, E) is a $\mathcal{S}\tau$ structure as shows in Example 3.3 and (G, E), (H, E) which defined as: $G(e_1)=\{x_1\}$, $G(e_2)=\{x_1\}$, $H(e_1)=\varnothing$, $H(e_2)=\{x_2\}$. Then (G, E), (H, E) are $\mathcal{S}\gamma$c sets, thus,

$$(G, E) \tilde{\cup} (H, E)=\{\{x_1\}, U\}=(K, E)$$

is not $\mathcal{S}\gamma$c.

Proposition 3.8. For each proper soft subset of a soft indiscrete structure (U, τ, E), the following statement are holds.

(1) Each proper soft subset of U is $\mathcal{S}\gamma$O(U) but not $\mathcal{S}$SO(U),
(2) $\mathcal{S}$PO(U)=$\mathcal{S}\gamma$O(U)=$\mathcal{S}\beta$O(U).

Proof. (1) Consider (F, E) is proper soft subset on a soft indiscrete structure (U, τ, E). Thus, cl(int(F,E))=∅ and int(cl(F, E))$\tilde{=}U$. Therefore each proper soft subset on U is $\mathcal{S}\gamma$O(U) but not $\mathcal{S}$SO(U). Since for each proper soft subset of a soft indiscrete structure (U, τ, E), int(cl(F, E))$\tilde{=}U$ and cl(int(cl(F,E)))$\tilde{=}U$. Then $\mathcal{S}$PO(U)=$\mathcal{S}\gamma$O(U)=$\mathcal{S}\beta$O(U).

Theorem 3.9. For any $\mathcal{S}\gamma$O(U) (L, E), the following statements are holds.

(1) If (G, E)∈$\mathcal{S}$O(U), then (L, E)$\tilde{\cap}$(G, E)∈$\mathcal{S}\gamma$O(U).

(2) If $(L, E)\in \mathcal{S}O(U)$, then $(L, E)\in \mathcal{S}SO(U)$.

Proof. (1) Assume that (L, E)∈$\mathcal{S}\gamma$O(U), (G, E)∈$\mathcal{S}$O(U). Hence,

$$(L, E)\tilde{\cap}(G, E)\tilde{\subseteq}[int\,(cl((L,E)))\,\tilde{\cup}cl\,(int((L,E)))]\tilde{\cap}(G,E)$$

$$=[int(cl((L, E)))\tilde{\cap}(G, E)]\,\tilde{\cup}[cl\,(int((L,E)))\tilde{\cap}(G,E)]$$

$$\tilde{\subseteq}int[cl((L,E))\tilde{\cap}(G,E)]\,\tilde{\cup}cl[(int(L,E))\tilde{\cap}(G,E)]$$

$$\tilde{\subseteq}int(cl[(L,E)\tilde{\cap}(G,E)])\,\tilde{\cup}cl(int[(L,E)\tilde{\cap}(G,E)]).$$

Therefore, (L, E)$\tilde{\cap}$(G, E) is a $\mathcal{S}\gamma$o set.
(2) Since (L, E) is a $\mathcal{S}\gamma$o set and $\mathcal{S}$c set, therefore

$$(L, E)\tilde{\subseteq}cl(int(L,E))\,\tilde{\cup}int(cl(L,E))=cl(int(L,E))\,\tilde{\cup}int(L,E)=cl(int(L,E)).$$

Thus (L, E)$\tilde{\subseteq}cl(int(L,E))$ and so (L, E) is $\mathcal{S}$so.

Corollary 3.10. For any $\mathcal{S}\gamma$C(U) (T, E), the following statements are hold:

(1) If (G, E)∈$\mathcal{S}$C(U), then (T, E)$\tilde{\cap}$(G, E)∈$\mathcal{S}\gamma$C(U).

(2) If $(T, E)\in \mathcal{S}$O(U), then $(T, E)\in \mathcal{S}SC(U)$.

Theorem 3.11. Each of $\mathcal{S}\gamma$O(U) and $\mathcal{S}\alpha$C(U) set is $\mathcal{S}$rc (where $\mathcal{S}$rc denotes of soft regular closed).

Proof. Assume that (T, E)∈$\mathcal{S}\gamma$O(U), (G, E)∈$\mathcal{S}\alpha$O(U) set. Then

$$cl\,(int(\mathrm{cl}(T,E)))\tilde{\subseteq}(T,E)\tilde{\subseteq}cl(int(T,E))\,\tilde{\cup}\,int(cl(T,E))\tilde{\subseteq}cl\,(int(cl(T,E))),$$

hence (T, E)=cl(int(cl(T,E))) which is $\mathcal{S}$C(U), therefore (T, E) is $\mathcal{S}$rc.

Corollary 3.12. Every $\mathcal{S}\gamma$C(U) and $\mathcal{S}\alpha$O(U) set is soft regular open.

Theorem 3.13. In a $\mathcal{S}\tau$ space (U, τ, E) over U, if (V, E) is $\mathcal{S}\gamma$O(U) and $\mathcal{S}\alpha$C(U), then

$$(V, E)=cl(int(V, E))\tilde{\cup}\, int(cl(V, E)).$$

Proof. Since, (V, E) is $\mathcal{S}\gamma$O(U), hence

$$(V, E)\tilde{\subseteq} cl(int(V, E))\tilde{\cup}\, int(cl(V, E)).$$

The other inclusion (V, E) is $\mathcal{S}$PC(U) and $\mathcal{S}$ SC(U), then

$$cl(int(V, E))\tilde{\subseteq}(V, E),\ in(cl(V, E))\tilde{\subseteq}(V, E),$$

respectively. Therefore

$$cl(int(V, E))\tilde{\cup}\, int(cl(V, E))\tilde{\subseteq}(V, E).$$

Then

$$(V, E)= cl(int(V, E))\tilde{\cup}\, int(cl(V, E)).$$

Definition 3.14. A $\mathcal{S}$-set (T, E) in a $\mathcal{S}\tau$ space (U, τ, E) is called:
(1) A soft nowhere dense if int(cl(T, E))=$\varnothing$.
(2) A soft dense ($\mathcal{S}d$) set if cl(T, E))=U.

Definition 3.15.A $\mathcal{S}\tau$ structure (U, τ, E) is called:
(1) Soft submaximal if all $\mathcal{S}d$ subset over U are $\mathcal{S}$o.
(2) Soft extremely disconnected ($\mathcal{S}$ED) if the soft closure of each $\mathcal{S}$o set is $\mathcal{S}$o.

Example 3.16. Let U=$\{x_1, x_2\}$, E=$\{e_1, e_2\}$ and

$$\tau=\{\varnothing, \tilde{U},\ (K_1, E), (K_2, E), (K_3, E), (K_4, E), (K_5, E), (K_6, E), (K_7, E), (K_8, E)\},$$

such that (K_1, E), (K_2, E), (K_3, E), (K_4, E), (K_5, E), (K_6, E), (K_7, E), (K_8, E) are $\mathcal{S}$-sets over U, which defined as: $K_1(e_1)$=$\varnothing$, $K_1(e_2)$=$\varnothing$, $K_2(e_1)$=$\varnothing$, $K_2(e_2)$=$\{x_1\}$, $K_3(e_1)$=$\varnothing$, $K_3(e_2)$=$\{x_2\}$, $K_4(e_1)$=$\varnothing$, $K_4(e_2)$=U, $K_5(e_1)$=$\{x_1\}$, $K_5(e_2)$=$\varnothing$, $K_6(e_1)$=$\{x_1\}$, $K_6(e_2)$=$\{x_1\}$, $K_7(e_1)$=$\{x_1\}$, $K_7(e_2)$=$\{x_2\}$, $K_8(e_1)$=$\{x_1\}$, $K_8(e_2)$=U. Thus, the $\mathcal{S}\tau$ structure (U, τ, E) is soft submaximal and soft extremely disconnected.

Proposition 3.17. In a $\mathcal{S}\tau$ space (U, τ, E), we have
(1) If each $\mathcal{S}$-set (F, E) of $\mathcal{S}\tau$ structure (U, τ, E) is a soft nowhere dense, then $\mathcal{S}$SO(U)=$\mathcal{S}\gamma$O(U).
(2) If (U, τ, E) is soft submaximal and $\mathcal{S}$ED space, then $\mathcal{S}\alpha$O(U)=$\mathcal{S}\gamma$O(U).

Proof. Obvious.

Theorem3.18. In a $\mathcal{S}\tau$ structure (U, τ, E) over U, then any (F, H)$\in\mathcal{S}\gamma$O(U) is $\mathcal{S}$PO(U) if one of the following conditions hold:
(1) (U, τ, E) is SED.
(2) (F, H) is soft dense over U.

Proof. (1) Since, (F, H)$\in\mathcal{S}\gamma$O(U) and (U, τ, E) is $\mathcal{S}$ED, therefore

$$(F, H)\tilde{\subseteq} cl(int(F, H))\tilde{\cup} int(cl(F, H))\tilde{\subseteq} int(cl(int(F, H)))\tilde{\cup}\, int(cl(F, H)),$$

then (F, H)∈$\mathcal{S}$PO(U).
(2) Since, (F, H)∈$\mathcal{S}\gamma$O(U) and $(F, H)^c$ is soft dense, therefore int(F, H)=∅ and

$$(F, H)\tilde{\subseteq}cl(int(F, H))\tilde{\cup}\ int(cl(F, H))=int(cl(F, H)).$$

Hence (F, H) is $\mathcal{S}$PO(U).

4. Soft γ-continuous mappings

We define soft γ-continuous and soft γ-irresolute mappings. We also study some of their properties with the help of $\mathcal{S}\gamma$o sets in this section.

Definition 4.1. Let (U, τ, E), (Y, σ, E) be two $\mathcal{S}\tau$ structures. A mapping f: (U, τ, E)→(Y, σ, E) is called a soft γ-continuous if $f^{-1}(G, E)$ is $\mathcal{S}\gamma$o in (U, τ, E), for every $\mathcal{S}$o set (G,E) of (Y, σ, E).

According to Definition 4.1, we have

$$\begin{array}{ccccc} \text{soft continuity} & \Rightarrow & \mathcal{S}\alpha\text{-continuity} & \Rightarrow & \mathcal{S}s\text{o-continuity} \\ & & \Downarrow & & \Downarrow \\ & & \text{Soft precontinuity} & \Rightarrow \mathcal{S}\gamma\text{-continuity} \Rightarrow & \mathcal{S}\beta\text{-continuity.} \end{array}$$

In the following examples, we will prove that the converses may not always have to be true.

Example 4.2. Let $U=\{x_1, x_2, x_3\}$, $E=\{e_1, e_2\}$ and $\tau=\{\varnothing, \tilde{U}, (K_1, E), (K_2, E), (K_3, E)\}$, $\sigma=\{\varnothing, \tilde{U}, (K_1, E), (K_3, E)\}$, where (K_1, E), (K_2, E), (K_3, E) are $\mathcal{S}$-sets which defined as: $K_1(e_1)=\{x_1\}$, $K_1(e_2)=\{x_1\}$, $K_2(e_1)=\{x_2\}$, $K_2(e_2)=\{x_2\}$, $K_3(e_1)=\{x_1, x_2\}$, $K_3(e_2)=\{x_1, x_2\}$. And the mapping f: (U, τ, E))→(Y, σ, E), which defined as follows: $f(x_1)=x_1$, $f(x_2)=x_3$, $(x_3)=x_2$. Hence f is $\mathcal{S}\gamma$ -continuous but not soft precontinuous.

Example 4.3. Assume that $U=Y=\{x_1, x_2, x_3, x_4\}$ *and* $E=\{e_1, e_2\}$. Then
(1) The identity mapping from soft indiscrete space (U, τ, E) onto a soft discrete space (Y, σ, E) is $\mathcal{S}\gamma$-continuous but not $\mathcal{S}s$-continuous.
(2) The identity mapping from $\mathcal{S}\tau$ structure (U, τ, E) onto a soft discrete space (Y, σ, E) is $\mathcal{S}\beta$-continuous but not $\mathcal{S}\gamma$-continuous, where

$$\tau=\{\varnothing, \tilde{U}, (K_1, E), (K_2, E), (K_3, E)\} \text{ and } (K_1, E), (K_2, E), (K_3, E)$$

are $\mathcal{S}$-sets which defined as follows: $K_1(e_1)=\{x_1\}$, $K_1(e_2)=\{x_1\}$, $K_2(e_1)=\{x_2, x_3\}$, $K_2(e_2)=\{x_2, x_3\}$, $K_3(e_1)=\{x_1, x_2, x_3\}$, $K_3(e_2)=\{x_1, x_2, x_3\}$.

Theorem 4.4. For a soft mapping f: (U, τ, E)→(Y, σ, E), the statements that follow are equivalent:
(1) f is $\mathcal{S}\gamma$-continuous.
(2) For each soft point (u_e, E) over U and every $\mathcal{S}$o (G, E) containing $f(u_e, E)=(f(u)_e, E)$ over Y, there exists a $\mathcal{S}\gamma$o set (F, E) over U containing (u_e, E), where $f(F, E)\tilde{\subseteq}(G, E)$.
(3) The inverse image of every $\mathcal{S}$c set in Y is $\mathcal{S}\gamma$c in U.
(4) $int\ (cl(f^{-1}(G, E)))\tilde{\cap}cl(int(f^{-1}(G, E)))\tilde{\subseteq}f^{-1}(cl(G, E))$ for every $\mathcal{S}$-set (G, E) over Y.
(5) $f^{-1}(int(F, E))\tilde{\subseteq}int\ (cl(f^{-1}(F, E)))\tilde{\cup}cl(int(f^{-1}(F, E)))$ for every $\mathcal{S}$-set (F, E) over U.
(6) If f is bijective, then $int(f(F, E))\tilde{\subseteq}f(int(cl(F, E)))\tilde{\cup}f(cl(int(F, E)))$ for every $\mathcal{S}$-set (F, E)

over U.

(7) If f is bijective, then $f(int(cl(F, E)))\tilde{\cap}f(cl(int(F, E)))\tilde{\subseteq}cl(f(F, E))$ for every $\mathcal{S}$-set (F, E) over U.

Proof. (1)⟹(2): Suppose that (G, E) is a $\mathcal{S}$o set over Y containing $f(u_e, E)$=$(f(u)_e, E)$, therefore $f^{-1}(G, E)\in\mathcal{S}\gamma O(U)$. Take a $\mathcal{S}$-set (F, E)=$f^{-1}(G, E)$ which containing (u_e, E). Therefore $(F, E)\tilde{\subseteq}(G, E)$.

(2)⇒(3): Assume that (G, E)∈$\mathcal{S}$C(Y). Then $(\tilde{Y}-(G, E))\in\mathcal{SS}(Y)$. Since f is soft γ-continuous, $f^{-1}(\tilde{Y}-(G, E))\in\mathcal{S}\gamma O(U)$. Hence [$\check{U}$-$f^{-1}(G, E)$]$\in\mathcal{S}\gamma O(U)$. Therefore $f^{-1}(G, E)$]$\in\mathcal{S}\gamma C(U)$.

(3)⇒(4): Consider (G, E) is a $\mathcal{S}$-set over Y, therefore $f^{-1}(cl(G, E))$ $\mathcal{S}\gamma C(U)$, and

$$f^{-1}(cl(G, E))\tilde{\subseteq}int(cl(f^{-1}(cl(G, E))))\tilde{\cap}cl(int(f^{-1}(cl(G, E))))$$
$$\tilde{\subseteq}in(cl(f^{-1}(G, E)))\tilde{\cap}cl(int(f^{-1}(G, E))).$$

(4)⇒(5): By replacing $\tilde{Y}-(G, E)$ instead of (G, E) in (4), we have

$$int(cl(f^{-1}((\tilde{Y}-(G, E))))\tilde{\cap}cl(int(f^{-1}(\tilde{Y}-(G, E))))\tilde{\subseteq}f^{-1}(cl(\tilde{Y}-(G, E)))$$

and therefore

$$f^{-1}(int(G, E))\tilde{\subseteq}int(cl(f^{-1}(G, E)))\tilde{\cap}cl(int(f^{-1}(G, E))).$$

(5)⇒(6): Follows directly by replacing (F, E) instead of $f^{-1}(G, E)$ in (5) and applying the bijection of f.

(6)⇒(7): By the complementation of (6) and applying the bijective of f, we have

$$f(int(cl(F, E)^c))\tilde{\cap}f(cl(int(F, E)^c))\tilde{\subseteq}cl(f(F, E)^c),$$

we obtain the replacing (F, E) instead of $(F, E)^c$.

(7)⇒(1): Assume that (G, E) is a $\mathcal{S}$-set over Y, put (W, E)=$(W, E)^c$ by (7), we have

$$f(f^{-1}(int(cl(W, E))))\tilde{\cap}f(f^{-1}(cl(int(W, E))))\tilde{\subseteq}cl(f(f^{-1}(W, E))))\tilde{\subseteq}cl(W, E)\tilde{=}(W, E).$$

So, $in(cl(f^{-1}(W, E)))\tilde{\cap}cl(int(f^{-1}(W, E)))$ implies $f^{-1}(W, E)$ is a $\mathcal{S}\gamma$c over U and therefore, $f^{-1}(W, E)$ is a $\mathcal{S}\gamma$o set over U.

Theorem 4.5. For a $\mathcal{S}$γ-continuous mapping f: (U, τ, E)→(Y, σ, E), the following statements are hold:

(1) If each $\mathcal{S}$-set (F, E) over U is $\mathcal{S}$αC(U), then $f^{-1}(F, E)$=cl$(int(f^{-1}(F, E)))\tilde{\cup}int(cl(f^{-1}(F, E)))$,

(2) If each $\mathcal{S}$-set over U is a soft nowhere dense, then f is a $\mathcal{S}s$-continuous function,

(3) If (U, τ, E) is SED, then f is a soft precontinuous function.

Proof. (1) Consider f: (U, τ, E)→(Y, σ, E) is a $\mathcal{S}$γ-continuous mapping and (G, E) is $\mathcal{S}$o set over Y, then $f^{-1}(G, E)$ is $\mathcal{S}\gamma O(U)$. Since (U, τ, E) is $\mathcal{S}$ED, by Theorem 3.18, $f^{-1}(G, E)$ is $\mathcal{S}PO(U)$. (2) and (3) are obvious.

In the following example, we will prove that the composition of two $\mathcal{S}$γ-continuous functions may not always have to be $\mathcal{S}$γ-continuous.

Example 4.6. Suppose that U=Z=$\{u_1, u_2, u_3\}$, Y=$\{u_1, u_2, u_3, u_4\}$ *and* E=$\{e_1, e_2\}$. Then τ_U=$\{\varnothing, \tilde{U}, (F, E)\}$ is a $\mathcal{S}\tau$ structure over U, τ_Y=$\{\varnothing, \tilde{Y}, (G, E)\}$ is a $\mathcal{S}\tau$ structure over Y and τ_Z=$\{\varnothing, \tilde{Z}, (H_1, E), (H_2, E)\}$ is a $\mathcal{S}\tau$ structure over Z, where (F, E) is a $\mathcal{S}$-set over U, (G, E) is a $\mathcal{S}$-set over Y and (K_1, E), (K_2, E)

are $\mathcal{S}$-sets over Z which defined as: $F(e_1)=\{u_1\}$, $F(e_2)=\{u_1\}$, $(e_1)=\{u_1, u_3\}$, $G(e_2)=\{u_1, u_3\}$, $K_1(e_1)=\{u_3\}$, $K_1(e_2)=\{u_3\}$, $K_2(e_1)=\{u_1, u_2\}$, $K_2(e_2)=\{u_1, u_2\}$. If the identity function I: (U, τ_U, E)→(Y, τ_Y, E) and f: (Y, τ_Y, E)→(Z, τ_Z, E), which defines as follows: $f(u_1)=u_1$, $f(u_2)=f(u_4)=u_2$, $f(u_3)=u_3$, since I and f are $\mathcal{S}$γ-continuous, but (foI) is not $\mathcal{S}$γ-continuous, since $(foI)^{-1}(K_1, E)=\{\{u_3\}, \{u_3\}\}$ is not a $\mathcal{S}\gamma o$ set over U.

Janki and Sreeja define a homomorphism in soft topological structure in [13].

Definition 4.7. A bijection function f: (U, τ, E)→(Y, σ, E) is called soft γ-homeomorphism (resp. $\mathcal{S}$γ-homeomorphism) if f is a $\mathcal{S}$γ -continuous (resp. $\mathcal{S}$γ-irresolute) and f^{-1}: (U, τ, E)→(Y, σ, E) is $\mathcal{S}$γ-continuous (resp. $\mathcal{S}$γ-irresolute).

Definition 4.8. For a $\mathcal{S}\tau$ structure (U, τ_U, E), we define:
(1) $\mathcal{S}$γ-h(U, τ_U, E)={f: f: (U, τ_U, E)→(U, τ_U, E) is a bijection soft γ-continuous, f^{-1}: (U, τ_U, E)→(U, τ_U, E) is soft γ-continuous}.
(2) $\mathcal{S}$γr-h(U, τ_U, E)={f: f: (U, τ_U, E)→(U, τ_U, E) is a bijection soft γ-irresolute, f^{-1}: (U, τ_U, E)→(U, τ_U, E) is soft γ-irresolute}.

Theorem 4.9. For a $\mathcal{S}\tau s$(U, τ_U, E), $\mathcal{S}$-h(U, τ_U, E)$\tilde{\subseteq}$ $\mathcal{S}$γr-h(U, τ_U, E)$\tilde{\subseteq}$ $\mathcal{S}$γ-h(Uτ_U, E), such that $\mathcal{S}$-h(U, τ_U, E)={f: f: (U, τ_U, E)→(U, τ_U, E) is a soft -homeomorphism}.

Proof. First, we prove that each soft-homeomorphism f: (U, τ, E)→(Y, σ, E) is $\mathcal{S}$γr-homeomorphism. Assume that (G, E)∈$\mathcal{S}\gamma O(Y)$. Then

$$(G, E)\tilde{\subseteq} cl(int(G, E))\tilde{\cup} int(cl(G, E)),$$

hence,

$$f^{-1}(G, E)\tilde{\subseteq} f^{-1}(cl(int(G, E))\tilde{\cup} int(cl(G, E)))\tilde{\subseteq} cl(intf^{-1}(G, E))\tilde{\cup} int(cl(f^{-1}(G, E))),$$

and so $f^{-1}(G, E)\in \mathcal{S}\gamma O(U)$. Thus, f is soft γ-irresolute.

By the similar way, f^{-1} is soft γ-irresolute. Thus, $\mathcal{S}$h(U, τ_U, E)$\tilde{\subseteq}$ $\mathcal{S}$γr-h(U, τ_U, E). Finally, it is obvious that $\mathcal{S}$γr-h(U, τ_U, E)$\tilde{\subseteq}$$\mathcal{S}$γ-h(U, τ_U, E). Since every $\mathcal{S}$γ-irresolute function is $\mathcal{S}$γ-continuous.

5. Conclusions

The $\mathcal{S}$-set model has been applied to many fields from the theoretical point of view of $\mathcal{S}$-sets is discussed and investigated again. Here, we introduce the class of $\mathcal{S}$γo sets as generalization to each of soft semi open and soft preopen soft.

Also, we construct a group of related topics like soft γ-continuity, $\mathcal{S}$γ-irresolute and soft γ-homeomorphism. Some related properties of these new soft of discussed with help of some examples. Moreover, the classes proposed in this paper, can be extended in the field of fuzzy $\mathcal{S}$-sets.

Acknowledgments

This research received funding from Taif University, researchers supporting and project number (TURSP-2020/207)Taif University, Saudi Arabi.

This work was financially supported by the Academy of Scientific Research & Technology (ASRT), Egypt, Grant No.6635 under the project Science Up. (ASRT) is the second affiliation of this research.

Conflict of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

References

1. C. G. Aras, A. Sonmez, H. Çakall, On soft mappings, *arXiv*, 2013. Available from: https://arxiv.org/abs/1305.4545.
2. I. Arockiarani, A. A. Lancy, Generalized soft gβ-closed sets and soft gsβ-closed sets in soft topological spaces, *Int. J. Math. Arch.*, **4** (2013), 1–7.
3. S. Bayramov, C. G. Aras, Soft locally compact spaces and soft paracompact spaces, *J. Math. Syst. Sci.*, **3** (2013), 122.
4. B. Chen, Soft semi-open sets and related properties in soft topological spaces, *Appl. Math. Inf. Sci.*, **7** (2013), 287–294.
5. F. Feng, Y. B. Jun, U. Zhao, Soft semirings, *Comput. Math. Appl.*, **56** (2008), 2621–2628. https://doi.org/10.1016/j.camwa.2008.05.011
6. S. Hussain, B. Ahmad, Some properties in soft topological spaces, *Comput. Math. Appl.*, **62** (2011), 4058–4067. https://doi.org/10.1016/j.camwa.2011.09.051
7. J. Mahanta, P. K. Das, On soft topological space via semiopen and semiclosed soft sets, *arXiv*, 2012. Available from: https://arxiv.org/abs/1203.4133.
8. P. K. Maji, R. Biswas, A. R. Roy, Soft set theory, *Comput. Math. Appl.*, **45** (2003), 555–562. https://doi.org/10.1016/S0898-1221(03)00016-6
9. D. Molodtsov, Soft set theory–First results, *Comput. Math. Appl.*, **37** (1999), 19–31. https://doi.org/10.1016/S0898-1221(99)00056-5
10. M. Shabir, M. Naz, On soft topological spaces, *Comput. Math. Appl.*, **61** (2011), 1786–1799. https://doi.org/10.1016/j.camwa.2011.02.006
11. Y. Yumak, A. K. Kaymakei, Soft β-open sets and their applications, *arXiv*, 2013. Available from: https://arxiv.org/abs/1312.6964.
12. T. M. Al-shami, E. A. Abo-Tabl, Soft α-separation axioms and α-fixed soft points, *AIMS Math.*, **6** (2021), 5675–5694. https://doi.org/10.3934/math.2021335
13. C. Janki, D. Sreeja, A new class of homomorphism in soft topological spaces, *Int. J. Sc. Res.*, **3** (2014), 810–814.