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Research article

Uniform regularity of the isentropic Navier-Stokes-Maxwell system

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Abstract: It is well known that Navier-Stokes-Maxwell system can be derived from the Vlasov-Maxwell-Boltzmann system. In this paper, the uniform regularity of strong solutions to the isentropic compressible Navier-Stokes-Maxwell system are proved. Here our result is obtained by using the bilinear commutator and product estimates.

Keywords: Navier-Stokes-Maxwell; isentropic; uniform regularity
Mathematics Subject Classification: 35B25, 35Q30, 35Q35

1. Introduction

In this paper, we consider the following isentropic compressible Navier-Stokes-Maxwell system [1,2]:

$$\partial_t\rho + \operatorname{div}(\rho u) = 0, \text{ in } \mathbb{T}^3 \times (0,\infty), \tag{1.1}$$

$$\partial_t(\rho u) + \operatorname{div}(\rho u \otimes u) + \nabla p - \mu\Delta u - (\lambda+\mu)\nabla \div u = j \times b, \text{ in } \mathbb{T}^3 \times (0,\infty), \tag{1.2}$$

$$\epsilon\partial_t E - \operatorname{rot} b + j = 0, j := E + u \times b, \text{ in } \mathbb{T}^3 \times (0,\infty), \tag{1.3}$$

$$\partial_t b + \operatorname{rot} E = 0, \text{ in } \mathbb{T}^3 \times (0,\infty), \tag{1.4}$$

$$\operatorname{div} b = 0, \text{ in } \mathbb{T}^3 \times (0,\infty), \tag{1.5}$$

$$(\rho, u, E, b)(\cdot, 0) = (\rho_0, u_0, E_0, b_0)(\cdot), \text{ in } \mathbb{T}^3. \tag{1.6}$$

Here, ρ is the electron density, u is the velocity, E and b represent electronic and magnetic fields respectively. The pressure is $p := a\rho^\gamma$ with constants $a > 0$ and $\gamma > 1$. j is the electric current expressed by Ohm's law. The force term $j \times B$ in the Navier-Stokes equations comes from Lorentz force under a quasi-neutrality assumption of the net charge carried by the fluid. The third equation is the Ampère-Maxwell equation for an electric field E and the fourth equation is Faraday's law. The viscosity coefficients μ and λ of the fluid satisfy $\mu > 0$ and $\lambda + \frac{2\mu}{3} \geq 0$. ϵ is the dielectric constant. The Navier-Stokes-Maxwell system is a plasma physical model that describes the motion of charged particles in electromagnetic field, which can be derived from the Vlasov-Maxwell-Boltzmann system. It includes many classical models. For example, when $j = 0$, (1.1) and (1.2) reduce to the well-known isentropic compressible Navier-Stokes system, Gong-Li-Liu-Zhang [3] and Huang [4] showed the local well-posedness of strong solutions.

When $\inf \rho_0 > 0$, the problem of Navier-Stokes-Maxwell system has attracted much attention. Jiang-Li [5–7] studied the vanishing limit of dielectric constant ϵ_1. Fan-Li-Nakamura [8–10] considered the vanishing limits of dielectric constant ϵ_1 or the Mach number ϵ_2. Chen-Li-Zhang [11] and Mi-Gao [12] established the long-time asymptotic behavior of the smooth solutions.

Before stating our main results, we recall the local existence of smooth solutions to the problem (1.1)–(1.6). Since the system (1.1)–(1.6) is a parabolic-hyperbolic one, the results in [13] imply that

Proposition 1.1. *([13]). Let* $\rho_0, u_0, E_0, b_0 \in H^3$ *and* $\frac{1}{C_0} \leq \rho_0$*, for a positive constant* C_0*. Then the problem* (1.1)–(1.6) *has a unique smooth solution* (ρ, u, E, b) *satisfying*

$$\rho, E, b \in C^\ell([0,T); H^{3-\ell}), u \in C^\ell([0,T); H^{3-2\ell}), \ell = 0,1; \frac{1}{C} \leq \rho, \tag{1.7}$$

for some $0 < T \leq \infty$*.*

The aim of this paper is to prove uniform regularity estimates in (λ, μ, ϵ). We will prove the following

Theorem 1.1. *Let* $0 < \mu < 1, 0 < \lambda + \mu < 1, 0 < \epsilon < 1, 0 < \frac{1}{C_0} \leq \rho_0, \rho_0, u_0, E_0, b_0 \in H^3(\mathbb{T}^3)$*. Let* (ρ, u, E, b) *be the unique local smooth solutions to the problem* (1.1)–(1.6)*. Then*

$$\|(\rho, u, \sqrt{\epsilon}E, b)(\cdot, t)\|_{H^3} \leq C \;\; and \;\; \|E(\cdot, t)\|_{L^2} + \|E\|_{L^2(0,t;H^3)} \leq C \;\; in \;\; [0, T] \tag{1.8}$$

hold true for some positive constants C *and* T_0 $(\leq T)$ *independent of* λ, μ *and* $\epsilon > 0$*.*

We define

$$\begin{aligned} M(t) : \;\; = \;\; & 1 + \sup_{0 \leq \tau \leq t} \Big\{ \|(\rho, u, \sqrt{\epsilon}E, b, p)(\cdot, \tau)\|_{H^3} + \|\partial_t u(\cdot, \tau)\|_{L^2} \\ & + \|E(\cdot, \tau)\|_{L^2} + \left\| \frac{1}{\rho}(\cdot, \tau) \right\|_{L^\infty} \Big\} + \|E_t\|_{L^2(0,t;L^2)}. \end{aligned} \tag{1.9}$$

We can prove

Theorem 1.2. *For any* $t \in [0, 1]$*, it holds that*

$$M(t) \leq C_0(M_0) \exp(t^{\frac{1}{3}} C(M)) \tag{1.10}$$

for some nondecreasing continuous functions $C_0(\cdot)$ *and* $C(\cdot)$*.*

It follows from (1.10) that

$$M(t) \leq C. \tag{1.11}$$

In the following proofs, we will use the bilinear commutator and product estimates due to Kato-Ponce [14]:

$$\|D^s(fg) - fD^s g\|_{L^p} \leq C(\|\nabla f\|_{L^{p_1}}\|D^{s-1}g\|_{L^{q_1}} + \|g\|_{L^{p_2}}\|D^s f\|_{L^{q_2}}), \tag{1.12}$$

$$\|D^s(fg)\|_{L^p} \leq C(\|f\|_{L^{p_1}}\|D^s g\|_{L^{q_1}} + \|D^s f\|_{L^{p_2}}\|g\|_{L^{q_2}}), \tag{1.13}$$

with $s > 0$ and $\dfrac{1}{p} = \dfrac{1}{p_1} + \dfrac{1}{q_1} = \dfrac{1}{p_2} + \dfrac{1}{q_2}$.

We only need to show Theorem 1.2.

2. Proof of Theorem 1.2

First, multiplying (1.1) by ρ^{q-1} and integrating the resulting equation yields

$$\frac{1}{q}\frac{\mathrm{d}}{\mathrm{d}t}\int \rho^q \mathrm{d}x = \left(1 - \frac{1}{q}\right)\int \rho^q \mathrm{div}\, u \mathrm{d}x \leq \|\mathrm{div}\, u\|_{L^\infty}\int \rho^q \mathrm{d}x,$$

from which it follows that

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\rho\|_{L^q} \leq \|\mathrm{div}\, u\|_{L^\infty}\|\rho\|_{L^q},$$

thus, one can have

$$\|\rho\|_{L^q} \leq \|\rho_0\|_{L^q} \exp\left(\int_0^t \|\mathrm{div}\, u\|_{L^\infty} \mathrm{d}\tau\right), \tag{2.1}$$

and

$$\|\rho\|_{L^\infty} \leq \|\rho_0\|_{L^\infty} \exp(tC(M)). \tag{2.2}$$

by taking $q \to +\infty$.

It follows from (1.1) that

$$\partial_t \frac{1}{\rho} + u \cdot \nabla\frac{1}{\rho} - \frac{1}{\rho}\mathrm{div}\, u = 0. \tag{2.3}$$

Multiplying (2.3) by $\left(\frac{1}{\rho}\right)^{q-1}$, and integrating the resulting equation yields that

$$\frac{1}{q}\frac{\mathrm{d}}{\mathrm{d}t}\int \left(\frac{1}{\rho}\right)^q \mathrm{d}x = \left(1 + \frac{1}{q}\right)\int \left(\frac{1}{\rho}\right)^q \mathrm{div}\, u \mathrm{d}x \leq \left(1 + \frac{1}{q}\right)\left\|\frac{1}{\rho}\right\|_{L^q}^q \|\mathrm{div}\, u\|_{L^\infty},$$

or equivalently,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\|\frac{1}{\rho}\right\|_{L^q}^q \leq (1 + q)\left\|\frac{1}{\rho}\right\|_{L^q}^q \|\mathrm{div}\, u\|_{L^\infty},$$

from which it follows that

$$\left\|\frac{1}{\rho}\right\|_{L^q}^q \leq \left\|\frac{1}{\rho_0}\right\|_{L^q}^q \exp\left((1 + q)\int_0^t \|\mathrm{div}\, u\|_{L^\infty} \mathrm{d}\tau\right),$$

and

$$\left\|\frac{1}{\rho}\right\|_{L^\infty} \leq \left\|\frac{1}{\rho_0}\right\|_{L^\infty} \exp(tC(M)) \tag{2.4}$$

by taking $q \to +\infty$. It follows from (2.2) and (2.4) that

$$\|p\|_{L^\infty} + \left\|\frac{1}{p}\right\|_{L^\infty} \leq C_0(M_0)\exp(tC(M)). \tag{2.5}$$

It is easy to verify that

$$\frac{\mathrm{d}}{\mathrm{d}t}\int |u|^2\mathrm{d}x = 2\int u\partial_t u\mathrm{d}x \leq 2\|u\|_{L^2}\|\partial_t u\|_{L^2} \leq C(M),$$

which implies

$$\|u\|_{L^2} \leq C_0(M_0)\exp(tC(M)). \tag{2.6}$$

Multiplying (1.3) by and E, (1.4) by b, integrating with respect to x respectively, and summing up the results, then it follows that

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int(\epsilon|E|^2 + |b|^2)\mathrm{d}x + \int |E|^2\mathrm{d}x = \int(b\times u)E\mathrm{d}x \leq \|u\|_{L^\infty}\|b\|_{L^2}\|E\|_{L^2} \leq C(M),$$

which implies

$$\sqrt{\epsilon}\|E(\cdot,t)\|_{L^2} + \|b(\cdot,t)\|_{L^2} \leq C_0(M_0)\exp(tC(M)). \tag{2.7}$$

Applying D^3 to (1.3) and (1.4), multiplying by D^3E and D^3b, respectively, and summing up the results, one can observe that

$$\begin{aligned}&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int(\epsilon|D^3E|^2 + |D^3b|^2)\mathrm{d}x + \int |D^3E|^2\mathrm{d}x = \int D^3(b\times u)D^3E\mathrm{d}x\\ \leq\ & C\|b\|_{H^3}\|u\|_{H^3}\|D^3E\|_{L^2} \leq C(M) + \frac{1}{2}\|D^3E\|_{L^2}^2,\end{aligned}$$

which yields

$$\sqrt{\epsilon}\|D^3E(\cdot,t)\|_{L^2} + \|D^3b(\cdot,t)\|_{L^2} + \|D^3E\|_{L^2(0,t;L^2)} \leq C_0(M_0)\exp(tC(M)). \tag{2.8}$$

Differentiating (1.3) and (1.4) with respect to t, multiplying by E_t and b_t, respectively, and summing up the results, one can deduce that

$$\begin{aligned}&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int(\epsilon|E_t|^2 + |b_t|^2)\mathrm{d}x + \int |E_t|^2\mathrm{d}x = \int \partial_t(b\times u)\cdot E_t\mathrm{d}x\\ \leq\ & (\|b_t\|_{L^2}\|u\|_{L^\infty} + \|b\|_{L^\infty}\|u_t\|_{L^2})\|E_t\|_{L^2} \leq \frac{1}{2}\|E_t\|_{L^2}^2 + C(M)\|b_t\|_{L^2}^2 + C(M),\end{aligned}$$

which implies

$$\int(\epsilon|E_t|^2 + |b_t|^2)\mathrm{d}x + \int_0^t\int |E_t|^2\mathrm{d}x\mathrm{d}\tau \leq C_0(M_0)\exp(tC(M)). \tag{2.9}$$

It is clear that

$$E = E_0 + \int_0^t E_t\mathrm{d}s$$

and hence

$$\|E(\cdot,t)\|_{L^2} \le C_0(M_0)\exp(\sqrt{t}C(M)). \tag{2.10}$$

It is obvious that

$$\frac{1}{\gamma p}\partial_t p + \frac{1}{\gamma p}u\cdot\nabla p + \operatorname{div} u = 0. \tag{2.11}$$

Similarly, applying D^3 to (2.11), multiplying the equation by $D^3 p$ and integrating with respect to x, then it follows from (2.11), (1.12) and (1.13) that

$$\begin{aligned}
&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int \frac{1}{\gamma p}(D^3 p)^2 \mathrm{d}x + \int D^3 p D^3 \operatorname{div} u \mathrm{d}x \\
= \;& \frac{1}{2}\int (D^3 p)^2\left[\operatorname{div}\left(\frac{u}{\gamma p}\right) - \frac{1}{\gamma p^2}\partial_t p\right]\mathrm{d}x - \int \left(D^3\left(\frac{1}{\gamma p}\partial_t p\right) - \frac{1}{\gamma p}D^3\partial_t p\right)D^3 p \mathrm{d}x \\
& - \int \left(D^3\left(\frac{u}{\gamma p}\cdot\nabla p\right) - \frac{u}{\gamma p}\cdot\nabla D^3 p\right)D^3 p\mathrm{d}x \\
\le \;& C\|D^3 p\|_{L^2}^2 \left\|\operatorname{div}\left(\frac{u}{\gamma p}\right) - \frac{1}{\gamma p^2}\partial_t p\right\|_{L^\infty} \\
& + C\|\partial_t p\|_{L^\infty}\left\|D^3\left(\frac{1}{\gamma p}\right)\right\|_{L^2}\|D^3 p\|_{L^2} + C\left\|\nabla\frac{1}{\gamma p}\right\|_{L^\infty}\|D^2\partial_t p\|_{L^2}\|D^3 p\|_{L^2} \\
& + C\|\nabla p\|_{L^\infty}\left\|D^3\left(\frac{u}{\gamma p}\right)\right\|_{L^2}\|D^3 p\|_{L^2} + C\left\|\nabla\frac{u}{\gamma p}\right\|_{L^\infty}\|D^3 p\|_{L^2}^2 \\
\le \;& C(M) + C(M)\|\partial_t p\|_{L^\infty} + C(M)\|D^2\partial_t p\|_{L^2} \\
\le \;& C(M) + C(M)\|u\cdot\nabla p + \gamma p \operatorname{div} u\|_{L^\infty} + C(M)\|D^2(u\cdot\nabla p + \gamma p\operatorname{div} u)\|_{L^2} \\
\le \;& C(M),
\end{aligned} \tag{2.12}$$

where we have used the estimate [15]:

$$\left\|D^3\frac{1}{p}\right\|_{L^2} \le C(M)\|D^3 p\|_{L^2} \le C(M). \tag{2.13}$$

It is obvious that

$$\int_0^t\int |\partial_t u|^2 \mathrm{d}x\mathrm{d}\tau \le t\sup\int |\partial_t u|^2\mathrm{d}x \le tC(M). \tag{2.14}$$

Applying D^2 to (1.2), multiplying the resulting equality by $D^2\partial_t u$ and integrating with respect to x, by (1.12)–(1.13) and some direct calculations, we obtain

$$\begin{aligned}
&\frac{\mu}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int |D^3 u|^2\mathrm{d}x + \frac{\lambda+\mu}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int (D^2\operatorname{div} u)^2\mathrm{d}x + \int \rho|D^2\partial_t u|^2\mathrm{d}x \\
= \;& -\int D^2\nabla p\cdot D^2\partial_t u\mathrm{d}x - \int D^2(\rho u\cdot\nabla u)\cdot D^2\partial_t u\mathrm{d}x - \int [D^2(\rho\partial_t u) - \rho D^2\partial_t u]D^2\partial_t u\mathrm{d}x \\
& + \int D^2(j\times b)D^2\partial_t u\mathrm{d}x \\
\le \;& C\|D^3 p\|_{L^2}\|D^2\partial_t u\|_{L^2} + C\|\rho\|_{H^2}\|u\|_{H^3}^2\|D^2\partial_t u\|_{L^2}
\end{aligned}$$

$$
\begin{aligned}
& +C(\|\nabla\rho\|_{L^\infty}\|D\partial_t u\|_{L^2} + \|\partial_t u\|_{L^\infty}\|D^2\rho\|_{L^2})\|D^2\partial_t u\|_{L^2} + \|D^2(j\times b)\|_{L^2}\|D^2\partial_t u\|_{L^2} \\
\le\ & C(M)\|D^2\partial_t u\|_{L^2} + C(M)(\|D\partial_t u\|_{L^2} + \|\partial_t u\|_{L^\infty})\|D^2\partial_t u\|_{L^2} + C(M)\|D^2E\|_{L^2}\|D^2\partial_t u\|_{L^2} \\
\le\ & C(M)\|D^2\partial_t u\|_{L^2} + C(M)(\|\partial_t u\|_{L^2}^{\frac12}\|D^2\partial_t u\|_{L^2}^{\frac12} + \|\partial_t u\|_{L^2} + \|\partial_t u\|_{L^2}^{\frac14}\|D^2\partial_t u\|_{L^2}^{\frac34})\|D^2\partial_t u\|_{L^2} \\
& +C(M)\|E\|_{L^2}^{\frac13}\|D^3E\|_{L^2}^{\frac23}\|D^2\partial_t u\|_{L^2} \\
\le\ & C(M)\|D^2\partial_t u\|_{L^2} + C(M)(\|D^2\partial_t u\|_{L^2}^{\frac12} + \|D^2\partial_t u\|_{L^2}^{\frac34})\|D^2\partial_t u\|_{L^2} + C(M)\|D^3E\|_{L^2}^{\frac23}\|D^2\partial_t u\|_{L^2} \\
\le\ & \frac12\int\rho|D^2\partial_t u|^2\mathrm{d}x + C(M) + C(M)\|D^3E\|_{L^2}^{\frac43},
\end{aligned}
$$

which gives rises to

$$
\int_0^t\int|D^2\partial_t u|^2\mathrm{d}x\mathrm{d}\tau \le C_0(M_0)\exp(t^{\frac13}C(M)). \tag{2.15}
$$

Applying D^3 to (1.2), multiplying the resulting equation by D^3u and integrating with respect to x, and it follows from (1.1), (1.12) and (1.13) that

$$
\begin{aligned}
& \frac12\frac{\mathrm{d}}{\mathrm{d}t}\int\rho|D^3u|^2\mathrm{d}x + \mu\int|D^4u|^2\mathrm{d}x + (\lambda+\mu)\int(D^3\mathrm{div}\,u)^2\mathrm{d}x + \int D^3\nabla p\cdot D^3u\mathrm{d}x \\
=\ & -\int(D^3(\rho\partial_t u) - \rho D^3\partial_t u)D^3u\mathrm{d}x - \int(D^3(\rho u\cdot\nabla u) - \rho u\cdot\nabla D^3u)D^3u\mathrm{d}x \\
& +\int D^3(j\times b)D^3u\mathrm{d}x \\
\le\ & C(\|\nabla\rho\|_{L^\infty}\|D^2\partial_t u\|_{L^2} + \|\partial_t u\|_{L^\infty}\|D^3\rho\|_{L^2})\|D^3u\|_{L^2} \\
& +C(\|\nabla u\|_{L^\infty}\|D^3(\rho u)\|_{L^2} + \|\nabla(\rho u)\|_{L^\infty}\|D^3u\|_{L^2})\|D^3u\|_{L^2} + \|D^3(j\times b)\|_{L^2}\|D^3u\|_{L^2} \\
\le\ & C(M) + C(M)(\|D^2\partial_t u\|_{L^2} + \|\partial_t u\|_{L^\infty}) + C(M)\|D^3E\|_{L^2} \\
\le\ & C(M) + \|D^2\partial_t u\|_{L^2}^2 + C(M)\|D^3E\|_{L^2}.
\end{aligned} \tag{2.16}
$$

Summing (2.12) and (2.16) up, one can deduce that

$$
\begin{aligned}
& \frac12\frac{\mathrm{d}}{\mathrm{d}t}\int\left(\frac{1}{\gamma p}(D^3p)^2 + \rho|D^3u|^2\right)\mathrm{d}x + \mu\int|D^4u|^2\mathrm{d}x + (\lambda+\mu)\int(D^3\mathrm{div}\,u)^2\mathrm{d}x \\
& +\int(D^3pD^3\mathrm{div}\,u + D^3\nabla p\cdot D^3u)\mathrm{d}x \\
\le\ & C(M) + \|D^2\partial_t u\|_{L^2}^2 + C(M)\|D^3E\|_{L^2}.
\end{aligned} \tag{2.17}
$$

Noting that the last term of LHS of (2.17) is zero, it follows from (2.15) that

$$
\|D^3p\|_{L^2} + \|D^3u\|_{L^2} \le C_0(M_0)\exp(t^{\frac13}C(M)). \tag{2.18}
$$

On the other hand, it follows from (1.2) that

$$\begin{aligned}\|\partial_t u\|_{L^2} &= \left\|\frac{1}{\rho}(j\times b+\mu\Delta u+(\lambda+\mu)\nabla\mathrm{div}\, u-\nabla p-\rho u\cdot\nabla u)\right\|_{L^2}\\ &\le C_0(M_0)\exp(t^{\frac{1}{3}}C(M)).\end{aligned}\tag{2.19}$$

By the aid of the following estimate [15]:

$$\|D^3\rho\|_{L^2}\le C(1+\|p\|_{L^\infty})^3\|f\|_{W^{3,\infty}(I)}\|D^3p\|_{L^2}\tag{2.20}$$

with $\rho=f(p):=\left(\frac{p}{a}\right)^{\frac{1}{\gamma}}$, and

$$I\subset\left(\frac{1}{C_0(M_0)}\exp(-tC(M)),C_0(M_0)\exp(tC(M))\right),$$

we have

$$\|D^3\rho\|_{L^2}\le C_0(M_0)\exp(t^{\frac{1}{3}}C(M)).\tag{2.21}$$

Combining (2.4)–(2.10), (2.18), (2.19) with (2.21), we conclude that (1.10) holds true.
This completes the proof. □

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Conflict of interest

We declare that we have no conflict of interest.

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