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Research article

A new procedure for unit root to long-memory process change-point monitoring

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Abstract: In this paper, we propose a Dickey-Fuller difference statistic to sequentially detect the change-point that shift from an unit root process to a long-memory process. The limiting distribution of monitoring statistic under the unit root process null hypothesis as well as its consistency under the alternative hypothesis are proved. Simulations indicate that the new method can control the empirical size well even for the heavy-tailed unit root process when using the sieve bootstrap method computing its critical values. In particular, it performs significantly better than the available method in the literature under the alternative hypothesis. Finally, we illustrate the new monitoring procedure by a set of foreign exchange rate data.

Keywords: change-point monitoring; Dickey-Fuller statistic; long-memory process; unit root process
Mathematics Subject Classification: 62F03, 62L10

1. Introduction

Since the seminal work of [15], detecting change-point that shifts between the short memory ($I(0)$) process and the unit root ($I(1)$) process becomes a popular topic in economics and statistics. A number of testing procedures have been studied to distinguish this classical framework. For surveys, we refer the reader to [1,3,10,14,18,20] among many others. Because the $I(0)$ and $I(1)$ process can be regarded as a special case of the more flexible long-memory (I(d), $0 \leq d < 3/2$) process, detecting change-point in the $I(d)$ process has also received considerable attention in the past decades. Hassler and Scheithauer [11] applied the ratio tests and the LBI tests of [1] to detect the change-point that shifts from $I(0)$ to $I(d)$ process. Sibbertsen and Kruse [21], Hassler and Meller [12], Caporin and Gupta [2] studied the change in persistence in $I(d)$ process. Lavancier et al. [17] and Iacone and Lazarová [13] studied the long-memory parameter change-point detection problem.

All of the works above are offline tests, i.e., detecting the change-point in a fixed historical sample. As many economic and financial data arrive steadily and cheaply, it is desirable to know whether a newly arrived data can be described by the current model or indicate that a change in the stochastic structure has taken place. This leads to the development of online tests, which is also an important issue in change-point analysis. We refer the reader to [6, 8, 9, 16, 19, 23] for the most recent works about this topic. However, the online monitoring study about the change-point in the $I(d)$ process are very limited. Chen et al. [4] applied a variance ratio statistic to monitor change-point that shifts from $I(0)$ to $I(d)$ process. Chen et al. [5] extended the variance ratio statistic to persistent change-point monitoring problem in the $I(d)$ process. Chen et al. [7] proposed a two stage moving ratio statistic to monitor long-memory parameter change-points.

In this paper, we propose a DF difference statistic to monitor $I(1)$ to $I(d)$ process change-point. Although this type of change-point can be regarded as a special case of the long-memory parameter change-point of [6], studying this problem has its own interesting for the unit root process plays an important role in economics. We will show that the DF difference statistic has significantly higher empirical power and shorter average run length (ARL) than the two stage moving ratio statistic in this special case. Moreover, the new proposed monitoring procedure still work for change-point that shifts from heavy-tailed unit root process to heavy-tailed long-memory process, while [7] only considered light tail case. Detecting change-point in a heavy-tailed long-memory process also is an important issue for many economic and finical data are heavy-tail.

The rest of the paper is organized as follows. Section 2 shows the model and the proposed new monitoring statistic. The limiting results and their proofs are gathered in Section 3. In Section 4, we check the finite sample performances of new monitoring procedure via simulation. Section 5 concludes the paper.

2. Model and monitoring statistic

We consider the following time series model:

$$Y_t = \mu_t + \varepsilon_t,\ t = 1, 2, \ldots, \tag{2.1}$$

where $\mu_t = E(Y_t)$ is a deterministic component, and the random component ε_t is a long-memory process written as $\varepsilon_t \sim I(d)$, in which $d \in (0, 0.5) \cup (0.5, 1.5)$ is unknown long-memory parameter. More specifically, ε_t are stationary and nonstationary long-memory process respectively, if $0 < d < 0.5$ and $0.5 < d < 1.5$ respectively, and it becomes an unit root process if $d = 1$. We do not give any assumption about the structure of ε_t but assume its innovation process $\{u_t\}$ has zero mean and finite moment of order $q > 2$.

Suppose we have observed the first m samples $Y_1, Y_2, \ldots, Y_m$ that follows the model (2.1) with $d = 1$, namely, they are unit root process. From the $(m+1)$th new observations $Y_{m+1}, Y_{m+2}, \cdots$, we sequentially detect whether the new observed samples still follows an unit root process or there occurs an unit root to long-memory process change-point. The test hypotheses are

$$H_0 : Y_t \sim I(1),\ t = m + 1, \ldots, T,$$

vs.

$$H_1 : \qquad Y_t \sim I(1),\ t = m + 1, \ldots, k^*,$$

$$Y_t \sim I(d),\ t = k^* + 1, \ldots, T, \text{with } 0 < d < 1, d \neq 0.5,$$

where $k^* > m$ denotes the unknown change-point, m denotes the training sample size, and T denotes the prespecified largest monitoring sample size.

According to [20], we realize that the DF ratio statistic is a powerful statistic to test $I(1)$ to $I(0)$ change-point. A nature idea is whether the DF ratio statistic can be extended to monitor this type of change-point? Unfortunately, a lot of unreported simulations indicate that this does not work. Luckily, we find the following statistic:

$$G_m(n) = T\left|\hat{\rho}_m(m) - \tilde{\rho}_m(n)\right|, \quad m < n \leq T, \tag{2.2}$$

where n denotes the sample size of currently available full observations, and

$$\hat{\rho}_m(m) = \frac{\sum_{t=1}^{m} \hat{\varepsilon}_{t-1}\hat{\varepsilon}_t}{\sum_{t=1}^{m} \hat{\varepsilon}_{t-1}^2},$$

$$\tilde{\rho}_m(n) = \frac{\sum_{t=n-m+1}^{n} \tilde{\varepsilon}_{t-1}\tilde{\varepsilon}_t}{\sum_{t=n-m+1}^{n} \tilde{\varepsilon}_{t-1}^2},$$

in which $\hat{\varepsilon}_t = Y_t - \hat{\mu}_t$ with $\hat{\mu}_t$ denotes the OLS estimator of μ_t in the model (2.1) based on the historical samples $Y_1, \ldots, Y_m$, and $\tilde{\varepsilon}_t = Y_t - \tilde{\mu}_t$ with $\tilde{\mu}_t$ denotes the OLS estimator of μ_t in the model (2.1) based on the newest m samples $Y_{n-m+1}, \ldots, Y_n$. Because the statistic $G_m(n)$ can be considered as a difference of two DF statistics, we call it DF difference statistic.

3. Main results

In this section we derive the limiting distribution of monitoring statistic $G_m(n)$. To save space, we only consider the deterministic component $\mu_t = \beta_0 + \beta_1 t$ case in the model (2.1), that is, the observed time series Y_t is an $I(d)$ process with time trend. This together with the constant mean ($\beta_1 \equiv 0$) case are the most popular studied deterministic components in the $I(d)$ process. Here, β_0 and β_1 are unknown parameters.

Theorem 3.1. *Let $m = [T\tau], n = [Ts], 0 < \tau \leq s < 1$, and Y_t was produced by model (2.1) with $\mu_t = \beta_0 + \beta_1 t$, then under the null hypothesis H_0, as $m \to \infty$, we have*

$$G_m(n) \Rightarrow \frac{1}{2}\left|\frac{\xi(\tau,\tau)^2 + \tau}{\int_0^\tau \xi(r,\tau)^2 dr} - \frac{\xi(s,\tau)^2 - \xi(s-\tau,\tau)^2 + \tau}{\int_{s-\tau}^s \xi(r,\tau)^2 dr}\right|,$$

where $\Rightarrow$ denotes the weak convergence, and $\xi(s,\tau) = W(s) - sD(s,\tau)$ with $W(s)$ denotes the standard Wiener process, and

$$D(s,\tau) = 12\tau^{-4}\left\{\frac{s^2 + (s-\tau)^2}{2}\int_{s-\tau}^s W(r)dr - s\int_{s-\tau}^s \int_{s-\tau}^r W(v)dvdr\right\}.$$

Proof. In order to easily prove the consistency of statistic $G_m(n)$, which will be showed in Theorem 3.2, we derive the null distribution of statistic $G_m(n)$ under the case of $0.5 < d < 1.5$. According to the OLS estimation, if $\mu_t = \beta_0 + \beta_1 t$, we have

$$\tilde{\varepsilon}_t = Y_t - \tilde{\beta}_0 - \tilde{\beta}_1 t, \quad t = n-m+1, \ldots, n,$$

where

$$\begin{aligned}
\tilde{\beta}_0 &= M^{-1}\left\{\left(\sum_{t=n-m+1}^{n} t^2\right)\left(\sum_{t=n-m+1}^{n} Y_t\right) - \left(\sum_{t=n-m+1}^{n} tY_t\right)\left(\sum_{t=n-m+1}^{n} t\right)\right\},\\
\tilde{\beta}_1 &= M^{-1}\left\{m\sum_{t=n-m+1}^{n} tY_t - \left(\sum_{t=n-m+1}^{n} t\right)\left(\sum_{t=n-m+1}^{n} Y_t)\right)\right\},\\
&= \beta_1 + M^{-1}\left\{m\sum_{t=n-m+1}^{n} t\varepsilon_t - \left(\sum_{t=n-m+1}^{n} t\right)\left(\sum_{t=n-m+1}^{n} \varepsilon_t\right)\right\}\\
M &= m\sum_{t=n-m+1}^{n} t^2 - \left(\sum_{t=n-m+1}^{n} t\right)^2.
\end{aligned}$$

Let $e_t = \varepsilon_t - \varepsilon_{t-1}$. If $\varepsilon_t \sim I(d), 0.5 < d < 1.5$, then $e_t \sim I(d-1)$ is a stationary long-memory process. Since $m \to \infty$ implies $T \to \infty$, according to the ergodic theory of stationary process and [22], as $m \to \infty$, we have

$$\begin{aligned}
T^{\frac{1}{2}-d}\varepsilon_{[T\tau]} &= T^{\frac{1}{2}-d}\sum_{t=1}^{[T\tau]} e_t \Rightarrow \omega W_{d-1}(\tau),\\
\frac{1}{T}\sum_{t=1}^{m} e_t^2 &= \frac{[T\tau]}{T}\frac{1}{[T\tau]}\sum_{t=1}^{[T\tau]} e_t^2 \xrightarrow{p} \tau E(e_t^2) = \tau\omega^2.
\end{aligned}$$

$$\begin{aligned}
T^{-3}\sum_{t=n-m+1}^{n} t^2 &\longrightarrow \frac{s^3 - (s-\tau)^3}{3},\\
T^{-2}\sum_{t=n-m+1}^{n} t &\longrightarrow \frac{s^2 - (s-\tau)^2}{2},
\end{aligned}$$

where ω^2 denotes the long-run variance of $\{e_t\}$, $W_{d-1}(\tau)$ denotes the type I fractional Brownian motion with long-memory parameter $d-1$, and $W_0(\tau) \equiv W(\tau)$ becomes a standard Wiener process.

$$\begin{aligned}
T^{-\frac{1}{2}-d}\sum_{t=n-m+1}^{n} \varepsilon_t &= T^{-1}\sum_{t=[Ts]-[T\tau]+1}^{[Ts]} T^{\frac{1}{2}-d_0}\sum_{i=1}^{t} e_i \Rightarrow \omega\int_{s-\tau}^{s} W_{d-1}(r)dr,\\
T^{-\frac{3}{2}-d}\sum_{t=n-m+1}^{n} t\varepsilon_t &= \frac{[Ts]+1}{T^{\frac{3}{2}+d}}\sum_{t=[Ts]-[T\tau]+1}^{[Ts]} \varepsilon_t - T^{-\frac{3}{2}-d}\sum_{t=[Ts]-[T\tau]+1}^{[Ts]}\sum_{j=[Ts]-[T\tau]+1}^{t} \varepsilon_j
\end{aligned}$$

$$\Rightarrow \quad \omega\left\{s\int_{s-\tau}^{s} W_{d-1}(r)dr - \int_{s-\tau}^{s}\int_{s-\tau}^{r} W_{d-1}(v)dvdr\right\}.$$

Thus,

$$\begin{aligned} T^{\frac{3}{2}-d}\left(\tilde{\beta}_1-\beta_1\right) &= (T^{-4}M)^{-1}\left\{\frac{m}{T^{\frac{5}{2}+d}}\sum_{t=n-m+1}^{n} t\varepsilon_t - (T^{-2}\sum_{t=n-m+1}^{n} t)T^{-\frac{1}{2}-d}\sum_{t=n-m+1}^{n}\varepsilon_t\right\} \\ &\Rightarrow 12\omega\tau^{-4}\left\{\frac{s^2+(s-\tau)^2}{2}\int_{s-\tau}^{s} W_{d-1}(r)dr - s\int_{s-\tau}^{s}\int_{s-\tau}^{r} W_{d-1}(v)dvdr\right\} \\ &:= \omega D_{d-1}(s,\tau), \end{aligned}$$

with

$$D_{d-1}(s,\tau) = 12\tau^{-4}\left\{\frac{s^2+(s-\tau)^2}{2}\int_{s-\tau}^{s} W_{d-1}(r)dr - s\int_{s-\tau}^{s}\int_{s-\tau}^{r} W_{d-1}(v)dvdr\right\}.$$

Let

$$\begin{aligned} z_t &= \tilde{\varepsilon}_t - \tilde{\varepsilon}_{t-1} \\ &= Y_t - \tilde{\beta}_0 - \tilde{\beta}_1 t - Y_{t-1} + \tilde{\beta}_0 + \tilde{\beta}_1(t-1) \\ &= e_t + (\tilde{\beta}_1 - \beta_1) \end{aligned}$$

then,

$$T^{\frac{1}{2}-d}\sum_{t=1}^{[Ts]} z_t \quad \Rightarrow \quad \omega W_{d-1}(s) - \omega s D_{d-1}(s,\tau).$$

From $\tilde{\varepsilon}_t^2 = (\tilde{\varepsilon}_{t-1}+z_t)^2 = \tilde{\varepsilon}_{t-1}^2 + 2\tilde{\varepsilon}_{t-1}z_t + z_t^2$, we have

$$\sum_{t=n-m+1}^{n}\tilde{\varepsilon}_{t-1}z_t = \frac{1}{2}\tilde{\varepsilon}_n^2 - \frac{1}{2}\tilde{\varepsilon}_{n-m}^2 - \frac{1}{2}\sum_{t=n-m+1}^{n} z_t^2.$$

Hence,

$$\begin{aligned} &T(\tilde{\rho}_m(n)-1) \\ &= \frac{T^{1-2d}\sum\limits_{t=n-m+1}^{n}\tilde{\varepsilon}_{t-1}z_t}{T^{-2d}\sum\limits_{t=n-m+1}^{n}\tilde{\varepsilon}_{t-1}^2} \\ &= \frac{\left(T^{\frac{1}{2}-d}\sum\limits_{j=1}^{n} z_j\right)^2 - \left(T^{\frac{1}{2}-d}\sum\limits_{j=1}^{n-m} z_j\right)^2 - T^{1-2d}\sum\limits_{t=n-m+1}^{n} z_t^2}{2T^{-1}\sum\limits_{t=n-m+1}^{n}\left(T^{\frac{1}{2}-d}\sum_{j=1}^{t-1} z_j\right)^2} \\ &\Rightarrow \frac{(W_{d-1}(s)-sD_{d-1}(s,\tau))^2 - (W_{d-1}(s-\tau)-(s-\tau)D_{d-1}(s,\tau))^2 + \tau\tilde{O}_p(T^{2-2d})}{2\int_{s-\tau}^{s} W_{d-1}(r) - rD_{d-1}(s,\tau)^2 dr}, \end{aligned}$$

(3.1)

where $\tilde{O}_p(T^{2-2d})$ denotes $\lim_{T\to\infty} P(\tilde{O}_p(T^{2-2d})/T^{2-2d} = 1) = 1$.

Similar argument gives that

$$T(\hat{\rho}_m(m) - 1) \quad \Rightarrow \quad \frac{(W_{d-1}(\tau) - \tau D_{d-1}(\tau,\tau))^2 + \tau \tilde{O}_p(T^{2-2d})}{2\int_0^\tau W_{d-1}(r) - rD_{d-1}(\tau,\tau)^2 dr}. \tag{3.2}$$

Combining (3.1), (3.2) and $d = 1$, we complete the proof of Theorem 3.1.

Theorem 3.2. *If the time series $\{Y_t\}$ in the model (2.1) occurs a change-point that shifts from an unit root process to a long-memory process at $k^*, m < k^* < n$, then as $m \to \infty$, we have*

$$G_m(n) = \begin{cases} O_p(1), & if\ 1 \le d < 1.5, \\ O_p(T^{2-2d}), & if\ d < 1. \end{cases}.$$

Proof. We continue use the notations in the proof of Theorem 3.1. Since $k^* > m$, the time series $Y_1, \ldots, Y_m$ are unit root process. This implies that the result (3.2) in the proof of Theorem 3.1 is still hold, that is $T(\hat{\rho}_m - 1) = O_p(1)$. According to the ergodic theory of stationary process, we have

$$T^{-1} \sum_{t=n-m+1}^{n} z_t^2 \xrightarrow{p} \tau\omega^2,$$

this together with (3.1) give that

$$T(\tilde{\rho}_m(n) - 1) = \begin{cases} O_p(1), & if\ 1 \le d < 1.5, \\ O_p(T^{2-2d}), & if\ d < 1. \end{cases}$$

This completes the proof of Theorem 3.2.

Theorem 3.2 indicates that the DF difference statistic $G_m(n)$ will diverge to infinity under the alternative hypothesis H_1 when $n > k^*$. Thus, for a given critical value c at the nominal level α, we can say that there occurs a change-point that shifts from $I(1)$ to $I(d), d < 1$ process if $G_m(n) > c$. In addition, since the two stage moving ratio statistic $\Xi_T(n)$ of [7] will diverge to infinity only when $n > m + k^*$, we guess that the statistic $G_m(n)$ has shorter delay time than the statistic $\Xi_T(n)$. However, we can see that this statistic is not consistent for those change-points that shift from $I(1)$ to $I(d), 1 < d < 1.5$ process. In order to monitor this type of change-point, we can make first order difference on the observed data. In the differenced data, the original change-point becomes a short- to long-memory process. And then we can use the variance ratio statistic of [5] to monitor this change-point.

Finally, we discuss how to determine the critical value of DF difference statistic $G_m(n)$. No doubt, its critical value will heavily depend on the historical sample size m and the largest monitoring sample size T. The traditional way is to construct an approximate curve of critical values which are obtained via direct simulation based on all of the necessary values of m and T. Obviously, it is hard work to provide approximate curves for all possible m and T. To overcome this drawback, we recommend using the sieve bootstrap method proposed by [7] to calculate the critical values of monitoring statistic $G_m(n)$ based on the historical samples $Y_1, \cdots, Y_m$. Simulations in the next section will show that the sieve bootstrap method performs well even if the moment condition $q > 2$ of innovation process is not satisfied. This is another reason why we recommend using the sieve bootstrap method to calculate the critical value of statistic $G_m(n)$.

4. Monte Carlo simulation and empirical application

4.1. Monte Carlo simulation

In this section, we evaluate the finite sample performance of new proposed DF difference monitoring statistic $G_m(n)$, and compare it with the two stage moving ratio statistic $\Xi_T(n)$ of [7]. Since many economic and financial data are heavy-tailed, checking whether the statistic $G_m(n)$ is still work in this case is an interesting question. So, we use the following data-generating process (DGP) to generate the simulation data.

$$y_t = \begin{cases} r_0 + r_1 t + \varepsilon 1_t, & t = 1, \ldots, k^*, \\ r_0 + r_1 t + \varepsilon 1_{[Tk^*]} + \varepsilon 2_t, & t = k^* + 1, \ldots, T, \end{cases} \tag{4.1}$$

where $\varepsilon 1_t$ follows a FARIMA(0,1,0) model, and $\varepsilon 2_t$ follows a FARIMA(0,d,0) model with d varying among $\{0, 0.2, 0.4, 0.6\}$. We assume the innovation process in these two FARIMA process follows a stable distribution with tail index κ. We assume the change-point location $k^* = T$ under the null hypothesis, and k^* varying among $\{[0.25T], [0.5T], [0.75T]\}$ under the alternative hypothesis. We set the tail index κ varying among $\{2, 1.8, 1.5, 1.2\}$, and the largest monitoring sample size T varying among $\{200, 400, 1000\}$. Throughout this section, we fix the sieve bootstrap frequency $B = 300$, the training sample size $m = 0.25T$, and all simulations are obtained by 1000 replications at $\alpha = 5\%$ nominal level.

Table 1 reports the empirical sizes of two monitoring statistics. We can see that the empirical sizes of statistic $G_m(n)$ near to the test level in all cases. The effects of tail index κ and largest monitoring sample size T on the empirical size have no significant regularity. The statistic $\Xi_T(n)$ shows obvious size distortion when $T = 200$, but it decreases quickly as T increases. Note that for DGP (4.1), the limiting results derived in the Theorems 3.1 and 3.2 only hold if $\kappa = 2$, we can still obtain nice result for $\kappa < 2$ cases mainly resort to the sieve bootstrap. Because it is a data deriving method. We say that the sieve bootstrap method proposed by [7] is a feasible way to approximate the critical values of two statistics even for heavy-tailed unit root process. In addition, we guess that the statistic $G_m(n)$ still converge to some unknown distribution even if the moment q of innovation u_t in the model (2.1) smaller than 2. However, this is still an open problem.

Table 1. Empirical sizes of monitoring statistics $G_m(n)$ and $\Xi_T(n)$ at 5% nominal level.

	$G_m(n)$				$\Xi_T(n)$			
$T\backslash\kappa$	2	1.8	1.5	1.2	2	1.8	1.5	1.2
200	0.059	0.046	0.035	0.022	0.129	0.096	0.084	0.080
400	0.050	0.069	0.075	0.071	0.078	0.068	0.053	0.061
1000	0.048	0.050	0.048	0.051	0.057	0.059	0.047	0.055

Tables 2 and 3 report the empirical powers and ARLs of two statistics, respectively. To save space, we only show the results for $T = 200, 400$. Three conclusions can be conclude from these two tables. First, the DF difference monitoring statistic $G_m(n)$ has higher empirical power and shorter ARL than the statistic $\Xi_T(n)$ in the most cases. This indicates that the newly proposed statistic have a significant advantage when monitoring unit root to long-memory process change-point. Second, the empirical power increases as T, k^* or change size increases. This verifies the consistency, which have been proved in the Theorem 3.2, of statistic $G_m(n)$. Third, a smaller tail index reflects a smaller empirical power and larger ARL, in general. This is a conceivable conclusion, since the change-point in a heavier tailed data is less likely to be detected.

Table 2. Empirical powers of monitoring statistics $G_m(n)$ and $\Xi_T(n)$ at 5% nominal level.

			$G_m(n)$				$\Xi_T(n)$			
T	k^*	$d\backslash\kappa$	2	1.8	1.5	1.2	2	1.8	1.5	1.2
200	0.25	0	0.997	0.989	0.920	0.623	0.522	0.357	0.211	0.137
		0.2	0.991	0.930	0.699	0.274	0.415	0.275	0.161	0.107
		0.4	0.919	0.692	0.351	0.115	0.294	0.191	0.116	0.087
		0.6	0.680	0.296	0.192	0.076	0.206	0.135	0.092	0.076
	0.5	0	0.996	0.970	0.886	0.480	0.702	0.567	0.386	0.257
		0.2	0.968	0.869	0.555	0.174	0.601	0.470	0.298	0.204
		0.4	0.872	0.553	0.241	0.071	0.461	0.340	0.225	0.160
		0.6	0.552	0.212	0.141	0.056	0.347	0.231	0.151	0.119
	0.75	0	0.969	0.881	0.652	0.214	0.664	0.553	0.367	0.245
		0.2	0.893	0.627	0.247	0.068	0.554	0.433	0.281	0.207
		0.4	0.644	0.258	0.092	0.042	0.421	0.315	0.215	0.159
		0.6	0.307	0.086	0.053	0.031	0.290	0.211	0.144	0.126
400	0.25	0	1.000	1.000	0.985	0.949	0.846	0.708	0.442	0.281
		0.2	1.000	0.997	0.889	0.414	0.667	0.545	0.292	0.198
		0.4	1.000	0.956	0.315	0.156	0.446	0.341	0.189	0.140
		0.6	0.968	0.459	0.144	0.103	0.238	0.169	0.117	0.090
	0.5	0	1.000	1.000	0.982	0.921	0.944	0.848	0.610	0.445
		0.2	1.000	0.996	0.781	0.308	0.815	0.694	0.458	0.317
		0.4	0.997	0.897	0.244	0.133	0.624	0.515	0.313	0.225
		0.6	0.917	0.344	0.125	0.096	0.386	0.292	0.191	0.139
	0.75	0	1.000	0.999	0.927	0.748	0.893	0.785	0.556	0.373
		0.2	1.000	0.978	0.456	0.177	0.761	0.623	0.408	0.273
		0.4	0.982	0.648	0.152	0.104	0.560	0.440	0.264	0.189
		0.6	0.705	0.157	0.102	0.091	0.313	0.255	0.156	0.126

4.2. Empirical application

In this section, we illustrate our proposed DF difference monitoring procedure by a set of foreign exchange rate data between the RMB and the U.S. dollars from May 1 in 2009 to February 17 in 2010. The data are download from the U.S. Federal Reserve Bank official website. Figure 1 shows the raw data of a total of 200 exchanges rates. Under the same parametric assumptions as in the previous simulation section, we find that the monitoring procedure stops at the 155th observation. This indicates that there occurs an unit root to long-memory process change-point. This result is coincide with the results of [20], who have studied this data set via the DF ratio offline test and found that it exits an unit root to short memory process change-point. In fact, we can estimate an I(1) to I(0) change point at the 125th observation via the ratio estimator of [15].

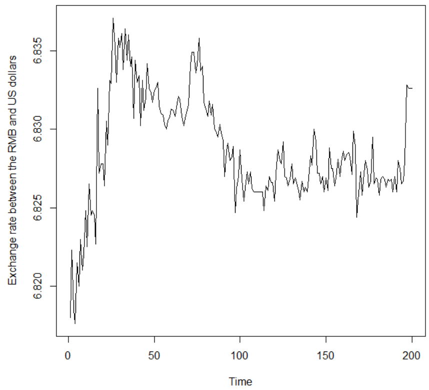

Figure 1. Foreign exchange rate data between the RMB and the U.S. dollars from May 1 in 2009 to February 17 in 2010.

Table 3. ARLs of monitoring statistics $G_m(n)$ and $\Xi_T(n)$ at 5% nominal level.

			$G_m(n)$				$\Xi_T(n)$			
T	k^*	$d\backslash\kappa$	2	1.8	1.5	1.2	2	1.8	1.5	1.2
200	0.25	0	36.0	43.2	53.3	78.4	87.5	94.4	92.8	82.1
		0.2	41.0	55.4	74.3	86.7	98.7	102	96.3	85.8
		0.4	52.0	74.7	85.2	87.3	104	103	99.6	86.8
		0.6	74.3	84.1	91.3	86.8	109	104	103	93.6
	0.5	0	34.6	40.2	46.8	58.6	52.6	54.4	57.4	55.7
		0.2	38.4	47.4	58.2	58.7	55.9	56.8	58.3	55.3
		0.4	45.9	57.0	57.4	56.3	57.9	58.4	59.3	55.4
		0.6	54.5	54.0	58.0	53.2	61.6	60.6	57.9	54.2
	0.75	0	31.3	35.4	37.3	34.4	35.8	36.1	34.5	28.3
		0.2	33.3	35.7	33.3	29.8	34.4	34.1	31.3	26.2
		0.4	33.4	29.7	28.5	28.8	32.8	32.1	28.6	22.9
		0.6	29.5	28.1	27.8	26.1	28.7	27.0	23.3	17.6
400	0.25	0	62.2	75.2	86.1	100	136	149	166	158
		0.2	67.1	80.9	127	158	162	171	175	161
		0.4	74.4	111	151	149	193	194	181	168
		0.6	101	156	136	147	214	205	189	167
	0.5	0	60.3	70.8	81.2	92.3	95.8	99.8	108	109
		0.2	64.6	76.8	106	108	101	106	113	108
		0.4	71.0	95.9	97.7	86.4	113	115	117	110
		0.6	88.0	106	70.0	72.8	123	121	120	100
	0.75	0	55.7	64.8	71.0	72.2	79.0	79.3	78.7	70.0
		0.2	59.7	69.2	63.6	63.3	77.7	77.3	75.6	63.1
		0.4	65.6	68.3	63.7	61.0	75.7	74.5	69.0	53.8
		0.6	68.2	65.2	58.8	57.7	69.7	67.6	58.9	42.0

5. Conclusions

It is more difficult to monitor a decreasing memory parameter change-point than monitor an increasing memory parameter change-point in a long-memory process. In this paper, we have proposed a DF difference statistic to sequentially detect unit root to long-memory process change-point. We showed that for this special type of decreasing memory parameter change-point, the new monitoring procedure can significantly improve the test power and shorten the delay time. Furthermore, the derived null distribution only holds if the innovation of unit root process has finite moment of order $q > 2$, but simulations indicate that the new monitoring procedure can also be used to monitor changes from heavy-tailed unit root to long memory process. While detecting change-point in heavy-tailed long-memory process also is an important issue for many economic and financial time series are heavy-tail, the corresponding asymptotic properties need for further study.

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Conflict of interest

The authors declare that they have no competing interests.

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