Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 7(4): 5844–5859.
DOI:10.3934/math.2022324
Received: 01 October 2021
Revised: 05 December 2021
Accepted: 04 January 2022
Published: 12 January 2022

Research article

Characterizations of intra-regular LA-semihyperrings in terms of their hyperideals

Warud Nakkhasen*

Department of Mathematics, Faculty of Science, Mahasarakham University, Maha Sarakham 44150, Thailand

* **Correspondence:** Email: warud.n@msu.ac.th.

Abstract: The purpose of this article is to investigate the class of intra-regular LA-semihyperrings. Then, characterizations of intra-regular LA-semihyperrings by the properties of many types of their hyperideals are obtained. Moreover, we present a construction of LA-semihyperrings from ordered LA-semirings.

Keywords: LA-semihypergroup; LA-semihyperring; intra-regular LA-semihyperring
Mathematics Subject Classification: 16Y60, 20M17, 20N20

1. Introduction

The algebraic structure of left almost semigroups (for short, LA-semigroups), which is a generalization of commutative semigroups, was first introduced by Kazim and Naseeruddin [20] in 1972. An Abel-Grassmann groupoid (for short, AG-groupoid) is another name for it [33]. A non-associative and a non-commutative algebraic structure that lies midway between a groupoid and a commutative semigroup is known as an LA-semigroup. Regularities are interesting and important properties to examine in LA-semigroups. In 2010, Khan and Asif [21] characterized intra-regular LA-semigroups by the properties of their fuzzy ideals. Later, Abdullah et al. [3] discussed characterizations of regular LA-semigroups using interval valued (α,β)-fuzzy ideals. Also, Khan et al. [22] characterized right regular LA-semigroups using their fuzzy left ideals and fuzzy right ideals. In 2016, Khan et al. [25] characterized the class of (m,n)-regular LA-semigroups by their (m,n)-ideals. Some characterizations of weakly regular LA-semigroups by using the smallest ideals and fuzzy ideals of LA-semigroups are investigated by Yousafzai et al. [40]. In addition, Sezer [36] have used the concept of soft sets to characterize regular, intra-regular, completely regular, weakly regular and quasi-regular LA-semigroups. Now, many mathematicians have investigated various characterizations of LA-semigroups (see, e.g., [2, 9, 41]). Furthermore, some mathematicians have considered the notion of left almost semirings (for short, *LA*-semirings), that is a generalization of left almost rings (for short, *LA*-rings) [37], to have different features. In 2021, the left almost structures are now widely studied such as Elmoasry [13] studied the concepts of rough prime and rough fuzzy prime ideals in *LA*-semigroups, Massouros and Yaqoob [26] investigated the theory of left and right almost groups and focused on more general structures, and Rehman et al. [34] introduced the notion of neutrosophic *LA*-rings and discussed various types of ideals and establish several results to better understand the characteristic behavior of neutrosophic *LA*-rings. In addition, the concept of left almost has been investigated in various algebraic structures (for example, in ordered *LA*-semigroups [4, 18, 46], in ordered *LA*-Γ-semigroups [8], in gamma *LA*-rings and gamma *LA*-semigroups [24], in *LA*-polygroups [7, 42, 44]).

Marty [28] introduced the concept of hyperstructures, as a generalization of ordinary algebraic structures. The composition of two elements in an ordinary algebraic structure is an element, but in an algebraic hyperstructure, the composition of two elements is a nonempty set. Many authors have developed on the concept of hyperstructures (see, e.g., [1, 12, 38]). Rehman et al. [35] introduced the concept of left almost hypergroups (for short, *LA*-hypergroups) and gave the examples of *LA*-hypergroups. Moreover, they introduced the concept of *LA*-hyperrings and characterized *LA*-hyperrings by their hyperideals and hypersystems. Next, the concept of weak *LA*-hypergroups was investigated by Nawaz et al. [30]. In 2020, Hu et al. [17] extended the notion of neutrosophic to *LA*-hypergroups and strong pure *LA*-semihypergroups. The concept of left almost semihypergroups (for short, *LA*-semihypergroups) is a generalization of *LA*-semigroups and commutative semihypergroups developed by Hila and Dine [16]. An *LA*-semihypergroups is a non-associative and non-commutative hyperstructure midway between a hypergroupoid and a commutative semihypergroup. Yaqoob et al. [43] have characterized intra-regular *LA*-semihypergroups by using the properties of their left and right hyperideals. Then, Gulistan et al. [14] defined the class of regular *LA*-semihypergroups in terms of $(\in_\Gamma, \in_\Gamma \vee q_\Delta)$-cubic (resp., left, right, two-sided, bi, generalized bi, interior, quasi) hyperideals of *LA*-semihypergroups. Furthermore, Khan et al. [19] investigated some properties of fuzzy left hyperideals and fuzzy right hyperideals in regular and intra-regular *LA*-semihypergroups. Meanwhile, the notion of ordered *LA*-semihypergroups which is a generalization of *LA*-semihypergroups was introduced by Yaqoob and Gulistan [45]. Also, Azhar et al. discussed some results related with fuzzy hyperideals and generalized fuzzy hyperideals of ordered *LA*-semihypergroups [5, 15].

It is known that every semiring can be considered to be a semihyperring. This implies that some results in intra-regular semihyperrings generalized the results in intra-regular semirings. The class of intra-regular semihyperrings was investigated by Nakkhasen and Pibaljommee [32] in 2019. Afterward, Nawaz et al. [31] introduced the notion of left almost semihyperrings (for short, *LA*-semihyperrings), which is a generalization of *LA*-semirings. Recently, Nakkhasen [29] characterized some classes of regularities in *LA*-semihyperrings, that is, weakly regular *LA*-semihyperrings and regular *LA*-semihyperrings by the properties of their hyperideals. In this paper, we are interested in the class of intra-regular *LA*-semihyperrings. Then, we give some characterizations of intra-regular *LA*-semihyperrings by means of their hyperideals. In addition, we show how ordered *LA*-semirings can be used to create *LA*-semihyperrings.

2. Preliminaries

First, we will review some fundamental notions and properties that are needed for this study. Let H be a nonempty set. Then, the mapping $\circ : H \times H \to \mathcal{P}^*(H)$ is called a *hyperoperation* (see, e.g., [10, 11, 39]) on H where $\mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\emptyset\}$ denotes the set of all nonempty subsets of H. A *hypergroupoid* is a nonempty set H together with a hyperopartion $\circ$ on H. If $x \in H$ and A, B are two nonempty subsets of H, then we denote

$$A \circ B = \bigcup_{a\in A, b\in B} a \circ b, A \circ x = A \circ \{x\} \text{ and } x \circ B = \{x\} \circ B.$$

A hypergroupoid $(H, \circ)$ is called an *LA-semihypergroup* [16] if for all $x, y, z \in H, (x \circ y) \circ z = (z \circ y) \circ x$. This law is known as a left invertive law. For any nonempty subsets A, B and C of an *LA*-semihypergroup $(H, \circ)$, we have that $(A \circ B) \circ C = (C \circ B) \circ A$.

A hyperstructure $(S, +, \cdot)$ is called an *LA-semihyperring* [31] if it satisfies the following conditions:

(i) $(S, +)$ is an *LA*-semihypergroup;
(ii) $(S, \cdot)$ is an *LA*-semihypergroup;
(iii) $x \cdot (y + z) = x \cdot y + x \cdot z$ and $(y + z) \cdot x = y \cdot x + z \cdot x$ for all $x, y, z \in S$.

Example 2.1. Let $\mathbb{Z}$ be the set of all integers. The hyperoperations $\ominus$ and $\odot$ on $\mathbb{Z}$ are defined by $x \ominus y = \{y - x\}$ and $x \odot y = \{xy\}$ for all $x, y \in \mathbb{Z}$, respectively. We have that $(\mathbb{Z}, \ominus, \odot)$ is an *LA*-semihyperrings.

Example 2.2. [35] Let $S = \{a, b, c\}$ be a set with the hyperoperations $+$ and $\cdot$ on S defined as follows:

$+$	a	b	c
a	$\{a\}$	$\{a, b, c\}$	$\{a, b, c\}$
b	$\{a, b, c\}$	$\{b, c\}$	$\{b, c\}$
c	$\{a, b, c\}$	$\{a, b, c\}$	$\{a, b, c\}$

$\cdot$	a	b	c
a	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a, b, c\}$	$\{c\}$
c	$\{a\}$	$\{a, b, c\}$	$\{a, b, c\}$

Then, $(S, +, \cdot)$ is an *LA*-semihyperring.

Throughout this paper, we say an *LA*-semihyperring S instead of an *LA*-semihyperring $(S, +, \cdot)$ and we write xy instead of $x \cdot y$ for any $x, y \in S$.

The concepts listed below will be considered in this research, as they occurred in [31]. For any *LA*-semihyperring S, the medial law $(xy)(zw) = (xz)(yw)$ holds for all $x, y, z, w \in S$. An element e of an *LA*-semihyperring S is called a *left identity* (resp., *pure left identity*) if for all $x \in S$, $x \in ex$ (resp., $x = ex$). We have that $S^2 = S$, for any *LA*-semihyperring S with a left identity e. If an *LA*-semihyperring S contains a pure left identity e, then it is unique. In an *LA*-semihyperring S with a pure left identity e, the paramedial law $(xy)(zw) = (wy)(zx)$ holds for all $x, y, z, w \in S$. An element a of an *LA*-semihyperring S with a left identity (resp., pure left identity) e is called a *left invertible* (resp., *pure left invertible*) if there exists $x \in S$ such that $e \in xa$ (resp., $e = xa$). An *LA*-semihyperring S is called a *left invertible* (resp., *pure left invertible*) if every element of S is a left invertible (resp., pure left invertible). We observe that if an element e is a pure left identity of an *LA*-semihyperring S, then e is also a left identity, but the converse is not true in general, see in [29].

Lemma 2.1. [31] *If S is an LA-semihyperring with a pure left identity e, then $x(yz) = y(xz)$ for all $x, y, z \in S$.*

Let S be an LA-semihyperring. Then, the following law holds $(AB)(CD) = (AC)(BD)$ for all nonempty subsets A, B, C, D of S. If an LA-semihyperring S contains the pure left identity e, then $(AB)(CD) = (DB)(CA)$ and $A(BC) = B(AC)$ for every nonempty subsets A, B, C, D of S.

Let S be an LA-semihyperring and a nonempty subset A of S such that $A + A \subseteq A$. Then:

(*i*) A is called a *left hyperideal* [31] of S if $SA \subseteq A$;
(*ii*) A is called a *right hyperideal* [31] of S if $AS \subseteq A$;
(*iii*) A is called a *hyperideal* [31] of S if it is both a left and a right hyperideal of S;
(*iv*) A is called a *quasi-hyperideal* [31] of S if $SA \cap AS \subseteq A$;
(*v*) A is called a *bi-hyperideal* [31] of S if $AA \subseteq A$ and $(AS)A \subseteq A$.

Example 2.3. Let $S = \{a, b, c, d\}$. Define hyperoperations $+$ and $\cdot$ on S by the following tables:

$+$	a	b	c	d
a	$\{a\}$	$\{a,b\}$	$\{c\}$	$\{d\}$
b	$\{a,b\}$	$\{a,b\}$	$\{c\}$	$\{d\}$
c	$\{c\}$	$\{c\}$	$\{c\}$	$\{d\}$
d	$\{d\}$	$\{d\}$	$\{d\}$	$\{d\}$

$\cdot$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
c	$\{a\}$	$\{a\}$	$\{a\}$	$\{a,b\}$
d	$\{a\}$	$\{a\}$	$\{a,b\}$	$\{a,b\}$

We can see that $(S, +, \cdot)$ is an LA-semihyperring. Consider $A = \{a, b, c\}$ and $B = \{a, c\}$. It is easy to see that A is a quasi-hyperideal of S. In addition, B is a bi-hyperideal of S, but it is not a quasi-hyperideal of S because $SB \cap BS = \{a, b\} \nsubseteq B$.

A nonempty subset G of an LA-semihyperring S is called a *generalized bi-hyperideal* of S if $G+G \subseteq G$ and $(GS)G \subseteq G$. Obviously, every bi-hyperideal of an LA-semihyperring S is a generalized bi-hyperideal, but the converse is not true in general. We can show this with the following example.

Example 2.4. From Example 2.3, consider $G = \{a, c, d\}$. It is not difficult to show that G is a generalized bi-hyperideal of S. But G is not a bi-hyperideal of S, because $c \cdot d = \{a, b\} \nsubseteq G$.

An *ordered LA-semiring* is a system $(S, +, \cdot, \leq)$ consisting of a nonempty set S such that $(S, +, \cdot)$ is an LA-semiring, $(S, \leq)$ is a partially ordered set, and for every $a, b, x \in S$ the following conditions are satisfied: (*i*) if $a \leq b$, then $a + x \leq b + x$ and $x + a \leq x + b$; (*ii*) if $a \leq b$, then $a \cdot x \leq b \cdot x$ and $x \cdot a \leq x \cdot b$. For an ordered LA-semiring $(S, +, \cdot, \leq)$ and $x \in S$, we denote $(x] = \{s \in S \mid s \leq x\}$.

In 2014, Amjad and Yousafzai [6] have shown that every ordered LA-semigroup $(S, \cdot, \leq)$ can be considered as an LA-semihypergroup $(S, \circ)$ where a hyperoperation $\circ$ on S defined by

$$a \circ b = \{x \in S \mid x \leq a \cdot b\} = (a \cdot b] \text{ for all } a, b \in S.$$

Now, we apply this idea to construct an LA-semihyperring from an ordered LA-semiring as the following lemma.

Lemma 2.2. *Let $(S, +, \cdot, \leq)$ be an ordered LA-semiring. Then $(S, \oplus, \odot)$ is an LA-semihyperring where the hyperoperations $\oplus$ and $\odot$ on S are defined by letting $a, b \in S$,*

$$a \oplus b = \{x \in S \mid x \leq a + b\} = (a + b] \text{ and } a \odot b = \{x \in S \mid x \leq a \cdot b\} = (a \cdot b].$$

Proof. By the Example in [6], it follows that $(S, \oplus)$ and $(S, \odot)$ are LA-semihypergroups. Next, we will show that the hyperoperation $\odot$ is distributive with respect to the hyperoperation $\oplus$ on S. First, we claim that $a \odot (b \oplus c) = (a \cdot (b + c)]$. Let $t \in a \odot (b \oplus c)$. Then, $t \in a \odot x$ for some $x \in b \oplus c$. So, $t \leq a \cdot x \leq a \cdot (b + c)$, then $t \in (a \cdot (b + c)]$. Hence, $a \odot (b \oplus c) \subseteq (a \cdot (b + c)]$. Let $s \in (a \cdot (b + c)]$. Then, $s \leq a \cdot (b + c)$, and so

$$s \in a \odot (b + c) \subseteq \bigcup_{x \in b \oplus c} a \odot x = a \odot (b \oplus c).$$

That is, $(a \cdot (b + c)] \subseteq a \odot (b \oplus c)$. It follows that $a \odot (b \oplus c) = (a \cdot (b + c)]$. Next, we show that $(a \odot b) \oplus (a \odot c) = (a \cdot b + a \cdot c]$. Let $t \in (a \odot b) \oplus (a \odot c)$. Then $t \in x \oplus y$ for some $x \in a \odot b$ and $y \in a \odot c$. This implies that $t \leq x + y \leq a \cdot b + a \cdot c$. Thus, $t \in (a \cdot b + a \cdot c]$. Hence, $(a \odot b) \oplus (a \odot c) \subseteq (a \cdot b + a \cdot c]$. Let $s \in (a \cdot b + a \cdot c]$. Then

$$s \in a \cdot b \oplus a \cdot c \subseteq \bigcup_{x \in a \odot b, y \in a \odot c} x \oplus y = (a \odot b) \oplus (a \odot c).$$

Hence, $(a \cdot b + a \cdot c] \subseteq (a \odot b) \oplus (a \odot c)$. Therefore, $(a \odot b) \oplus (a \odot c) = (a \cdot b + a \cdot c]$. Since $(a \cdot (b + c)] = (a \cdot b + a \cdot c]$, we obtain that $a \odot (b \oplus c) = (a \odot b) \oplus (a \odot c)$. Similarly, we can show that $(b \oplus c) \odot a = (b \odot a) \oplus (c \odot a)$. Consequently, $(S, \oplus, \odot)$ is an LA-semihyperring. □

Example 2.5. Let $S = \{a, b, c\}$ be a set with two binary operations $+$ and $\cdot$ on S defined as follows:

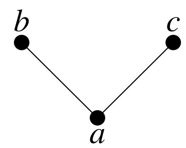
$+$	a	b	c
a	a	a	a
b	a	a	c
c	a	a	a

$\cdot$	a	b	c
a	a	a	a
b	a	a	c
c	a	a	a

Then, $(S, +, \cdot)$ is an LA-semiring [27]. We define an order relation $\leq$ on S by

$$\leq := \{(a, a), (b, b), (c, c), (a, b), (a, c)\}.$$

The figure of $\leq$ on S is given by



It is a routine matter to check that $(S, +, \cdot, \leq)$ is an ordered LA-semiring. We obtain that its associated LA-semihyperring $(S, \oplus, \odot)$ where $\oplus$ and $\odot$ are defined by Lemma 2.2 as follows:

$\oplus$	a	b	c
a	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a, c\}$
c	$\{a\}$	$\{a\}$	$\{a\}$

$\odot$	a	b	c
a	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a, c\}$
c	$\{a\}$	$\{a\}$	$\{a\}$

Now, we can see that $A = \{a, b\}$ is a left hyperideal of S, but it is not a right hyperideal of S because $b \odot c = \{a, c\} \nsubseteq A$.

Lemma 2.3. [29] *Let S be an LA-semihyperring with a pure left identity e. Then every right hyperideal of S is a hyperideal of S.*

Lemma 2.4. [29] *Every left (resp., right) hyperideal of an LA-semihyperring S is a quasi-hyperideal of S.*

Lemma 2.5. *Every left (resp., right) hyperideal of an LA-semihyperring S is a bi-hyperideal of S.*

Proof. Let B be a left hyperideal of an LA-semihyperring S. Then, $BB \subseteq SB \subseteq B$, and so $(BS)B \subseteq SB \subseteq B$. Thus, B is a bi-hyperideal of S. For the case right hyperideals, we can prove similarly. □

Lemma 2.6. [29] *Let S be an LA-semihyperring with a left identity e such that $(xe)S \subseteq xS$ for all $x \in S$. Then every quasi-hyperideal of S is a bi-hyperideal of S.*

Lemma 2.7. [29] *If S is an LA-semihyperring with a pure left identity e, then for every $a \in S$, a^2S is a hyperideal of S such that $a^2 \subseteq a^2S$.*

Lemma 2.8. *If S is an LA-semihyperring with a left identity e, then for every $a \in S$, Sa is a left hyperideal of S such that $a \in Sa$.*

Proof. Assume that S is an LA-semihyperring with a left identity e. Let $a \in S$. Then, $a \in ea \subseteq Sa$ and $Sa + Sa = (S + S)a \subseteq Sa$. Now, by using paramedial law and left invertive law, we have

$$S(Sa) \subseteq (eS)(Sa) = (aS)(Se) = ((Se)S)a \subseteq Sa.$$

It follows that Sa is a left hyperideal of S. □

Let J be a finite nonempty subset of $\mathbb{N}$ such that $J = \{j_1, j_2, j_3, \ldots, j_n\}$, where $j_1, j_2, j_3, \ldots, j_n \in \mathbb{N}$. For any $a \in S$, we denote

$$\sum_{i \in J} a_i = (\cdots((a_{j_1} + a_{j_2}) + a_{j_3}) + \cdots) + a_{j_n}.$$

For any nonempty subsets A and B of LA-semihyperring S and $a \in S$, we denote

$$\Sigma A = \{t \in S \mid t \in \sum_{i \in I} a_i, a_i \in A \text{ and } I \text{ is a finite nonempty subset of } \mathbb{N}\},$$
$$\Sigma AB = \{t \in S \mid t \in \sum_{i \in I} a_i b_i, a_i \in A, b_i \in B \text{ and } I \text{ is a finite nonempty subset of } \mathbb{N}\},$$
$$\Sigma a = \Sigma\{a\}.$$

Remark 2.1. Let A and B be any nonempty subsets of an LA-semihyperring S. Then the following statements hold:

(i) $A \subseteq \Sigma A$;
(ii) $A(\Sigma B) \subseteq \Sigma AB$ and $(\Sigma A)B \subseteq \Sigma AB$.

Lemma 2.9. *Let A be any nonempty subset of an LA-semihyperring S. If $A + A \subseteq A$, then $\Sigma aA = aA$ and $\Sigma Aa = Aa$ for all $a \in S$.*

3. Characterizations of intra-regular *LA*-semihyperrings

In this section, we apply the concept of intra-regular *LA*-rings, defined in [23], to define the notion of intra-regular *LA*-semihyperrings and study some of its properties. Finally, we give some characterizations of intra-regular *LA*-semihyperrings by the properties of many types of hyperideals of *LA*-semihyperrings.

Definition 3.1. An *LA*-semihyperring S is said to be *intra-regular* if for every $a \in S$, $a \in \Sigma(Sa^2)S$.

Example 3.1. Let $S = \{a, b, c\}$ be a set with the hyperoperations $+$ and $\cdot$ on S defined as follows:

$+$	a	b	c
a	$\{a\}$	$\{a, b, c\}$	$\{a, b, c\}$
b	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$
c	$\{a, b, c\}$	$\{a, b, c\}$	$\{a, b, c\}$

$\cdot$	a	b	c
a	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a, b, c\}$	$\{c\}$
c	$\{a\}$	$\{a, b, c\}$	$\{a, b, c\}$

Then, $(S, +, \cdot)$ is an *LA*-semihyperring [31]. Now, we can see that S is intra-regular.

However, the set $S = \{a, b, c, d, e\}$ with two hyperoperations $\oplus$ and $\odot$ on S as defined in Example 2.5 is not intra-regular, because $b \notin \{a\} = \Sigma(S \odot b^2) \odot S$.

Proposition 3.1. *Every left (resp., right) hyperideal of an intra-regular LA-semihyperring S is a hyperideal of S.*

Proof. Let S be an intra-regular *LA*-semihyperring and $x \in S$. Assume that L is a left hyperideal of S and $a \in L$. Then, $a \in \Sigma(Sa^2)S$. Now, by using Remark 2.1 and left invertive law, we have

$$ax \subseteq (\Sigma(Sa^2)S)x \subseteq \Sigma((Sa^2)S)x = \Sigma(xS)(Sa^2) \subseteq \Sigma SL \subseteq \Sigma L \subseteq L.$$

Thus, L is a right hyperideal of S, and so L is a hyperideal of S. Suppose that R is a right hyperideal of S and $r \in R$. Then,

$$xr \subseteq (\Sigma(Sx^2)S)r \subseteq \Sigma((Sx^2)S)r = \Sigma(rS)(Sx^2) \subseteq \Sigma RS \subseteq \Sigma R \subseteq R.$$

Hence, R is a left hyperideal of S. It follows that R is a hyperideal of S. □

Proposition 3.2. *If S is an intra-regular LA-semihyperring with a pure left identity e, then $\Sigma I^2 = I$ for every left hyperideal I of S.*

Proof. Assume that S is an intra-regular *LA*-semihyperring with a pure left identity e. Let I be a left hyperideal of S. Then, $\Sigma I^2 \subseteq I$. Let $a \in I$. By using left invertive law, medial law and Lemma 2.1, we have

$$\begin{aligned} a \in \Sigma(Sa^2)S &= \Sigma(S(aa))S = \Sigma(a(Sa))S = \Sigma(a(Sa))(eS) = \Sigma(ae)((Sa)S) \\ &= \Sigma(Sa)((ae)S) = \Sigma(Sa)((Se)a) \subseteq \Sigma(SI)(SI) \subseteq \Sigma II = \Sigma I^2. \end{aligned}$$

Thus, $I \subseteq \Sigma I^2$. Therefore, $\Sigma I^2 = I$. □

A (resp., left, right) hyperideal P of an *LA*-semihyperring S is called *semiprime* if for any $a \in S$, $a^2 \subseteq P$ implies $a \in P$.

Proposition 3.3. *Every hyperideal of an intra-regular LA-semihyperring is semiprime.*

Proof. Assume that S is an intra-regular LA-semihyperring. Let I be a hyperideal of S and $a \in S$ such that $a^2 \subseteq I$. Then, $a \in \Sigma(Sa^2)S \subseteq \Sigma(SI)S \subseteq \Sigma IS \subseteq \Sigma I = I$. Hence, I is semiprime. □

Proposition 3.4. *Let S be an LA-semihyperring S with a pure left identity e. If S satisfies $L \cup R = \Sigma LR$, for every left hyperideal L and every right hyperideal R of S such that R is semiprime, then S is intra-regular.*

Proof. Let $a \in S$. By Lemma 2.8 and Lemma 2.7, we have that Sa is a left hyperideal and a^2S is a right hyperideal of S such that $a \in Sa$ and $a^2 \subseteq a^2S$, respectively. Thus, by the given assumption, $a \in a^2S$. Now, by using left invertive law, medial law and Lemma 2.1, we have

$$\begin{aligned} a \in Sa \cup a^2S &= \Sigma(Sa)(a^2S) = \Sigma(Sa)((aa)S) \subseteq \Sigma(Sa)((aS)S) = \Sigma(aS)((Sa)S) \\ &= \Sigma(a(Sa))(SS) = \Sigma(a(Sa))S = \Sigma(S(aa))S = \Sigma(Sa^2)S. \end{aligned}$$

This shows that S is intra-regular. □

Next, we give characterizations of intra-regular LA-semihyperrings by means of (resp., left, right) hyperideals, quasi-hyperideals, bi-hyperideals and generalized bi-hyperideals of LA-semihyperrings as show by the following theorems.

Theorem 3.1. *Let S be an LA-semihyperring with a pure left identity e. Then S is intra-regular if and only if $L = L^3$, for every left hyperideal L of S.*

Proof. Assume that S is intra-regular. Let L be any left hyperideal of S. Then, $L^3 = (LL)L \subseteq (SL)L \subseteq LL \subseteq L$. Now, let $a \in L$. By Lemma 2.7, a^2S is a hyperideal of S such that $a^2 \subseteq a^2S$. Thus, by given assumption and Proposition 3.3, we have that a^2S is semiprime, and so $a \in a^2S$. Thus, by using left invertive law and Lemma 2.1, we have

$$\begin{aligned} a \in a^2S &= (aa)S = (Sa)a \subseteq (S(a^2S))a = (a^2(SS))a = ((aa)S)a \\ &= ((Sa)a)a \subseteq ((SL)L)L \subseteq (LL)L = L^3. \end{aligned}$$

Hence, $L \subseteq L^3$. Therefore, $L = L^3$.

Conversely, assume that $L = L^3$, for every left hyperideal L of S. Let $a \in S$. By Lemma 2.8, Sa is a left hyperideal of S such that $a \in Sa$. Then, by the given assumption and using medial law, we have

$$a \in Sa = ((Sa)(Sa))(Sa) = ((SS)(aa))(Sa) \subseteq (Sa^2)S \subseteq \Sigma(Sa^2)S.$$

This shows that S is intra-regular. □

Theorem 3.2. *Let S be a pure left invertible LA-semihyperring with a pure left identity e. Then the following conditions are equivalent:*

(*i*) *S is intra-regular;*
(*ii*) *$L \cap R \subseteq \Sigma LR$, where L and R are any left and right hyperideals of S, respectively.*

Proof. $(i) \Rightarrow (ii)$ Assume that S is intra-regular. Let L be a left hyperideal and R be a right hyperideal of S, and let $a \in L \cap R$. Then, by using left invertive law and Lemma 2.1, we have

$$a \in \Sigma(Sa^2)S = \Sigma(S(aa))S = \Sigma(a(Sa))S = \Sigma(S(Sa))a \subseteq \Sigma(S(SL))R \subseteq \Sigma LR.$$

Hence, $L \cap R \subseteq \Sigma LR$.

$(ii) \Rightarrow (i)$ Assume that (ii) holds. Let $a \in S$. Since S is a pure left invertible, there exists $x \in S$ such that $e = xa$. By Lemma 2.7, a^2S is both a left and a right hyperideal of S such that $a^2 \subseteq a^2S$. Then, by using left interive law, Lemma 2.1 and given assumption, we have

$$\begin{aligned} a^2 &\subseteq a^2S \cap a^2S \subseteq \Sigma(a^2S)(a^2S) = \Sigma a^2((a^2S)S) \\ &= \Sigma a^2((SS)a^2) = \Sigma(aa)(Sa^2) = \Sigma((Sa^2)a)a. \end{aligned}$$

Now, by using left invertive law and Remark 2.1, we have

$$\begin{aligned} a = ea = (xa)a = (aa)x &\subseteq (\Sigma((Sa^2)a)a)x \subseteq \Sigma(((Sa^2)a)a)x \\ &= \Sigma(xa)((Sa^2)a) = \Sigma e((Sa^2)a) = \Sigma(Sa^2)a \subseteq \Sigma(Sa^2)S. \end{aligned}$$

Therefore, S is intra-regular. □

Theorem 3.3. *Let S be a pure left invertible LA-semihyperring with a pure left identity e. Then the following statements are equivalent:*

(i) *S is intra-regular;*
(ii) *$L \cap R = \Sigma RL$, for every left hyperideal L and every right hyperideal R of S.*

Proof. $(i) \Rightarrow (ii)$ Assume that S is intra-regular. Let L and R be a left hyperideal and a right hyperideal of S, respectively. It is easy to see that $\Sigma RL \subseteq L \cap R$. On the other hand, let $a \in L \cap R$. Then, $a \in \Sigma(Sa^2)S$. By using left invertive law, paramedial law and Lemma 2.1, we have

$$\begin{aligned} a \in \Sigma(Sa^2)S &= \Sigma(S(aa))S = \Sigma(a(Sa))S = \Sigma(S(Sa))a = \Sigma((eS)(Sa))a \\ &= \Sigma((aS)(Se))a \subseteq \Sigma((RS)S)L \subseteq \Sigma RL. \end{aligned}$$

Hence, $L \cap R \subseteq \Sigma RL$. Therefore, $L \cap R = \Sigma RL$.

$(ii) \Rightarrow (i)$ This proof is similar to the proof of $(ii) \Rightarrow (i)$ in Theorem 3.2, because a^2S is both a left hyperideal and a right hyperideal of S. □

Theorem 3.4. *Let S be a pure left invertible LA-semihyperring with a pure left identity e such that $(xe)S \subseteq xS$ for all $x \in S$. Then the following statements are equivalent:*

(i) *S is intra-regular;*
(ii) *$G \cap I = (GI)G$, for every generalized bi-hyperideal G and every hyperideal I of S;*
(iii) *$B \cap I = (BI)B$, for every bi-hyperideal B and every hyperideal I of S;*
(iv) *$Q \cap I = (QI)Q$, for every quasi-hyperideal Q and every hyperideal I of S.*

Proof. $(i) \Rightarrow (ii)$ Assume that S is intra-regular. Let G be a generalized bi-hyperideal and I be a hyperideal of S, and let $a \in G \cap I$. Then, $a \in \Sigma(Sa^2)S$. Now, by using left invertive law and Lemma 2.1, we have

$$a \in \Sigma(Sa^2)S = \Sigma(S(aa))S = \Sigma(a(Sa))S = \Sigma(S(Sa))a.$$

Consider,

$$\begin{aligned} S(Sa) &\subseteq S(S(\Sigma(Sa^2)S)) \subseteq \Sigma S(S((Sa^2)S)) = \Sigma S((Sa^2)(SS)) \\ &= \Sigma(Sa^2)(S(SS)) \subseteq \Sigma(S(aa))S = \Sigma(a(Sa))S \\ &= \Sigma(S(Sa))a = (\Sigma S(Sa))a \subseteq Sa. \end{aligned} \tag{3.1}$$

Then, by using (3.1), medial law, Lemma 2.1 and Lemma 2.9, we have

$$S(Sa) \subseteq (\Sigma S(Sa))a \subseteq (\Sigma Sa)a = (Sa)a = (Sa)(ea) = (Se)(aa) = a((Se)a) \subseteq a(Sa) \subseteq S(Sa).$$

It follows that $S(Sa) = a(Sa)$. Thus, $a \in \Sigma(S(Sa))a = \Sigma(a(Sa))a = (a(Sa))a \subseteq (G(SI))G \subseteq (GI)G$. Hence, $G \cap I \subseteq (GI)G$. On the other hand, $(GI)G \subseteq (SI)S \subseteq I$ and $(GI)G \subseteq (GS)G \subseteq G$, that is, $(GI)G \subseteq G \cap I$. Therefore, $G \cap I = (GI)G$.

$(ii) \Rightarrow (iii)$ Since every bi-hyperideal is a generalized bi-hyperideal of S, it follows that (iii) holds.

$(iii) \Rightarrow (iv)$ By Lemma 2.6, we have that every quasi-hyperideal of S is a bi-hyperideal. Hence, (iv) holds.

$(iv) \Rightarrow (i)$ Let L be a left hyperideal and R be a right hyperideal of S. By Lemma 2.3 and Lemma 2.4, we have that R is a hyperideal and L is a quasi-hyperideal of S, respectively. By assumption, $L \cap R = (LR)L \subseteq (SR)L \subseteq RL \subseteq \Sigma RL$. On the other hand, $\Sigma RL \subseteq L \cap R$. Therefore, $L \cap R = \Sigma RL$. By Theorem 3.3, we have that S is intra-regular. □

Theorem 3.5. *Let S be a pure left invertible LA-semihyperring with a pure left identity e such that $(xe)S \subseteq xS$ for all $x \in S$. Then the following statements are equivalent:*

(i) *S is intra-regular;*

(ii) *$R \cap G \subseteq \Sigma GR$, for every generalized bi-hyperideal G and every right hyperideal R of S;*

(iii) *$R \cap B \subseteq \Sigma BR$, for every bi-hyperideal B and every right hyperideal R of S;*

(iv) *$R \cap Q \subseteq \Sigma QR$, for every quasi-hyperideal Q and every right hyperideal R of S.*

Proof. $(i) \Rightarrow (ii)$ Assume that S is intra-regular. Let R be a right hyperideal and G be a generalized bi-hyperideal of S, and let $a \in R \cap G$. Then, $a \in \Sigma(Sa^2)S$. Since $S(Sa) \subseteq Sa$, left invertive law, medial law and Lemma 2.1, we obtain that

$$\begin{aligned} a \in \Sigma(Sa^2)S &= \Sigma(S(aa))S = \Sigma(a(Sa))S = \Sigma(S(Sa))a \subseteq \Sigma(Sa)a \\ &= \Sigma(Sa)(ea) = \Sigma(Se)(aa) = \Sigma a((Se)a) \\ &= \Sigma a((ae)S) \subseteq \Sigma G((RS)S) \subseteq \Sigma GR. \end{aligned}$$

Hence, $R \cap G \subseteq \Sigma GR$.

$(ii) \Rightarrow (iii)$ Since every bi-hyperideal is a generalized bi-hyperideal of S, it follows that (iii) holds.

$(iii) \Rightarrow (iv)$ By Lemma 2.6, we have that every quasi-hyperideal of S is a bi-hyperideal. Hence, (iv) holds.

$(iv) \Rightarrow (i)$ Let L be a left hyperideal and R be a right hyperideal of S. By Lemma 2.4, L is a quasi-hyperideal of S. By assumption, $L \cap R \subseteq \Sigma LR$. Therefore, S is intra-regular by Theorem 3.2. □

Theorem 3.6. *Let S be a pure left invertible LA-semihyperring with a pure left identity e such that $(xe)S \subseteq xS$ for all $x \in S$. Then the following conditions are equivalent:*

(*i*) *S is intra-regular;*
(*ii*) *$R \cap G \subseteq \Sigma RG$, for every generalized bi-hyperideal G and every right hyperideal R of S;*
(*iii*) *$R \cap B \subseteq \Sigma RB$, for every bi-hyperideal B and every right hyperideal R of S;*
(*iv*) *$R \cap Q \subseteq \Sigma RQ$, for every quasi-hyperideal Q and every right hyperideal R of S.*

Proof. (*i*) ⇒ (*ii*) Assume that S is intra-regular. Let G be a generalized bi-hyperideal and R be a right hyperideal of S. Let $a \in R \cap G$. Then, $a \in \Sigma(Sa^2)S$. Thus, by using left invertive law and Lemma 2.1, we have $a \in \Sigma(Sa^2)S = \Sigma(S(aa))S = \Sigma(a(Sa))S = \Sigma(S(Sa))a$. Since $S(Sa) = a(Sa)$, we have

$$a \in \Sigma(S(Sa))a = \Sigma(a(Sa))a \subseteq \Sigma(RS)G \subseteq \Sigma RG.$$

This implies that $R \cap G \subseteq \Sigma RG$.

(*ii*) ⇒ (*iii*) Since every bi-hyperideal is a generalized bi-hyperideal of S, it turns out that (*iii*) holds.

(*iii*) ⇒ (*iv*) By Lemma 2.6, we have that every quasi-hyperideal of S is a bi-hyperideal. So, (*iv*) holds.

(*iv*) ⇒ (*v*) Let L and R be a left hyperideal and a right hyperideal of S, respectively. By Lemma 2.4, L is also a quasi-hyperideal of S. By hypothesis, $L \cap R \subseteq \Sigma RL$. Otherwise, $\Sigma RL \subseteq L \cap R$. Hence, $L \cap R = \Sigma RL$. Therefore, S is intra-regular by Theorem 3.3. □

Theorem 3.7. *Let S be a pure left invertibleLA-semihyperring with a pure left identity e such that $(xe)S \subseteq xS$ for all $x \in S$. Then the following statements are equivalent:*

(*i*) *S is intra-regular;*
(*ii*) *$L \cap G \subseteq \Sigma LG$, for every generalized bi-hyperideal G and every left hyperideal L of S;*
(*iii*) *$L \cap B \subseteq \Sigma LB$, for every bi-hyperideal B and every left hyperideal L of S;*
(*iv*) *$L \cap Q \subseteq \Sigma LQ$, for every quasi-hyperideal Q and every left hyperideal L of S.*

Proof. (*i*) ⇒ (*ii*) Assume that S is intra-regular. Let L be a left hyperideal and G be a generalized bi-hyperideal of S, and let $a \in L \cap G$. Then, $a \in \Sigma(Sa^2)S$. Now, by using left invertive law and Lemma 2.1, we have

$$a \in \Sigma(Sa^2)S = \Sigma(S(aa))S = \Sigma(a(Sa))S = \Sigma(S(Sa))a \subseteq \Sigma(S(SL))G \subseteq \Sigma LG.$$

This implies that $L \cap G \subseteq \Sigma LG$.

(*ii*) ⇒ (*iii*) Since every bi-hyperideal is a generalized bi-hyperideal of S, it follows that (*iii*) holds.

(*iii*) ⇒ (*iv*) By Lemma 2.6, we have that every quasi-hyperideal of S is a bi-hyperideal. Hence, (*iv*) holds.

(*iv*) ⇒ (*i*) Let L be a left hyperideal and R be a right hyperideal of S. By Lemma 2.4, R is also a quasi-hyperideal of S. By assumption, $L \cap R \subseteq \Sigma LR$. Therefore, S is intra-regular by Theorem 3.2. □

Theorem 3.8. *Let S be a pure left invertible LA-semihyperring with a pure left identity e such that $(xe)S \subseteq xS$ for all $x \in S$. Then the following statements are equivalent:*

(*i*) *S is intra-regular;*
(*ii*) *$L \cap G \subseteq \Sigma GL$, for every generalized bi-hyperideal G and every left hyperideal L of S;*

(*iii*) $L \cap B \subseteq \Sigma BL$, *for every bi-hyperideal B and every left hyperideal L of S;*
(*iv*) $L \cap Q \subseteq \Sigma QL$, *for every quasi-hyperideal Q and every left hyperideal L of S.*

Proof. (*i*) $\Rightarrow$ (*ii*) Assume that S is intra-regular. Let G be a generalized bi-hyperideal and L be a left hyperideal of S and let $a \in L \cap G$. Then, $a \in \Sigma(Sa^2)S$. Thus, by using $S(Sa) \subseteq Sa$, left invertive law, medial law and Lemma 2.1, we have

$$\begin{aligned} a \in \Sigma(Sa^2)S &= \Sigma(a(Sa))S = \Sigma(S(Sa))a \subseteq \Sigma(Sa)a = \Sigma(Sa)(ea) = \Sigma(Se)(aa) \\ &= \Sigma a((Se)a) \subseteq \Sigma a(Sa) \subseteq \Sigma G(SL) \subseteq \Sigma GL. \end{aligned}$$

Hence, $L \cap G \subseteq \Sigma GL$.

(*ii*) $\Rightarrow$ (*iii*) Since every bi-hyperideal of S is a generalized bi-hyperideal, it follows that (*iii*) holds.

(*iii*) $\Rightarrow$ (*iv*) The implication holds from Lemma 2.6.

(*iv*) $\Rightarrow$ (*i*) Let L and R be a left hyperideal and a right hyperideal of S, respectively. By Lemma 2.4, R is also a quasi-hyperideal of S. By the given assumption, we have $L \cap R \subseteq \Sigma RL$. On the other hand, $\Sigma RL \subseteq L \cap R$. Therefore, $L \cap R = \Sigma RL$. By Theorem 3.3, we obtain that S is intra-regular. □

Theorem 3.9. *Let S be a pure left invertible LA-semihyperring with a pure left identity e such that* $(xe)S \subseteq xS$ *for all* $x \in S$. *Then the following conditions are equivalent:*

(*i*) *S is intra-regular;*
(*ii*) $L \cap G \cap R \subseteq \Sigma(LG)R$, *for every generalized bi-hyperideal G, every left hyperideal L and every right hyperideal R of S;*
(*iii*) $L \cap B \cap R \subseteq \Sigma(LB)R$, *for every bi-hyperideal B, every left hyperideal L and every right hyperideal R of S;*
(*iv*) $L \cap Q \cap R \subseteq \Sigma(LQ)R$, *for every quasi-hyperideal Q, every left hyperideal L and every right hyperideal R of S.*

Proof. (*i*) $\Rightarrow$ (*ii*) Assume that S is intra-regular. Let G be a generalized bi-hyperideal, L be a left hyperideal and R be a right hyperideal of S, and let $a \in L \cap G \cap R$. Then, $a \in \Sigma(Sa^2)S$. We note that $S(Sa) = a(Sa)$. Then, by using left invertive law, medial law, paramedial law and Lemma 2.1, we have

$$\begin{aligned} a \in \Sigma(Sa^2)S &= \Sigma(a(Sa))S = \Sigma(S(Sa))a = \Sigma(a(Sa))a = \Sigma(a(Sa))(ea) = \Sigma(S(aa))(ea) \\ &= \Sigma(ae)((aa)S) = \Sigma(aa)((ae)S) \subseteq \Sigma(LG)((RS)S) \subseteq \Sigma(LG)R. \end{aligned}$$

Hence, $L \cap G \cap R \subseteq \Sigma(LG)R$.

(*ii*) $\Rightarrow$ (*iii*) Since every bi-hyperideal is a generalized bi-hyperideal of S, it follows that (*iii*) holds.

(*iii*) $\Rightarrow$ (*iv*) By Lemma 2.6, we have that every quasi-hyperideal of S is a bi-hyperideal. Hence, (*iv*) holds.

(*iv*) $\Rightarrow$ (*i*) Let L be a left hyperideal and R be a right hyperideal of S. By Lemma 2.4, L is a quasi-hyperideal of S. By assumption, $L \cap R = L \cap L \cap R \subseteq \Sigma(LL)R \subseteq \Sigma(SL)R \subseteq \Sigma LR$. By Theorem 3.2, we obtain that S is intra-regular. □

Theorem 3.10. *Let S be a pure left invertible LA-semihyperring with a pure left identity e such that* $(xe)S \subseteq xS$ *for all* $x \in S$. *Then the following statements are equivalent:*

(*i*) *S is intra-regular;*

(*ii*) *$L \cap G \cap R \subseteq \Sigma(RG)L$, for every generalized bi-hyperideal G, every left hyperideal L and every right hyperideal R of S;*
(*iii*) *$L \cap B \cap R \subseteq \Sigma(RB)L$, for every bi-hyperideal B, every left hyperideal L and every right hyperideal R of S;*
(*iv*) *$L \cap Q \cap R \subseteq \Sigma(RQ)L$, for every quasi-hyperideal Q, every left hyperideal L and every right hyperideal R of S.*

Proof. (i) $\Rightarrow$ (ii) Assume that S is intra-regular. Let G be a generalized bi-hyperideal, L be a left hyperideal and R be a right hyperideal of S. Let $a \in L \cap G \cap R$. Then, $a \in \Sigma(Sa^2)S$. Since $S(Sa) \subseteq (\Sigma S(Sa))a \subseteq Sa$ and by Lemma 2.9, we have $S(Sa) \subseteq (\Sigma S(Sa))a \subseteq (\Sigma Sa)a = (Sa)a$. By the given assumption, left invertive law, medial law, paramedial law and Lemma 2.1, we have

$$\begin{aligned} a \in \Sigma(Sa^2)S &= \Sigma(a(Sa)S) = \Sigma(S(Sa))a \subseteq \Sigma((Sa)a)a = \Sigma((Sa)(ea))a = \Sigma((ae)(aS))a \\ &= \Sigma(((aS)e)a)a \subseteq \Sigma(((RS)S)G)L \subseteq \Sigma(RG)L. \end{aligned}$$

This shows that, $L \cap G \cap R \subseteq \Sigma(RG)L$.

(ii) $\Rightarrow$ (iii) Since every bi-hyperideal of S is a generalized bi-hyperideal, which implies that (iii) holds.

(iii) $\Rightarrow$ (iv) The proof follows from Lemma 2.6.

(iv) $\Rightarrow$ (v) Let L be a left hyperideal and R be a right hyperideal of S. Also, L is a quasi-hyperideal of S by Lemma 2.4. By assumption, we have that $L \cap R = L \cap L \cap R \subseteq \Sigma(RL)L \subseteq \Sigma(RS)L \subseteq \Sigma RL$. Otherwise, $\Sigma RL \subseteq L \cap R$. Hence, $L \cap R = \Sigma RL$. Therefore, S is intra-regular by Theorem 3.3. □

The following theorem, we can prove similarly.

Theorem 3.11. *Let S be a pure left invertible LA-semihyperring with a pure left identity e such that $(xe)S \subseteq xS$ for all $x \in S$. Then the following conditions are equivalent:*

(*i*) *S is intra-regular;*
(*ii*) *$R \cap G \subseteq \Sigma(RG)R$, for every generalized bi-hyperideal G and every right hyperideal R of S;*
(*iii*) *$R \cap B \subseteq \Sigma(RB)R$, for every bi-hyperideal B every right hyperideal R of S;*
(*iv*) *$R \cap Q \subseteq \Sigma(RQ)R$, for every quasi-hyperideal Q and every right hyperideal R of S.*

4. Conclusions

In 2018, the concept of *LA*-semihyperrings was introduced by Nawaz et al. [31] as a generalization of *LA*-semirings. In Section 2, we have shown that some *LA*-semihyperring can be constructed from an ordered *LA*-semiring as shown in Lemma 2.2. This means that the *LA*-semihyperring is also a generalization of an ordered *LA*-semiring. In Section 3, we applied the concept of intra-regular *LA*-rings, appeared in [23], to define the concept of intra-regular *LA*-semihyperrings and discussed some of its properties. Finally, we characterized the class of intra-regular *LA*-semihyperrings by using (resp., left, right) hyperideals, quasi-hyperideals, bi-hyperideals and generalized bi-hyperideals of *LA*-semihyperrings were shown in Theorem 3.1 - Theorem 3.11. In our future study, we can consider the characterizations of the class of both regular and intra-regular *LA*-semihyperrings based on different types of hyperideals of *LA*-semihyperrings.

Acknowledgments

This research was financially supported by Faculty of Science, Mahasarakham University (Grant year 2020).

Conflict of interest

The author declares no conflict of interest.

References

1. A. Alsubie, A. Al-Masarwah, MBJ-neutrosophic hyper BCK-ideals in hyper BCK-algebras, *AIMS Math.*, **6** (2021), 6107–6121. http://dx.doi.org/10.3934/math.2021358
2. I. Ahmad, S. Rahman, M. Iqbal, Amanullah, A note on left abelian distributive *LA*-semigroups, *Punjap Univ. J. Math.*, **52** (2020), 47–63.
3. S. Abdullah, S. Aslam, N. U. Amin, *LA*-semigroups characterized by the properties of interval valued (α, β)-fuzzy ideals, *J. Appl. Math. Inform.*, **32** (2014), 405–426. https://doi.org/10.14317/JAMI.2014.405
4. M. A. Ansari, Roughness in generalized (m, n) bi-ideals in ordered *LA*-semigroups, *Int. J. Math. Comput. Sc.*, **14** (2019), 371–386.
5. M. Azhar, M. Gulistan, N. Yaqoob, S. Kadry, On fuzzy ordered *LA*-semihypergroups, *Int. J. Anal. Appl.*, **16** (2018), 276–289. https://doi.org/10.28924/2291-8639-16-2018-276
6. V. Amjad, F. Yousafzai, On pure *LA*-semihypergroups, *Konuralp J. Math.*, **2** (2014), 53–63.
7. N. Abughazalah, N. Yaqoob, A. Bashir, Cayley graphs over *LA*-groups and *LA*-polygroups, *Math. Probl. Eng.*, **2021** (2021), 4226232, 9 pages. https://doi.org/10.1155/2021/4226232
8. A. Basar, A note on (m, n)-Γ-ideals of ordered *LA*-Γ-semigroups, *Konuralp J. Math.*, **7** (2019), 107–111.
9. S. I. Batool, I. Younas, M. Khan, N. Yaqoob, A new technique for the construction of confusion component based on inverse *LA*-semigroups and its application in stenography, *Multimed. Tools Appl.*, **80** (2021), 28857–28877. https://doi.org/10.1007/s11042-021-11090-w
10. P. Corsini, *Prolegomena of hypergroup theory*, Aviani Editore, 1993.
11. P. Corsini, V. Leoreanu, *Applications of hyperstructure theory*, Springer Science & Business Media, 2003.
12. B. Davvaz, S. Subiono, M. A. Tahan, Calculus of meet plus hyperalgebra (tropical semihyperrings), *Commun. Algebra*, **48** (2020), 2143–2159. https://doi.org/10.1080/00927872.2019.1710178
13. A. Elmoasy, On rough fuzzy prime ideals in left almost semigroups, *Int. J. Anal. Appl.*, **19** (2021), 455–464.
14. M. Gulistan, M. Khan, N. Yaqoob, M. Shahzad, Structural properties of cubic sets in regular *LA*-semihypergroups, *Fuzzy Inform. Eng.*, **9** (2017), 93–116. https://doi.org/10.1016/j.fiae.2017.03.005

15. M. Gulistan, N. Yaqoob, S. Kadry, M. Azhar, On generalized fuzzy sets in ordered *LA*-semihypergroups, *P. Est. Acad. Sci.*, **68** (2019), 43–54. https://doi.org/10.3176/proc.2019.1.06
16. K. Hila, J. Dine, On hyperideals in left almost semihypergroups, *International Scholarly Research Notices*, **2011** (2011), 953124. https://doi.org/10.5402/2011/953124
17. M. Hu, F. Smarandache, X. Zhang, On neutrosophic extended triplet *LA*-hypergroups and strong pure *LA*-semihypergroups, *Symmetry*, **12** (2020), 163. https://doi.org/10.3390/sym12010163
18. W. Jantanan, R. Chinram, P. Petchkaew, On (m,n)-quasi-gamma-ideals in ordered *LA*-gamma-semigroups, *J. Math. Comput. Sci.*, **11** (2021), 3377–3390. https://doi.org/10.28919/jmcs/5705
19. A. Khan, M. Farooq, M. Izhar, B. Davvaz, Fuzzy hyperideals of left almost semihypergroups, *Int. J. Anal. Appl.*, **15** (2017), 155–171.
20. M. A. Kazim, M. Neseeruddin, On almost semigroups, *Rort. Math.*, **36** (1977), 41–47.
21. M. Khan, T. Asif, Characterizations of intra-regular left almost semigroups by their fuzzy ideals, *J. Math. Res.*, **2** (2010), 87–96.
22. M. Khan, Y. B. Jun, F. Yousafzai, Fuzzy ideals in right regular *LA*-semigroups, *Hacet. J. Math. Stat.*, **44** (2015), 569–586.
23. N. Kausar, M. Munir, B. Islam, M. Alesemi, Salahuddin, M. Gulzar, Ideals in *LA*-rings, *Ital. J. Pure Appl. Math.*, **44** (2020), 731–744.
24. W. A. Khan, A. Taouti, A. Salami, Z. Hussain, On gamma *LA*-rings and gamma *LA*-semirings, *Eur. J. Pure Appl. Math.*, **14** (2021), 989–1001. https://doi.org/10.29020/nybg.ejpam.v14i3.4034
25. W. Khan, F. Yousafzai, M. Khan, On generalized ideals of left almost semigroups, *Eur. J. Pure Appl. Math.*, **9** (2016), 277–291.
26. C. G. Massouros, N. Yaqoob, On theory of left/right almost groups and hypergroups with their relevant enumerations, *Mathematics*, **9** (2021), 1828. https://doi.org/10.3390/math9151828
27. D. Mruduladevi, G. Shobhalatha, T. Padma Praveen, Congruences on la-semirings and variant of semigroup, *Int. J. Math. Trends Technol.*, **40** (2016), 180–182.
28. F. Marty, Sur une generalization de la notion de group, *8th Congress Mathematics Scandinaves*, Stockholm, 1934.
29. W. Nakkhasen, Left almost semihyperrings characterized by their hyperideals, *AIMS Math.*, **6** (2021), 13222–13234. https://doi.org/10.3934/math.2021764
30. S. Nawaz, M. Gulistan, S. Khan, Weak *LA*-hypergroups; neutrosophy, enumeration and redox reaction, *Neutrosophic Sets Sy.*, **36** (2020), 352–368.
31. S. Nawaz, I. Rehman, M. Gulistan, On left almost semihyperrings, *Int. J. Anal. Appl.*, **16** (2018), 528–541. https://doi.org/10.28924/2291-8639-16-2018-528
32. W. Nakkhasen, B. Pibaljommee, Intra-regular semihyperrings, *J. Discret. Math. Sci. C.*, **22** (2019), 1019–1034. https://doi.org/10.1080/09720529.2019.1649818
33. P. V. Protić, N. Stevanović, AG-test and some general properties of Abel-Grassmann's groupoids, *Pure Math. Appl.*, **4** (1995), 371–383.
34. I. Rehman, A. Razzaque, M. I. Faraz, Neutrosophic set approach to study the characteristic behavior of left almost rings, *Neutrosophic Sets Sy.*, **46** (2021), 24–36.

35. I. Rehman, N. Yaqoob, S. Nawaz, Hyperideals and hypersystems in *LA*-hyperrings, *Songklanakarin J. Sci. Technol.*, **39** (2017), 651–657. https://doi.org/10.14456/sjst-psu.2017.80

36. A. S. Sezer, Certain characterizations of *LA*-semigroups by soft sets, *J. Intell. Fuzzy Syst.*, **27** (2014), 1035–1046. https://doi.org/10.3233/IFS-131064

37. T. Shah, I. Rehman, On *LA*-rings of finitely nonzero functions, *Int. J. Contemp. Math. Sciences*, **5** (2010), 209–222.

38. J. Tang, X. Y. Xie, Z. Gu, A study on weak hyperfilters of ordered semihypergroups, *AIMS Math.*, **6** (2021), 4319–4330. https://doi.org/10.3934/math.2021256

39. T. Vougiouklis, *Hyperstructures and their representations*, Handronic press, 1994.

40. F. Yousafzai, A. Iampam, J. Tang, Study on smallest (fuzzy) ideals of *LA*-semigroups, *Thai J. Math.*, **16** (2018), 549–561.

41. I. Younas, Q. Mushtaq, A. Rafiq, Presentation of inverse *LA*-semigroups, *Maejo Int. J. Sci. Technol.*, **14** (2020), 242–251.

42. N. Yaqoob, Approximations in left almost polygroups, *J. Intell. Fuzzy Syst.*, **36** (2019), 517–526.

43. N. Yaqoob, P. Corsini, F. Yousafzai, On intra-regular left almost semihypergroups with pure left identity, *J. Math.*, **2013** (2013), 510790. https://doi.org/10.1155/2013/510790

44. N. Yaqoob, I. Cristea, M. Gulistan, S. Nawaz, Left almost polygroups, *Ital. J. Pure Appl. Math.*, **39** (2018), 465–474.

45. N. Yaqoob, M. Gulistan, Partially ordered left almost semihypergroups, *J. Egyptian Math. Soc.*, **23** (2015), 231–235. https://doi.org/10.1016/j.joems.2014.05.012

46. P. Yiarayong, On generalizations of quasi-prime ideals of an ordered left almost semigroups, *Afrika Mathematatika*, **32** (2021), 969–982. https://doi.org/10.1007/s13370-021-00873-x

AIMS Press