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Research article

A study of behaviour for fractional order diabetes model via the nonsingular kernel

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Abstract: A susceptible diabetes comorbidity model was used in the mathematical treatment to explain the predominance of mellitus. In the susceptible diabetes comorbidity model, diabetic patients were divided into three groups: susceptible diabetes, uncomplicated diabetics, and complicated diabetics. In this research, we investigate the susceptible diabetes comorbidity model and its intricacy via the Atangana-Baleanu fractional derivative operator in the Caputo sense (ABC). The analysis backs up the idea that the aforesaid fractional order technique plays an important role in predicting whether or not a person will develop diabetes after a substantial immunological assault. Using the fixed point postulates, several theoretic outcomes of existence and Ulam's stability are proposed for the susceptible diabetes comorbidity model. Meanwhile, a mathematical approach is provided for determining the numerical solution of the developed framework employing the Adams type predictor–corrector algorithm for the ABC-fractional integral operator. Numerous mathematical representations correlating to multiple fractional orders are shown. It brings up the prospect of employing this structure to generate framework regulators for glucose metabolism in type 2 diabetes mellitus patients.

Keywords: fractional diabetes model; Atangana-Baleanu fractional derivative operator; fixed point theory; Ulam's stability; Adams type predictor–corrector method
Mathematics Subject Classification: 46S40, 47H10, 54H25

1. Introduction

Epidemiology is the investigation of how illnesses disseminate in a live entity in relation to its surroundings [1]. The epidemiology of an illness can be studied using numerical simulations. Several studies have attempted to predict and simulate the transmission of contagious ailments in the past, including measles [2], rubella [3], HIV [4], dengue fever [5], tuberculosis [6], and more recently, ebola [7] and the Zika virus [8].

Mathematical simulation is being employed to investigate not only the transmission of contagious ailments, but also increasingly non-communicable diseases, as research advances. Medications and other environmental ailments can often be modeled [9]. This is possible owing to the characteristics of how it spreads, namely via intimate communication as the media spreads.

Hyperglycemia is a non-communicable ailment with a variety of "dispersion" characteristics, the first being the influence of interpersonal contacts on dietary modification. Hyperglycemia is a long-term illness characterized by elevated plasma glucose concentrations. An individual is declared to have hyperglycemia if their fasting blood glucose level is greater than 126 mg/dL or if their blood sugar level is greater than 200 mg/dL two hours after eating. The pancreatic enzymes that generate the hormone glucagon are unable to function properly. Without insulin, the body's systems are unable to accept and convert glycogen into vitality, resulting in extreme tiredness.

Scientific tools have been shown to be an effective instrument in gaining a better knowledge of hyperglycemia patterns. Different formulations depending on insulin levels and concentrations have been employed to describe the glycemic relationship. Considering specific settings and assumptions, all of these systems are viable. However, while these approaches may indeed be beneficial in research, they all have restrictions with respect to estimating blood sugar levels in a real therapeutic scenario owing to the underlying demand for continuously modified inputs about parameter estimates such as glucose concentrations and insulin accessibility [10].

Diabetics without problems prefer to live an unpleasant lifestyle in their everyday pursuits. The connection involving diabetics with an unsustainable diet and normal volunteers, reported by Hill et al. [11], can result in behavioural "dissemination." Prevalence is the effect of lifestyle "spread." Diabetic predominance develops as high pervasiveness rises. The ratio of susceptible individuals in a particular individual group termed susceptible should also be determined in calculating the proportion of probable interaction ($\mathbb{S}$). So, the $\mathbb{DC}$ model, according to Boutayeb et al. [12] can be written as

$$\begin{cases} \frac{d\mathbb{D}}{d\tau} = \mathbf{I} - (\lambda + \mu)\mathbb{D} + \eta\mathbb{C}, \\ \frac{d\mathbb{C}}{d\tau} = \lambda\mathbb{D} - (\eta + \delta + \nu + \mu)\mathbb{C}. \end{cases} \tag{1.1}$$

The $\mathbf{D}\mathbb{C}$ (1.1) model is converted into the susceptible diabetes complexity model attributable to the susceptible person category ($\mathbb{SDC}$). If is the proportion of association generating prevalence denoted by β, then $\beta\mathbb{SD}$ is the number of instances caused by behavioral determinants. So each person who has subsequently been diagnosed with hyperglycemia is presumed to be free of problems, resulting in an $\beta\mathbb{SD}$ rise in the number of people in the $\mathbb{D}$ category. Simultaneously, the number of people in the $\mathbb{S}$ class falls by about $\beta\mathbb{SD}$. Spontaneous fatalities are definitely a possibility in the $\mathbb{S}$. The spontaneous rate of death in the S part is the same as the natural mortality rate in the $\mathbb{D}$ and $\mathbb{C}$ compartments individually. As a result of normal mortality, the amount of persons in the $\mathbb{S}$ category dropped by $\mu\mathbb{S}$.

Population growth is represented by γ and the prevalence of genetic abnormalities is indicated by ρ. The proportion of healthy individuals born is $\gamma\mathbb{S} + \gamma(1-\rho)(\mathbb{D}+\mathbb{C})$, while the number of people born with hereditary diseases is $\gamma\rho(\mathbb{D}+\mathbb{C})$. The number of people in the $\mathbb{S}$ category grows as more people are born healthy, while the number of individuals in the $\mathbb{D}$ segment grows as more people are born with hereditary diseases.

As a result, the $\mathbb{SDC}$ model was used to calculate the mellitus prevalence that considers behaviour and genetic predisposition as determinants of prevalence while not excluding people with impairments from the demographic. The $\mathbb{SDC}$ framework is summarized as follows:

$$\begin{cases} \frac{d\mathbb{S}}{d\tau} = \gamma\mathbb{S} + \gamma(1-\rho)(\mathbb{D}+\mathbb{C}) - \beta\mathbb{S}\mathbb{D} - \mu\mathbb{S}, \\ \frac{d\mathbb{D}}{d\tau} = \gamma\mathbb{S}(\mathbb{D}+\mathbb{C}) - (\lambda+\mu)\mathbb{D} + \gamma\mathbb{C}, \\ \frac{d\mathbb{C}}{d\tau} = \lambda\mathbb{D} - (\mu+\delta+\eta)\mathbb{D} + \gamma\mathbb{C}. \end{cases} \tag{1.2}$$

supplemented with the initial settings $\mathbb{S}(0) > 0, \mathbb{D} > 0$ and $\mathbb{C}(0) > 0$. The values $\gamma, \beta, \eta, \delta, \lambda, \mu, \rho > 0$ and $\rho \in [0, 1]$, respectively represents the birth rate, interaction rate, recovery rate of complications, death rate due complications, occurrence rate of complications, natural mortality rate, and the proportion of genetic disorder's birth. The $\mathbb{SDC}$ (1.2) model is a first-order nonlinear differential equation system.

Fractional calculus has been shown to be a superb tool for depicting the hereditary features of various structures in recent decades, see [13–17]. Additionally, fractional differential frameworks have been implemented in numerous domains of real-world phenomena, including bifurcation, chaos, thermodynamics, finance, and epidemics, having various sorts of fractional techniques involving Coimbra, Davison, and Essex, Riez, Caputo, Hadamard, Riemann-Liouville, Katugumpola, Caputo–Fabrizio, and fractal–fractional, see [18–27]. This combination has recently gained a lot of importance, primarily since fractional differential equations have turned out to be incredible instruments for presenting a few extremely complicated marvels in a variety of diverse and limitless scientific domains; reviewers are directed to [28–35]. The ABC-FD operator is among the most widely used operators. The implementation of such a fractional operator is influenced by the observation that it eliminates the redundancy encountered in the Caputo fractional derivative. The Atangana-Baleanu derivative [36] is a fractional derivative having a nonsingular and nonlocal kernel that is used to simulate physical and biological phenomena and became the pioneer to employ a fractional-order derivative in the component of a non-singular having the Mittag-Leffler function in the kernel. In several real-world situations, the ABC-fractional derivative yields more accurate results [37]. Additionally, employing the Atangana-Baleanu derivative to describe the transmission dynamics involving delay is a novelty in the research. The infection will be controlled by the order of the fractional operator. Recently, Ghanbari et al. [38] expounded the estimates for immune and tumor cells in immunogenetic tumour model pertaining to the ABC-FD operator. Ahmad et al. [39] proposed the analysis of the fractional mathematical model of the rotavirus epidemic with the effects of breastfeeding and vaccination under the ABC-FD operator. Rahman et al. [40] established the solution of a nonlinear fractional mathematical model of tuberculosis (TB) disease with incomplete treatment employing ABC-FD.

Owing to the aforementioned phenomena, no articles have examined the mathematical model of $\mathbb{SDC}$ having multiple fractional derivatives. The ABC-fractional derivative has been incorporated into the $\mathbb{SDC}$ model, which is the manuscript's innovation. As a result, we are concerned about addressing gaps by analyzing the $\mathbb{SDC}$ model [12] under the ABC-fractional derivative with order. Consequently,

the classical model (1.2) is expanded to fractional-order systems by inserting the ABC fractional operator ${}^{ABC}_{\tau}\mathbf{D}^{\phi}_{a_1}$ for the classical time derivative $d/d\tau$.

The ABC-FD of the improved $\mathbb{SDC}$ transmission model suggests the following model:

$$\begin{cases} {}^{ABC}_{\tau}\mathbf{D}^{\phi}_{a_1}\mathbb{S}(\tau) = \gamma\mathbb{S} + \gamma(1-\rho)(\mathbb{D}+\mathbb{C}) - \beta\mathbb{S}\mathbb{D} - \mu\mathbb{S}, \\ {}^{ABC}_{\tau}\mathbf{D}^{\phi}_{a_1}\mathbb{D}(\tau) = \gamma\mathbb{S}(\mathbb{D}+\mathbb{C}) - (\lambda+\mu)\mathbb{D} + \gamma\mathbb{C}, \\ {}^{ABC}_{\tau}\mathbf{D}^{\phi}_{a_1}\mathbb{C}(\tau) = \lambda\mathbb{D} - (\mu+\delta+\eta)\mathbb{D} + \gamma\mathbb{C}. \end{cases} \tag{1.3}$$

subject to the ICs $(\mathbb{S},\mathbb{D},\mathbb{C}) = (\mathbb{S}_0,\mathbb{D}_0,\mathbb{C}_0)$. The explanations of all the characteristics are presented above. Recently, Saleem et al. [41] established the Caputo Fabrizio fractional order model for control of glucose in insulin therapies for diabetes, Singh et al. [42] obtained the solution of fractional diabetes model with exponential law and Dubey et al. [43] presented the mathematical model of diabetes and its complication involving fractional operator without singular kernal.

The primary goal of this publication is to examine the factors that impact the transmission of this genetic disease and slow it down or, in the worst-case scenario, make it epidemic, as measured by the number of replicates. For the present work, the prominent fixed point theorems are used to prove the existence and uniqueness of the results. To illustrate the stability evaluation, the framework of diverse Ulam's stability is offered. Furthermore, we apply Alkahtani et al. [44] unique mathematical approach to obtain the estimated solutions of the $\mathbb{S},\mathbb{D},\mathbb{C}$ for various fractional orders.

2. Preliminaries

This portion highlights the most important and pertinent topics utilized in this article.

Definition 2.1. ([36]) For $\phi \in [0,1]$ and let $f_1 \in H_1(x_1,y_1),\ x_1 < y_1$, then the ABC-FD of a mapping f_1 of order ϕ is stated as follows:

$${}^{ABC}_{\tau}\mathbf{D}^{\phi}_{a_1}f_1(\tau) = \frac{\mathbb{A}(\phi)}{1-\phi}\int_{a_1}^{\tau} E_{\phi}\Big(-\frac{\phi}{1-\phi}(\tau-\mathbf{u})^{\phi}\Big)\frac{d}{d\tau}f_1(\mathbf{u})d\mathbf{u}, \quad 0 < a_1 < \tau, \tag{2.1}$$

where $\mathbb{A}(\phi) = 1-\phi+\frac{\phi}{\Gamma(\phi)}$ represents the normalization function, satisfying the property $\mathbb{A}(0) = \mathbb{A}(1) = 1$ and E_{ϕ} signifies the Mittag-Leffler as a special function in the kernel is presented as

$$E_{\phi}(z_1) = \sum_{\ell=0}^{\infty}\frac{z_1^{\ell}}{\Gamma(\phi\ell+1)}, \ \ z_1,\phi \in \mathbb{C}, \Re(\phi) > 0. \tag{2.2}$$

Definition 2.2. ([36]) Let $f_1 \in H_1(x_1,y_1),\ x_1 < y_1$, then the ABC fractional integral of a mapping f_1 of order ϕ is stated as follows:

$${}^{ABC}_{\tau}\mathbf{I}^{\phi}_{a_1}f_1(\tau) = \frac{1-\phi}{\mathbb{A}(\phi)}f_1(\tau) + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_{a_1}^{\tau}(\tau-\mathbf{u})^{\phi-1}f_1(\mathbf{u})d\mathbf{u}, \quad 0 < \tau < a_1. \tag{2.3}$$

Evidently, if $\phi = 0$ and $\phi = 1$, then ones attain the initial mapping and classical Riemann-integral, respectively.

In order to prove our findings, we demonstrate some important results from fixed point F_p theory.

Lemma 2.3. *([45])(**Contraction mapping**) Suppose there be a Banach space ϖ, then the map $\mathbf{T}$: $\varpi \mapsto \varpi$ is contraction, if*

$$\left\|\mathbf{T}x_1 - \mathbf{T}y_1\right\| \leq \mathbb{L}\|x_1 - y_1\|, \quad \forall x_1, y_1 \in \varpi, \mathbb{L} \in (0,1). \tag{2.4}$$

Lemma 2.4. *([45])(**Banach's fixed point theorem**) Suppose there be a non-empty closed subset $\tilde{U}$ of a Banach space $\mathbb{E}$. Then any contraction mapping $\mathbf{Q}$ from $\tilde{U}$ into itself has a unique F_p.*

Lemma 2.5. *([45])(**Krasnoselskii's fixed point theorem**) Suppose there be a non-empty, closed, convex subset $\tilde{U}$ of a Banach space $\mathbb{E}$. Assume that there be two maps $\mathcal{T}_1, \mathcal{T}_2$ such that (a) $\mathcal{T}_1 x_1 + \mathcal{T}_2 x_2 \in \tilde{U}$, $\forall x_1, x_2 \in \tilde{U}$; (c) $\mathcal{T}_1$ is compact and continuous; (d) $\mathcal{T}_2$ is a contraction mapping. Then there exists $z_1 \in \tilde{U}$ such that $\mathcal{T}_1 z_1 + \mathcal{T}_2 z_1 = z_1$.*

3. Existence consequences of susceptible diabetes complicated mathematical model

In what follows, we investigate the existence and uniqueness of findings for the fractional $\mathbb{SDC}$ system utilizing of Lemma 2.4 and 2.5 F_p consequences.

Throughout this investigation, we express the *ABC*-fractional diabetes system (1.3) as follows:

$$\begin{cases} {}^{ABC}_{\tau}\mathbf{D}_0^{\phi}\Upsilon(\tau) = \Psi(\tau, \Upsilon(\tau)), \\ \Upsilon(0) = \Upsilon_0 \geq 0, \ \ 0 < \tau < \mathcal{T} < \infty, \end{cases} \tag{3.1}$$

where $\Upsilon(\tau) = (G_1, G_2, G_3)$ denotes the system parameters and a vector mapping Ψ is continuous such that

$$\Psi \ = \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} = \begin{bmatrix} \gamma\mathbb{S} + \gamma(1-\rho)(\mathbb{D}+\mathbb{C}) - \beta\mathbb{S}\mathbb{D} - \mu\mathbb{S} \\ \gamma\mathbb{S}(\mathbb{D}+\mathbb{C}) - (\lambda+\mu)\mathbb{D} + \gamma\mathbb{C} \\ \lambda\mathbb{D} - (\mu+\delta+\eta)\mathbb{D} + \gamma\mathbb{C} \end{bmatrix} \tag{3.2}$$

supplemented with initial settings $\Upsilon_0 = (\mathbb{S}_0, \mathbb{D}_0, \mathbb{C}_0)$. Implementing the Definition 2.2, we have the following formulation

$$\Upsilon(\tau) = \Upsilon_0(\tau) + \frac{1-\phi}{\mathbb{A}(\phi)}\Psi(\tau, \Upsilon(\tau)) + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^{\tau}(\tau - \mathbf{u})^{\phi-1}\Psi(\mathbf{u}, \Upsilon(\mathbf{u}))d\mathbf{u}.$$

Now, introducing a $\mathbf{B}_s$ by utilizing $\Omega = [0, \mathcal{T}]$ as $\mathbf{V} = \mathbb{C}(\Omega, \mathbb{R}^3_+)$ induced by the norm presented as

$$\left\|\Upsilon\right\| = \left\|\mathbb{S}\right\| + \left\|\mathbb{D}\right\| + \left\|\mathbb{C}\right\|,$$

where $\sup\limits_{\tau\in\Omega}\{\left|\Upsilon(\tau)\right|\} = \sup\limits_{\tau\in\Omega}\{\left|\mathbb{S}(\tau)\right|\} + \sup\limits_{\tau\in\Omega}\{\left|\mathbb{D}(\tau)\right|\} + \sup\limits_{\tau\in\Omega}\{\left|\mathbb{C}(\tau)\right|\}$.

3.1. Uniqueness analysis

The significance and validity of the fractional $\mathbb{SDC}$ model (1.3) will be studied in this section using Banach's F_p postulate and the ABC derivative operator.

Theorem 3.1. *Let there be a continuous quadratic vector mapping* $\Psi : \Omega \times \mathbb{R}^3 \mapsto \mathbb{R}$ *such that:*
$(\boldsymbol{A}_1)$ $\exists$ *a positive constant* $\mathbb{L}_\Psi > 0$ *such that*

$$\Big|\Psi(\tau, \Upsilon_1(\tau)) - \Psi(\tau, \Upsilon_2(\tau))\Big| \leq \mathbb{L}_\Psi \Big|\Upsilon_1(\tau) - \Upsilon_2(\tau)\Big|, \ \ \forall \Upsilon_1, \Upsilon_2 \in \mathbf{V}, \ \forall \mathbf{t} \in \Omega.$$

If

$$\Big((1-\phi)\Gamma(\phi) + \mathbf{T}^\phi\Big)\mathbb{L}_\Psi < \mathbb{A}(\phi)\Gamma(\phi), \tag{3.3}$$

then the fractional system (1.3) *has a only one solution on* Ω.

Proof. We previously transformed the IVP (3.1) (that is analogous to the ABC-fractional diabetes framework (1.3)) into a F_p formulation $\Upsilon = \mathbf{T}\Upsilon$. Further, we suppose a map $\mathbf{T} : \mathbf{V} \mapsto \mathbf{V}$ described as

$$(\mathbf{T}\Upsilon)(\tau) = \Upsilon_0(\tau) + \frac{1-\phi}{\mathbb{A}(\phi)}\Psi(\tau, \Upsilon(\tau)) + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau - \mathbf{u})^{\phi-1}\Psi(\mathbf{u}, \Upsilon(\mathbf{u}))d\mathbf{u}. \tag{3.4}$$

Evidently, the IVP (3.1) has a result if and only if the map $\mathbf{T}$ has F_{ps}.
Considering there is a non-negative number $\mathbb{K}_1$ such that $\sup_{\tau\in\Omega}\Big|\Psi(\tau, 0)\Big| = \mathbb{K}_1 < +\infty$. Now, proposing a bounded, closed, and convex subset $\mathbb{B}_{r_1}$ of $\mathbf{V}$, where $\mathbb{B}_{r_1} = \{\Upsilon \in \mathbf{V} : \Big\|\Upsilon\Big\| \leq r_1\}$, where r_1 is selected in such a way that

$$r_1 \geq \frac{\Big\|\Upsilon_0\Big\|\mathbb{A}(\phi)\Gamma(\phi) + \Big((1-\phi)\Gamma(\phi) + \mathcal{T}^\phi_{\max}\Big)\mathbb{K}_1}{\mathbb{A}(\phi)\Gamma(\phi) - [(1-\phi)\Gamma(\phi) - \mathcal{T}^\phi_{\max}]\mathbb{L}_\Psi}. \tag{3.5}$$

The proof is divided into two parts.
Case I: We demonstrate that $\mathbf{T}\mathbb{B}_{r_1} \subset \mathbb{B}_{r_1}$.
for any $\Upsilon \in \mathbb{B}_{r_1}$, we have

$$\begin{aligned}
\Big|(\mathbf{T}\Upsilon)(\tau)\Big| \quad &\leq \Big\|\Upsilon_0\Big\| + \frac{1-\phi}{\mathbb{A}(\phi)}\Big|\Psi(\tau), \Upsilon(\tau)\Big| + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}\Big|\Psi(\mathbf{u}, \Upsilon(\mathbf{u}))\Big|d\mathbf{u} \\
&\leq \Big\|\Upsilon_0\Big\| + \frac{1-\phi}{\mathbb{A}(\phi)}\Big[\Big|\Psi(\tau), \Upsilon(\tau) - \Psi(\tau, 0)\Big| + \Big|\Psi(\tau, 0)\Big|\Big] \\
&\quad + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}\Big[\Big|\Psi(\mathbf{u}, \Upsilon(\mathbf{u})) - \Psi(\mathbf{u}, 0)\Big| + \Big|\Psi(\mathbf{u}, 0)\Big|\Big]d\mathbf{u} \\
&\leq \Big\|\Upsilon_0\Big\| + \frac{1-\phi}{\mathbb{A}(\phi)}[\mathbb{L}_\Psi r_1 + \mathbb{K}_1] + \frac{\phi[\mathbb{L}_\Psi r_1 + \mathbb{K}_1]}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}d\mathbf{u} \\
&\leq \Big\|\Upsilon_0\Big\| + \Big(\frac{1-\phi}{\mathbb{A}(\phi)} + \frac{\mathcal{T}^\phi_{\max}}{\mathbb{A}(\phi)\Gamma(\phi)}\Big)[\mathbb{L}_\Psi r_1 + \mathbb{K}_1] \leq r_1,
\end{aligned} \tag{3.6}$$

which illustrates that $\mathbf{T}\mathbb{B}_1 \subset \mathbb{B}_{r_1}$.
Case II: To prove $\mathbf{T}$ is contraction, for this, for every $\Upsilon_1, \Upsilon_2 \in \mathbb{B}_{r_1}$ and for any $\tau \in \Omega$, we find

$$\Big|(\mathbf{T}\Upsilon_1)(\tau) - \mathbf{T}\Upsilon_2)(\tau)\Big|$$

$$\leq \frac{1-\phi}{\mathbb{A}(\phi)}\Big[\Big|\Psi(\tau,\Upsilon_1(\tau))-\Psi(\tau,\Upsilon_2(\tau))\Big|+$$
$$+\frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau(\tau-\mathbf{u})^{\phi-1}\Big|\Psi(\mathbf{u},\Upsilon_1(\mathbf{u}))-\Psi(\mathbf{u},\Upsilon_2(\mathbf{u}))\Big|d\mathbf{u}$$
$$\leq \frac{1-\phi}{\mathbb{A}(\phi)}\Big[\Big|\Upsilon_1(\tau)-\Upsilon_2(\tau)\Big|+$$
$$+\frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau(\tau-\mathbf{u})^{\phi-1}\Big|\Upsilon_1(\mathbf{u})-\Upsilon_2(\mathbf{u})\Big|d\mathbf{u}$$
$$\leq \Big(\frac{(1-\phi)\Gamma(\phi)+\mathcal{T}_{\max}^{\phi}}{\mathbb{A}(\phi)\Gamma(\phi)}\Big)\mathbb{L}_\Psi\Big\|\Upsilon_1-\Upsilon_2\Big\|. \tag{3.7}$$

Clearly, observe that $\left(\frac{(1-\phi)\Gamma(\phi)+\mathcal{T}_{\max}^{\phi}}{\mathbb{A}(\phi)\Gamma(\phi)}\right)<1$, using the fact of Lemma 2.3, deduce $\mathbf{T}$ is contraction. So, $\mathbf{T}$ has a unique F_p. This means that fractional order $\mathbb{SDC}$ model (1.3) has only one elucidation on Ω. □

3.2. Existence consequence using Krasnoselskii's fixed point theorem

Theorem 3.2. *Suppose the hypothesis* $(\boldsymbol{A}_1)$ *satisfies and* $(\boldsymbol{A}_2)$ *there exists positive constants* Ω_Ψ *and* Ψ_Ψ *such that*

$$\Big|\Psi(\tau,\Upsilon(\tau))\Big|\leq\Omega_\Psi\Big|\Upsilon(\tau)\Big|+\Psi_\Psi,\ \ \forall\ \Upsilon\in\mathbf{V}\ \textit{and}\ \forall\ \tau\in\Omega.$$

Then there exists at least one solution of the fractional $\mathbb{SDC}$ *model* (1.3)*, given that* $(1-\phi)\mathbb{L}_\Psi<\mathbb{A}(\phi)$.

Proof. Suppose a mapping $\mathbf{T}:\mathbf{V}\mapsto\mathbf{V}$ defined by $(\mathbf{T}\Upsilon)(\tau)=(\tau\Upsilon)(\tau)+(\mathbf{T}_2\Upsilon)(\tau),\ \ \Upsilon\in\mathbf{V},\ \tau\in\Omega$, where

$$(\tau\Upsilon)(\tau)=\Upsilon_0+\frac{1-\phi}{\mathbb{A}(\phi)}\Psi(\tau,\Upsilon(\tau)), \tag{3.8}$$

$$(\mathbf{T}_2\Upsilon)(\tau)=\frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau(\tau-\mathbf{u})^{\phi-1}\Psi(\mathbf{u},\Upsilon(\mathbf{u}))d\mathbf{u}. \tag{3.9}$$

Assume that there be a closed convex set having radius $\mathbb{B}_{r_2}=\{\Upsilon\in\mathbf{V}:\|\Upsilon\|\leq r_2\}$ can be expressed as

$$r_2\geq\frac{\Big\|\Upsilon_0\Big\|+\left(\frac{1-\phi}{\mathbb{A}(\phi)}+\frac{\mathcal{T}_{\max}^{\phi}}{\mathbb{A}(\phi\Gamma(\phi))}\right)\Psi_\Psi}{1-\left(\frac{1-\phi}{\mathbb{A}(\phi)}+\frac{\mathcal{T}_{\max}^{\phi}}{\mathbb{A}(\phi\Gamma(\phi))}\right)\Omega_\Psi}. \tag{3.10}$$

the proof is consists of four cases.

Case I. We evaluate that $\mathbf{T}_1\Upsilon_1+\mathbf{T}_2\Upsilon_2\in\mathbb{B}_{r_1}$, for every $\Upsilon_1,\Upsilon_2\in\mathbb{B}_{r_2}$.
In view of the operator (3.8), we have

$$\Big|(\mathbf{T}_1\Upsilon_1)(\tau)-(\mathbf{T}_2\Upsilon_2)(\tau)\Big|$$

$$\leq \left\|\Upsilon_0\right\| + \frac{1-\phi}{\mathbb{A}(\phi)}\left|\Psi(\tau), \Upsilon_1(\tau)\right| + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}\left|\Psi(\mathbf{u}, \Upsilon_2(\mathbf{u}))\right|d\mathbf{u}$$

$$\leq \left\|\Upsilon_0\right\| + \frac{1-\phi}{\mathbb{A}(\phi)}[\Omega_\Psi r_2 + \Psi_\Psi] + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}[\Omega_\Psi r_2 + \Psi_\Psi]d\mathbf{u}$$

$$\leq \left\|\Upsilon_0\right\| + \left(\frac{1-\phi}{\mathbb{A}(\phi)} + \frac{\mathcal{T}_{\max}^{\phi}}{\mathbb{A}(\phi\Gamma(\phi))}\right)\Psi_\Psi + \left(\frac{1-\phi}{\mathbb{A}(\phi)} + \frac{\mathcal{T}_{\max}^{\phi}}{\mathbb{A}(\phi\Gamma(\phi))}\right)r_2\Omega_\Psi$$

$$\leq r_2, \tag{3.11}$$

Thus, we conclude that $\left\|\mathbf{T}_1\Upsilon_1 + \mathbf{T}_2\Upsilon_2\right\| \leq r_2$. Then $\mathbf{T}_1\Upsilon_1 + \mathbf{T}_2\Upsilon_2 \in \mathbb{B}_{r_2}$ for all $\Upsilon_1, \Upsilon_2 \in \mathbb{B}_{r_2}$.
Case II. Further, we prove that $\mathbf{T}_1$ is a contraction mapping. For any $\Upsilon_1, \Upsilon_2 \in \mathbb{B}_{r_2}$, we have

$$\begin{aligned}
&\left|(\mathbf{T}_1\Upsilon_1)(\tau) - (\mathbf{T}_2\Upsilon_2)(\tau)\right| \\
&\leq \frac{1-\phi}{\mathbb{A}(\phi)}\left|\Psi(\tau), \Upsilon_1(\tau) - \Psi(\tau), \Upsilon_2(\tau)\right| \\
&\leq \mathbb{L}_\Psi\frac{1-\phi}{\mathbb{A}(\phi)}\left|\Upsilon_1(\tau) - \Upsilon_2(\tau)\right|.
\end{aligned} \tag{3.12}$$

Implies that

$$\left\|\mathbf{T}_1\Upsilon_1 - \mathbf{T}_2\Upsilon_2\right\| \leq \mathbb{L}_\Psi\frac{1-\phi}{\mathbb{A}(\phi)}\left\|\Upsilon_1 - \Upsilon_2\right\|.$$

Since $\mathbb{L}_\Psi\frac{1-\phi}{\mathbb{A}(\phi)} < 1$, which shows that $\mathbf{T}_1$ is contraction mapping.
Case III. Now, to prove $\mathbf{T}_2$ is continuous and compact.
For this, suppose that there be a sequence Υ_{n_1} such that $\Upsilon_1 \mapsto \Upsilon \in \mathbf{V}$. Then, for any $\tau \in \Omega$, we have

$$\left|(\mathbf{T}_2\Upsilon_{n_1})(\tau) - (\mathbf{T}_2\Upsilon)(\tau)\right| \leq \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}\left|\Psi(\mathbf{u}, \Upsilon_{n_1}(\mathbf{u})) - \Psi(\mathbf{u}, \Upsilon(\mathbf{u}))\right|d\mathbf{u}$$

$$\leq \frac{\mathcal{T}_{\max}^{\phi}}{\mathbb{A}(\phi)\Gamma(\phi)}\left\|\Psi(., \Upsilon_{n_1}(.)) - \Psi(., \Upsilon(.))\right\|. \tag{3.13}$$

Since Ψ is continuous and $\mathbf{T}_2$ is also continuous. Then we have $\left\|\mathbf{T}_2\Upsilon_n - \mathbf{T}_2\Upsilon\right\| \mapsto 0,\ as\ n_1 \mapsto \infty$.
Further, we show that $\mathbf{T}_2$ is uniformly bounded on $\mathbb{B}_{r_2}$ ($\mathbf{T}_2$ *is relatively compact*). For any $\Upsilon \in \mathbb{B}_{r_2}$ and $\tau \in \mathbb{J}$, we have

$$\left|(\mathbf{T}_2\Upsilon)\right| \leq \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}\left|\Psi(\mathbf{u}, \Upsilon(\mathbf{u}))\right|d\mathbf{u} \leq \frac{\mathcal{T}_{\max}^{\phi}}{\mathbb{A}(\phi)\Gamma(\phi)}[\Omega_\Lambda r_2 + \Psi_\Lambda], \tag{3.14}$$

which proves that $\mathbb{T}_2$ is uniformly bounded on $\mathbb{B}_{r_2}$.
Case IV. To prove $\mathbf{T}_2$ is equi-continuous, for this let $\sigma_1, \sigma_2 \in \Omega$ having $0 \leq \sigma_1 \leq \sigma_2 \leq \mathcal{T}$ and $\Upsilon \in \mathbb{B}_{r_2}$, then we have

$$\left|(\mathbf{T}_2\Upsilon)(\sigma_2) - (\mathbf{T}_2\Upsilon)(\sigma_1)\right|$$

$$\leq \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\left|\int_0^{\sigma_2}(\sigma_2-\mathbf{u})^{\phi-1}\Psi(\mathbf{u},\Upsilon_1(\mathbf{u}))d\mathbf{u}-\int_0^{\sigma_1}(\sigma_{21}-\mathbf{u})^{\phi-1}\Psi(\mathbf{u},\Upsilon_1(\mathbf{u}))d\mathbf{u}\right|$$

$$\leq \frac{\phi[\Omega_\lambda r_1+\Psi_\Psi]}{\mathbb{A}(\phi)\Gamma(\phi)}\left|\int_0^{\sigma_2}(\sigma_2-\mathbf{u})^{\phi-1}d\mathbf{u}-\int_0^{\sigma_1}(\sigma_2-\mathbf{u})^{\phi-1}d\mathbf{u}\right|$$

$$\leq \frac{[\Omega_\lambda r_1+\Psi_\Psi]}{\mathbb{A}(\phi)\Gamma(\phi)}(2|\sigma_2-\sigma_1|^{\phi}). \tag{3.15}$$

Evidently, above expression is free of $\Upsilon \in \mathbb{B}_{r_2}$, the right side of the above variant (3.15) approaches to zero as $\sigma_2 \mapsto \sigma_1$. Thus, by Arzelá-Ascoli theorem , $\mathbf{T}_1\mathbb{B}_{r_2}$ is relatively compact and $\mathbf{T}_2$ is completely continuous. So, using the fact of Lemma 2.5, which deduce that fractional $\mathbb{SDC}$ model (1.3) has at least one solution on Ω. □

4. Stability analysis

This subsection discusses various fractional $\mathbb{SDC}$ system (1.3) necessary requirements that correlate to the hypotheses of the four assortments of Ulam's stability: **UH**, **GUH**, **UHR**, and **GUHR** stability. We shall start by stating Ulam's stability postulate, which would be applied throughout this segment. For $\zeta > 0$ and let there is a positive real number such that a continuous mapping $\mathcal{U}_\Psi : \Omega \mapsto \mathbb{R}^+$. Assume that

$$\left|{}^{ABC}_{\tau}\mathbf{D}^{\phi}_{0}\chi(\tau)-\Psi(\tau,\chi(\tau))\right| \leq \zeta, \quad \forall\ \tau \in \Omega, \tag{4.1}$$

$$\left|{}^{ABC}_{\tau}\mathbf{D}^{\phi}_{0}\chi(\tau)-\Psi(\tau,\chi(\tau))\right| \leq \zeta\mathcal{U}_\Psi, \quad \forall\ \tau \in \Omega, \tag{4.2}$$

$$\left|{}^{ABC}_{\tau}\mathbf{D}^{\phi}_{0}\chi(\tau)-\Psi(\tau,\chi(\tau))\right| \leq \mathcal{U}_\Psi, \quad \forall\ \tau \in \Omega, \tag{4.3}$$

where $\zeta = \max(\zeta_\vartheta)^{\bar{T}}, \ \ for \ \ \vartheta = 1,2,3$. Next, we are presenting the concepts of various sorts of stability as follows:

Definition 4.1. ([46]) We say that the fractional $\mathbb{SDC}$ model (1.3) is **UH** stable if $\exists$ a real number $\mathbb{C}_\Psi > 0$ such that for each $\zeta > 0$ and for every result $\chi \in \mathbf{V}$ of (4.1), $\exists$ a result $\Upsilon \in \mathbf{V}$ of the fractional $\mathbb{SDC}$ model (1.3) having

$$\left|\chi(\tau)-\Upsilon(\tau)\right| \leq \zeta\mathbb{C}_\Psi, \quad \tau \in \Omega, \tag{4.4}$$

where $\zeta = \max(\zeta_\vartheta^{\bar{T}})$ and $\mathbb{C}_\Psi = \max(\mathbb{C}_{\Psi_\vartheta})^{\bar{T}}, \ \ for \ \ \vartheta = 1,2,3$.

Definition 4.2. ([46]) We say that the fractional $\mathbb{SDC}$ model (1.3) is **GUH** stable if $\exists$ a mapping $\mathcal{U}_\Psi \in \mathbb{C}(\mathbb{R}^+,\mathbb{R}^+)$ having $\mathcal{U}_\Psi = 0$ such that for all $\zeta > 0$ and for every result $\chi \in \mathbf{V}$ of (4.2), $\exists$ a result $\Upsilon \in \mathbf{V}$ of the fractional $\mathbb{SDC}$ model (1.3) having

$$\left|\chi(\mathbf{t})-\Upsilon(\tau)\right| \leq \mathcal{U}_\Psi(\zeta), \quad \tau \in \Omega, \tag{4.5}$$

where $\zeta = \max(\zeta_\vartheta^{\bar{T}})$ and $\mathcal{U}_\Psi = \max(\mathcal{U}_{\Psi_\vartheta})^{\bar{T}}, \ \ for \ \ \vartheta = 1,2,3$.

Definition 4.3. ([46]) We say that the fractional $\mathbb{SDC}$ model (1.3) is **UHR** stable regarding to $\mathcal{U}_\Psi \in \mathbb{C}(\Omega, \mathbb{R}^+)$ if $\exists$ a real constant $K_{\mathcal{U}_\Psi} > 0$ such that for each $\zeta > 0$ and for every result $\chi \in \mathbf{V}$ of (4.2), $\exists$ a result $\Upsilon \in \mathbf{V}$ of the fractional $\mathbb{SDC}$ model (1.3) having

$$\left|\chi(\tau) - \Upsilon(\tau)\right| \leq K_{\mathcal{U}_\Psi}\zeta\mathcal{U}_\Psi(\tau), \quad \tau \in \Omega, \tag{4.6}$$

where $\zeta = \max(\zeta_\vartheta^{\bar{T}})$ and $K_{\mathcal{U}_\Psi} = \max(K_{\mathcal{U}_{\Psi_\vartheta}})^{\bar{T}}$ *and* $\mathcal{U}_\Psi = \max(K_{\mathcal{U}_{\Psi_\vartheta}})^{\bar{T}}$ *for* $\vartheta = 1, 2, 3$.

Definition 4.4. We say that the fractional $\mathbb{SDC}$ model (1.3) is **GUHR** stable regarding to $\mathcal{U}_\Psi \in \mathbb{C}(\Omega, \mathbb{R}^+)$ if $\exists$ a real constant $K_{\mathcal{U}_\Psi} > 0$ such that for each result $\chi \in \mathbf{V}$ of (4.3), $\exists$ a result $\Upsilon \in \mathbf{V}$ of the fractional $\mathbb{SDC}$ model (1.3) having

$$\left|\chi(\tau) - \Upsilon(\tau)\right| \leq K_{\mathcal{U}_\Psi}\mathcal{U}_\Psi(\tau), \quad \tau \in \Omega, \tag{4.7}$$

where $K_{\mathcal{U}_\Psi} = \max(K_{\mathcal{U}_{\Psi_\vartheta}})^{\bar{T}}$ *and* $\mathcal{U}_\Psi = \max(K_{\mathcal{U}_{\Psi_\vartheta}})^{\bar{T}}$ *for* $\vartheta = 1, 2, 3$.

Remark 1. Clearly, we observe that inequality (4.4) implies to inequality (4.5), inequality (4.6) implies to inequality (4.7) and inequality (4.6) implies to inequality (4.4) when $\mathcal{U}_\lambda(.) = 1$.

Remark 2. A mapping $\chi \in \mathbf{V}$ is a result of (4.1) if and only if $\exists$ a mapping $w \in \mathbf{V}$ (influenced by χ) such that the subsequent assertions hold:
(a) $|\omega(\tau)| \leq \zeta, \ \ \omega = \max(\omega_\vartheta)^{\bar{T}}, \ \ \forall\, \tau \in \Omega,$
(b) ${}_\tau^{ABC}\mathbf{D}_0^\phi\chi(\tau) = \Psi(\tau, \chi(\tau)) + \omega(\tau), \ \ \forall\, \tau \in \Omega.$

Remark 3. A mapping $\chi \in \mathbf{V}$ is a result of (4.2) if and only if $\exists$ a mapping $v \in \mathbf{V}$ (influenced by χ) such that the subsequent assertions hold:
(a) $|v(\tau)| \leq \zeta\mathcal{U}_\Psi(\tau), \ \ v = \max(v_\vartheta)^{\bar{T}}, \ \ \forall\, \tau \in \Omega,$
(b) ${}_\tau^{ABC}\mathbf{D}_0^\phi\chi(\tau) = \Psi(\tau, \chi(\tau)) + v(\tau), \ \ \forall\, \tau \in \Omega.$

4.1. Consequences of **UH** *and* **GUH** *stability*

Lemma 4.5. *For* $0 < \phi \leq 1$, *if* $\chi \in \mathbf{V}$ *is a result of* (4.1)*, then* χ *is a response of the subsequent variant:*

$$\left|\chi(\tau) - \mathcal{R}_\chi(\tau) - \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau - \mathbf{u})^{\phi-1}\Psi(\mathbf{u}, \chi(\mathbf{u}))d\mathbf{u}\right| \leq \left(\frac{1-\phi}{\mathbb{A}(\phi)} - \frac{\mathcal{T}_{\max}^\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\right)\zeta, \tag{4.8}$$

where $\mathcal{R}_\chi = \chi_0 + \frac{1-\phi}{\mathbb{A}(\phi)}\Psi(\tau, \chi(\tau))$.

Proof. Assume that χ be a solution of (4.1). Using the fact of Remark 2-(b), we have

$$\begin{cases} {}_\tau^{ABC}\mathbf{D}_0^\phi\chi(\tau) = \Psi(\tau, \chi(\tau)) + \omega(\tau), \ \ \tau \in \Omega \\ \chi(0) = \chi_0 \geq 0, \ \ 0 < \tau < \mathcal{T} < \infty, \end{cases} \tag{4.9}$$

Then the estimated solution of (4.9) can be expressed as

$$\chi(\tau) = \chi_0 \quad + \quad \frac{1-\phi}{\mathbb{A}(\phi)}\Psi(\tau, \chi(\tau)) + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau - \mathbf{u})^{\phi-1}\Psi(\mathbf{u}, \chi(\mathbf{u}))d\mathbf{u}$$

$$\leq \frac{1-\phi}{\mathbb{A}(\phi)}\omega(\tau) + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}\omega(\mathbf{u})d\mathbf{u}. \tag{4.10}$$

Using the fact of Remark 2-(a), we have

$$\begin{aligned}
&\left|\chi(\tau) - \mathcal{R}_\chi(\tau) + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}\Psi(\mathbf{u},\chi(\mathbf{u}))d\mathbf{u}\right| \\
&\leq \frac{1-\phi}{\mathbb{A}(\phi)}\left|\omega(\tau)\right| + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}\left|\omega(\mathbf{u})\right|d\mathbf{u} \\
&\leq \left(\frac{1-\phi}{\mathbb{A}(\phi)} + \frac{\mathcal{T}_{\max}^{\phi}}{\mathbb{A}(\phi)\Gamma(\phi)}\right)\zeta.
\end{aligned} \tag{4.11}$$

Which concludes the variant (4.8). □

Theorem 4.6. *Suppose that there be a continuous mapping* $\Psi : \Omega\times\mathbb{R}\mapsto\mathbb{R}$ *such that for every* $\Upsilon\in\mathbf{V}$. *Under the assumption of* ($\boldsymbol{A}_1$) *and* (3.3)*, then the fractional* $\mathbb{SDC}$ *system* (1.3) *is* **UH** *stable on* Ω.

Proof. Assume that $\zeta > 0$ and let $\chi\in\mathbf{V}$ be any response of (4.1). Let $\Upsilon\in\mathbf{V}$ be the only result of the system (3.1), we have

$$\begin{cases} {}^{ABC}_{\tau}\mathbf{D}_0^{\phi}\Upsilon(\tau) = \Psi(\tau,\Upsilon(\tau)), \quad \tau\in\Omega \\ \Upsilon(0) = \Upsilon_0, \end{cases} \tag{4.12}$$

where

$$\Upsilon(\tau) = \Upsilon_0 + \frac{1-\phi}{\mathbb{A}(\phi)}\Psi(\tau,\Upsilon(\tau)) + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}\Psi(\mathbf{u},\Upsilon(\mathbf{u}))d\mathbf{u}. \tag{4.13}$$

In view of Lemma 4.5 and the hypothesis of ($\mathbf{A}_1$), we have

$$\begin{aligned}
\left|\chi(\tau)-\Upsilon(\tau)\right| &\leq \left|\chi(\tau) - \mathcal{R}_\Upsilon(\tau) - \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}\Psi(\mathbf{u},\chi(\mathbf{u}))d\mathbf{u}\right| \\
&\leq \left|\chi(\tau) - \mathcal{R}_\chi(\tau) - \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}\Psi(\mathbf{u},\Upsilon(\mathbf{u}))d\mathbf{u}\right| \\
&\quad + \frac{\phi}{\mathbb{A}(\mathbf{u})\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}\left|\Psi(\mathbf{u},\chi(\mathbf{u})) - \Psi(\mathbf{u},\Upsilon(\mathbf{u}))\right|d\mathbf{u} \\
&\leq \left(\frac{1-\phi}{\mathbb{A}(\phi)} + \frac{\mathcal{T}_{\max}^{\phi}}{\mathbb{A}(\phi)\Gamma(\phi)}\right)\zeta + \frac{\phi\mathbb{L}_\Psi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}\left|\chi(\mathbf{u})-\Upsilon(\mathbf{u})\right|d\mathbf{u}
\end{aligned}$$

$$\leq \Big(\frac{1-\phi}{\mathbb{A}(\phi)} + \frac{\mathcal{T}^{\phi}_{\max}}{\mathbb{A}(\phi)\Gamma(\phi)}\Big)\zeta + \frac{\mathcal{T}^{\phi}_{\max}\mathbb{L}_{\Psi}}{\mathbb{A}(\phi)\Gamma(\phi)}\big|\chi(\mathbf{u}) - \Upsilon(\mathbf{u})\big|. \tag{4.14}$$

It follows that $\big|\chi(\tau) - \Upsilon(\tau)\big| \leq \mathbb{C}_{\Psi}\zeta$, where

$$\mathbb{C}_{\Psi} = \frac{(1-\phi)\Gamma(\phi) + \mathcal{T}^{\phi}_{\max}}{\mathbb{A}(\phi)\Gamma(\phi) - \mathcal{T}^{\phi}_{\max}\mathbb{L}_{\Psi}}. \tag{4.15}$$

Consequently, the fractional $\mathbb{SDC}$ model (1.3) is **UH** stable. □

Corollary 1. *Setting $\mathcal{F}_{\Psi}(\zeta) = \mathbb{C}_{\Psi}\zeta$ in Theorem 4.6 such that $\mathcal{F}_{\Psi}(0) = 0$, then the fractional $\mathbb{SDC}$ system* (1.3) *is* **GUH** *stable.*

4.2. Consequences of **UHR** *and* **GUHR** *stability*

In order to prove our next result, we have the subsequent hypothesis:
($\mathbf{A}_3$) *exists* an increasing mapping $\mathcal{F}_{\Psi} \in \mathbf{V}$ and $\exists\, \lambda_{\mathcal{F}_{\Psi}} > 0$, such that, for any $\tau \in \Omega$, then the subsequent integral inequality can be written as:

$$ {}^{AB}_{0}\mathcal{I}^{\phi}_{\tau}\mathcal{F}_{\Psi} \leq \lambda_{\mathcal{F}_{\Psi}}\mathcal{F}_{\Psi}(\tau). \tag{4.16}$$

Lemma 4.7. *For $0 < \phi \leq 1$, if $\chi \in \mathbf{V}$ is a response of* (4.2)*, then χ is a result of the subsequent variant:*

$$\Big|\chi(\tau) - \mathcal{R}_{\chi}(\tau) - \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_{0}^{\tau}(\tau-\mathbf{u})^{\phi-1}\Psi(\mathbf{u},\chi(\mathbf{u}))d\mathbf{u}\Big| \leq \zeta\lambda_{\mathcal{F}_{\Psi}}\mathcal{F}_{\Psi}(\tau), \tag{4.17}$$

where $\mathcal{R}_{\chi} = \chi_0 + \frac{1-\phi}{\mathbb{A}(\phi)}\Psi(\tau,\chi(\tau))$.

Proof. Assume that χ be a result of (4.2). Using the fact of Remark 3-(b), we have

$$\begin{cases} {}^{ABC}_{\tau}\mathbf{D}^{\phi}_{0}\chi(\tau) = \Psi(\tau,\chi(\tau)) + \nu(\tau), \quad \tau \in \Omega \\ \chi(0) = \chi_0 \geq 0, \end{cases} \tag{4.18}$$

Then the estimated solution of (4.18) can be expressed as

$$\begin{aligned} \chi(\tau) = \chi_0 \quad &+ \quad \frac{1-\phi}{\mathbb{A}(\phi)}\Psi(\tau,\chi(\tau)) + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_{0}^{\tau}(\tau-\mathbf{u})^{\phi-1}\Psi(\mathbf{u},\chi(\mathbf{u}))d\mathbf{u} \\ &\leq \frac{1-\phi}{\mathbb{A}(\phi)}\nu(\tau) + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_{0}^{\tau}(\tau-\mathbf{u})^{\phi-1}\nu(\mathbf{u})d\mathbf{u}. \end{aligned} \tag{4.19}$$

Using the fact of Remark 3-(a), we have

$$\Big|\chi(\tau) - \mathcal{R}_{\chi}(\tau) - \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_{0}^{\tau}(\tau-\mathbf{u})^{\phi-1}\Psi(\mathbf{u},\chi(\mathbf{u}))d\mathbf{u}\Big|$$

$$\leq \frac{1-\phi}{\mathbb{A}(\phi)}\big|\nu(\tau)\big| + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}\big|\nu(\mathbf{u})\big|d\mathbf{u}$$
$$\leq \zeta\lambda_{\mathcal{F}_\Psi}\mathcal{F}_\Psi(\tau), \tag{4.20}$$

which concludes the variant (4.17). □

Theorem 4.8. *Suppose that there be a continuous mapping* $\Psi : \Omega\times\mathbb{R}\mapsto\mathbb{R}$ *such that for every* $\Upsilon\in\mathbf{V}$. *Under the assumption of* $(\boldsymbol{A}_1)$ *and* (3.1)*, then the fractional* $\mathbb{SDC}$ *model* (1.3) *is* **GHR** *stable on* Ω.

Proof. Assume that $\zeta>0$ and let $\chi\in\mathbf{V}$ be any response of (4.3). Let $\Upsilon\in\mathbf{V}$ be the only result of the system (1.3). By means of Lemma 4.7, $(\mathbf{A}_1)$ and $\mathbf{A}_3$), we have

$$\begin{aligned}
\big|\chi(\tau)-\Upsilon(\tau)\big| &\leq \left|\chi(\tau)-\mathcal{R}_\Upsilon(\tau)-\frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau(\tau-\mathbf{u})^{\phi-1}\Psi(\mathbf{u},\chi(\mathbf{u}))d\mathbf{u}\right|\\
&\leq \left|\Upsilon(\tau)-\mathcal{R}_\chi(\tau)-\frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau(\tau-\mathbf{u})^{\phi-1}\Psi(\mathbf{u},\chi(\mathbf{u}))d\mathbf{u}\right|\\
&\quad+\frac{\phi}{\mathbb{A}(\mathbf{u})\Gamma(\phi)}\int_0^\tau(\tau-\mathbf{u})^{\phi-1}\big|\Psi(\mathbf{u},\chi(\mathbf{u}))-\Psi(\mathbf{u},\Upsilon(\mathbf{u}))\big|d\mathbf{u}\\
&\leq \left(\frac{1-\phi}{\mathbb{A}(\phi)}+\frac{\mathcal{T}^\phi_{\max}}{\mathbb{A}(\phi)\Gamma(\phi)}\right)\zeta+\frac{\phi\mathbb{L}_\Psi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau(\tau-\mathbf{u})^{\phi-1}\big|\chi(\mathbf{u})-\Upsilon(\mathbf{u})\big|d\mathbf{u}\\
&\leq \lambda_{\mathcal{F}_\Psi}\mathcal{F}_\Psi(\tau)\zeta+\frac{\mathcal{T}^\phi_{\max}\mathbb{L}_\Psi}{\mathbb{A}(\phi)\Gamma(\phi)}\big|\chi(\mathbf{u})-\Upsilon(\mathbf{u})\big|.
\end{aligned}\tag{4.21}$$

This produces the variant

$$\big|\chi(\tau)-\Upsilon(\tau)\big|\leq\mathbb{K}_{\mathcal{F}_\Psi}\zeta\mathcal{F}_\Psi(\tau), \tag{4.22}$$

where

$$\mathbb{K}_{\mathcal{F}_\Psi}=\frac{\lambda_{\mathcal{F}_\Psi}}{1-\frac{\mathcal{T}^\phi_{\max}\mathbb{L}_\Psi}{\mathbb{A}(\phi)\Gamma(\phi)}}. \tag{4.23}$$

Consequently, the fractional $\mathbb{SDC}$ system (1.3) is **UH** stable. □

Corollary 2. *Setting* $\zeta=1$ *into Theorem 4.8, then the fractional* $\mathbb{SDC}$ *system* (1.3) *is* **GUHR** *stable.*

5. Numerical configuration of $\mathbb{SDC}$ model

The $\mathbb{SDC}$ model was designed and simulated using the novel computational approach proposed in the [] article, which makes use of ABC-FDs. To accomplish this, we revisit the $\mathbb{SDC}$ model in the

shape of (1.3) and (3.1).
By implementing Definition 1.1 on both sides of (3.1), we have

$$\begin{aligned}
\mathbb{S}(\tau) &= \mathbb{S}_0 + \frac{1-\phi}{\mathbb{A}(\phi)}G_1(\tau,\mathbb{S},\mathbb{D},\mathbb{C}) + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbb{S})^{\phi-1}G_1(\tau,\mathbb{S},\mathbb{D},\mathbb{C})d\mathbf{u},\\
\mathbb{D}(\tau) &= \mathbb{D}_0 + \frac{1-\phi}{\mathbb{A}(\phi)}G_2(\tau,\mathbb{S},\mathbb{D},\mathbb{C}) + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}G_2(\tau,\mathbb{S},\mathbb{D},\mathbb{C})d\mathbf{u},\\
\mathbb{C}(\tau) &= \mathbb{C}_0 + \frac{1-\phi}{\mathbb{A}(\phi)}G_2(\tau,\mathbb{S},\mathbb{D},\mathbb{C}) + \frac{\phi}{\mathbb{A}(\phi)\Gamma(\phi)}\int_0^\tau (\tau-\mathbf{u})^{\phi-1}G_2(\tau,\mathbb{S},\mathbb{D},\mathbb{C})d\mathbf{u}.
\end{aligned} \tag{5.1}$$

By means of Adams type predictor-corrector method demonstrated by [44] to find the estimated findings of the right side of the model (1.3). The initial iteration of the algorithm is based on the supposition that the result is in a closed interval $[0,\mathcal{T}]$, this interval interacted with simply providing $\hbar = \mathcal{T}/\mathcal{N}, \tau_\ell = \hbar\ell \ \ (\ell = 0,1,2,...,\mathcal{N})$. Finally, the corrector approaches of changeable order integral version of ABC-FD are presented as:

$$\begin{aligned}
\mathbb{S}_{\ell+1}(\tau) &= \mathbb{S}_0 + \frac{(1-\phi)\hbar^\phi}{\mathbb{A}(\phi)\Gamma(\phi+2)}G_1(\tau_{\ell+1},\mathbb{S}^{\mathbf{p}}_{\ell+1},\mathbb{M}^{\mathbf{p}}_{\ell+1},\mathbb{C}^{\mathbf{p}}_{\ell+1}) + \frac{\phi\hbar^\phi}{\mathbb{A}(\phi)\Gamma(\phi+2)}\sum_{\vartheta=0}^{\ell}\Xi_{\vartheta,\ell+1}G_1(\tau_\vartheta,\mathbb{S}_\vartheta,\mathbb{D}_\vartheta,\mathbb{C}_\vartheta),\\
\mathbb{D}_{\ell+1}(\tau) &= \mathbb{D}_0 + \frac{(1-\phi)\hbar^\phi}{\mathbb{A}(\phi)\Gamma(\phi+2)}G_2(\tau_{\ell+1},\mathbb{S}^{\mathbf{p}}_{\ell+1},\mathbb{M}^{\mathbf{p}}_{\ell+1},\mathbb{C}^{\mathbf{p}}_{\ell+1}) + \frac{\phi\hbar^\phi}{\mathbb{A}(\phi)\Gamma(\phi+2)}\sum_{\vartheta=0}^{\ell}\Xi_{\vartheta,\ell+1}G_2(\tau_\vartheta,\mathbb{S}_\vartheta,\mathbb{D}_\vartheta,\mathbb{C}_\vartheta),\\
\mathbb{C}_{\ell+1}(\tau) &= \mathbb{C}_0 + \frac{(1-\phi)\hbar^\phi}{\mathbb{A}(\phi)\Gamma(\phi+2)}G_3(\tau_{\ell+1},\mathbb{S}^{\mathbf{p}}_{\ell+1},\mathbb{M}^{\mathbf{p}}_{\ell+1},\mathbb{C}^{\mathbf{p}}_{\ell+1}) + \frac{\phi\hbar^\phi}{\mathbb{A}(\phi)\Gamma(\phi+2)}\sum_{\vartheta=0}^{\ell}\Xi_{\vartheta,\ell+1}G_3(\tau_\vartheta,\mathbb{S}_\vartheta,\mathbb{D}_\vartheta,\mathbb{C}_\vartheta),
\end{aligned} \tag{5.2}$$

where

$$\Xi_{\vartheta,\ell+1} = \begin{cases} \ell^{\phi+1} - (\ell-\phi)(\ell+1)^\phi, & \vartheta = 0,\\ (\ell-\vartheta+2)^\phi + (\ell-\vartheta)^\phi - 2(\ell-\vartheta+1)^{\phi+1}, & 1 \le \vartheta \le \ell. \end{cases} \tag{5.3}$$

Also, the predictor terms $\mathbb{S}^{\mathbf{p}}_{\ell+1}, \mathbb{D}^{\mathbf{p}}_{\ell+1}$ are stated as

$$\begin{aligned}
\mathbb{S}^{\mathbf{p}}_{\ell+1}(\tau) &= \mathbb{S}_0 + \frac{(1-\phi)}{\mathbb{A}(\phi)\Gamma(\phi+2)}G_1(\tau_\ell,\mathbb{S}_\ell,\mathbb{M}_\ell,\mathbb{C}_\ell) + \frac{\phi}{\mathbb{A}(\phi)\Gamma^2(\phi)}\sum_{\vartheta=0}^{\ell}\omega_{\vartheta,\ell+1}G_1(\tau_\vartheta,\mathbb{S}_\vartheta,\mathbb{D}_\vartheta,\mathbb{C}_\vartheta),\\
\mathbb{D}^{\mathbf{p}}_{\ell+1}(\tau) &= \mathbb{D}_0 + \frac{(1-\phi)}{\mathbb{A}(\phi)\Gamma(\phi+2)}G_2(\tau_\ell,\mathbb{S}_\ell,\mathbb{M}_\ell,\mathbb{C}_\ell) + \frac{\phi}{\mathbb{A}(\phi)\Gamma^2(\phi)}\sum_{\vartheta=0}^{\ell}\omega_{\vartheta,\ell+1}G_2(\tau_\vartheta,\mathbb{S}_\vartheta,\mathbb{D}_\vartheta,\mathbb{C}_\vartheta),\\
\mathbb{C}^{\mathbf{p}}_{\ell+1}(\tau) &= \mathbb{C}_0 + \frac{(1-\phi)}{\mathbb{A}(\phi)\Gamma(\phi+2)}G_3(\tau_\ell,\mathbb{S}_\ell,\mathbb{M}_\ell,\mathbb{C}_\ell) + \frac{\phi}{\mathbb{A}(\phi)\Gamma^2(\phi)}\sum_{\vartheta=0}^{\ell}\omega_{\vartheta,\ell+1}G_3(\tau_\vartheta,\mathbb{S}_\vartheta,\mathbb{D}_\vartheta,\mathbb{C}_\vartheta),
\end{aligned} \tag{5.4}$$

where

$$\omega_{\vartheta,\ell+1} = \frac{\hbar^\phi}{\phi}((\ell-\vartheta+1)^\phi - (\ell-\vartheta)^\phi), \ \ 0 \le \vartheta \le \ell. \tag{5.5}$$

5.1. Numerical findings and description

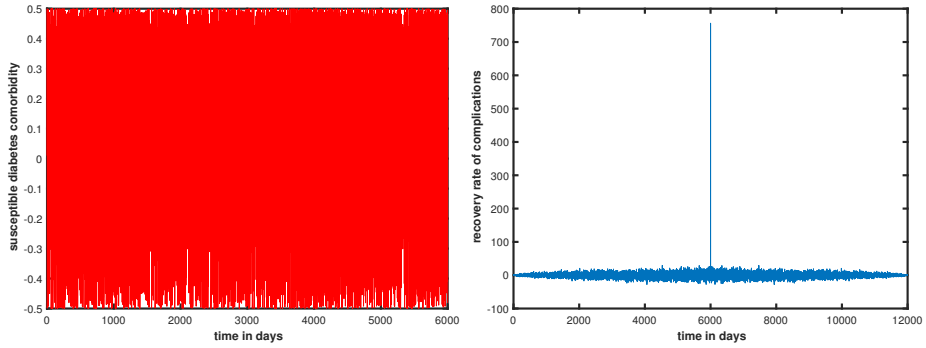
Diabetes mellitus is characterized not just by an unsustainable diet, but also by hereditary abnormalities passed down from parents with a history of diabetes. Children of diabetics are at risk of developing a hereditary condition that enables the pancreas to malfunction.

Figure 1(a) illustrates the genetic factors involved in the $\mathbb{SDC}$ model 1.3 in the absence of treatment, and Figure 1(b) represents the recovery rate of complications in parents as well as their children due to hereditary effects. Figure 2(a) shows the diabetes with and without complications involved in the $\mathbb{SDC}$ model 1.3 in the absence of treatment and Figure 2(b) emphasize the recovery rate of complications in parents as well as their children due to insulin therapy. The numerical findings for $\mathbb{S}(\tau)$, $\mathbb{D}(\tau)$, and $\mathbb{C}(\tau)$, are computed for different fractional order $\phi = 1, 0.9, 0.8, 0.7$ and 0.6 in this section. The analysis to obtain of the non-integer $\mathbb{SDC}$ model is determined by employing the Adams-type predictor–corrector introduced by [44], with the change in parameters as mentioned in Table 1. The pattern of the proportion of susceptible with difficulties according to the period for multiple orders of fractional derivative is visualized in Figure 3(a) and with the proportion of genetic disorder's birth is presented in Figure 3(b). Figure 4(a) depicts the effect of the order of the ABC-derivative on the size of diabetics over time without complications, while the proportion of individuals born with highly genetic disorders is presented in Figure 4(b). Figure 5(a) exhibits the influence of the rate at which people with diabetes with comorbidity are transformed into the number of severely impaired mellitus, with consequences with respect to time. Despite the fact that Figure 5(b) depicts the proportion of genetic disorders inherited from parents.

The $\mathbb{SDC}$ model visual behavior demonstrates that the arbitrary order has a substantial influence on the system. Figures 1–5 show the distinct differences at $\phi = 1, 0.9, 0.8, 0.7, 0.6$. The model depicts a novel feature of $\phi = 0.9, 0.8, 0.7, 0.6$ which was previously unnoticed while modeling with $\phi = 1$. Hence, the Adams-type predictor–corrector method is a cutting-edge and powerful computational method for solving non-integer order differential equations. As a result, it can be concluded that Adams-type predictor–corrector technique is a reasonable, straightforward, and more sophisticated computational process for analyzing linear and non-linear problems as compared to the method applied in [46].

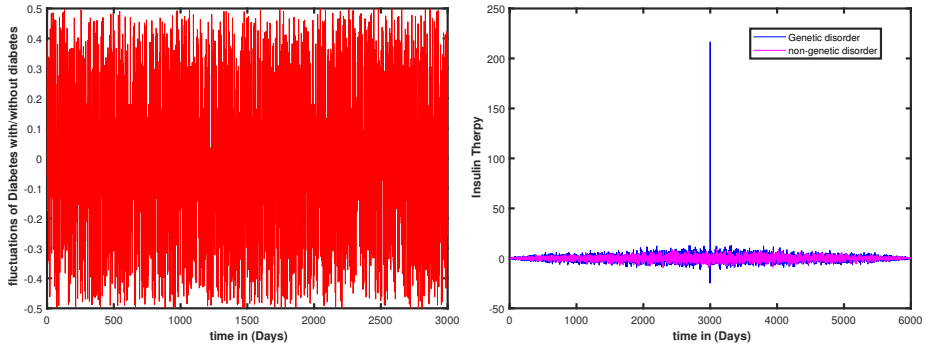
Table 1. Table of parameterized settings considered in problem (1.3) for simulations.

Population/*parameters*	Explanation
$\mathbb{N} = 500$	Overall Population
$\mathbb{S}_0 = 289.8$	Susceptible Population
$\mathbb{D}_0 = 9.65$	diabetics without complication
$\mathbb{C}_0 = 11.05$	diabetics with complication
$\eta = 0.01623$	birth rate
$\beta = 0.16263$	interaction rate
$\gamma = 0.37141$	recovery rate of complications
$\delta = 0.0068$	death rate due complications
$\lambda = 0.67758$	occurrence rate of complications
$\mu = 0.00764$	natural mortality rate
$\mathbf{p} = 0.077$	proportion of genetic disorder's birth



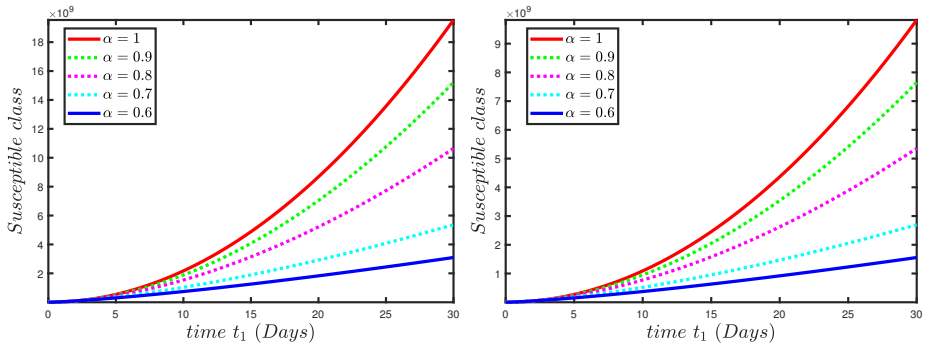
(a) **(b)**

Figure 1. (a) The effects of the susceptible diabetes comorbidity for model 1.3 in the absence of treatment. (b) Recovery rate effects for model 1.3 after considering a healthy lifestyle.



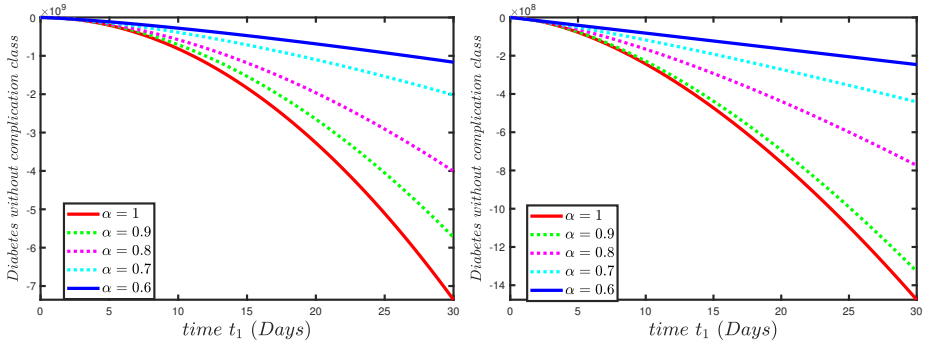
(a) **(b)**

Figure 2. (a) The fluctuating effects of diabetes with/without complications for model 1.3 in the absence of treatment. (b) The effects of insulin therapy on recovery rate for model 1.3.



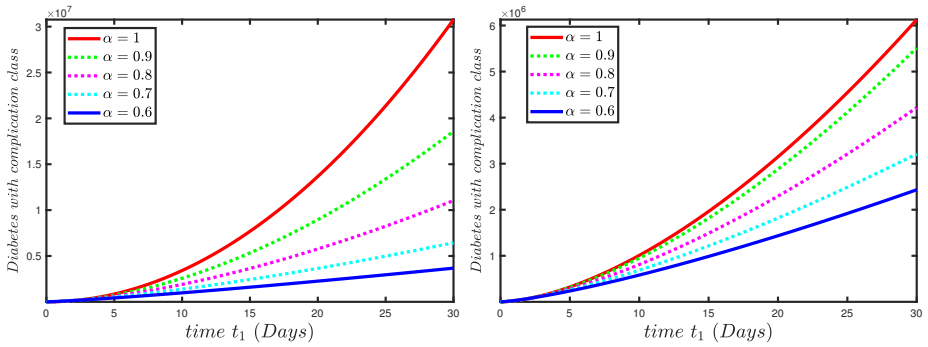
(a) **(b)**

Figure 3. (a) Simulation of the approximate results of the Susceptible class $\mathbb{S}(\tau)$ for various fractional order $\phi = 1,\ 0.9,\ 0.8,\ 0.7,\ 0.6$ when $\mathbf{p} = 1/3$. (b) Simulation of the approximate results of the Susceptible class $\mathbb{S}(\tau)$ for various fractional order $\phi = 1,\ 0.9,\ 0.8,\ 0.7,\ 0.6$ when $\mathbf{p} = 0.97$.



(a) **(b)**

Figure 4. (a) Simulation of the approximate results of the diabetes without complications class $\mathbb{D}(\tau)$ for various fractional order $\phi = 1,\ 0.9,\ 0.8,\ 0.7,\ 0.6$ when $\mathbf{p} = 1/3$. (b) Simulation of the approximate results of the diabetes without complications class $\mathbb{D}(\tau)$ for various fractional order $\phi = 1,\ 0.9,\ 0.8,\ 0.7,\ 0.6$ when $\mathbf{p} = 0.97$.



(a) **(b)**

Figure 5. (a) Simulation of the approximate results of the diabetes with complications class $\mathbb{C}(\tau)$ for various fractional order $\phi = 1,\ 0.9,\ 0.8,\ 0.7,\ 0.6$ when $\mathbf{p} = 1/3$. (b) Simulation of the approximate results of the diabetes with complications class $\mathbb{C}(\tau)$ for various fractional order $\phi = 1,\ 0.9,\ 0.8,\ 0.7,\ 0.6$ when $\mathbf{p} = 0.97$.

6. Conclusions

We investigated a fractional $\mathbb{SDC}$ system via the ABC-derivative interpretation in this article. Using Banach's and Krasnoselskii's F_p theorems, the existence findings of the responses for the suggested framework (1.3) were explored. Ulam's stability was used to determine the stability of the systems, which included: **UH**, **GUH**, **UHR**, and **GUHR** stability. The estimated findings for the various fractional orders are illustrated using the unique mathematical methodology, particularly the Adams-type predictor–corrector methodology. The dynamic behaviour of the $\mathbb{SDC}$ system was investigated. Ultimately, the highly achieved specific projected scheme for multiple fractional derivative orders reveals that several modifications in the fractional derivative order had no effect on the function's behaviour, only the simulation studies that were performed. This research would seem to be a novel approach to investigating the mathematical model of $\mathbb{SDC}$ with a fractional ABC

derivative. The researcher could expand on this work by developing and applying the $\mathbb{SDC}$ model to various kinds of fractional-order derivative operators.

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Conflict of interest

The authors declare that they have no competing interests.

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