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Research article

Estimations for aggregate amount of claims in a risk model with arbitrary dependence between claim sizes and inter-arrival times

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Abstract: This paper considers a compound risk model, in which the individual claim sizes and their inter-arrival times can be arbitrarily dependent. We mainly investigate the claim sizes are extended negatively dependent. When the claim sizes have consistently-varying-tailed distributions, we obtain precise large deviations of the aggregate amount of claims in the above dependent compound risk model.

Keywords: precise large deviations; compound sums; extended negative dependence; consistently varying distribution
Mathematics Subject Classification: 60F10, 91B05, 91G05

1. Introduction

In this paper, we consider a compound renewal risk model. Let the individual claim sizes $\{X_i, i \geq 1\}$ be a sequence of dependent nonnegative random variables (r.v.s) with common distribution F_X and finite mean $\mu_X > 0$ and the inter-arrival times of events $\{\theta_i, i \geq 1\}$ be a sequence of independent and identically distributed (i.i.d.) nonnegative r.v.s with common distribution F_θ and finite mean $\beta^{-1} > 0$. Let Z_n be the number of claims caused by the nth event, $n \geq 1$. Assume that $\{Z_i, i \geq 1\}$ is a sequence of i.i.d. positive integer r.v.s with common distribution F_Z and finite mean $\mu_Z > 0$. Suppose that $\{Z_k, k \geq 1\}$ are independent of $\{X_k, k \geq 1\}$ and $\{\theta_k, k \geq 1\}$, and $\{X_k, k \geq 1\}$ and $\{\theta_k, k \geq 1\}$ may be arbitrarily dependent. The number of events up to time $t \geq 0$ is a renewal counting process

$$N(t) = \sup\left\{n \geq 1, \sum_{i=1}^{n} \theta_i \leq t\right\}. \tag{1.1}$$

Let $\theta(t) = E(N(t))$, $t \geq 0$. Since $\{\theta_k, k \geq 1\}$ are i.i.d., by the elementary renewal theorem we get $\theta(t)/t \to \beta$ as $t \to \infty$. The number of claims up to time $t \geq 0$ is a compound renewal counting process

$$\Lambda(t) = \sum_{k=1}^{N(t)} Z_k. \tag{1.2}$$

Since $\{Z_i, i \geq 1\}$ and $\{\theta_i, i \geq 1\}$ are independent, the mean function of $\{\Lambda(t), t \geq 0\}$ is $\lambda(t) = E(\Lambda(t)) = \mu_Z \theta(t), t \geq 0$ and $\lambda(t)/t \to \mu_Z \beta$ as $t \to \infty$. Thus, the aggregate amount of claims up to time $t \geq 0$ is denoted by

$$S_{\Lambda(t)} = \sum_{k=1}^{\Lambda(t)} X_k, \quad t \geq 0, \tag{1.3}$$

which is called a compound sum.

For the compound sum $S_{\Lambda(t)}, t \geq 0$, the early studies are focus on the independent structure,where $\{X_i, i \geq 1\}$ is a sequence of i.i.d. r.v.s and independent of $\{\theta_i, i \geq 1\}$. We refer the reader to Tang et al. [12], Aleškevičienė et al. [1], Konstantinides and Loukissas [9], Yang et al. [17], Wang et al. [14], among others.

In recent years, more and more researchers are interested in dependent compound renewal risk model. They add some dependent structures on r.v.s $\{X_i, i \geq 1\}$, $\{\theta_i, i \geq 1\}$ and $\{Z_i, i \geq 1\}$. For example, Yang et al. [16] considered $\{X_i, i \geq 1\}$, $\{\theta_i, i \geq 1\}$ and $\{Z_i, i \geq 1\}$ are extended negatively dependent r.v.s, respectively. Konstantinides and Loukissas [10] and Chen et al. [5] considered the case that $\{X_i, i \geq 1\}$ are negatively dependent r.v.s. Wang and Chen [13] investigated there was a pairwise negatively quadrant dependence structure or the upper tail asymptotical dependence structure in the claim sizes $\{X_i, i \geq 1\}$. In the above results, they need $\{X_i, i \geq 1\}$ and $\{\theta_i, i \geq 1\}$ are independent.

However, in order to reflect the insurance practice, some researchers put some dependence structures between $\{X_i, i \geq 1\}$ and $\{\theta_i, i \geq 1\}$. For the renewal risk model, Chen and Yuen [4] introduced a size-dependent risk model and Bi and Zhang [2] considered a regression-type size-dependent risk model. They obtained some results of precise large deviation of the aggregate amount of claims. Guo et al. [8] and Zhou et al. [18] obtained the precise large deviations for the compound sum $S_{\Lambda(t)}, t \geq 0$ for the dependence structures of Chen and Yuen [4] and Bi and Zhang [2], respectively.

In this paper, we will still investigate the precise large deviations of $S_{\Lambda(t)}, t \geq 0$. We will drop the independent assumption or dependence structures between the claim sizes $\{X_i, i \geq 1\}$ and the inter-arrival times of events $\{\theta_i, i \geq 1\}$ under the condition $\overline{F_\theta}(t) = o\left(\overline{F_X}(t)\right)$ as $t \to \infty$.

The rest of the paper is organized as follows. In Section 2, we present some necessary preliminaries and main results. In Section 3 the proofs of main results are given.

2. Preliminaries and main results

Without special statement, in this paper a limit is taken as $t \to \infty$. For a real-valued number a, let $a^+ = \max\{0, a\}$ and $a_- = -\min\{0, a\}$. $\lfloor a \rfloor$ denotes the integer part of a. For two positive functions $a(\cdot)$ and $b(\cdot)$, we write $a(t) \lesssim b(t)$ if $\limsup a(t)/b(t) \leq 1$; write $a(t) \gtrsim b(t)$ if $\liminf a(t)/b(t) \geq 1$; write $a(t) \sim b(t)$ if $\lim a(t)/b(t) = 1$; and write $a(t) = o(b(t))$ if $\lim a(t)/b(t) = 0$. For two positive bivariate functions $g(\cdot, \cdot)$ and $h(\cdot, \cdot)$, we write $g(x, t) \lesssim h(x, t)$ holds uniformly for $x \in \Delta \neq \phi$ as $t \to \infty$, if

$$\limsup_{t \to \infty} \sup_{x \in \Delta} \frac{g(x, t)}{h(x, t)} \leq 1;$$

write $g(x,t) \gtrsim h(x,t)$ holds uniformly for $x \in \Delta \neq \phi$ as $t \to \infty$, if

$$\liminf_{t\to\infty} \inf_{x\in\Delta} \frac{g(x,t)}{h(x,t)} \geq 1;$$

write $g(x,t) \sim h(x,t)$ holds uniformly for $x \in \Delta \neq \phi$ as $t \to \infty$, if

$$\lim_{t\to\infty} \sup_{x\in\Delta} \left| \frac{g(x,t)}{h(x,t)} - 1 \right| = 0.$$

Here we recall some subclasses of the heavy-tailed distribution class which we consider in our paper. A distribution V on $(-\infty,\infty)$ with the tail $\overline{V} = 1 - V$ is said to have a dominatedly varying tail, denoted by $V \in \mathscr{D}$, if

$$\limsup_{t\to\infty} \frac{\overline{V}(yt)}{\overline{V}(t)} < \infty,$$

for any fixed $y > 0$. A slightly smaller class is $\mathscr{C}$. A distribution V on $(-\infty,\infty)$ is said to have a consistently varying tail, denoted by $V \in \mathscr{C}$, if

$$\lim_{y\searrow 1} \liminf_{x\to\infty} \frac{\overline{V}(xy)}{\overline{V}(x)} = \lim_{y\nearrow 1} \limsup_{x\to\infty} \frac{\overline{V}(xy)}{\overline{V}(x)} = 1.$$

A distribution V on $(-\infty,\infty)$ is said to have a long tail, denoted by $V \in \mathscr{L}$, if

$$\lim_{x\to\infty} \frac{\overline{V}(x-y)}{\overline{V}(x)} = 1,$$

for any fixed $y \in (-\infty,\infty)$. It is well-known that the following inclusion relationships hold:

$$\mathscr{C} \subset \mathscr{L} \cap \mathscr{D} \subset \mathscr{L},$$

(see, e.g., Cline and Samorodnitsky [3], Embrechts et al. [7] and Xun et al. [15]).

In this paper, we will consider the claim sizes have the following dependence structure. According to Liu [11], a sequence of real-valued r.v.s $\xi_1, \xi_2, \ldots$ is said to be upper extended negatively dependent UEND if there exists some positive constant M, such that for each $k = 1, 2, \ldots$ and all $y_1, \ldots, y_k \in (-\infty,\infty)$

$$P\left(\bigcap_{i=1}^{k} \{\xi_i > y_i\}\right) \leq M \prod_{i=1}^{k} P(\xi_i > y_i);$$

they are said to be lower extended negatively dependent (LEND) if there exists some positive constant M, such that for each $k = 1, 2, \ldots$ and all $y_1, \ldots, y_k \in (-\infty,\infty)$

$$P\left(\bigcap_{i=1}^{k} \{\xi_i \leq y_i\}\right) \leq M \prod_{i=1}^{k} P(\xi_i \leq y_i);$$

and the r.v.s $\xi_1, \xi_2, \ldots$ are said to be extended negatively dependent END if they are both UEND and LEND.

The following is the main result of this paper, which gives the precise large deviations for the compound sum $S_{\Lambda(t)}, t \geq 0$.

Theorem 2.1. *Consider the compound sum (1.3). Assume that the claim sizes $\{X_i, i \geq 1\}$ are END r.v.s with common distribution $F_X \in \mathscr{C}$, $E(X_1^r) < \infty$ for some $r > 1$, $\overline{F_\theta}(t) = o\left(\overline{F_X}(t)\right)$ and $\overline{F_Z}(t) = o\left(\overline{F_X}(t)\right)$. Then for any $0 < \gamma < \Gamma < \infty$, it holds uniformly for all $x \in [\gamma\lambda(t), \Gamma\lambda(t)]$ that*

$$P(S_{\Lambda(t)} - \mu_X \lambda(t) > x) \sim \lambda(t)\overline{F_X}(x). \tag{2.1}$$

When $Z_i \equiv 1$, $i \geq 1$, then $S_{\Lambda(t)} = S_{N(t)}$, $t \geq 0$. From Theorem 2.1 the following corollary can be obtained.

Corollary 2.1. *Assume that the claim sizes $\{X_i, i \geq 1\}$ are END r.v.s with common distribution $F_X \in \mathscr{C}$, $E(X_1^r) < \infty$ for some $r > 1$ and $\overline{F_\theta}(t) = o\left(\overline{F_X}(t)\right)$. Then for any $0 < \gamma < \Gamma < \infty$, it holds uniformly for all $x \in [\gamma t, \Gamma t]$ that*

$$P(S_{N(t)} - \mu_X \beta t > x) \sim \beta t\overline{F_X}(x). \tag{2.2}$$

Remark 2.1. *When $\{X_i, i \geq 1\}$ are i.i.d. r.v.s, Chen et al. [6] obtained (2.2) under the condition $0 < \mu_X < \infty$, $F_x \in \mathscr{C}$ and $\overline{F_\theta}(t) = o\left(\overline{F_X}(t)\right)$. Thus Corollary 2.1 extends the result of Chen et al. [6] to the dependent claim sizes $\{X_i, i \geq 1\}$.*

3. Proofs of main results

Before giving the proof of Theorem 2.1, we first give two lemmas. The following lemma comes from Theorem 2.1 of Liu [11].

Lemma 3.1. *Let $\{\xi_i, i \geq 1\}$ be a sequence of END nonnegative r.v.s with common distribution $V \in \mathscr{C}$ and finite mean $\mu > 0$. If $E(\xi_1)^r < \infty$ for some $r > 1$, then for any $\gamma > 0$*

$$P\left(\sum_{i=1}^{n} \xi_i - n\mu > x\right) \sim n\overline{V}(x)$$

holds uniformly for all $x \geq \gamma n$ as $n \to \infty$.

The following lemma is Corollary 3.1 of Chen et al. [6].

Lemma 3.2. *Let $\xi_1, \xi_2, ...$ be i.i.d. copies of real-valued r.v. ξ with mean 0. Suppose that $P(|\xi| > x) = o\left(\overline{V}(x)\right)$ for some $V \in \mathscr{C}$. Then for any given $\gamma > 0$, it holds uniformly for all $x \geq \gamma n$ as $n \to \infty$ that*

$$P\left(\left|\sum_{i=1}^{n} \xi_k\right| > x\right) = o\left(n\overline{V}(x)\right).$$

Now we turn to the proof of Theorem 2.1. In the proof, unless otherwise stated, a limit is understood as being valid uniformly for all $\gamma\lambda(t) \leq x \leq \Gamma\lambda(t)$ as $t \to \infty$ for any $0 < \gamma < \Gamma < \infty$.

Proof of Theorem 2.1. For any $0 < \gamma < \Gamma < \infty$, since $\lambda(t) \sim \mu_Z \beta t$, for any small $0 < \epsilon < \max\left\{\frac{1}{\beta\mu_Z}, \frac{\gamma}{\mu_X \mu_Z \beta}\right\}$, there exists $t_0 > 0$ such that for any $t > t_0$,

$$\left|\frac{\lambda(t)}{t} - \mu_Z \beta\right| < \epsilon. \tag{3.1}$$

We first prove

$$P(S_{\Lambda(t)} - \mu_X\lambda(t) > x) \gtrsim \lambda(t)\overline{F_X}(x). \tag{3.2}$$

For any $\mu_Z\epsilon\beta < \delta_1 < 1$, it holds for all $x > 0$ and $t > 0$ that

$$\begin{aligned} P(S_{\Lambda(t)} - \mu_X\lambda(t) > x) &\geq P\left(\sum_{k=1}^{(1-\delta_1)\lambda(t)} X_k - \mu_X\lambda(t) > x, \Lambda(t) \geq (1-\delta_1)\lambda(t)\right) \\ &\geq P\left(\sum_{k=1}^{(1-\delta_1)\lambda(t)} X_k - \mu_X\lambda(t) > x\right) - P\left(\Lambda(t) < (1-\delta_1)\lambda(t)\right) \\ &=: I_1(x,t) - I_2(t). \end{aligned} \tag{3.3}$$

Applying Lemma 3.1, it holds that

$$\begin{aligned} I_1(x,t) &= P\left(\sum_{k=1}^{\lfloor(1-\delta_1)\lambda(t)\rfloor} X_k - \lfloor(1-\delta_1)\lambda(t)\rfloor\mu_X > x + \mu_X\lambda(t) - \mu_X\lfloor(1-\delta_1)\lambda(t)\rfloor\right) \\ &\sim \lfloor(1-\delta_1)\lambda(t)\rfloor\overline{F_X}\left(x + \mu_X\lambda(t) - \mu_X\lfloor(1-\delta_1)\lambda(t)\rfloor\right) \\ &\geq ((1-\delta_1)\lambda(t) - 1)\overline{F_X}\left(\left(1 + \frac{\mu_X\delta_1}{\gamma}\right)x + \mu_X\right), \end{aligned} \tag{3.4}$$

where the second step uses the facts $\gamma + \delta_1\mu_X > 0$, and for any $0 < \gamma' < \frac{\gamma+\mu_X\delta_1}{1-\delta_1}$, it holds that $x + \mu_X\lambda(t) - \mu_X\lfloor(1-\delta_1)\lambda(t)\rfloor \geq \gamma'\lfloor(1-\delta_1)\lambda(t)\rfloor$. Since $F_X \in \mathscr{C} \subset \mathscr{L}$, we have

$$I_1(x,t) \gtrsim ((1-\delta_1)\lambda(t) - 1)\overline{F_X}\left(\left(1 + \frac{\mu_X\delta_1}{\gamma}\right)x\right).$$

By $F_X \in \mathscr{C}$ and letting $\epsilon \downarrow 0$ and $\delta_1 \downarrow 0$ we have

$$I_1(x,t) \gtrsim \lambda(t)\overline{F_X}(x). \tag{3.5}$$

For $I_2(t)$, taking $\mu_Z\epsilon\beta < \epsilon_1 < \delta_1$ and then split $I_2(t)$ as follows:

$$\begin{aligned} I_2(t) &= P\left(\sum_{j=1}^{N(t)} Z_j < (1-\delta_1)\lambda(t), N(t) \geq \frac{\lambda(t)}{\mu_Z}(1-\epsilon_1)\right) \\ &\quad + P\left(\sum_{j=1}^{N(t)} Z_j < (1-\delta_1)\lambda(t), N(t) < \frac{\lambda(t)}{\mu_Z}(1-\epsilon_1)\right) \\ &=: K_1(t) + K_2(t). \end{aligned} \tag{3.6}$$

For $K_1(t)$, it holds that for all $x > 0$ and $t > 0$ that

$$\begin{aligned} K_1(t) &\leq P\left(\sum_{j=1}^{\left\lfloor\frac{\lambda(t)}{\mu_Z}(1-\epsilon_1)\right\rfloor} Z_j < (1-\delta_1)\lambda(t)\right) \\ &\leq P\left(\sum_{j=1}^{\left\lfloor\frac{\lambda(t)}{\mu_Z}(1-\epsilon_1)\right\rfloor} (Z_j - \mu_Z) < (\epsilon_1 - \delta_1)\lambda(t) + \mu_Z\right). \end{aligned}$$

By Lemma 3.2 and $F \in \mathscr{C} \subset \mathscr{L} \bigcap \mathscr{D}$ it holds that

$$\begin{aligned} K_1(t) &= o\left(\lambda(t)\overline{F_X}\left(\lambda(t)(\delta_1-\epsilon_1)-\mu_Z\right)\right) \\ &= o\left(\lambda(t)\overline{F_X}\left(\Gamma^{-1}(\delta_1-\epsilon_1)x-\mu_Z\right)\right) \\ &= o\left(\lambda(t)\overline{F_X}(x)\right), \end{aligned} \tag{3.7}$$

where in the first step Lemma 3.2 can be used because of for any $0 < \gamma^* < \frac{(\delta_1-\epsilon_1)\mu_Z}{1-\epsilon_1}$, it holds that $(\delta_1-\epsilon_1)\lambda(t)-\mu_Z \geq \gamma^* \left\lfloor \frac{\lambda(t)}{\mu_Z}(1-\epsilon_1)\right\rfloor$.

For $K_2(t)$, by Lemma 3.2 and $F_X \in \mathscr{C}$ we have

$$\begin{aligned} P\left(N(t) < \frac{\lambda(t)}{\mu_Z}(1-\epsilon_1)\right) &\leq P\left(\sum_{j=1}^{\frac{\lambda(t)}{\mu_Z}(1-\epsilon_1)+1} \theta_j > t\right) \\ &\leq P\left(\sum_{j=1}^{\frac{\lambda(t)}{\mu_Z}(1-\epsilon_1)+1} \left(\theta_j - \frac{1}{\beta}\right) > t - \frac{\frac{\lambda(t)}{\mu_Z}(1-\epsilon_1)+1}{\beta}\right) \\ &= o\left(\lambda(t)\overline{F_X}\left(t - \frac{\lambda(t)}{\beta\mu_Z} + \frac{\epsilon_1\lambda(t)}{\beta\mu_Z} - \frac{1}{\beta}\right)\right) \\ &= o\left(\lambda(t)\overline{F_X}\left(\frac{\epsilon_1\lambda(t)}{\beta\mu_Z}\right)\right) \\ &= o\left(\lambda(t)\overline{F_X}\left((\Gamma\beta\mu_Z)^{-1}\epsilon_1 x\right)\right) \\ &= o\left(\lambda(t)\overline{F_X}(x)\right), \end{aligned} \tag{3.8}$$

where in the third step Lemma 3.2 is used, which is due to the fact that for any $0 < \hat{\gamma} < \frac{\epsilon_1-\mu_Z\epsilon\beta}{\beta(1-\epsilon_1)}$, it holds that $t - \frac{\lambda(t)}{\beta\mu_Z} + \frac{\epsilon_1\lambda(t)}{\beta\mu_Z} - \frac{1}{\beta} > \hat{\gamma}\left(\frac{\lambda(t)}{\mu_Z}(1-\epsilon_1)+1\right)$.

By (3.7) and (3.8), we have

$$I_2(t) = o\left(\lambda(t)\overline{F_X}(x)\right). \tag{3.9}$$

Using (3.5), (3.9) and (3.3), we know that (3.2) holds.

Next we will prove

$$P(S_{\Lambda(t)} - \mu_X\lambda(t) > x) \lesssim \lambda(t)\overline{F_X}(x). \tag{3.10}$$

For any fixed $\mu_Z\epsilon\beta < \delta_2 < \frac{\gamma}{\mu_X}$, it holds for any $x > 0$ and $t > 0$ that

$$\begin{aligned} P(S_{\Lambda(t)} - \mu_X\lambda(t) > x) &= P(S_{\Lambda(t)} - \mu_X\lambda(t) > x, \Lambda(t) \leq (1+\delta_2)\lambda(t)) \\ &\quad + P(S_{\Lambda(t)} - \mu_X\lambda(t) > x, \Lambda(t) > (1+\delta_2)\lambda(t)) \\ &\leq P\left(\sum_{k=1}^{(1+\delta_2)\lambda(t)} X_k - \mu_X\lambda(t) > x\right) + P(\Lambda(t) > (1+\delta_2)\lambda(t)) \\ &=: J_1(x,t) + J_2(t). \end{aligned} \tag{3.11}$$

By Lemma 3.1, we have

$$J_1(x,t) = P\left(\sum_{k=1}^{\lfloor(1+\delta_2)\lambda(t)\rfloor} X_k - \lfloor(1+\delta_2)\lambda(t)\rfloor\mu_X > x + \mu_X\lambda(t) - \mu_X\lfloor(1+\delta_2)\lambda(t)\rfloor\right)$$

$$
\begin{aligned}
&\sim \lfloor(1+\delta_2)\lambda(t)\rfloor \overline{F_X}(x+\mu_X\lambda(t)-\mu_X\lfloor(1+\delta_2)\lambda(t)\rfloor) \\
&\leq ((1+\delta_2)\lambda(t))\overline{F_X}\left(\left(1-\frac{\mu_X\delta_2}{\gamma}\right)x\right), \qquad (3.12)
\end{aligned}
$$

where in the second step Lemma 3.1 can be used because of for any $0 < \tilde{\gamma} < \frac{\gamma-\mu_X\delta_2}{1+\delta_2}$, it holds that $x+\mu_X\lambda(t)-\mu_X\lfloor(1+\delta_2)\lambda(t)\rfloor \geq \tilde{\gamma}\lfloor(1+\delta_2)\lambda(t)\rfloor$. Since $F_X \in \mathscr{C} \subset \mathscr{L} \bigcap \mathscr{D}$, by letting $\epsilon \downarrow 0$ and $\delta_2 \downarrow 0$, it holds that

$$
J_1(x,t) \lesssim \lambda(t)\overline{F_X}(x). \qquad (3.13)
$$

For $J_2(t)$, taking $\mu_Z\epsilon\beta < \epsilon_2 < \delta_2$ it holds for any $x > 0$ and $t > 0$ that

$$
\begin{aligned}
J_2(t) &= P\left(\sum_{j=1}^{N(t)} Z_j > (1+\delta_2)\lambda(t), N(t) \leq \frac{\lambda(t)}{\mu_Z}(1+\epsilon_2)\right) \\
&\quad +P\left(\sum_{j=1}^{N(t)} Z_j > (1+\delta_2)\lambda(t), N(t) > \frac{\lambda(t)}{\mu_Z}(1+\epsilon_2)\right) \\
&=: K_3(t)+K_4(t). \qquad (3.14)
\end{aligned}
$$

For $K_3(t)$, it holds for any $x > 0$ and $t > 0$ that

$$
\begin{aligned}
K_3(t) &\leq P\left(\sum_{j=1}^{\left\lfloor \frac{\lambda(t)}{\mu_Z}(1+\epsilon_2)\right\rfloor} (Z_j-\mu_Z) > (1+\delta_2)\lambda(t)-\mu_Z\left\lfloor \frac{\lambda(t)}{\mu_Z}(1+\epsilon_2)\right\rfloor\right) \\
&\leq P\left(\sum_{j=1}^{\left\lfloor \frac{\lambda(t)}{\mu_Z}(1+\epsilon_2)\right\rfloor} (Z_j-\mu_Z) > (\delta_2-\epsilon_2)\lambda(t)\right).
\end{aligned}
$$

Thus

$$
\begin{aligned}
K_3(t) &= o\left(\lambda(t)\overline{F_X}\left(\lambda(t)(\delta_2-\epsilon_2)\right)\right) \\
&= o\left(\lambda(t)\overline{F_X}\left(\Gamma^{-1}(\delta_2-\epsilon_2)x\right)\right) \\
&= o\left(\lambda(t)\overline{F_X}(x)\right), \qquad (3.15)
\end{aligned}
$$

where in the first step Lemma 3.2 can be used because of for any $0 < \bar{\gamma} < \frac{(\delta_2-\epsilon_2)\mu_Z}{1+\epsilon_2}$, it holds that $(\delta_2-\epsilon_2)\lambda(t) \geq \bar{\gamma}\left\lfloor \frac{\lambda(t)}{\mu_Z}(1+\epsilon_2)\right\rfloor$. The last step can be verified by $F_X \in \mathscr{C} \subset \mathscr{D}$.

For $K_4(t)$, by Lemma 3.2 and $F_X \in \mathscr{C}$ we have

$$
\begin{aligned}
P\left(N(t) > \frac{\lambda(t)}{\mu_Z}(1+\epsilon_2)\right) &\leq P\left(\sum_{j=1}^{\left\lfloor \frac{\lambda(t)}{\mu_Z}(1+\epsilon_2)\right\rfloor} \theta_j \leq t\right) \\
&\leq P\left(\sum_{j=1}^{\left\lfloor \frac{\lambda(t)}{\mu_Z}(1+\epsilon_2)\right\rfloor} \left(\theta_j-\frac{1}{\beta}\right) \leq t-\frac{\frac{\lambda(t)}{\mu_Z}(1+\epsilon_2)-1}{\beta}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= o\left(\lambda(t)\overline{F_X}\left(\frac{\lambda(t)}{\beta\mu_Z}+\frac{\epsilon_2\lambda(t)}{\beta\mu_Z}-t-\frac{1}{\beta}\right)\right)\\
&= o\left(\lambda(t)\overline{F_X}\left(\frac{\epsilon_2\lambda(t)}{\beta\mu_Z}\right)\right)\\
&= o\left(\lambda(t)\overline{F_X}\left((\Gamma\beta\mu_Z)^{-1}\epsilon_2 x\right)\right)\\
&= o\left(\lambda(t)\overline{F_X}(x)\right),
\end{aligned}
\tag{3.16}
$$

where in the third step Lemma 3.2 is used, which is due to the fact that for any $0<\breve{\gamma}<\frac{\epsilon_2-\mu_Z\epsilon\beta}{\beta(1+\epsilon_2)}$, it holds that $\frac{\frac{\lambda(t)}{\mu_Z}(1+\epsilon_2)-1}{\beta}-t\geq\breve{\gamma}\left(\frac{\lambda(t)}{\mu_Z}(1+\epsilon_2)\right)$.

By (3.15) and (3.16), we have

$$J_2(t)=o\left(\lambda(t)\overline{F_X}(x)\right). \tag{3.17}$$

Therefore, by (3.13), (3.17) and (3.11), we know that (3.10) holds. □

Proof of Corollary 2.1. By Theorem 2.1 for any $0<\gamma_1<\Gamma_1<\infty$, it holds uniformly for $x\in[\gamma_1\theta(t),\Gamma_1\theta(t)]$ that

$$P\left(S_{N(t)}-\mu_X\theta(t)>x\right)\sim\theta(t)\overline{F_X}(x). \tag{3.18}$$

For any $0<\gamma<\Gamma<\infty$, since $\theta(t)\sim\beta t$, taking $0<\gamma_1<\frac{\gamma}{\beta}$ and $\frac{\Gamma}{\beta}<\Gamma_1<\infty$, when t is sufficiently large, it holds that

$$\gamma_1\theta(t)<\gamma t\leq x\leq\Gamma t<\Gamma_1\theta(t).$$

Since $\theta(t)\sim\beta t$, for any small enough $0<\epsilon<1$, when t is sufficiently large it holds that

$$0<\beta t-\epsilon\leq\theta(t)\leq\beta t+\epsilon. \tag{3.19}$$

Thus by (3.18), (3.19) and $F\in\mathscr{C}\subset\mathscr{L}$ it holds uniformly for $x\in[\gamma t,\Gamma t]$ that

$$
\begin{aligned}
P\left(S_{N(t)}-\mu_X\beta t>x\right) &\leq P\left(S_{N(t)}-\mu_X\theta(t)>x-\mu_X\epsilon\right)\\
&\sim \theta(t)\overline{F_X}(x-\mu_X\epsilon)\\
&\sim \beta t\overline{F_X}(x)
\end{aligned}
$$

and

$$
\begin{aligned}
P\left(S_{N(t)}-\mu_X\beta t>x\right) &\geq P\left(S_{N(t)}-\mu_X\theta(t)>x+\mu_X\epsilon\right)\\
&\sim \theta(t)\overline{F_X}(x+\mu_X\epsilon)\\
&\sim \beta t\overline{F_X}(x).
\end{aligned}
$$

This completes the proof of Corollary 2.1. □

4. Conclusions

In this paper we use the probability limiting theory to investigate the aggregate amount of claims of a compound risk model. When the claims have heavy-tailed distributions we give the precise large deviations of the aggregate amount of claims. Under some technical conditions we drop the independent assumption or dependence structures between the claim sizes and the inter-arrival times of events, which expands the use range of the main results.

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Conflict of interest

The authors declare no conflict of interest.

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