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Research article

Approximate solution for the nonlinear fractional order mathematical model

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Abstract: Health organizations are working to reduce the outbreak of infectious diseases with the help of several techniques so that exposure to infectious diseases can be minimized. Mathematics is also an important tool in the study of epidemiology. Mathematical modeling presents mathematical expressions and offers a clear view of how variables and interactions between variables affect the results. The objective of this work is to solve the mathematical model of MERS-CoV with the simplest, easiest and most proficient techniques considering the fractional Caputo derivative. To acquire the approximate solution, we apply the Adomian decomposition technique coupled with the Laplace transformation. Also, a convergence analysis of the method is conducted. For the comparison of the obtained results, we apply another semi-analytic technique called the homotopy perturbation method and compare the results. We also investigate the positivity and boundedness of the selected model. The dynamics and solution of the MERS-CoV compartmental mathematical fractional order model and its transmission between the human populace and the camels are investigated graphically for $\theta = 0.5,\ 0.7,\ 0.9,\ 1.0$. It is seen that the recommended schemes are proficient and powerful for the given model considering the fractional Caputo derivative.

Keywords: Adomian decomposition method; convergent series; Laplace transformation; Caputo fractional derivative; homotopy perturbation method; invariant set
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1. Introduction

In various countries, coronaviruses have been tested, and in December 2012, the Middle East respiration syndrome coronavirus (MERS-CoV) was found in Saudi Arabia [1–3]. This virus was found in animals in the Middle Eastern countries and after that, it was very alarming and threatening when the same virus was also tested in human beings [4]. It was also observed that this disease spreads through already contaminated individuals [5]. For many years, MERS-CoV has caused numerous fatality and disease cases. Researchers examined MERS-CoV for the most part in the riding camels, which became a reservoir source to spread this virus. It was observed that the transmission rate of diseases from animals to humans is low while this transmission rate is high from one human to another. From animal to human transmission has been discussed in [6–8].

To study infectious diseases, mathematical modeling is the best tool. With the help of this, one can study the transmission and spread of disease among the same species and between different species [9–13]. Mathematical modeling can give the best understanding of the transmission dynamics of every epidemic. Fractional order mathematical models help to understand the transmission of diseases, the spread of disease, and its effect on the population that an integer-order mathematical model can not completely describe. For the last many years, fractional-order differential equations have been used to model real-world problems with a larger degree of accuracy.

The uses of differential equations with fractional order are remarkable in physical and medical sciences, finance and different disciplines of mathematics [14–21]. Numerous forms of fractional-order operators are introduced in partial differential equations modeling and their applications have been found in several fields like biological sciences, chemical, engineering, and statistical sciences. Many investigations in mathematical models have been made by using fractional operators [22–30].

In the presented work, we study a mathematical model presented in [31]. Here MERS-CoV transmission elements are presented mathematically between human and camel. The population is divided into seven compartments as follows: the susceptible class $\mathring{S}(t)$, the exposed population $\mathring{E}(t)$, $\mathring{I}(t)$ as the infected class, the asymptomatic population class $\mathring{A}(t)$, the hospitalized class $\mathring{H}(t)$, the recovery class $\mathring{R}(t)$ and camel class $\mathring{C}(t)$ which is the reservoir for MERS-Cov.

$$\begin{cases} \mathrm{D}_t(\mathring{S}(t)) = \Lambda' - \tau\mathring{I}\mathring{S} - \eta\psi\mathring{A}\mathring{S} - \zeta\rho\mathring{H}\mathring{S} - \beta\mathring{C}\mathring{S} - \dot{\pi}\mathring{S}, \\ \mathrm{D}_t(\mathring{E}(t)) = \tau\mathring{I}\mathring{S} + \eta\psi\mathring{A}\mathring{S} + \zeta\rho\mathring{H}\mathring{S} + \beta\mathring{C}\mathring{S} - (\mu + \dot{\pi})\mathring{E}, \\ \mathrm{D}_t(\mathring{I}(t)) = \mu\nu\mathring{E} - (\gamma + \lambda)\mathring{I} - (\dot{\pi} + \pi')\mathring{I}, \\ \mathrm{D}_t(\mathring{A}(t)) = \mu(1 - \nu)\mathring{E} - (\kappa + \dot{\pi})\mathring{A}, \\ \mathrm{D}_t(\mathring{H}(t)) = \gamma\mathring{I} + \kappa\mathring{A} - (\varrho + \dot{\pi})\mathring{H}, \\ \mathrm{D}_t(\mathring{R}(t)) = \lambda\mathring{I} + \varrho\mathring{H} - \dot{\pi}\mathring{R}, \\ \mathrm{D}_t(\mathring{C}(t)) = \epsilon\mathring{I} + \varepsilon\mathring{A} - \omega\mathring{C}, \end{cases} \tag{1.1}$$

where Λ' is the rate of a newborn, $\tau, \eta, \zeta,$ *and* β are the disorder transmission rates, ψ shows the asymptomatic patient rate, μ represents the continuation to infectious class, γ is the average rate of symptomatic individuals hospitalized, the mortality rate is $\dot{\pi}$, infectious individual retrieved without hospitalization at a rate λ, ρ is the approximate rate of translatability of a hospitalized patient, the recovery rate of a hospitalized individual is shown by ϱ, the camel lifetime rate is ω, ϵ is the rate of symptomatic individuals and ε is the number of asymptomatic infected individuals spread virus from population $\mathring{C}$.

By taking the Caputo fractional derivative of Model (1.1), we get

$$\begin{cases} {}^{C}\mathrm{D}_t^{\theta}(\mathring{S}(t)) = \Lambda^{'} - \tau\mathring{I}\mathring{S} - \eta\psi\mathring{A}\mathring{S} - \zeta\rho\mathring{H}\mathring{S} - \beta\mathring{C}\mathring{S} - \dot{\pi}\mathring{S}, \\ {}^{C}\mathrm{D}_t^{\theta}(\mathring{E}(t)) = \tau\mathring{I}\mathring{S} + \eta\psi\mathring{A}\mathring{S} + \zeta\rho\mathring{H}\mathring{S} + \beta\mathring{C}\mathring{S} - (\mu + \dot{\pi})\mathring{E}, \\ {}^{C}\mathrm{D}_t^{\theta}(\mathring{I}(t)) = \mu\nu\mathring{E} - (\gamma + \lambda)\mathring{I} - (\dot{\pi} + \pi^{'})\mathring{I}, \\ {}^{C}\mathrm{D}_t^{\theta}(\mathring{A}(t)) = \mu(1 - \nu)\mathring{E} - (\kappa + \dot{\pi})\mathring{A}, \\ {}^{C}\mathrm{D}_t^{\theta}(\mathring{H}(t)) = \gamma\mathring{I} + \kappa\mathring{A} - (\varrho + \dot{\pi})\mathring{H}, \\ {}^{C}\mathrm{D}_t^{\theta}(\mathring{R}(t)) = \lambda\mathring{I} + \varrho\mathring{H} - \dot{\pi}\mathring{R}, \\ {}^{C}\mathrm{D}_t^{\theta}(\mathring{C}(t)) = \epsilon\mathring{I} + \varepsilon\mathring{A} - \omega\mathring{C} \end{cases} \tag{1.2}$$

with the initial conditions

$$\begin{aligned} \mathring{S}(0) &= \mathrm{M}_1, \mathring{E}(0) = \mathrm{M}_2, \mathring{I}(0) = \mathrm{M}_3, \mathring{A}(0) = \mathrm{M}_4, \\ \mathring{H}(0) &= \mathrm{M}_5, \mathring{R}(0) = \mathrm{M}_6, \mathring{C}(0) = \mathrm{M}_7, \end{aligned}$$

where each initial value is either equal to zero or greater than zero.

For a better study of the model's behavior, various fractional operators have been given in the literature like Riemann-Liouville (RL), truncated derivative, Katugampola, Atangana-Koca, Caputo, Caputo-Fabrizio Atangana-Baleanu, Atangana beta-derivative, Atangana bi-order, and many other operators. Fractional-order differential equations describe the dynamical systems with memory. Especially, nonlinear systems describing different phenomena can be modeled with fractional derivatives [32–34]. The selected model is studied using Caputo fractional differential operator. The applications of the Caputo fractional operator in the mathematical modeling of real-world problems are remarkable. One can see the references [13, 24, 26] in order to study the details. In Section 2, some existing results and important definitions are given which will be helpful to prove our main findings. In Section 3, the selected model's positivity and boundedness are investigated. Section 4 represents the construction of the analytical solution of the selected fractional mathematical model by using the Laplace Adomian decomposition method (LADM) and convergence analysis of the method. Section 5 represents the analytical solution of the selected fractional model via homotopy petrubation method (HPM) while in Sections 6 and 7, discussion about the plotted graphs and concluding remarks are given, respectively.

2. Preliminaries

Here, we give some definitions from [35, 36].

Definition 2.1. *Let the function $F \in L^1([0,\infty), R)$, the RL fractional integral of order θ is given as*

$$I_t^{\theta}F(t) = \frac{1}{\Gamma(\theta)}\int_0^t \frac{F(\eta)}{(t-\eta)^{1-\theta}}d\eta, \quad \theta > 0, \tag{2.1}$$

existence of the right-hand side integral holds true.

Definition 2.2. *Let F be the function; then, the following mathematical form exits and is called the Caputo fractional order derivative:*

$$^{C}\mathrm{D}_t^{\theta}F(t) = \frac{1}{\Gamma(n-\theta)}\int_0^t (t-\eta)^{n-\theta-1}F^{(n)}(\eta)d\eta$$

Also, $n = [\theta] + 1$ *and if* θ *belongs to* $(0,1)$*, then we have*

$$^{C}\mathrm{D}_t^{\theta}F(t) = \frac{1}{\Gamma(1-\theta)}\int_0^t \frac{F'(\eta)}{(t-\eta)^{\theta}}d\zeta.$$

Lemma 2.1. *The following result holds true for fractional differential equations:*

$$I^{\theta}[^{C}\mathrm{D}^{\theta}q](t) = q(t) + \gamma_0 + \gamma_1 t + \gamma_2 t^2 + \cdots + \gamma_{n-1}t^{n-1}$$

Definition 2.3. *The Laplace transform of the Caputo fractional derivative can be defined as*

$$\mathcal{L}[^{C}\mathrm{D}_t^{\theta}g(\mathrm{t})] = s^{\theta}G(s) - \sum_{j=0}^{m-1} s^{\theta-j-1}g^{k}(0), \quad \theta \in [m-1, m], m \in \mathbb{N}.$$

Definition 2.4. *The construction of homotopy for an equation* $V(r,q) : \Omega \times [0 \times 1] \to \mathcal{R}$ *is*

$$H(V,q) = (1-q)[\mathrm{L}(V) - \mathrm{L}(v_0)] + q[\mathrm{L}(V) + \mathrm{M}(V) - f(r)] = 0,$$

where L and M are, respectively, the linear and nonlinear parts of the equation. r belongs to Ω *and q belongs to* $[0,1]$ *which is the embedding parameter.*

3. Boundedness and positivity of the model

Here, the boundedness and positivity of the solution of System (1.1) are presented. We follow the theorems given in [37–40] to obtain the desired results.

Theorem 3.1. *There exists a domain in which all positive solutions of the system have the ultimate upper bound.*

Proof. Let

$$\mathring{N} = \mathring{S} + \mathring{E} + \mathring{I} + \mathring{A} + \mathring{R} + \mathring{H};$$

then

$$\frac{d\mathring{N}(t)}{dt} = \Lambda' - \pi'\mathring{I} - \dot{\pi}\mathring{N}$$

$\mathring{N}(0) = \mathring{N}_0 \geq 0$, which implies

$$\mathring{N}(t) \leq \mathring{N}(t) \leq \mathring{N}_0 e^{-\theta t} + \frac{\Lambda'}{\dot{\pi}}(1 - e^{-\theta t})$$

$\mathring{N}(t) \leq \frac{\Lambda'}{\dot{\pi}}$ as $t \to \infty$, so the feasible region of the considered model is

$$\Theta = \{(\mathring{S}, \mathring{E}, \mathring{I}, \mathring{A}, \mathring{R}, \mathring{H} \in \mathbb{R}_+^6 : \mathring{N} \leq \frac{\Lambda'}{\dot{\pi}}, \mathring{C} \in \mathbb{R}^+ : \mathring{C} \leq \frac{\Lambda'(\epsilon + \varepsilon)}{\dot{\pi}\omega}\}$$

□

Theorem 3.2. *The set* $\mathbb{R}^7_+$ *is a positive invariant set with respect to the given System* (1.1).

Proof. From (1.1), we have

$$\frac{dV}{dt} = \mathcal{M}(V(t)),\ V(0) = V_0 \geq 0,$$

$$\mathcal{M}(V(t)) = \Big(\mathcal{M}_1(V(t)), \mathcal{M}_2(V(t)), \mathcal{M}_3(V(t)), \mathcal{M}_4(V(t)), \mathcal{M}_5(V(t)), \mathcal{M}_6(V(t)), \mathcal{M}_7(V(t))\Big)^T.$$

We observed that

$$\begin{cases} \dfrac{d\mathring{S}}{dt}|_{\mathring{S}=0} = \Lambda^{'} \geq 0, \\ \dfrac{d\mathring{E}}{dt}|_{\mathring{E}=0} = \tau\mathring{I}\mathring{S} + \eta\psi\mathring{A}\mathring{S} + \zeta\rho\mathring{H}\mathring{S} + \beta\mathring{C}\mathring{S} \geq 0, \\ \dfrac{d\mathring{I}}{dt}|_{\mathring{I}=0} = \mu\nu\mathring{E} \geq 0, \\ \dfrac{d\mathring{A}}{dt}|_{\mathring{A}=0} = \mu(1-\nu)\mathring{E} \geq 0, \\ \dfrac{d\mathring{H}}{dt}|_{\mathring{H}=0} = \gamma\mathring{I} + \kappa\mathring{A} \geq 0, \\ \dfrac{d\mathring{R}}{dt}|_{\mathring{R}=0} = \lambda\mathring{I} + \varrho\mathring{H} \geq 0, \\ \dfrac{d\mathring{C}}{dt}|_{\mathring{C}=0} = \epsilon\mathring{I} + \varepsilon\mathring{A} \geq 0. \end{cases}$$

Hence, $\mathbb{R}^7_+$ is an invariant set. □

For the further study of the qualitative analysis of the model one can see [31], in which stability analysis of the model, uniqueness and existence of the solution are investigated.
Many non-linear mathematical models have been solved to obtain numerical and analytical solutions. Mathematicians introduced numerical and analytical solutions schemes like the finite element scheme, finite-difference scheme, variational iteration method, Homotpy perturbabtion method, Laplace Adomian decomposition and many more. The Laplace Adomian decomposition method and homotopy scheme are easy and reliable for many nonlinear fractional models to obtain the approximate solutions [11, 12, 41–43].

4. Construction of the approximate solution for Model (1.2) via the LADM

The LADM is a strong mathematical tool in which the Adomian decomposition method is coupled with the Laplace transform. One can convert a differential equation to an algebraic equation using a Laplace transform. Also, the decomposition of nonlinear terms in the terms of the Adomain polynomials is an advantage. We follow a general approach by considering the Laplace transform of the understudied Model (1.2) with the following initial conditions:

$$\begin{cases} \mathcal{L}\left[\ {}^{C}\mathrm{D}_t^{\theta}(\mathring{S}(t))\right] = \mathcal{L}\left[\Lambda' - \tau\mathring{I}\mathring{S} - \eta\psi\mathring{A}\mathring{S} - \zeta\rho\mathring{H}\mathring{S} - \beta\mathring{C}\mathring{S} - \dot{\pi}\mathring{S}\right], \\ \mathcal{L}\left[\ {}^{C}\mathrm{D}_t^{\theta}(\mathring{E}(t))\right] = \mathcal{L}\left[\tau\mathring{I}\mathring{S} + \eta\psi\mathring{A}\mathring{S} + \zeta\rho\mathring{H}\mathring{S} + \beta\mathring{C}\mathring{S} - (\mu + \dot{\pi})\mathring{E}\right], \\ \mathcal{L}\left[\ {}^{C}\mathrm{D}_t^{\theta}(\mathring{I}(t))\right] = \mathcal{L}\left[\mu\nu\mathring{E} - (\gamma + \lambda)\mathring{I} - (\dot{\pi} + \pi')\mathring{I}\right], \\ \mathcal{L}\left[\ {}^{C}\mathrm{D}_t^{\theta}(\mathring{A}(t))\right] = \mathcal{L}\left[\mu(1 - \nu)\mathring{E} - (\kappa + \dot{\pi})\mathring{A}\right], \\ \mathcal{L}\left[\ {}^{C}\mathrm{D}_t^{\theta}(\mathring{H}(t))\right] = \mathcal{L}\left[\gamma\mathring{I} + \kappa\mathring{A} - (\varrho + \dot{\pi})\mathring{H}\right], \\ \mathcal{L}\left[\ {}^{C}\mathrm{D}_t^{\theta}(\mathring{R}(t))\right] = \mathcal{L}\left[\lambda\mathring{I} + \varrho\mathring{H} - \dot{\pi}\mathring{R}\right], \\ \mathcal{L}\left[\ {}^{C}\mathrm{D}_t^{\theta}(\mathring{C}(t))\right] = \mathcal{L}\left[\epsilon\mathring{I} + \varepsilon\mathring{A} - \omega\mathring{C}\right]. \end{cases} \tag{4.1}$$

By the initial conditions, Eq (4.1) becomes

$$\begin{cases} \mathcal{L}[\mathring{S}(t)] = \dfrac{\mathrm{M}_1}{s} + \dfrac{1}{s^{\theta}}\mathcal{L}\left[\Lambda' - \tau\mathring{I}\mathring{S} - \eta\psi\mathring{A}\mathring{S} - \zeta\rho\mathring{H}\mathring{S} - \beta\mathring{C}\mathring{S} - \dot{\pi}\mathring{S}\right], \\ \mathcal{L}[\mathring{E}(t)] = \dfrac{\mathrm{M}_2}{s} + \dfrac{1}{s^{\theta}}\mathcal{L}\left[\tau\mathring{I}\mathring{S} + \eta\psi\mathring{A}\mathring{S} + \zeta\rho\mathring{H}\mathring{S} + \beta\mathring{C}\mathring{S} - (\mu + \dot{\pi})\mathring{E}\right], \\ \mathcal{L}[\mathring{I}(t)] = \dfrac{\mathrm{M}_3}{s} + \dfrac{1}{s^{\theta}}\mathcal{L}\left[\mu\nu\mathring{E} - (\gamma + \lambda)\mathring{I} - (\dot{\pi} + \pi')\mathring{I}\right], \\ \mathcal{L}[\mathring{A}(t)] = \dfrac{\mathrm{M}_4}{s} + \dfrac{1}{s^{\theta}}\mathcal{L}\left[\mu(1 - \nu)\mathring{E} - (\kappa + \dot{\pi})\mathring{A}\right], \\ \mathcal{L}[\mathring{H}(t)] = \dfrac{\mathrm{M}_5}{s} + \dfrac{1}{s^{\theta}}\mathcal{L}\left[\gamma\mathring{I} + \kappa\mathring{A} - (\varrho + \dot{\pi})\mathring{H}\right], \\ \mathcal{L}[\mathring{R}(t)] = \dfrac{\mathrm{M}_6}{s} + \dfrac{1}{s^{\theta}}\mathcal{L}\left[\lambda\mathring{I} + \varrho\mathring{H} - \dot{\pi}\mathring{R}\right], \\ \mathcal{L}[\mathring{C}(t)] = \dfrac{\mathrm{M}_7}{s} + \dfrac{1}{s^{\theta}}\mathcal{L}\left[\epsilon\mathring{I} + \varepsilon\mathring{A} - \omega\mathring{C}\right]. \end{cases} \tag{4.2}$$

The solutions in an infinite series form for $\mathring{S}, \mathring{E}, \mathring{I}, \mathring{A}, \mathring{H}, \mathring{R}$ *and* $\mathring{C}$ are given below

$$\begin{aligned} &\mathring{S}(t) = \sum_{n=0}^{\infty}\mathring{S}_n(t), \quad \mathring{E}(t) = \sum_{n=0}^{\infty}\mathring{E}_n(t), \quad \mathring{I}(t) = \sum_{n=0}^{\infty}\mathring{I}_n(t), \quad \mathring{A}(t) = \sum_{n=0}^{\infty}\mathring{A}_n(t) \\ &\mathring{H}(t) = \sum_{n=0}^{\infty}\mathring{H}_n(t), \quad \mathring{R}(t) = \sum_{n=0}^{\infty}\mathring{R}_n(t), \quad \mathring{C}(t) = \sum_{n=0}^{\infty}\mathring{C}_n(t), \end{aligned} \tag{4.3}$$

where

$$\mathring{I}(t)\mathring{S}(t) = \sum_{n=0}^{\infty}\mathrm{W_n(t)}, \mathring{A}(t)\mathring{S}(t) = \sum_{n=0}^{\infty}\mathrm{X_n(t)}, \mathring{H}(t)\mathring{S}(t) = \sum_{n=0}^{\infty}\mathrm{Y_n(t)}, \mathring{C}(t)\mathring{S}(t) = \sum_{n=0}^{\infty}\mathrm{Z_n(t)}$$

are the nonlinear terms of the Adomian polynomials where

$$
\begin{aligned}
\mathrm{W_n(t)} &= \frac{1}{n!}\frac{d^n}{d\Lambda^n}\left[\sum_{i=0}^{n}\Lambda^i \mathring{I}_i(t)\sum_{i=0}^{n}\lambda^i \mathring{S}_i(t)\right]_{\Lambda=0}\\
\mathrm{X_n(t)} &= \frac{1}{n!}\frac{d^n}{d\Lambda^n}\left[\sum_{i=0}^{n}\Lambda^i \mathring{A}_i(t)\sum_{i=0}^{n}\Lambda^i \mathring{S}_i(t)\right]_{\Lambda=0}\\
\mathrm{Y_n(t)} &= \frac{1}{n!}\frac{d^n}{d\Lambda^n}\left[\sum_{i=0}^{n}\Lambda^i \mathring{H}_i(t)\sum_{i=0}^{n}\Lambda^i \mathring{S}_i(t)\right]_{\Lambda=0}\\
\mathrm{Z_n(t)} &= \frac{1}{n!}\frac{d^n}{d\Lambda^n}\left[\sum_{i=0}^{n}\Lambda^i \mathring{C}_i(t)\sum_{i=0}^{n}\Lambda^i \mathring{S}_i(t)\right]_{\Lambda=0}.
\end{aligned}
\tag{4.4}
$$

From Eqs (4.3), (4.4) and (4.2) and by comparing the terms on both sides, we obtain

$$
\begin{cases}
\mathcal{L}[\mathring{S}_0(t)] = \dfrac{\mathrm{M}_1}{s}, \mathcal{L}[\mathring{E}_0(t)] = \dfrac{\mathrm{M}_2}{s}, \mathcal{L}[\mathring{I}_0(t)] = \dfrac{\mathrm{M}_3}{s}, \mathcal{L}[\mathring{A}_0(t)] = \dfrac{\mathrm{M}_4}{s},\\
\mathcal{L}[\mathring{H}_0(t)] = \dfrac{\mathrm{M}_5}{s}, \mathcal{L}[\mathring{R}_0(t)] = \dfrac{\mathrm{M}_6}{s}, \mathcal{L}[\mathring{C}_0(t)] = \dfrac{\mathrm{M}_7}{s},\\
\mathcal{L}[\mathring{S}_1(t)] = \dfrac{1}{s^\theta}\mathcal{L}\left[\Lambda' - \tau\mathring{I}_0\mathring{S}_0 - \eta\psi\mathring{A}_0\mathring{S}_0 - \zeta\rho\mathring{H}_0\mathring{S}_0 - \beta\mathring{C}_0\mathring{S}_0 - \dot{\pi}\mathring{S}_0\right],\\
\mathcal{L}[\mathring{E}_1(t)] = \dfrac{1}{s^\theta}\mathcal{L}\left[\tau\mathring{I}_0\mathring{S}_0 + \eta\psi\mathring{A}_0\mathring{S}_0 + \zeta\rho\mathring{H}_0\mathring{S}_0 + \beta\mathring{C}_0\mathring{S}_0 - (\mu + \dot{\pi})\mathring{E}_0\right],\\
\mathcal{L}[\mathring{I}_1(t)] = \dfrac{1}{s^\theta}\mathcal{L}\left[\mu\nu\mathring{E}_0 - (\gamma + \lambda)\mathring{I}_0 - (\dot{\pi} + \pi')\mathring{I}_0\right],\\
\mathcal{L}[\mathring{A}_1(t)] = \dfrac{1}{s^\theta}\mathcal{L}\left[\mu(1-\nu)\mathring{E}_0 - (\kappa + \dot{\pi})\mathring{A}_0\right],\\
\mathcal{L}[\mathring{H}_1(t)] = \dfrac{1}{s^\theta}\mathcal{L}\left[\gamma\mathring{I}_0 + \kappa\mathring{A}_0 - (\varrho + \dot{\pi})\mathring{H}_0\right],\\
\mathcal{L}[\mathring{R}_1(t)] = \dfrac{1}{s^\theta}\mathcal{L}\left[\lambda\mathring{I}_0 + \varrho\mathring{H}_0 - \dot{\pi}\mathring{R}_0\right],\\
\mathcal{L}[\mathring{C}_1(t)] = \dfrac{1}{s^\theta}\mathcal{L}\left[\epsilon\mathring{I}_0 + \varepsilon\mathring{A}_0 - \omega\mathring{C}_0\right],\\
\qquad\vdots\\
\mathcal{L}[\mathring{S}_{n+1}(t)] = \dfrac{1}{s^\theta}\mathcal{L}\left[\Lambda' - \tau\mathring{I}_n\mathring{S}_n - \eta\psi\mathring{A}_n\mathring{S}_n - \zeta\rho\mathring{H}_n\mathring{S}_n - \beta\mathring{C}_n\mathring{S}_n - \dot{\pi}\mathring{S}_n\right],\\
\mathcal{L}[\mathring{E}_{n+1}(t)] = \dfrac{1}{s^\theta}\mathcal{L}\left[\tau\mathring{I}_n\mathring{S}_n + \eta\psi\mathring{A}_n\mathring{S}_n + \zeta\rho\mathring{H}_n\mathring{S}_n + \beta\mathring{C}_n\mathring{S}_n - (\mu + \dot{\pi})\mathring{E}_n\right],\\
\mathcal{L}[\mathring{I}_{n+1}(t)] = \dfrac{1}{s^\theta}\mathcal{L}\left[\mu\nu\mathring{E}_n - (\gamma + \lambda)\mathring{I}_n - (\dot{\pi} + \pi')\mathring{I}_n\right],\\
\mathcal{L}[\mathring{A}_{n+1}(t)] = \dfrac{1}{s^\theta}\mathcal{L}\left[\mu(1-\nu)\mathring{E}_n - (\kappa + \dot{\pi})\mathring{A}_n\right],\\
\mathcal{L}[\mathring{H}_{n+1}(t)] = \dfrac{1}{s^\theta}\mathcal{L}\left[\gamma\mathring{I}_n + \kappa\mathring{A}_n - (\varrho + \dot{\pi})\mathring{H}_n\right],\\
\mathcal{L}[\mathring{R}_{n+1}(t)] = \dfrac{1}{s^\theta}\mathcal{L}\left[\lambda\mathring{I}_n + \varrho\mathring{H}_n - \dot{\pi}\mathring{R}_n\right],\\
\mathcal{L}[\mathring{C}_{n+1}(t)] = \dfrac{1}{s^\theta}\mathcal{L}\left[\epsilon\mathring{I}_n + \varepsilon\mathring{A}_n - \omega\mathring{C}_n\right].
\end{cases}
\tag{4.5}
$$

By the Laplace inverse transform, we get

$$
\begin{aligned}
\mathring{S}_0(t) &= \mathcal{L}^{-1}\left[\frac{\mathrm{M}_1}{s}\right] = \mathrm{M}_1, \mathring{E}_0(t) = \mathcal{L}^{-1}\left[\frac{\mathrm{M}_2}{s}\right] = \mathrm{M}_2, \mathring{I}_0(t) = \mathcal{L}^{-1}\left[\frac{\mathrm{M}_3}{s}\right] = \mathrm{M}_3,\\
\mathring{A}_0(t) &= \mathcal{L}^{-1}\left[\frac{\mathrm{M}_4}{s}\right] = \mathrm{M}_4, \mathring{H}_0(t) = \left[\frac{\mathrm{M}_5}{s}\right] = \mathrm{M}_5,\\
\mathring{R}_0(t) &= \mathcal{L}^{-1}\left[\frac{\mathrm{M}_6}{s}\right] = \mathrm{M}_6, \mathring{C}_0(t) = \left[\frac{\mathrm{M}_7}{s}\right] = \mathrm{M}_7,\\
\mathring{S}_1(t) &= \left[\Lambda' - \tau\mathrm{M}_3\mathrm{M}_1 - \eta\psi\mathrm{M}_4\mathrm{M}_1 - \zeta\rho\mathrm{M}_5\mathrm{M}_1 - \beta\mathrm{M}_7\mathrm{M}_1 - \dot{\pi}\mathrm{M}_1\right]\frac{t^\theta}{\Gamma(\theta+1)},\\
\mathring{E}_1(t) &= \left[\tau\mathrm{M}_3\mathrm{M}_1 + \eta\psi\mathrm{M}_4\mathrm{M}_1 + \zeta\rho\mathrm{M}_5\mathrm{M}_1 + \beta\mathrm{M}_7\mathrm{M}_1 - (\mu+\dot{\pi})\mathrm{M}_2\right]\frac{t^\theta}{\Gamma(\theta+1)},\\
\mathring{I}_1(t) &= \left[\mu\nu\mathrm{M}_2 - (\gamma+\lambda)\mathrm{M}_3 - (\dot{\pi}+\pi')\mathrm{M}_3\right]\frac{t^\theta}{\Gamma(\theta+1)},\\
\mathring{A}_1(t) &= \left[\mu(1-\nu)\mathrm{M}_2 - (\kappa+\dot{\pi})\mathrm{M}_4\right]\frac{t^\theta}{\Gamma(\theta+1)},\\
\mathring{H}_1(t) &= \left[\gamma\mathrm{M}_3 + \kappa\mathrm{M}_4 - (\varrho+\dot{\pi})\mathrm{M}_5\right]\frac{t^\theta}{\Gamma(\theta+1)},\\
\mathring{R}_1(t) &= \left[\lambda\mathrm{M}_3 + \varrho\mathrm{M}_5 - \dot{\pi}\mathrm{M}_6\right]\frac{t^\theta}{\Gamma(\theta+1)},\\
\mathring{C}_1(t) &= \left[\epsilon\mathrm{M}_3 + \varepsilon\mathrm{M}_4 - \omega\mathrm{M}_7\right]\frac{t^\theta}{\Gamma(\theta+1)},\\
\mathring{S}_2(t) &= \Lambda'\frac{t^\theta}{\Gamma(\theta+1)} - \left[\tau(\mathrm{M}_3 s_1 + i_1\mathrm{M}_1) + \eta\psi(\mathrm{M}_4 s_1 + a_1\mathrm{M}_1)\right.\\
&\quad \left.+\zeta\rho(\mathrm{M}_5 s_1 + h_1\mathrm{M}_1) + \beta(\mathrm{M}_7 s_1 + c_1\mathrm{M}_1) + \dot{\pi}\mathrm{M}_1\right]\frac{t^{2\theta}}{\Gamma(2\theta+1)},\\
\mathring{E}_2(t) &= \left[\tau(\mathrm{M}_3 s_1 + i_1\mathrm{M}_1) + \eta\psi(\mathrm{M}_4 s_1 + a_1\mathrm{M}_1) + \zeta\rho(\mathrm{M}_5 s_1 + h_1\mathrm{M}_1)\right.\\
&\quad \left.+\beta(\mathrm{M}_7 s_1 + c_1\mathrm{M}_1) - (\mu+\dot{\pi})\mathrm{e}_1\right]\frac{t^{2\theta}}{\Gamma(2\theta+1)},\\
\mathring{\mathrm{I}}_2(t) &= \left[\mu\nu\mathrm{e}_1 - (\gamma+\lambda)\mathrm{i}_1 - (\dot{\pi}+\pi')\mathrm{i}_1\right]\frac{t^{2\theta}}{\Gamma(2\theta+1)},\\
\mathring{A}_2(t) &= \left[\mu(1-\nu)\mathrm{e}_1 - (\kappa+\dot{\pi})\mathrm{a}_1\right]\frac{t^{2\theta}}{\Gamma(2\theta+1)},\\
\mathring{H}_2(t) &= \left[\gamma\mathrm{i}_1 + \kappa\mathrm{a}_1 - (\varrho+\dot{\pi})\mathrm{h}_1\right]\frac{t^{2\theta}}{\Gamma(2\theta+1)},\\
\mathring{R}_2(t) &= \left[\lambda\mathrm{i}_1 + \varrho\mathrm{h}_1 - \dot{\pi}\mathrm{i}_1\right]\frac{t^{2\theta}}{\Gamma(2\theta+1)},\\
\mathring{C}_2(t) &= \left[\epsilon\mathrm{i}_1 + \varepsilon\mathrm{a}_1 - \omega\mathrm{c}_1\right]\frac{t^{2\theta}}{\Gamma(2\theta+1)}.
\end{aligned} \tag{4.6}
$$

In the same way, one can get the renaming terms, where

$$
\begin{aligned}
\mathrm{s}_1 &= \Lambda^{'} - \tau \mathrm{M}_3\mathrm{M}_1 - \eta\psi\mathrm{M}_4\mathrm{M}_1 - \zeta\rho\mathrm{M}_5\mathrm{M}_1 - \beta\mathrm{M}_7\mathrm{M}_1 - \dot{\pi}\mathrm{M}_1,\\
\mathrm{e}_1 &= \tau \mathrm{M}_3\mathrm{M}_1 + \eta\psi\mathrm{M}_4\mathrm{M}_1 + \zeta\rho\mathrm{M}_5\mathrm{M}_1 + \beta\mathrm{M}_7\mathrm{M}_1 - (\mu + \dot{\pi})\mathrm{M}_2,\\
\mathrm{i}_1 &= \mu\nu\mathrm{M}_2 - (\gamma + \lambda)\mathrm{M}_3 - (\dot{\pi} + \pi^{'})\mathrm{M}_3,\\
\mathrm{a}_1 &= \mu(1-\nu)\mathrm{M}_2 - (\kappa + \dot{\pi})\mathrm{M}_4,\\
\mathrm{h}_1 &= \gamma\mathrm{M}_3 + \kappa\mathrm{M}_4 - (\varrho + \dot{\pi})\mathrm{M}_5,\\
\mathrm{r}_1 &= \lambda\mathrm{M}_3 + \varrho\mathrm{M}_5 - \dot{\pi}\mathrm{M}_6,\\
\mathrm{c}_1 &= \epsilon\mathrm{M}_3 + \varepsilon\mathrm{M}_4 - \omega\mathrm{M}_7.
\end{aligned}
$$

We present the following theorem to show the convergence analysis:

Theorem 4.1. *Let Z be a Banach space and Y be a contractive nonlinear operator $Y : Z \to Z$ such that $\forall\, z, z' \in Z$, and$\|Y(z) - Y(z')\| \leq k\|y - y'\|, 0 < k < 1$ Then Y has a unique point z such that $Y(z) = z$ where $y = (\mathring{S}, \mathring{E}, \mathring{I}, \mathring{A}, \mathring{H}, \mathring{R}, \mathring{C})$. By using the Adomian decomposition method, Eq (4.6) yields*

$$z_n = z_{n-1},\ z_{n-1} = \sum_{i=1}^{n-1} z_i,\ n = 1, 2, 3...$$

and let $z_0 = \in \mathcal{S}_r(y)$ where

$$\mathcal{S}_r(z) = z' \in Z; \|z - z'\| < r$$

then, we have

$$(i)\ \ z_n \in \mathcal{S}_r(z)\ \ and\ \ (ii)\ \ \lim_{n\to\infty} y_n = z.$$

Proof. For (i), we use mathematical induction for $n = 1$; we have

$$\|z_0 - z\| = \|Y(z_0) - Y(z)\| \leq k\|z_0 - z\|.$$

Let the result holds true for $n - 1$; then,

$$\|z_0 - z\| \leq k^{n-1}\|z_0 - z\|.$$

We have

$$\|z_n - z\| = \|Y(z_{n-1}) - Y(z)\| \leq k\|z_{n-1} - z\| \leq k^n\|z_0 - z\|$$

i.e.,

$$\|z_n - z\| \leq k^n\|z_n - z\| \leq k^n r < r,$$

$$\implies z_n \in \mathcal{S}_r(z).$$

Now for (ii), since $\|z_n - z\| \leq k^n\|z_n - z\|$ and as $\lim_{n\to\infty} k_n = 0$ we therefore have $\lim_{n\to\infty} \|z_n - z\| = 0 \implies z_n = z.$ □

5. Construction of the approximate solution for Model (1.2) via the HPM

In this section, the HPM is being considered for Model (1.2) according to [42, 44]. By constructing the general homotopy technique for the considered model, we get

$$\begin{cases} (1-p)\left[{}^{C}\mathrm{D}_t^{\theta}(\mathring{S}(t)) - {}^{C}\mathrm{D}_t^{\theta}(\mathring{S}_0(t))\right] = p\left[{}^{C}\mathrm{D}_t^{\theta}(\mathring{S}(t)) - \Lambda' + \tau\mathring{I}\mathring{S} + \eta\psi\mathring{A}\mathring{S} + \zeta\rho\mathring{H}\mathring{S} + \beta\mathring{C}\mathring{S} + \dot{\pi}\mathring{S}\right], \\ (1-p)\left[{}^{C}\mathrm{D}_t^{\theta}(\mathring{E}(t)) - {}^{C}\mathrm{D}_t^{\theta}(\mathring{E}(t))\right] = p\left[{}^{C}\mathrm{D}_t^{\theta}(\mathring{E}_0(t)) - \tau\mathring{I}\mathring{S} - \eta\psi\mathring{A}\mathring{S} - \zeta\rho\mathring{H}\mathring{S} - \beta\mathring{C}\mathring{S} + (\mu + \dot{\pi})\mathring{E}\right], \\ (1-p)\left[{}^{C}\mathrm{D}_t^{\theta}(\mathring{I}(t)) - {}^{C}\mathrm{D}_t^{\theta}(\mathring{I}_0(t))\right] = p\left[{}^{C}\mathrm{D}_t^{\theta}(\mathring{I}(t)) - \mu\nu\mathring{E} + (\gamma + \lambda)\mathring{I} + (\dot{\pi} + \pi')\mathring{I}\right], \\ (1-p)\left[{}^{C}\mathrm{D}_t^{\theta}(\mathring{A}(t)) - {}^{C}\mathrm{D}_t^{\theta}(\mathring{A}_0(t))\right] = p\left[{}^{C}\mathrm{D}_t^{\theta}(\mathring{A}(t)) - \mu(1-\nu)\mathring{E} + (\kappa + \dot{\pi})\mathring{A}\right], \\ (1-p)\left[{}^{C}\mathrm{D}_t^{\theta}(\mathring{H}(t) - {}^{C}\mathrm{D}_t^{\theta}(\mathring{H}_0(t)))\right] = p\left[{}^{C}\mathrm{D}_t^{\theta}(\mathring{H}(t)) - \gamma\mathring{I} - \kappa\mathring{A} - (\varrho + \dot{\pi})\mathring{H}\right], \\ (1-p)\left[{}^{C}\mathrm{D}_t^{\theta}(\mathring{R}(t)) - {}^{C}\mathrm{D}_t^{\theta}(\mathring{R}_0(t))\right] = p\left[{}^{C}\mathrm{D}_t^{\theta}(\mathring{R}(t)) - \lambda\mathring{I} - \varrho\mathring{H} + \dot{\pi}\mathring{R}\right], \\ (1-p)\left[{}^{C}\mathrm{D}_t^{\theta}(\mathring{C}(t)) - {}^{C}\mathrm{D}_t^{\theta}(\mathring{C}_0(t))\right] = p\left[{}^{C}\mathrm{D}_t^{\theta}(\mathring{C}(t)) - \epsilon\mathring{I} - \varepsilon\mathring{A} + \omega\mathring{C}\right]. \end{cases} \tag{5.1}$$

If we put $p = 0$ in Eq (5.1), then the reduced system of equations is easy to handle and a solution can be easily obtained. If we put $p = 1$, then we get System (1.2). So, using Eq (4.3) in Eq (5.1) we get the following equations by equating the powers of p:

$$p^0 : \begin{cases} \mathring{S}_0(t) = \mathrm{M}_1,\ \mathring{E}_0(t) = \mathrm{M}_2,\ \mathring{I}_0(t) = \mathrm{M}_3, \\ \mathring{A}_0(t) = \mathrm{M}_4,\ \mathring{H}_0(t) = \mathrm{M}_5, \\ \mathring{R}_0(t) = \mathrm{M}_6,\ \mathring{C}_0(t) = \mathrm{M}_7; \end{cases} \tag{5.2}$$

similarly

$$p^1 : \begin{cases} \mathring{S}_1(t) = \left[\Lambda' - \tau\mathrm{M}_3\mathrm{M}_1 - \eta\psi\mathrm{M}_4\mathrm{M}_1 - \zeta\rho\mathrm{M}_5\mathrm{M}_1 - \beta\mathrm{M}_7\mathrm{M}_1 - \dot{\pi}\mathrm{M}_1\right]\dfrac{t^{\theta}}{\Gamma(\theta+1)}, \\ \mathring{E}_1(t) = \left[\tau\mathrm{M}_3\mathrm{M}_1 + \eta\psi\mathrm{M}_4\mathrm{M}_1 + \zeta\rho\mathrm{M}_5\mathrm{M}_1 + \beta\mathrm{M}_7\mathrm{M}_1 - (\mu + \dot{\pi})\mathrm{M}_2\right]\dfrac{t^{\theta}}{\Gamma(\theta+1)}, \\ \mathring{I}_1(t) = \left[\mu\nu\mathrm{M}_2 - (\gamma + \lambda)\mathrm{M}_3 - (\dot{\pi} + \pi')\mathrm{M}_3\right]\dfrac{t^{\theta}}{\Gamma(\theta+1)}, \\ \mathring{A}_1(t) = \left[\mu(1-\nu)\mathrm{M}_2 - (\kappa + \dot{\pi})\mathrm{M}_4\right]\dfrac{t^{\theta}}{\Gamma(\theta+1)}, \\ \mathring{H}_1(t) = \left[\gamma\mathrm{M}_3 + \kappa\mathrm{M}_4 - (\varrho + \dot{\pi})\mathrm{M}_5\right]\dfrac{t^{\theta}}{\Gamma(\theta+1)}, \\ \mathring{R}_1(t) = \left[\lambda\mathrm{M}_3 + \varrho\mathrm{M}_5 - \dot{\pi}\mathrm{M}_6\right]\dfrac{t^{\theta}}{\Gamma(\theta+1)}, \\ \mathring{C}_1(t) = \left[\epsilon\mathrm{M}_3 + \varepsilon\mathrm{M}_4 - \omega\mathrm{M}_7\right]\dfrac{t^{\theta}}{\Gamma(\theta+1)} \end{cases} \tag{5.3}$$

and

$$p^2:\begin{cases}\mathring{S}_2(t)=\Lambda^{'}\dfrac{t^{\theta}}{\Gamma(\theta+1)}-\left[\tau(\mathrm{M}_3s_1+i_1\mathrm{M}_1)+\eta\psi(\mathrm{M}_4s_1+a_1\mathrm{M}_1)\right.\\ \qquad\qquad \left.+\zeta\rho(\mathrm{M}_5s_1+h_1\mathrm{M}_1)+\beta(\mathrm{M}_7s_1+c_1\mathrm{M}_1)+\dot{\pi}\mathrm{M}_1\right]\dfrac{t^{2\theta}}{\Gamma(2\theta+1)},\\ \mathring{E}_2(t)=\left[\tau(\mathrm{M}_3s_1+i_1\mathrm{M}_1)+\eta\psi(\mathrm{M}_4s_1+a_1\mathrm{M}_1)+\zeta\rho(\mathrm{M}_5s_1+h_1\mathrm{M}_1)\right.\\ \qquad\qquad \left.+\beta(\mathrm{M}_7s_1+c_1\mathrm{M}_1)-(\mu+\dot{\pi})\mathrm{e}_1\right]\dfrac{t^{2\theta}}{\Gamma(2\theta+1)},\\ \mathring{\mathrm{I}}_2(t)=\left[\mu\nu\mathrm{e}_1-(\gamma+\lambda)\mathrm{i}_1-(\dot{\pi}+\pi^{'})\mathrm{i}_1\right]\dfrac{t^{2\theta}}{\Gamma(2\theta+1)},\\ \mathring{A}_2(t)=\left[\mu(1-\nu)\mathrm{e}_1-(\kappa+\dot{\pi})\mathrm{a}_1\right]\dfrac{t^{2\theta}}{\Gamma(2\theta+1)},\\ \mathring{H}_2(t)=\left[\gamma\mathrm{i}_1+\kappa\mathrm{a}_1-(\varrho+\dot{\pi})\mathrm{h}_1\right]\dfrac{t^{2\theta}}{\Gamma(2\theta+1)},\\ \mathring{R}_2(t)=\left[\lambda\mathrm{i}_1+\varrho\mathrm{h}_1-\dot{\pi}\mathrm{i}_1\right]\dfrac{t^{2\theta}}{\Gamma(2\theta+1)},\\ \mathring{C}_2(t)=\left[\epsilon\mathrm{i}_1+\varepsilon\mathrm{a}_1-\omega\mathrm{c}_1\right]\dfrac{t^{2\theta}}{\Gamma(2\theta+1)}.\end{cases} \tag{5.4}$$

6. Discussion

Here, we discuss the validation of the results for the established iterative scheme and the selected fractional derivative by the behavior of graphs. For this, the values given in the Table 1 were used. All the compartments were simulated in different fractional orders as well as integer order $\theta = 1$, against the values shown in the table.

Table 1. Values of the parameters.

Notation	Parameter values	Notation	Parameter values
M_1	30	M_2	25
M_3	20	M_4	15
M_5	10	M_6	9
M_7	7	$\Lambda^{'}$	0.8
ζ	0.5	ψ	0.2
τ	0.1	$\dot{\pi}$	0.009
$\pi^{'}$	0.01	λ	0.1
μ	0.09	ν	0.05
ϱ	0.7	ε	0.03
ϵ	0.06	ω	0.4
κ	0.2	β	0.001
γ	0.1	η	0.03
ρ	0.07	-	-

The graph of the susceptible population class $\mathring{S}(t)$ represents simulation against θ =

0.5, 0.7, 0.9, *and* 1.0 in Figure 1. At the start of the 1st week, a rapid decreasing behavior of the curves can be seen, and after the 1st-week the susceptible population increases with time.

The graphs of the exposed population class $\mathring{E}(t)$, and asymptotic population class $\mathring{A}(t)$ in Figures 2 and 4 respectively represent increasing behavior in the first week and then decrease as time increases. After some time the curves show constant behavior.

As the infected and hospitalized population decreases, the recovered population increases with time and then becomes constant. The fractional order dynamics of hospitalized and recovered classes can be observed in Figures 3, 5 and 6 respectively. Also, the curves of the graph of Class $\mathring{C}(t)$ represent decreasing behavior as time increases and then become constant (see in Figure 7). This means improvement can be expected in this population over time.

Simulation of the results of all compartments are calculated for $\theta = 0.5,\ 0.7,\ 0.8,\ 1$. It can be seen that the converging behavior of the curves for fractional orders is fast as compared to the integer order 1 due to the robust nature of the Caputo fractional derivative. The increasing and decreasing behavior of the curves is fast at small fractional orders as compared to large fractional orders.

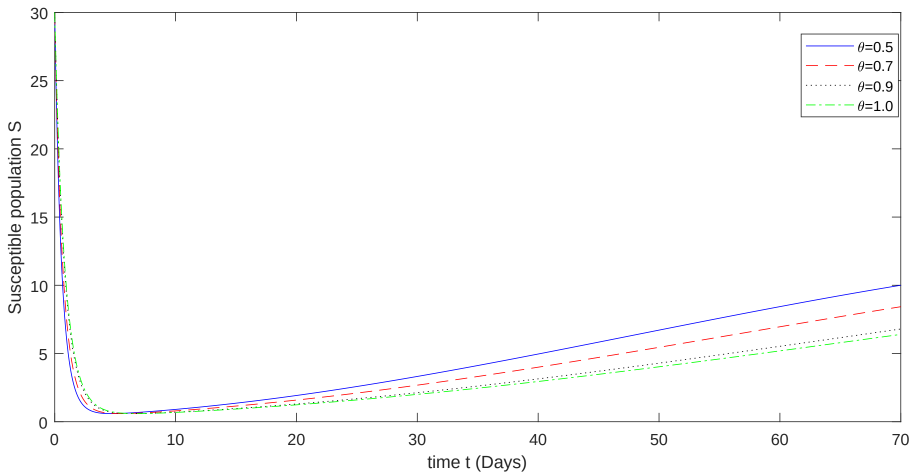

Figure 1. Graphical presentation of approximate values at different fractional orders for the susceptible class.

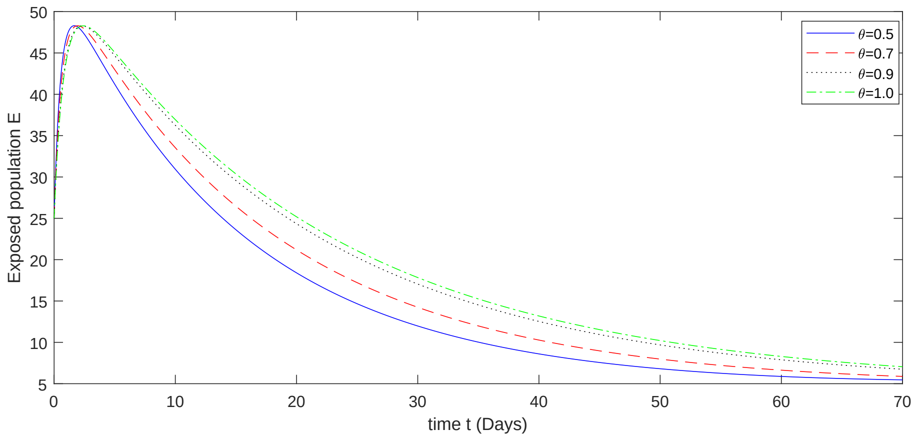

Figure 2. Graphical presentation of approximate values at different fractional orders for the exposed class.

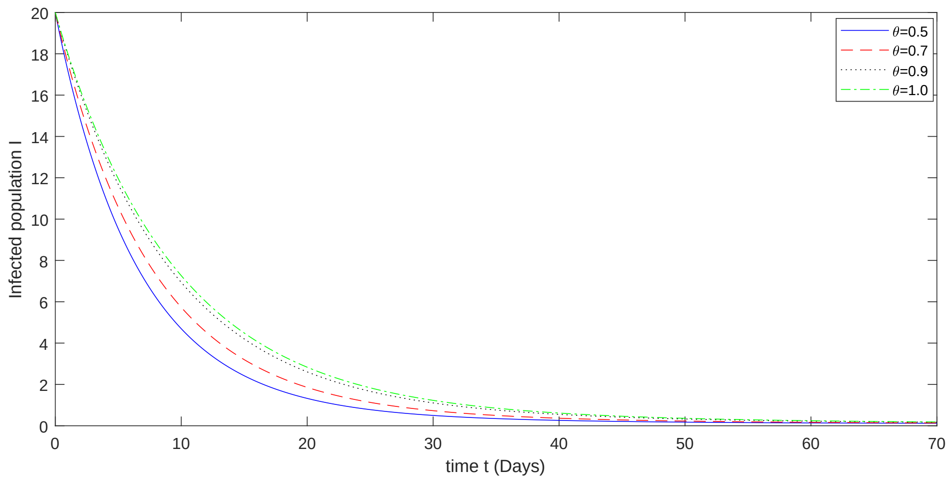

Figure 3. Graphical presentation of approximate values at different fractional orders for the infected class.

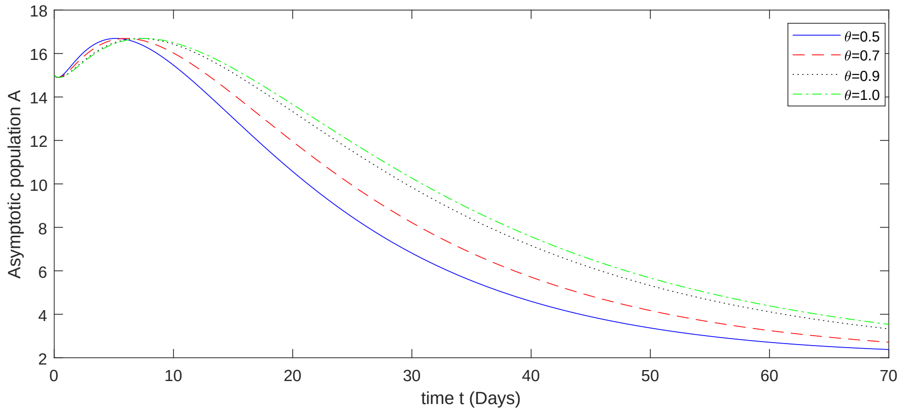

Figure 4. Graphical presentation of approximate values at different fractional order for the asymptomatic class.

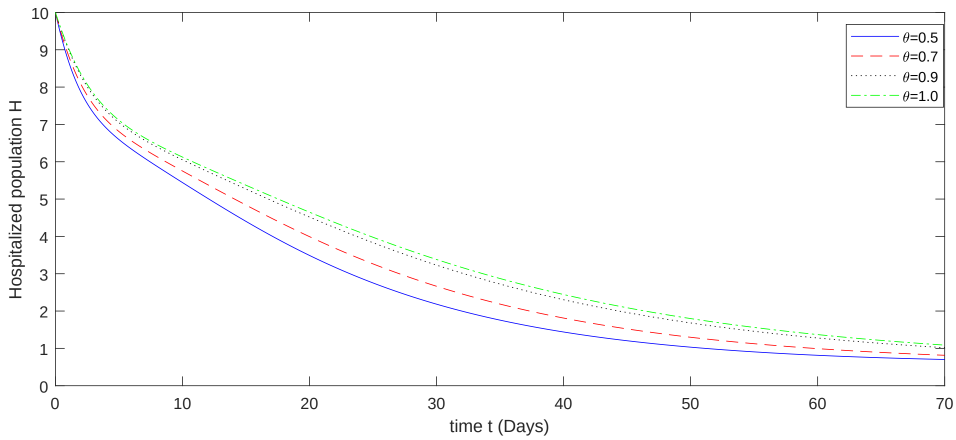

Figure 5. Graphical presentation of approximate values at different fractional orders for the hospitalized class.

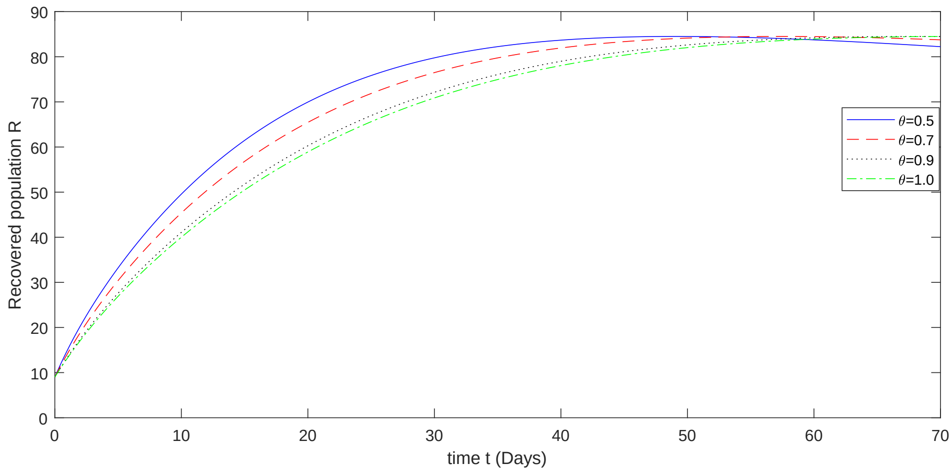

Figure 6. Graphical presentation of approximate values at different fractional orders for the recovered class.

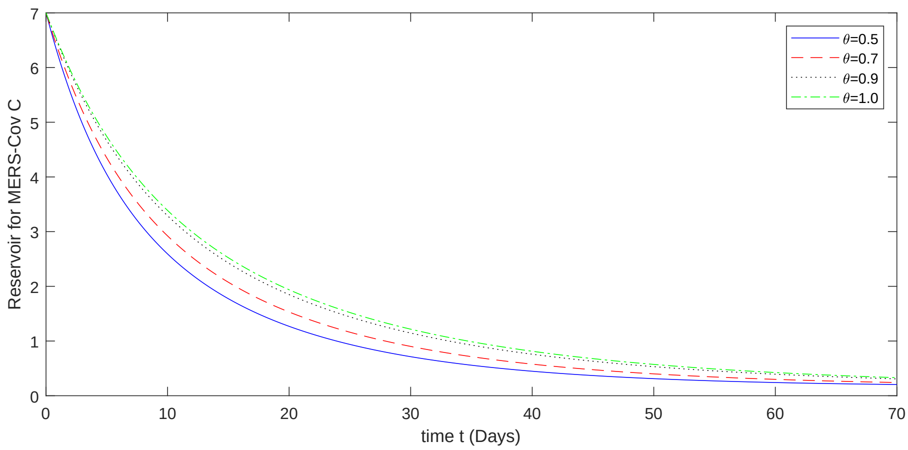

Figure 7. Graphical presentation of approximate values at different fractional orders for the Camel class.

Next, the comparison graphs are given to compare the different compartments of the model for $\theta = 1$. We compare the plotted compartments of the proposed model obtained via LADM and HPM techniques for the first three calculated terms. One can see the similarity in simulations, concluding that both schemes give the same results. For an efficient approximation for the nonlinear systems of ordinary differential equations and partial differential equation, the LADM is an effective technique. This technique is free of parameters, large or small, which is a big advantage of this method over the others. It deals with the nonlinearization directly while in contrast, some other procedures consider either linearization or discretization. This procedure demonstrates how the Laplace transform may be used to obtain the approximate solution of a nonlinear system of differential equations of fractional order by manipulating the decomposition method. Similarly, without the need for a linearization process, the HPM is an efficient and powerful method for obtaining the solution to a nonlinear system. This technique can take the ascendancy of the conventional perturbation technique while eliminating its limitations. Here in Figure 8, we compared the approximate solutions obtained via the LADM and HPM both methods. The graphical representation shows the similar approximate solution to the proposed model.

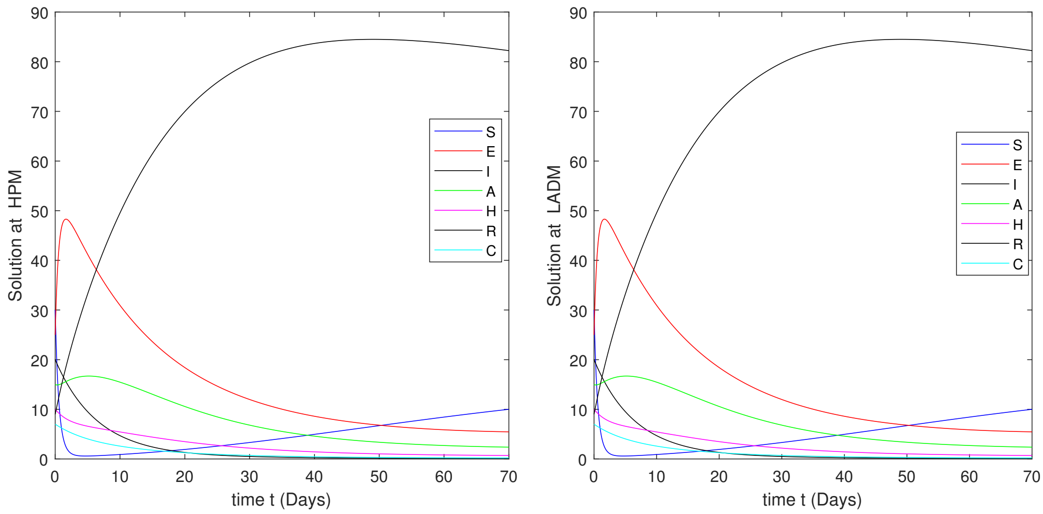

Figure 8. Comparison of approximate values of the LADM and HPM for the proposed model at $\theta = 1$.

7. Conclusions

The proposed model was investigated for approximate solutions via the LADM and HPM introducing a global operator that is a Caputo fractional derivative for the first three terms. The obtained results show that both methods give identical results for the problem under investigation. The results were also investigated graphically for the different values of θ. The traditional numerical methods need discretization, perturbation, transformation, or linearization to solve the nonlinear problems whereas, in the LADM, each term of the series is a generalized polynomial called the Adomian polynomial. Moreover, another advantage that it is free from rounding-off errors. Similarly, the HPM gives a very quick convergence of the solution series in most cases. It also avoids linearization or discretization and round-off errors. We can say that both methods are the best validation of a model of an arbitrary order derivative. To obtain the numerical solution, the fractional-order differential model can be selected for the larger accuracy in a more sophisticated way as compared to the differential model of integer-order. Different epidemic and infectious disease mathematical models can be studied via fractional order derivatives and the best choice of numerical techniques in order to obtain approximate solutions so that health care centers can control the spread of diseases. For numerical computations and implementation, Matlab15 is used.

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Conflict of interest

There does not exist conflict of interest.

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