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Research article

Robust stabilization for uncertain saturated systems with multiple time delays

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Abstract: This paper is concerned with the robust stabilization problem for uncertain saturated linear systems with multiple discrete delays. First of all, a new distributed-delay-dependent polytopic approach is proposed, and a new type of Lyapunov-Krasovskii functional is constructed. Then, by further incorporating some integral inequalities, both stabilization and robust stabilization conditions are proposed in terms of linear matrix inequalities under which the closed-loop systems are asymptotically stable for admissible initial conditions. Finally, a simulation example is given to illustrate the feasibility and advantages of the obtained results.

Keywords: regional stabilization; uncertain systems; multiple delays; actuator saturation; polytopic approach
Mathematics Subject Classification: 93D09

1. Introduction

Over the past decades, time-delay systems have received significant research attention due to the ubiquity of time delays in many dynamical systems, see, e.g., [1–29] and the references therein. In particular, in [1–19], some researchers have considered the systems with multiple time delays. The motivation for studying multiple time-delay systems is that multiple time delays are often encountered in many practical systems such as power systems [5–8], complex networks [11,12], neural networks [4, 13], multi-agent systems [18, 19]. For example, in [6, 7], multi-area power systems with time delays have been modeled as linear systems with multiple time delays. The main techniques for studying multiple time-delay systems are the characteristic roots approach [1,2], the state trajectory approach [3] and the Lyapunov-Krasovskii (L-K) approach [30–32]. Using the L-K approach, the obtained results can be expressed by linear matrix inequalities (LMIs), which can be easily solved by existing software. In particular, in [31, 32], to derive some less conservative stability criteria in the framework of LMIs, some cross-terms related to multiple different time delays have been introduced into the constructed L-K functionals. However, it is worth mentioning that the results proposed in [31, 32] are based on the free-weighting matrices technique and the Jensen inequality, which are still conservative to some extent.

On the other hand, it is well known that almost all practical feedback control systems are limited by the amplitude or rate of physical actuators for safety reasons or the limitations of the physical components. For example, a motor cannot generate infinite torque, and the amplifier output voltage cannot be unlimited. When an aircraft's vertical tail controls the horizontal steering, its steering angle has a maximum limit, and its rotation rate cannot be too fast; otherwise, the aircraft will roll over and cause an accident. Actuator saturation can cause instability and performance degradation of the overall system. During the past two decades, much effort has been focused on linear systems with actuator saturation [33–44]. One of the important research questions is related to global and semi-global stabilization [33–36]. Global stabilization is not possible if the open-loop system is exponentially unstable [33]. Furthermore, global stabilization cannot, in general, be achieved by linear feedback even for open-loop systems that are not exponentially unstable [35]. The low-gain design technique and the dynamic scheduling approach have been utilized in [36], and the parametric Lyapunov equation-based low-gain design was proposed in [37]. The local stabilization problem has been well discussed for open-loop systems that are unstable [33, 34, 38]. In particular, two dominant approaches to deal with saturation nonlinearities are the polytopic models [33, 38, 45] and the generalized sector condition [34, 46].

By utilizing the polytopic approach in [33], time-delay systems with actuator saturations have been transformed into the convex polytope of linear systems [39, 40]. Then, by incorporating the L-K approach, LMI-based sufficient conditions have been established in [39, 40], under which the local stability of closed-loop systems can be guaranteed. Based on the generalized sector condition, the asynchronous H_∞ control problem has been addressed in [41] for time-delayed switched systems with actuator saturation via the anti-windup scheme. By introducing auxiliary time-delay feedbacks, the delay-dependent polytopic approach was developed in [42–44,47] to reduce the potential conservatism. However, the results in [39–44, 48, 49] are mainly concerned with systems with a single time delay. To the best of our knowledge, the local stabilization problem for multiple time-delay systems with actuator saturation has not been considered, likely due to the mathematical complexity.

Motivated by the above discussions, in this paper, the problems of local stabilization and the corresponding estimate of the domain of attraction are considered for multiple time-delay systems with actuator saturation. First, a distributed-delay-dependent polytopic approach is proposed, and the saturation nonlinearly is represented by the convex combination of state feedback and auxiliary distributed-delay feedback. Then, based on the L-K approach, both the stabilization and robust stabilization conditions are established in terms of LMIs. Subsequently, the optimization problems regarding the maximization of the domain of attraction are discussed. The main contributions of this work are summarized below.

1) The results proposed in this paper are quite general since many factors are considered, such as norm-bounded uncertainties, multiple time delays, and actuator saturation. Therefore, the results obtained in this paper generalize to existing results in the literature.

2) Different from the L-K functionals used in [30–32, 34], a novel L-K functional is constructed in this paper. Specifically, the proposed functional contains augmented state vectors and some

interconnected terms about multiple delays, which will lead to less conservative results.

3) A new polytopic model is proposed to represent the saturation nonlinearity. In particular, some interconnected terms concerning multiple delays are utilized, which is useful in reducing the possible conservatism.

Notation: The superscript "T" denotes matrix transposition; I denotes the identity matrix of appropriate dimensions; $\lambda_M(P)$ denotes the maximum eigenvalue of matrix P; $\mathrm{Sym}(E)$ is a shorthand notation for matrix $E + E^T$; and $\mathbb{C}_{n,d} = \mathbb{C}([-d,0], \mathbb{R}^n)$ denotes the Banach space of continuous vector functions mapping interval $[-d,0]$ into $\mathbb{R}^n$ using the topology of uniform convergence. $\|\varphi(t)\|_c \triangleq \max_{t\in[-d,0]}\|\varphi(t)\|_2$ stands for the norm of a function $\varphi(t) \in \mathbb{C}_{n,d}$; $\mathbb{D}_m$ is the set of $m \times m$ diagonal matrices, where the diagonal elements are either 1 or 0; $e_{m,i} \in \mathbb{R}^{1\times m}$ denotes a row vector whose i-th element is 1 and the others are 0; $\mathbf{I}[1,N]$ denotes the set $\{1,2,\cdots,N\}$; and $co\{h_1, h_2, \cdots, h_m\}$ denotes the convex hull of the vectors.

2. Problem formulation

Consider the following uncertain saturated linear system with multiple discrete delays:

$$\dot{x}(t) = A_0(t)x(t) + \sum_{j=1}^{N} A_j(t)x(t-d_j) + B(t)\mathrm{sat}(u(t)), t > 0, \tag{1}$$

$$x(t) = \varphi(t), t \in [-d,0], d = \max_{1\le j\le N}\left\{d_j\right\}, \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the system state; $u(t) \in \mathbb{R}^m$ is the control input; the time delays d_j, $j \in \mathbf{I}[1,N]$ are known scalars; $\varphi(t) \in \mathbb{C}_{n,d}$ denotes the initial function; $\mathrm{sat}(u) = [\mathrm{sat}(u_1)\,\mathrm{sat}(u_2)\,\cdots\,\mathrm{sat}(u_m)]^T$ is the standard saturation function with $\mathrm{sat}(u_i) = \mathrm{sgn}(u_i)\min\{1,|u_i|\}$; and $A_0(t)$, $A_1(t)$, $\cdots$, $A_N(t)$ and $B(t)$ are time-varying matrices that satisfy $A_0(t) = A_0 + \Delta A_0(t)$, $A_j(t) = A_j + \Delta A_j(t)$ ($j \in \mathbf{I}[1,N]$) and $B(t) = B + \Delta B(t)$, where A_j and B are known constant matrices with appropriate dimensions.

Remark 1. Different from most existing literature concerning control systems with actuator saturation [32–41], the systems (1) and (2) considered in this paper contains multiple time delays. Note that multiple time delays are often encountered in many practical systems such as power systems [5–8], complex networks [11, 12], neural networks [4, 13, 29], and multi-agent systems [18, 19]. Therefore, the results obtained in this paper can be seen as an indispensable supplement to the existing literature.

Assumption 1. The uncertain matrices $\Delta A_0(t)$, $\Delta A_j(t)$ ($j \in \mathbf{I}[1,N]$), $\Delta A_N(t)$ and $\Delta B(t)$ satisfy

$$[\Delta A_0(t)\ \Delta A_1(t)\ \cdots\ \Delta A_N(t)\ \Delta B(t)] \overset{\Delta}{=} MF(t)[E_0\ E_1\ \cdots\ E_N\ E_B], \tag{3}$$

where M, E_0, E_1, $\cdots$, E_N and E_B are known real constant matrices, and $F(t)$ is an unknown time-varying matrix satisfying $F^T(t)F(t) \le I$.

This paper employs the following state feedback controller:

$$u(t) = Kx(t), \tag{4}$$

where $K \in \mathbb{R}^{m\times n}$ is the gain matrix to be designed.

Lemma 1. [38] Denote $\overleftrightarrow{m} \overset{\Delta}{=} m2^{m-1}$. Let $w \in \mathbb{R}^{\overleftrightarrow{m}}$ be such that $\|w\|_\infty \leqslant 1$, $D_k \in \mathbb{D}_m$, $k \in \mathbf{I}[1,2^m]$. The function f_m is defined as $f_m(0) = 0$ and

$$f_m(k) = \begin{cases} f_m(k-1)+1, & D_k + D_{k'} \neq I_m, \ \forall k' \in \mathbf{I}[\mathbf{1}, \mathbf{k}], \\ f_m(k'), & D_k + D_{k'} = I_m, \ \exists k' \in \mathbf{I}[\mathbf{1}, \mathbf{k}]. \end{cases}$$

Letting $\otimes$ denote the Kronecker product, for any $u \in \mathbb{R}^m$, the following holds:

$$\mathrm{sat}(u) \in co\{D_k u + \mathcal{D}_k^- w : k \in \mathbf{I}[1, 2^m]\},$$

where $\mathcal{D}_k^- \in \mathbb{R}^{m \times \overleftrightarrow{m}}$ is defined as $\mathcal{D}_k^- = e_{2^{m-1}, f_m(k)} \otimes D_k^-$ with $D_k^- = I - D_k$.

Lemma 2. [50] For a given symmetric positive definite R and any differentiable function x in $[-d, 0] \longrightarrow \mathbb{R}^n$, the following inequality holds:

$$\int_{-d}^{0} \dot{x}^T(\theta) R \dot{x}(\theta) \mathrm{d}\theta \geqslant \frac{1}{d} \begin{bmatrix} \Theta_0 \\ \Theta_1 \end{bmatrix}^T \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix} \begin{bmatrix} \Theta_0 \\ \Theta_1 \end{bmatrix},$$

where

$$\begin{aligned} \Theta_0 =& x(0) - x(-d), \\ \Theta_1 =& x(0) + x(-d) - \frac{2}{d} \int_{-d}^{0} x(\theta) \mathrm{d}\theta. \end{aligned}$$

In addtion, to represent the saturation nonlinearity in a delay-depdent framwork, we introduce the following lemma directly derived from Lemma 1.

Lemma 3. Let us define the following functional:

$$w(t) = U x(t) + \sum_{j=1}^{N} V_j \int_{t-d_j}^{t} x(s) \mathrm{d}s + \sum_{\hat{\imath}=1}^{N-1} \sum_{\hat{\jmath}=\hat{\imath}+1}^{N} V_{\hat{\imath}\hat{\jmath}} \int_{t-d_{\hat{\jmath}}}^{t-d_{\hat{\imath}}} x(s) \mathrm{d}s, \tag{5}$$

where U, $V_j \in \mathbb{R}^{\overleftrightarrow{m} \times n}$, $j \in \mathbf{I}[1, N]$, $V_{\hat{\imath}\hat{\jmath}} \in \mathbb{R}^{\overleftrightarrow{m} \times n}$, $\hat{\imath} \in \mathbf{I}[1, N-1]$, and $\hat{\jmath} \in \mathbf{I}[\hat{\imath}+1, N]$. If the constraint

$$\|w(t)\|_\infty \leqslant 1 \tag{6}$$

is satisfied, the saturation nonlinearity $\mathrm{sat}(u(t))$ can be written as follows:

$$\mathrm{sat}(u(t)) = \sum_{k=1}^{2^m} \lambda_k^t \Big[\left(D_k K + \mathcal{D}_k^- U\right) x(t) + \mathcal{D}_k^- \sum_{j=1}^{N} V_j \int_{t-d_j}^{t} x(s) \mathrm{d}s + \mathcal{D}_k^- \sum_{\hat{\imath}=1}^{N-1} \sum_{\hat{\jmath}=\hat{\imath}+1}^{N} V_{\hat{\imath}\hat{\jmath}} \int_{t-d_{\hat{\jmath}}}^{t-d_{\hat{\imath}}} x(s) \mathrm{d}s \Big], \tag{7}$$

where $\lambda_1^t \geq 0$, $\lambda_2^t \geq 0, \cdots, \lambda_{2^m}^t \geq 0$ and $\sum_{k=1}^{2^m} \lambda_k^t = 1$.

Proof. From (5) and Lemma 1, the (7) can be easily obtained. □

Using (1) and (7), one can obtain the following closed-loop system:

$$\dot{x}(t) = \sum_{k=1}^{2^m} \lambda_k^t \Big\{ [A_0(t) + B(t)(D_k K + \mathcal{D}_k^- U)]\, x(t) + \sum_{j=1}^{N} A_j(t) x(t-d_j)$$
$$+ \sum_{j=1}^{N} B(t)\mathcal{D}_k^- V_j \int_{t-d_j}^{t} x(s)\mathrm{d}s + \sum_{\hat{i}=1}^{N-1} \sum_{\hat{j}=\hat{i}+1}^{N} B(t)\mathcal{D}_k^- V_{\hat{i}\hat{j}} \int_{t-d_{\hat{j}}}^{t-d_{\hat{i}}} x(s)\mathrm{d}s \Big\} \triangleq \eta(t). \tag{8}$$

Remark 2. Compared with the traditional polytope model [38], it can be seen that the polytope model (7) contains the auxiliary time-delay feedback $\sum_{j=1}^{N} V_j \int_{t-d_j}^{t} x(s)\mathrm{d}s$ and $\sum_{\hat{i}=1}^{N-1} \sum_{\hat{j}=\hat{i}+1}^{N} V_{\hat{i}\hat{j}} \int_{t-d_{\hat{j}}}^{t-d_{\hat{i}}} x(s)\mathrm{d}s$ related to different time delays. Since the time delay information is sufficiently used in representing the saturation nonlinearity, it is expected that less conservative stabilization conditions can be achieved.

In this paper, we assume that the admissible initial conditions $\varphi(t)$ belong to the following set:

$$X_\rho \triangleq \{\varphi(t) \in \mathbb{C}_{n,d} : \|\varphi(t)\|_c \leqslant \rho_1, \|\dot{\varphi}(t)\|_c \leqslant \rho_2\}, \tag{9}$$

where ρ_1 and ρ_2 are some positive scalars to be optimized.

In this paper, we are interested in designing the state feedback controller (4) such that the closed-loop system (8) is locally stable with an estimate of the region of attraction that is as large as possible.

3. Main results

In this paper, we choose the following L-K functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{10}$$

where $V_1(t) = \xi^T(t)P\xi(t)$ and

$$V_2(t) = \sum_{j=1}^{N} \Big[\int_{t-d_j}^{t} x^T(s) S_j x(s)\mathrm{d}s + d_j \int_{-d_j}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R_j \dot{x}(s)\mathrm{d}s\mathrm{d}\theta \Big],$$
$$V_3(t) = \sum_{\hat{i}=1}^{N-1} \sum_{\hat{j}=\hat{i}+1}^{N} d_{\hat{j}\hat{i}} \Big[\int_{t-d_{\hat{j}}}^{t-d_{\hat{i}}} x^T(s) Q_{\hat{i}\hat{j}} x(s)\mathrm{d}s + \int_{-d_{\hat{j}}}^{-d_{\hat{i}}} \int_{t+\theta}^{t} \dot{x}^T(s) Z_{\hat{i}\hat{j}} \dot{x}(s)\mathrm{d}s\mathrm{d}\theta \Big],$$

with $P > 0$, $R_j > 0$, $S_j > 0$, $Q_{\hat{i}\hat{j}} > 0$, $Z_{\hat{i}\hat{j}} > 0$, $d_{\hat{j}\hat{i}} \triangleq d_{\hat{j}} - d_{\hat{i}}$ ($j \in \mathbf{I}[1,N]$, $\hat{i} \in \mathbf{I}[1,N-1]$, $\hat{j} \in \mathbf{I}[\hat{i}+1,N]$).

For convenience in subsequent presentation, we denote

$$\xi(t) \triangleq \Big[x^T(t) \int_{t-d_1}^{t} x^T(s)\mathrm{d}s \cdots \int_{t-d_N}^{t} x^T(s)\mathrm{d}s \int_{t-d_2}^{t-d_1} x^T(s)\mathrm{d}s \cdots \int_{t-d_N}^{t-d_1} x^T(s)\mathrm{d}s \int_{t-d_3}^{t-d_2} x^T(s)\mathrm{d}s$$
$$\cdots \int_{t-d_N}^{t-d_2} x^T(s)\mathrm{d}s \int_{t-d_4}^{t-d_3} x^T(s)\mathrm{d}s \cdots \int_{t-d_N}^{t-d_3} x^T(s)\mathrm{d}s \cdots \int_{t-d_N}^{t-d_{N-1}} x^T(s)\mathrm{d}s \Big]^T,$$
$$\zeta(t) \triangleq \Big[x^T(t)\; x^T(t-d_1) \cdots x^T(t-d_N) \int_{t-d_1}^{t} x^T(s)\mathrm{d}s \cdots \int_{t-d_N}^{t} x^T(s)\mathrm{d}s \int_{t-d_2}^{t-d_1} x^T(s)\mathrm{d}s \cdots$$

$$\int_{t-d_N}^{t-d_1} x^T(s)\mathrm{d}s \ \cdots \ \int_{t-d_N}^{t-d_{N-1}} x^T(s)\mathrm{d}s \ \dot{x}^T(t)\Big]^T, \ \hat{A} \triangleq [A_1 \ A_2 \ \cdots \ A_N]^T,$$

$$N_1 \triangleq (N^2+N+2)n/2, \ N_2 \triangleq N(N-1)n/2, \ N_3 \triangleq N(N+1)n/2, \ N_4 \triangleq (N^2+3N+2)n/2,$$

$$\tilde{d}_{\hat{\imath}\hat{\jmath}} \triangleq -12/(d_{\hat{\jmath}}-d_{\hat{\imath}})^2, \ \bar{d}_{\hat{\imath}\hat{\jmath}} \triangleq 2/(d_{\hat{\jmath}}-d_{\hat{\imath}}), \ \tilde{d}_j \triangleq -12/d_j^2, \ \bar{d}_j \triangleq 1/d_j,$$

$$\hat{\Phi}_2 \triangleq [I_1^T-I_2^T \ I_1^T-I_3^T \ \cdots \ I_1^T-I_{N+1}^T \ I_2^T-I_3^T \cdots \ I_2^T-I_{N+1}^T \ \cdots \ I_N^T-I_{N+1}^T]^T,$$

$$I'_j \triangleq [0_{n\times(j-1)n} \ I_n \ 0_{n\times(N-j)n}], \ I_t \triangleq [0_{n\times(t-1)n} \ I_n \ 0_{n\times(N+1-t)n}]^T,$$

where $j \in \mathbf{I}[1,N]$, $t \in \mathbf{I}[1,N+1]$.

First, we consider the stabilization problem for the system (1) without uncertainties, i.e., the system matrices in (1) satisfy $F(t)=0$.

Theorem 1. Let the constants $\delta \neq 0$ and d_j, $j \in \mathbf{I}[1,N]$ be given. System (1) without uncertainties can be asymptotically stabilized by controller (4) with $K = YX^{-T}$ if for any initial function $\varphi(t) \in \mathbb{C}_{n,d}$ satisfying $V(0) \leqslant 1$, there exist $N_1 \times N_1$ matrix $\bar{P} > 0$, $n \times n$ matrices $\bar{R}_j > 0$, $\bar{S}_j > 0$, $\bar{Q}_{\hat{\imath}\hat{\jmath}} > 0$, $\bar{Z}_{\hat{\imath}\hat{\jmath}} > 0$, $n \times n$ invertible matrix X, $m \times n$ matrix Y, and $\overleftrightarrow{m} \times n$ matrices G, H_j, $j \in \mathbf{I}[1,N]$, $H_{\hat{\imath}\hat{\jmath}}$, $\hat{\imath} \in \mathbf{I}[1,N-1]$, $\hat{\jmath} \in \mathbf{I}[\hat{\imath}+1,N]$, such that the following LMIs hold:

$$(\bar{\Omega}_{rs}^k)_{5\times5} + \mathrm{Sym}(\Phi_1^T\bar{P}\Phi_2) \triangleq \bar{\Omega}^k < 0, \ \forall k \in \mathbf{I}[1,2^m], \tag{11}$$

$$\begin{bmatrix} 1 & \bar{F}_l \\ * & \bar{P} \end{bmatrix} + \begin{bmatrix} 0 & 0_{1\times n} & 0_{1\times N_3} \\ * & \bar{\Sigma}_{11} & \bar{\vartheta} \\ * & * & \bar{\Xi} \end{bmatrix} \geqslant 0, \ \forall l \in \mathbf{I}[1,\overleftrightarrow{m}], \tag{12}$$

where $\bar{F}_l$ is the l-th row of $\bar{F} = [G \ H_1 \ H_2 \ \cdots \ H_N \ H_{12} \ \cdots \ H_{1N} \ H_{23} \ \cdots \ H_{2N} \ \cdots \ H_{(N-1)N}]$, $l \in \mathbf{I}[1,\overleftrightarrow{m}]$ and

$$\bar{\Omega}_{11}^k = \sum_{j=1}^{N}(\bar{S}_j - 4\bar{R}_j) + \mathrm{Sym}[A_0X^T + B(D_kY + \mathcal{D}_k^-G)],$$

$$\bar{\Omega}_{12}^k = [-2\bar{R}_1 + A_1X^T \ -2\bar{R}_2 + A_2X^T \cdots -2\bar{R}_N + A_NX^T],$$

$$\bar{\Omega}_{13}^k = [\bar{\beta}_1^k \ \bar{\beta}_2^k \ \cdots \ \bar{\beta}_N^k], \ \bar{\Omega}_{14}^k = [\bar{\gamma}_{12}^k \ \bar{\gamma}_{13}^k \ \cdots \ \bar{\gamma}_{1N}^k \ \bar{\gamma}_{23}^k \ \cdots \ \bar{\gamma}_{2N}^k \ \cdots \ \bar{\gamma}_{(N-1)N}^k],$$

$$\bar{\Omega}_{15}^k = -X^T + \delta XA_0^T + \delta(D_kY + \mathcal{D}_k^-G)^TB^T,$$

$$\bar{\Omega}_{22}^k = \begin{bmatrix} \bar{\alpha}_2 & -2\bar{Z}_{12} & \cdots & -2\bar{Z}_{1(N-1)} & -2\bar{Z}_{1N} \\ * & \bar{\alpha}_3 & \cdots & -2\bar{Z}_{2(N-1)} & -2\bar{Z}_{2N} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \bar{\alpha}_N & -2\bar{Z}_{(N-1)N} \\ * & * & * & * & \bar{\alpha}_{N+1} \end{bmatrix},$$

$$\bar{\Omega}_{23}^k = 6\mathrm{diag}\left\{\bar{d}_1\bar{R}_1 \ \bar{d}_2\bar{R}_2 \ \cdots \ \bar{d}_N\bar{R}_N\right\},$$

$$\bar{\Omega}_{24}^k = 3[\bar{\xi}_{12} \ \bar{\xi}_{13} \ \cdots \ \bar{\xi}_{1N} \ \bar{\xi}_{23} \ \cdots \ \bar{\xi}_{2N} \ \cdots \ \bar{\xi}_{(N-1)N}],$$

$$\bar{\Omega}_{25}^k = \delta X\hat{A}, \ \bar{\Omega}_{33}^k = \mathrm{diag}\left\{\tilde{d}_1\bar{R}_1 \ \tilde{d}_2\bar{R}_2 \ \cdots \ \tilde{d}_N\bar{R}_N\right\},$$

$$\bar{\Omega}_{34}^k = 0_{Nn\times N_2}, \ \bar{\Omega}_{35}^k = [\bar{\sigma}_1^k \ \bar{\sigma}_2^k \ \cdots \ \bar{\sigma}_N^k]^T,$$

$$\bar{\Omega}_{44}^k = \mathrm{diag}\left\{\tilde{d}_{12}\bar{Z}_{12} \ \cdots \ \tilde{d}_{1N}\bar{Z}_{1N} \ \tilde{d}_{23}\bar{Z}_{23} \ \cdots \ \tilde{d}_{2N}\bar{Z}_{2N} \ \cdots \ \tilde{d}_{(N-1)N}\bar{Z}_{(N-1)N}\right\},$$

$$\bar{\Omega}_{45}^{k} = [\bar{\mu}_{12}^{k} \ \cdots \ \bar{\mu}_{1N}^{k} \ \bar{\mu}_{23}^{k} \ \cdots \ \bar{\mu}_{2N}^{k} \ \cdots \ \bar{\mu}_{(N-1)N}^{k}]^{T},$$

$$\bar{\Omega}_{55}^{k} = -\delta \mathrm{Sym}(X) + \sum_{j=1}^{N} d_j^2 \bar{R}_j + \sum_{\hat{\imath}=1}^{N-1} \sum_{\hat{\jmath}=\hat{\imath}+1}^{N} d_{\hat{\imath}\hat{\jmath}}^2 \bar{Z}_{\hat{\imath}\hat{\jmath}},$$

$$\Phi_1 = \begin{bmatrix} I & 0_{n\times Nn} & 0 & 0 & \cdots & 0 & 0_{n\times n} \\ 0 & 0_{n\times Nn} & I & 0 & \cdots & 0 & 0_{n\times n} \\ 0 & 0_{n\times Nn} & 0 & I & \cdots & 0 & 0_{n\times n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0_{n\times Nn} & 0 & 0 & \cdots & I & 0_{n\times n} \end{bmatrix},$$

$$\Phi_2 = \begin{bmatrix} 0_{n\times N_4} & I \\ \hat{\Phi}_2 & 0_{n\times N_3} \end{bmatrix}, \bar{\Sigma}_{11} = 2\sum_{j=1}^{N} d_j \bar{R}_j + 2\sum_{\hat{\imath}=1}^{N-1} \sum_{\hat{\jmath}=\hat{\imath}+1}^{N} d_{\hat{\jmath}\hat{\imath}} \bar{Z}_{\hat{\imath}\hat{\jmath}},$$

$$\bar{\vartheta} = \left[\bar{\Sigma}_{12} \ \bar{\Sigma}_{13} \ \cdots \ \bar{\Sigma}_{1(N+1)} \ \bar{\Sigma}_1^{12} \ \bar{\Sigma}_1^{13} \ \cdots \ \bar{\Sigma}_1^{(N-1)N}\right],$$

$$\bar{\Xi} = \mathrm{diag}\left\{\bar{\Sigma}_{22} \ \cdots \ \bar{\Sigma}_{(N+1)(N+1)} \ \bar{\Sigma}_{12}^{12} \ \bar{\Sigma}_{13}^{13} \ \cdots \ \bar{\Sigma}_{(N-1)N}^{(N-1)N}\right\},$$

with

$$\bar{\alpha}_2 = -4\bar{R}_1 - \bar{S}_1 + \sum_{\hat{\jmath}=2}^{N} d_{\hat{\jmath}1} \bar{Q}_{1\hat{\jmath}} - 4\sum_{\hat{\jmath}=2}^{N} \bar{Z}_{1\hat{\jmath}},$$

$$\bar{\alpha}_{t+1} = -4\bar{R}_t - \bar{S}_t + \sum_{\hat{\jmath}=t+1}^{N} \left(d_{\hat{\jmath}t} \bar{Q}_{t\hat{\jmath}} - 4\bar{Z}_{t\hat{\jmath}}\right) - \sum_{\hat{\imath}=1}^{t-1} \left(d_{t\hat{\imath}} \bar{Q}_{\hat{\imath}t} + 4\bar{Z}_{\hat{\imath}t}\right), \ t \in \mathbf{I}[2, N-1],$$

$$\bar{\alpha}_{N+1} = -4\bar{R}_N - \bar{S}_N - \sum_{\hat{\imath}=1}^{N-1} d_{N\hat{\imath}} \bar{Q}_{\hat{\imath}N} - 4\sum_{\hat{\imath}=1}^{N-1} \bar{Z}_{\hat{\imath}N}, \ \bar{\beta}_j^k = 6\bar{d}_j \bar{R}_j + B\mathcal{D}_k^- H_j, \ \bar{\Sigma}_{1(j+1)} = -2\bar{R}_j,$$

$$\bar{\gamma}_{\hat{\imath}\hat{\jmath}}^k = B\mathcal{D}_k^- H_{\hat{\imath}\hat{\jmath}}, \ \bar{\xi}_{\hat{\imath}\hat{\jmath}} = \bar{d}_{\hat{\imath}\hat{\jmath}} \bar{Z}_{\hat{\imath}\hat{\jmath}} (I'_{\hat{\imath}} + I'_{\hat{\jmath}}), \ \bar{\sigma}_j^k = \delta B\mathcal{D}_k^- H_j, \ \bar{\mu}_{\hat{\imath}\hat{\jmath}}^k = \delta B\mathcal{D}_k^- H_{\hat{\imath}\hat{\jmath}}, \ \bar{\Sigma}_1^{\hat{\imath}\hat{\jmath}} = -2\bar{Z}_{\hat{\imath}\hat{\jmath}},$$

$$\bar{\Sigma}_{\hat{\imath}\hat{\jmath}}^{\hat{\imath}\hat{\jmath}} = \bar{Q}_{\hat{\imath}\hat{\jmath}} + \bar{d}_{\hat{\imath}\hat{\jmath}} \bar{Z}_{\hat{\imath}\hat{\jmath}}, \ \bar{\Sigma}_{(j+1)(j+1)} = \bar{d}_j (2\bar{R}_j + \bar{S}_j), \ j \in \mathbf{I}[1, N], \ \hat{\imath} \in \mathbf{I}[1, N-1], \ \hat{\jmath} \in \mathbf{I}[\hat{\imath}+1, N].$$

Proof. Differentiating $V(t)$ in (10) along the closed-loop system (8) yields

$$\begin{aligned} \dot{V}(t) =& 2\xi^T(t) P \dot{\xi}(t) + \sum_{j=1}^{N} \Big[x^T(t) S_j x(t) + d_j^2 \dot{x}^T(t) R_j \dot{x}(t) - x^T(t-d_j) S_j x(t-d_j) \\ & - d_j \int_{t-d_j}^{t} \dot{x}^T(s) R_j \dot{x}(s) \mathrm{d}s \Big] + \sum_{\hat{\imath}=1}^{N-1} \sum_{\hat{\jmath}=\hat{\imath}+1}^{N} d_{\hat{\jmath}\hat{\imath}} \Big\{ \Big[x^T(t-d_{\hat{\imath}}) Q_{\hat{\imath}\hat{\jmath}} x(t-d_{\hat{\imath}}) \\ & - x^T(t-d_{\hat{\jmath}}) Q_{\hat{\imath}\hat{\jmath}} x(t-d_{\hat{\jmath}}) \Big] + d_{\hat{\jmath}\hat{\imath}} \dot{x}^T(t) Z_{\hat{\imath}\hat{\jmath}} \dot{x}(t) - \int_{t-d_{\hat{\jmath}}}^{t-d_{\hat{\imath}}} \dot{x}^T(s) Z_{\hat{\imath}\hat{\jmath}} \dot{x}(s) \mathrm{d}s \Big\}. \end{aligned} \tag{13}$$

Using the Wirtinger-based inequality (Lemma 2) for $\int_{t-d_j}^{t} \dot{x}^T(s) R_j \dot{x}(s) \mathrm{d}s$ and $\int_{t-d_{\hat{\jmath}}}^{t-d_{\hat{\imath}}} \dot{x}^T(s) Z_{\hat{\imath}\hat{\jmath}} \dot{x}(s) \mathrm{d}s$, we have

$$-d_j \int_{t-d_j}^{t} \dot{x}^T(s) R_j \dot{x}(s) \mathrm{d}s \leqslant -[x(t) - x(t-d_j)]^T R_j [x(t) - x(t-d_j)] - 3\Psi_j^T R_j \Psi_j, \tag{14}$$

$$-d_{\hat{j}\hat{i}} \int_{t-d_{\hat{j}}}^{t-d_{\hat{i}}} \dot{x}^T(s) Z_{\hat{i}\hat{j}} \dot{x}(s) \mathrm{d}s \leqslant -[x(t-d_{\hat{i}}) - x(t-d_{\hat{j}})]^T Z_{\hat{i}\hat{j}} [x(t-d_{\hat{i}}) - x(t-d_{\hat{j}})] - 3\Gamma_{\hat{i}\hat{j}}^T Z_{\hat{i}\hat{j}} \Gamma_{\hat{i}\hat{j}}, \tag{15}$$

where

$$\Psi_j = x(t) + x(t-d_j) - 2\bar{d}_j \int_{t-d_j}^{t} x(s) \mathrm{d}s,$$
$$\Gamma_{\hat{i}\hat{j}} = x(t-d_{\hat{i}}) + x(t-d_{\hat{j}}) - \bar{d}_{\hat{i}\hat{j}} \int_{t-d_{\hat{j}}}^{t-d_{\hat{i}}} x(s) \mathrm{d}s.$$

For any matrices $T_1, T_2 \in \mathbb{R}^{n\times n}$, it follows from the closed-loop system (8) that

$$2[x^T(t)T_1 + \dot{x}^T(t)T_2][\eta(t) - \dot{x}(t)] = 0. \tag{16}$$

Adding the left side of (16) to $\dot{V}(t)$ in (13) and substituting (14) and (15) into (13), one can obtain the following inequality:

$$\dot{V}(t) \leqslant \sum_{k=1}^{2^m} \lambda_k^t \zeta^T(t) \Omega^k \zeta(t), \tag{17}$$

where

$$\Omega^k \overset{\Delta}{=} (\Omega_{rs}^k)_{5\times 5} + \mathrm{Sym}(\Phi_1^T P \Phi_2),$$

with

$$\Omega_{11}^k = \sum_{j=1}^{N} (S_j - 4R_j) + T_1[A_0 + B(D_k K + \mathcal{D}_k^- U)] + [A_0 + B(D_k K + \mathcal{D}_k^- U)]^T T_1^T,$$
$$\Omega_{12}^k = [-2R_1 + T_1 A_1 \; -2R_2 + T_1 A_2 \; \cdots \; -2R_N + T_1 A_N],\ \Omega_{13}^k = [\beta_1^k \, \beta_2^k \, \cdots \, \beta_N^k],$$
$$\Omega_{14}^k = [\gamma_{12}^k \, \gamma_{13}^k \, \cdots \, \gamma_{1N}^k \, \gamma_{23}^k \, \cdots \, \gamma_{2N}^k \, \cdots \, \gamma_{(N-1)N}^k],\ \Omega_{15}^k = -T_1 + [A_0 + B(D_k K + \mathcal{D}_k^- U)]^T T_2^T,$$
$$\Omega_{22}^k = \begin{bmatrix} \alpha_2 & -2Z_{12} & \cdots & -2Z_{1(N-1)} & -2Z_{1N} \\ * & \alpha_3 & \cdots & -2Z_{2(N-1)} & -2Z_{2N} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \alpha_N & -2Z_{(N-1)N} \\ * & * & * & * & \alpha_{N+1} \end{bmatrix},$$
$$\Omega_{23}^k = 6\mathrm{diag}\left\{\bar{d}_1 R_1 \, \bar{d}_2 R_2 \, \cdots \, \bar{d}_N R_N\right\},$$
$$\Omega_{24}^k = 3[\xi_{12} \, \xi_{13} \, \cdots \, \xi_{1N} \, \xi_{23} \, \cdots \, \xi_{2N} \, \cdots \, \xi_{(N-1)N}],$$
$$\Omega_{25}^k = \hat{A} T_2^T,\ \Omega_{33}^k = \mathrm{diag}\left\{\tilde{d}_1 R_1 \, \tilde{d}_2 R_2 \, \cdots \, \tilde{d}_N R_N\right\},$$
$$\Omega_{34}^k = 0_{Nn\times N_2},\ \Omega_{35}^k = [\sigma_1^k \, \sigma_2^k \, \cdots \, \sigma_N^k]^T,$$

$$\Omega_{44}^{k} = \mathrm{diag}\Big\{\tilde{d}_{12}Z_{12} \ \cdots \ \tilde{d}_{1N}Z_{1N} \ \tilde{d}_{23}Z_{23} \ \cdots \ \tilde{d}_{2N}Z_{2N} \ \cdots \ \tilde{d}_{(N-1)N}Z_{(N-1)N}\Big\},$$

$$\Omega_{45}^{k} = [\mu_{12}^{k} \ \cdots \ \mu_{1N}^{k} \ \mu_{23}^{k} \ \cdots \ \mu_{2N}^{k} \ \cdots \ \mu_{(N-1)N}^{k}]^{T},$$

$$\Omega_{55}^{k} = -\mathrm{Sym}(T_2) + \sum_{j=1}^{N} d_j^2 R_j + \sum_{\hat{i}=1}^{N-1}\sum_{\hat{j}=\hat{i}+1}^{N} d_{\hat{i}\hat{j}}^2 Z_{\hat{i}\hat{j}},$$

$$\Phi_1 = \begin{bmatrix} I & 0_{n\times Nn} & 0 & 0 & \cdots & 0 & 0_{n\times n} \\ 0 & 0_{n\times Nn} & I & 0 & \cdots & 0 & 0_{n\times n} \\ 0 & 0_{n\times Nn} & 0 & I & \cdots & 0 & 0_{n\times n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0_{n\times Nn} & 0 & 0 & \cdots & I & 0_{n\times n} \end{bmatrix}, \ \Phi_2 = \begin{bmatrix} 0_{n\times N_4} & I \\ \hat{\Phi}_2 & 0_{n\times N_3} \end{bmatrix},$$

$$\alpha_{t+1} = -4R_t - S_t + \sum_{\hat{j}=t+1}^{N}\left(d_{\hat{j}t}Q_{t\hat{j}} - 4Z_{t\hat{j}}\right) - \sum_{\hat{i}=1}^{t-1}(d_{t\hat{i}}Q_{\hat{i}t} + 4Z_{\hat{i}t}), \ t \in \mathbf{I}[2, N-1],$$

$$\alpha_{N+1} = -4R_N - S_N - \sum_{\hat{i}=1}^{N-1} d_{N\hat{i}}Q_{\hat{i}N} - 4\sum_{\hat{i}=1}^{N-1} Z_{\hat{i}N}, \ \alpha_2 = -4R_1 - S_1 + \sum_{\hat{j}=2}^{N} d_{\hat{j}1}Q_{1\hat{j}} - 4\sum_{\hat{j}=2}^{N} Z_{1\hat{j}},$$

$$\gamma_{\hat{i}\hat{j}}^{k} = T_1 B\mathcal{D}_k^{-} V_{\hat{i}\hat{j}}, \ \xi_{\hat{i}\hat{j}} = d_{\hat{i}\hat{j}}Z_{\hat{i}\hat{j}}(I'_{\hat{i}} + I'_{\hat{j}}), \ \mu_{\hat{i}\hat{j}}^{k} = T_2 B\mathcal{D}_k^{-} V_{\hat{i}\hat{j}}, \ \beta_j^{k} = 6\bar{d}_j R_j + T_1 B\mathcal{D}_k^{-} V_j,$$

$$\sigma_j^{k} = T_2 B\mathcal{D}_k^{-} V_j, \ \hat{i} \in \mathbf{I}[1, N-1], \ \hat{j} \in \mathbf{I}[\hat{i}+1, N], \ j \in \mathbf{I}[1, N].$$

Suppose that the following matrix inequality holds:

$$\Omega^{k} < 0.$$

Then, it is seen from (17) that $\dot{V}(t) < 0$ can be ensured. Furthermore, $\dot{V}(t) < 0$ means that

$$V(t) \leqslant V(0), \ t \geqslant 0. \tag{18}$$

Next, we will show that the condition described by (6) can be ensured. For the functional $V(t)$ defined in (10), using Jensen inequalities for the terms $d_{\hat{j}\hat{i}}\int_{t-d_{\hat{j}}}^{t-d_{\hat{i}}} x^T(s)Q_{\hat{i}\hat{j}}x(s)\mathrm{d}s$ and $d_{\hat{j}\hat{i}}\int_{-d_{\hat{j}}}^{-d_{\hat{i}}}\int_{t+\theta}^{t} \dot{x}^T(s)Z_{\hat{i}\hat{j}}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$, $\hat{i} \in \mathbf{I}[1, N-1]$, $\hat{j} \in \mathbf{I}[\hat{i}+1, N]$ and $d_j\int_{-d_j}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_j\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$, $j \in \mathbf{I}[1, N]$, it follows that

$$\begin{aligned} V(t) \geqslant & \sum_{j=1}^{N}\Big\{2x^T(t)R_j\Big[d_jx(t) - 2\int_{t-d_j}^{t} x(s)\mathrm{d}s\Big] + \bar{d}_j\int_{t-d_j}^{t} x^T(s)\mathrm{d}s\left(2R_j + S_j\right)\int_{t-d_j}^{t} x(s)\mathrm{d}s\Big\} \\ & + \sum_{\hat{i}=1}^{N-1}\sum_{\hat{j}=\hat{i}+1}^{N}\Big\{2d_{\hat{j}\hat{i}}x^T(t)Z_{\hat{i}\hat{j}}x(t) - \Big[4x^T(t) + \bar{d}_{\hat{i}\hat{j}}\int_{t-d_{\hat{j}}}^{t-d_{\hat{i}}} x^T(s)\mathrm{d}s\Big]Z_{\hat{i}\hat{j}}\int_{t-d_{\hat{j}}}^{t-d_{\hat{i}}} x(s)\mathrm{d}s\Big\} \\ & + \sum_{\hat{i}=1}^{N-1}\sum_{\hat{j}=\hat{i}+1}^{N}\int_{t-d_{\hat{j}}}^{t-d_{\hat{i}}} x^T(s)\mathrm{d}sQ_{\hat{i}\hat{j}}\int_{t-d_{\hat{j}}}^{t-d_{\hat{i}}} x(s)\mathrm{d}s + \xi^T(t)P\xi(t) = \xi^T(t)\Big(\begin{bmatrix} \Sigma_{11} & \vartheta \\ * & \Xi \end{bmatrix} + P\Big)\xi(t), \end{aligned} \tag{19}$$

where

$$\Sigma_{11} = 2\sum_{j=1}^{N} d_jR_j + 2\sum_{\hat{i}=1}^{N-1}\sum_{\hat{j}=\hat{i}+1}^{N} d_{\hat{j}\hat{i}}Z_{\hat{i}\hat{j}},$$

$$\vartheta = \left[\Sigma_{12}\ \Sigma_{13}\ \cdots\ \Sigma_{1(N+1)}\ \Sigma_1^{12}\ \Sigma_1^{13}\ \cdots\ \Sigma_1^{(N-1)N}\right],$$
$$\Xi = \mathrm{diag}\left\{\Sigma_{22}\ \cdots\ \Sigma_{(N+1)(N+1)}\ \Sigma_{12}^{12}\ \Sigma_{13}^{13}\ \cdots\ \Sigma_{(N-1)N}^{(N-1)N}\right\},$$

with

$$\Sigma_{1(j+1)} = -2R_j,\ \Sigma_1^{\hat{i}\hat{j}} = -2Z_{\hat{i}\hat{j}},\ \Sigma_{(j+1)(j+1)} = \bar{d}_j\left(2R_j + S_j\right),$$
$$\Sigma_{\hat{i}\hat{j}}^{\hat{i}\hat{j}} = Q_{\hat{i}\hat{j}} + \bar{d}_{\hat{i}\hat{j}} Z_{\hat{i}\hat{j}},\ j \in \mathbf{I}[1, N],\ \hat{i} \in \mathbf{I}[1, N-1],\ \hat{j} \in \mathbf{I}[\hat{i}+1, N].$$

Assume that the following matrix inequalities hold:

$$F_l^T F_l \leqslant \begin{bmatrix} \Sigma_{11} & \vartheta \\ * & \Xi \end{bmatrix} + P, l \in \mathbf{I}[1, \overleftrightarrow{m}], \tag{20}$$

where $F = [U\ V_1\ \cdots\ V_N\ V_{12}\ \cdots\ V_{1N}\ V_{23}\ \cdots\ V_{2N}\ \cdots\ V_{(N-1)N}]$, and F_l is the l - th row of matrix F, $l \in \mathbf{I}[1, \overleftrightarrow{m}]$. From (18)-(20), it can be seen that

$$|w_l(t)|^2 \leqslant V(t) \leqslant V(0), l \in \mathbf{I}[1, \overleftrightarrow{m}], \tag{21}$$

where $V_{(\hat{i}\hat{j})l}$ is the l-th row of matrix $V_{\hat{i}\hat{j}}$. For any $\varphi(t) \in \mathbb{C}_{n,d}$ satisfying $V(0) \leqslant 1$, it can be seen from (5) and (21) that $|w_l(t)| \leqslant 1$ holds for $l \in \mathbf{I}[1, \overleftrightarrow{m}]$, which implies that the assumption (6) can be guaranteed. Then, it can be concluded that the closed-loop system (8) without uncertainties is locally asymptotically stable for any initial function $\varphi(t) \in \mathbb{C}_{n,d}$ satisfying $V(0) \leqslant 1$.

To obtain LMI-based conditions, we set $T_2 \triangleq \delta T_1$, $\delta \neq 0$ in (16) and introduce the following new matrix variables:

$$\begin{cases} T_1^{-1} \triangleq X,\ KX^T \triangleq Y,\ \ UX^T \triangleq G,\ V_jX^T \triangleq H_j,\ \tilde{X}P\tilde{X}^T \triangleq \bar{P},\ XR_jX^T \triangleq \bar{R}_j,\ \ XS_jX^T \triangleq \bar{S}_j, \\ XQ_{\hat{i}\hat{j}}X^T \triangleq \bar{Q}_{\hat{i}\hat{j}},\ \ XZ_{\hat{i}\hat{j}}X^T \triangleq \bar{Z}_{\hat{i}\hat{j}},\ V_{\hat{i}\hat{j}}X^T \triangleq H_{\hat{i}\hat{j}},\ \ \tilde{X} = \mathrm{diag}\{X, X, \cdots, X\}. \end{cases} \tag{22}$$

Performing some congruence transformations as in [42] and noting (22), one can ascertain that $\Omega^k < 0$ is equivalent to linear matrix inequality (11), and nonlinear matrix inequality (20) is equivalent to linear matrix inequality (12). This completes the proof. □

Remark 3. In this paper, we are mainly concerned with the systems with multiple delays. In the L-K functional (10), N is required to be greater than 1. For the case that $N = 1$, the system (1) becomes the case with a single delay, which has been investigated in some existing literature.

Remark 4: For the case that $d_1 < d_2 < d_3 < \cdots < d_N$, we can choose the following L-K functional:

$$V(t) = \varpi^T(t)P\varpi(t) + \sum_{j=1}^{N}\Big[\int_{t-d_j}^{t-d_{j-1}} x^T(s)S_jx(s)\mathrm{d}s + (d_j - d_{j-1})\int_{-d_j}^{-d_{j-1}}\int_{t+\theta}^{t}\dot{x}^T(s)R_j\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\Big], \tag{23}$$

where

$$\varpi(t) \overset{\Delta}{=} \left[x^T(t)\ \int_{t-d_1}^{t} x^T(s)\mathrm{d}s\ \int_{t-d_2}^{t-d_1} x^T(s)\mathrm{d}s\ \int_{t-d_3}^{t-d_2} x^T(s)\mathrm{d}s\ \cdots\ \int_{t-d_N}^{t-d_{N-1}} x^T(s)\mathrm{d}s\right]^T,$$

and $P > 0$, $S_j > 0$, $R_j > 0$, $d_0 = 0$ ($j \in \mathbf{I}[1, N]$).

Correspondingly, under the constraint

$$\|Ux(t) + \sum_{j=1}^{N} V_j \int_{t-d_j}^{t-d_{j-1}} x(s)\mathrm{d}s\|_\infty \leqslant 1, \tag{24}$$

we have the following closed-loop system:

$$\dot{x}(t) = \sum_{k=1}^{2^m} \lambda_k^t \Big\{ \left[A_0(t) + B(t)(D_k K + \mathcal{D}_k^- U)\right] x(t) + \sum_{j=1}^{N} A_j(t)x(t-d_j) + \sum_{j=1}^{N} B(t)\mathcal{D}_k^- V_j \int_{t-d_j}^{t-d_{j-1}} x(s)\mathrm{d}s \Big\}, \tag{25}$$

where $\lambda_1^t \geq 0$, $\lambda_2^t \geq 0$, $\cdots$, $\lambda_{2^m}^t \geq 0$ and $\sum_{k=1}^{2^m} \lambda_k^t = 1$. Follow the derivation steps in Theorem 1, the corresponding results can be readily obtained.

Remark 5. First, different from the L-K functionals in [10, 15], some integral terms are introduced in $V_1(t)$. Second, by considering the relationships between multiple delays d_j ($j \in \mathbf{I}[1, N]$), this paper introduces the term $V_3(t)$ in the L-K functional (10). In addition, the Theorem 1 is slacker since the variables H_j ($j \in \mathbf{I}[1, N]$) and $H_{\hat{i}\hat{j}}$ ($\hat{i} \in \mathbf{I}[1, N-1]$, $\hat{j} \in \mathbf{I}[\hat{i}+1, N]$) are additionally introduced in the conditions (11) and (12). The matrices H_j and $H_{\hat{i}\hat{j}}$ can be seen as slack variables in this paper. These new techniques can result in less conservative stabilization criteria.

Remark 6. In proving Theorem 1, the stability criterion of the open-loop system can be represented as $\tilde{\Omega}^k < 0$, where $\tilde{\Omega}^k$ is obtained by setting $U = 0$, $K = 0$, $V_j = 0$, and $V_{\hat{i}\hat{j}} = 0$ in Ω^k as defined below in (15). For convenience of comparison, the stability criterion is referred to as Corollary 1.

Now, we consider system (1) with $F(t) \neq 0$.

Theorem 2. Let the constants $\delta \neq 0$ and d_j, $j \in \mathbf{I}[1, N]$ be given. The system (1) can be robustly stabilized by the controller (4) with $K = YX^{-T}$ if for any initial function $\varphi(t) \in \mathbb{C}_{n,d}$ satisfying $V(0) \leqslant 1$, there exist $N_1 \times N_1$ matrix $\bar{P} > 0$, $n \times n$ matrices $\bar{R}_j > 0$, $\bar{S}_j > 0$, $\bar{Q}_{\hat{i}\hat{j}} > 0$, $\bar{Z}_{\hat{i}\hat{j}} > 0$, X, $m \times n$ matrix Y, $\overleftrightarrow{m} \times n$ matrices G, H_j, $j \in \mathbf{I}[1, N]$, $H_{\hat{i}\hat{j}}$, $\hat{i} \in \mathbf{I}[1, N-1]$, $\hat{j} \in \mathbf{I}[\hat{i}+1, N]$, and a scalar $\varepsilon > 0$ such that for $\forall k \in \mathbf{I}[1, 2^m]$ and $\forall l \in \mathbf{I}[1, \overleftrightarrow{m}]$, the LMI (12) and the following LMI hold:

$$\begin{pmatrix} \bar{\Omega}^k & \varepsilon \bar{M} & (\bar{E}^k)^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{pmatrix} < 0, \tag{26}$$

where $\bar{\Omega}^k$ are defined in Theorem 1 and

$$\bar{M} = [M^T \; 0_{n \times \frac{N(N+3)}{2} n} \; \delta M^T]^T,$$

$$\bar{E}^k = [\bar{v}_0^k \; E_1 X^T \; \cdots \; E_N X^T \; \bar{v}_1^k \; \cdots \; \bar{v}_N^k \; \bar{v}_1 2^k \; \cdots \; \bar{v}_1 N^k \; \bar{v}_2 3^k \; \cdots \; \bar{v}_2 N^k \; \cdots \; \bar{v}_{(N-1)N}^k \; 0],$$

with

$$\bar{v}_0^k = E_0 X^T + E_{N+1}(D_k Y + \mathcal{D}_k^- G),\; \bar{v}_j^k = E_{N+1}\mathcal{D}_k^- H_j,\; j \in \mathbf{I}[1, N],$$

$$\bar{v}_{\hat{i}\hat{j}}^k = E_{N+1}\mathcal{D}_k^- H_{\hat{i}\hat{j}},\; \hat{i} \in \mathbf{I}[1, N-1],\; \hat{j} \in \mathbf{I}[\hat{i}+1, N].$$

Proof. Denote

$$\Delta\Omega^k \triangleq \begin{bmatrix} \Delta\Omega_{11}^k & \Delta\Omega_{12}^k & \Delta\Omega_{13}^k & \Delta\Omega_{14}^k & \Delta\Omega_{15}^k \\ * & 0 & 0 & 0 & \Delta\Omega_{25}^k \\ * & * & 0 & 0 & \Delta\Omega_{35}^k \\ * & * & * & 0 & \Delta\Omega_{45}^k \\ * & * & * & * & 0 \end{bmatrix}, \tag{27}$$

where

$$\begin{gathered} \Delta\Omega_{11}^k = \mathrm{Sym}[T_1\Delta A_0 + T_1\Delta B(D_kK + \mathcal{D}_k^- U)], \\ \Delta\Omega_{12}^k = [T_1\Delta A_1 \ T_2\Delta A_2 \ \cdots \ T_2\Delta A_N], \ \Delta\Omega_{13}^k = \left[\Delta\beta_1^k \ \Delta\beta_2^k \ \cdots \ \Delta\beta_N^k\right], \\ \Delta\Omega_{14}^k = [\Delta\gamma_{12}^k \ \Delta\gamma_{13}^k \ \cdots \ \Delta\gamma_{1N}^k \ \Delta\gamma_{23}^k \ \cdots \ \Delta\gamma_{2N}^k \ \cdots \ \Delta\gamma_{(N-1)N}^k], \\ \Delta\Omega_{15}^k = \Delta A_0^T T_2^T + [\Delta B(D_kK + \mathcal{D}_k^- U)]^T T_2^T, \\ \Delta\Omega_{25}^k = [\Delta A_1 T_2^T \ \Delta A_2 T_2^T \ \cdots \ \Delta A_N T_2^T]^T, \ \Delta\Omega_{35}^k = \left[\Delta\sigma_1^k \ \Delta\sigma_2^k \ \cdots \ \Delta\sigma_N^k\right]^T, \\ \Delta\Omega_{45}^k = [\Delta\mu_{12}^k \ \Delta\mu_{13}^k \ \cdots \ \Delta\mu_{1N}^k \ \Delta\mu_{23}^k \ \cdots \ \Delta\mu_{2N}^k \ \cdots \ \Delta\mu_{(N-1)N}^k]^T, \end{gathered}$$

with

$$\begin{aligned} \Delta\beta_j^k =& T_1\Delta B\mathcal{D}_k^- V_j, \ \Delta\sigma_j^k = T_2\Delta B\mathcal{D}_k^- V_j, \Delta\gamma_{\hat{i}\hat{j}}^k = T_1\Delta B\mathcal{D}_k^- V_{\hat{i}\hat{j}}, \\ \Delta\mu_{\hat{i}\hat{j}}^k =& T_2\Delta B\mathcal{D}_k^- V_{\hat{i}\hat{j}}, \ j \in \mathbf{I}[1, N], \ \hat{i} \in \mathbf{I}[1, N-1], \ \hat{j} \in \mathbf{I}[\hat{i}+1, N]. \end{aligned}$$

Here, we choose the same L-K functional (10); then, the LMI (12) and the following LMI hold:

$$\Omega^k + \Delta\Omega^k < 0. \tag{28}$$

Using the assumptions (3) and $T_2 \triangleq \delta T_1$, $\delta \neq 0$, it is easy to obtain that

$$\Delta\Omega^k = T_1\bar{M}F(t)E^k + (E^k)^T F^T(t)\bar{M}^T T_1^T, \tag{29}$$

where

$$E^k = [\nu_0^k \ E_1 \ \cdots \ E_N \ \nu_1^k \ \cdots \ \nu_N^k \ \nu_{12}^k \ \cdots \ \nu_{1N}^k \ \nu_{23}^k \ \cdots \ \nu_{2N}^k \ \cdots \ \nu_{(N-1)N}^k \ 0],$$

with
$\nu_0^k = E_0 + E_B(D_kK + \mathcal{D}_k^- U)$, $\nu_j^k = E_B\mathcal{D}_k^- V_j$, $\nu_{\hat{i}\hat{j}}^k = E_{N+1}\mathcal{D}_k^- V_{\hat{i}\hat{j}}$, $j \in \mathbf{I}[1, N]$, $\hat{i} \in \mathbf{I}[1, N-1]$, $\hat{j} \in \mathbf{I}[\hat{i}+1, N]$.

From (28) and (29), one can obtain that

$$\Omega^k + T_1\bar{M}F(t)E^k + (E^k)^T F^T(t)\bar{M}^T T_1^T < 0. \tag{30}$$

Performing congruence transformation, one can ascertain that the nonlinear matrix inequality (30) is equivalent to the following matrix inequality:

$$\bar{\Omega}^k + \bar{M}F(t)\bar{E}^k + (\bar{E}^k)^T F^T(t)\bar{M}^T < 0. \tag{31}$$

Using the well-known Schur complement, it is clear that (31) is equivalent to the LMI (26). The proof is completed. □

Remark 7. Note that the Wirtinger integral inequality is a special case of the Bessel-Legendre inequality [51]. Therefore, the obtained conditions in this paper remain conservative to a certain extent. By modifying the sector condition and using the Bessel-Legendre inequality, some more effective conditions are expected to be obtained.

To obtain a larger estimate of the domain of attraction X_ρ when designing a controller, we discuss the estimate and maximization of the domain of attraction. Noting the L-K functional (10) and (22) and using the Jensen integral inequality to estimate the first term $\xi^T(0)P\xi(0)$ of $V(0)$, it can be seen that the domain of attraction X_ρ can be bounded by the following inequality:

$$V(0) \leqslant \Psi_1\rho_1^2 + \Psi_2\rho_2^2, \tag{32}$$

where

$$\Psi_1 \triangleq \lambda_M(\tilde{\Lambda}_0) + \sum_{j=1}^{N} d_j\Big[\lambda_M(d_j\tilde{\Lambda}_j + X^{-1}\bar{S}_jX^{-T})\Big] + \sum_{\hat{i}=1}^{N-1}\sum_{\hat{j}=\hat{i}+1}^{N} d_{\hat{j}\hat{i}}^2\Big[\lambda_M(\tilde{\Lambda}_{\hat{i}\hat{j}}) + \lambda_M(X^{-1}\bar{Q}_{\hat{i}\hat{j}}X^{-T})\Big],$$

$$\Psi_2 \triangleq (1/2)\Big[\sum_{j=1}^{N} d_j^3\lambda_M(X^{-1}\bar{R}_jX^{-T}) + \sum_{\hat{i}=1}^{N-1}\sum_{\hat{j}=\hat{i}+1}^{N} d_{\hat{j}\hat{i}}^2(d_{\hat{j}} + d_{\hat{i}})\lambda_M(X^{-1}\bar{Z}_{\hat{i}\hat{j}}X^{-T})\Big],$$

with $\tilde{\Lambda}_{\tilde{j}} = X^{-1}\Lambda_{\tilde{j}}X^{-T}$, $\tilde{j} \in \mathbf{I}[0,N]$, $\tilde{\Lambda}_{\hat{i}\hat{j}} = X^{-1}\Lambda_{\hat{i}\hat{j}}X^{-T}$, $\hat{i} \in \mathbf{I}[1,N-1]$, $\hat{j} \in \mathbf{I}[\hat{i}+1,N]$, and the LMI constraint $\bar{P} \leqslant \mathrm{diag}\{\Lambda_0\ \Lambda_1\ \cdots\ \Lambda_N\ \Lambda_{12}\ \cdots\ \Lambda_{1N}\ \Lambda_{23}\cdots\Lambda_{2N}\ \cdots\ \Lambda_{(N-1)N}\} \triangleq \Lambda$.

As in [42], we introduce the matrix inequality $X^{-1}X^{-T} \leqslant rI$, which can be guaranteed by the LMI

$$\begin{pmatrix} rI & I \\ I & X + X^T - I \end{pmatrix} \geqslant 0. \tag{33}$$

Meanwhile, we define the following LMIs:

$$\begin{cases} \Lambda_{\tilde{j}} \leqslant p_{\tilde{j}}I,\ \tilde{j} \in \mathbf{I}[0,N],\ \bar{P} \leqslant \Lambda,\ \bar{R}_j \leqslant r_jI,\ \bar{S}_j \leqslant s_jI,\ j \in \mathbf{I}[1,N], \\ \Lambda_{\hat{i}\hat{j}} \leqslant p_{\hat{i}\hat{j}}I,\ \bar{Q}_{\hat{i}\hat{j}} \leqslant q_{\hat{i}\hat{j}}I,\ \bar{Z}_{\hat{i}\hat{j}} \leqslant z_{\hat{i}\hat{j}}I, \hat{i} \in \mathbf{I}[1,N-1],\ \hat{j} \in \mathbf{I}[\hat{i}+1,N]. \end{cases} \tag{34}$$

Then, it can be seen from (32) that the maximization of the estimate of the domain of attraction X_ρ for system (1) without uncertainties in Theorem 1 can be formulated as the following optimization problem:

Prob. 1.

$$\min_{\bar{P},\,\bar{R}_j,\,\bar{S}_j,\,\bar{Q}_{\hat{i}\hat{j}},\,\bar{Z}_{\hat{i}\hat{j}},\,\Lambda_{\tilde{j}},\,\Lambda_{\hat{i}\hat{j}},\,X,\,Y,\,G,\,H_j,\,H_{\hat{i}\hat{j}},\,r,\,p_{\tilde{j}},\,p_{\hat{i}\hat{j}},\,r_j,\,s_j,\,q_{\hat{i}\hat{j}},\,z_{\hat{i}\hat{j}}} \lambda,\ s.t.\ \text{LMIs (11), (12), (33) and (34) hold.}$$

Additionally, the maximization of the estimate of the domain of attraction X_ρ for system (1) in Theorem 2 can be formulated as the following optimization problem:

Prob. 2.

$$\min_{\bar{P},\,\bar{R}_j,\,\bar{S}_j,\,\bar{Q}_{\hat{i}\hat{j}},\,\bar{Z}_{\hat{i}\hat{j}},\,\Lambda_{\tilde{j}},\,\Lambda_{\hat{i}\hat{j}},\,X,\,Y,\,G,\,H_j,\,H_{\hat{i}\hat{j}},\,\varepsilon,\,r,\,p_{\tilde{j}},\,p_{\hat{i}\hat{j}},\,r_j,\,s_j,\,q_{\hat{i}\hat{j}},\,z_{\hat{i}\hat{j}}} \lambda,\ s.t.\ \text{LMIs (12), (26), (33) and (34) hold,}$$

where $\lambda = e * r + p_0 + \sum_{j=1}^{N} d_j(s_j + d_jp_j + 0.5d_j^2r_j) + \sum_{\hat{i}=1}^{N-1}\sum_{\hat{j}=\hat{i}+1}^{N} d_{\hat{i}\hat{j}}^2[p_{\hat{i}\hat{j}} + q_{\hat{i}\hat{j}} + 0.5(d_{\hat{j}} + d_{\hat{i}})z_{\hat{i}\hat{j}}]$ and e is a weighting parameter.

By solving the optimization problems Prob. 1 and Prob. 2, the values of Ψ_1 and Ψ_2 can be easily obtained. Then, we can determine the admissible bounds ρ_1 and ρ_2 of the set X_ρ by the equation $\Psi_1\rho_1^2 + \Psi_2\rho_2^2 = 1$. In particular, when $\rho_1 = \rho_2 \triangleq \rho$, the maximum admissible scalar ρ can be obtained by $\rho_{\max} \triangleq 1/\sqrt{\Psi_1 + \Psi_2}$. For any initial conditions $\varphi(t)$ in the set X_ρ satisfying the equation $\Psi_1\rho_1^2+\Psi_2\rho_2^2 = 1$, it can be seen that the constraint $V(0) \leqslant 1$ can be guaranteed.

Remark 8. In this section, by using the novel L-K functional (12) and the Wirtinger-based integral inequality, two sufficient conditions are established by LMIs under which the closed-loop system (6) is asymptotically stable for the case without uncertainties and robustly asymptotically stable for the case with uncertainties. In addition, the corresponding optimization problems are proposed to maximize the estimate of the domain of attraction.

4. Numerical examples

Example 1. [31, 32] Consider the time-delay system (1), where $N = 2$ and

$$A_0 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\ A_1 = \begin{bmatrix} -1 & 0.6 \\ -0.4 & -1 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & -0.6 \\ -0.6 & 0 \end{bmatrix},\ B = \begin{bmatrix} 10 \\ 10 \end{bmatrix}. \tag{35}$$

First, we show the effectiveness of the proposed method. For the case $d_1 = 5$, $d_2 = 6$, it is clear from Figure 1 that the open-loop system (1) is not stable. By solving Prob.1 in this paper ($\delta = 1$, $e = 5\times10^9$), one can obtain the scalar $\rho_{\max} = 32.2055$ associated with the set X_ρ. Meanwhile, we have $U = [-0.0032\ -0.0092]$, $V_1 = [-0.0001845\ -0.0002640]$, $V_2 = [-0.000000009548\ -0.000000009128]$, $V_{12} = [0.0040\ 0.0067]$ and the controller gain matrix $K = [0.0078\ -0.0228]$. In Figure 2, we plot the state responses of the closed-loop system and the signal $w(t)$, where $\varphi(t) = [24\ 20]^T \in X_\rho$. In Figure 3, we plot the state trajectories of the closed-loop systems. Figure 2 shows that the closed-loop system (1) is stable and that the functional $w(t)$ defined in (5) satisfies the constraints $\|w(t)\|_\infty < 1$. From Figure 3, it can be seen that the trajectories starting on the periphery of the ball never leave this ball and converge to the origin.

By solving Prob.1 with $\delta = 1$, $e = 5 * 10^9$, we obtained the scalars $\rho_{\max}$ related to the set X_ρ for different d_1 and d_2, which are listed in Table 1. In Table 1, we also list the scalars $\rho_{\max}$ obtained by Case 1 and Case 2, where Case 1 is obtained by setting $V_j = 0$, $S_j = 0$ in LMIs (11) and (12), and Case 2 is obtained by setting $V_{\hat{i}\hat{j}} = 0$, $Q_{\hat{i}\hat{j}} = 0$ and $Z_{\hat{i}\hat{j}} = 0$ in LMIs (11) and (12). From Table 1, it is clear that Prob. 1 can provide a larger estimate of the domain of attraction. Noting that Case 1 and Case 2 are based on the traditional techniques that address saturations and time delays, it can be concluded that our proposed result is effective in reducing the possible conservatism.

In Table 2, we list the admissible ranges of the time delay d_2 for different d_1 obtained by Corollary 1 in this paper and the results in [31, 32]. It is clear from Table 2 that Corollary 1 can provide the larger range of the time delay d_2, which shows that our proposed stability analysis approach is less conservative than that in [31, 32].

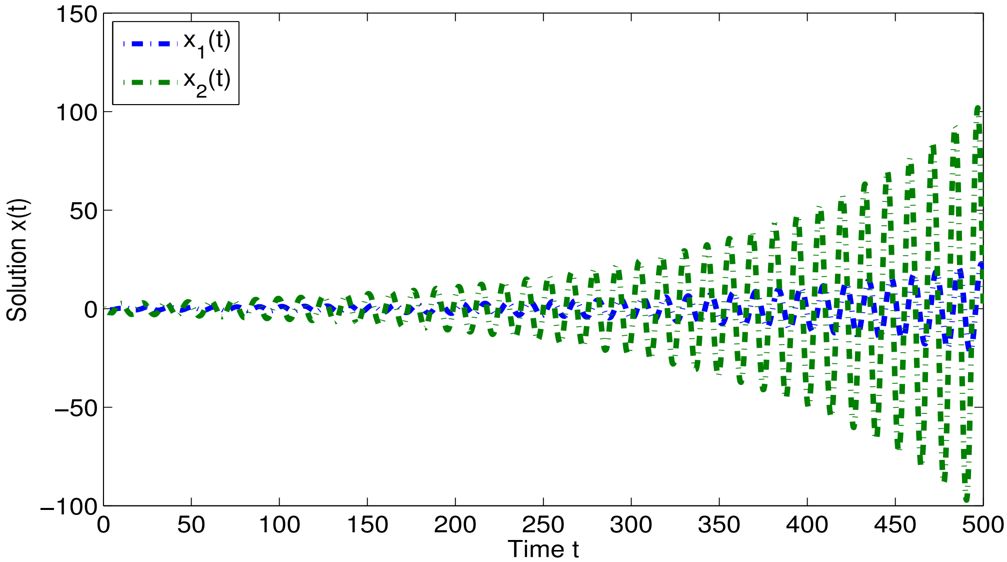

Figure 1. State responses of the open-loop system ($d_1 = 5$, $d_2 = 6$).

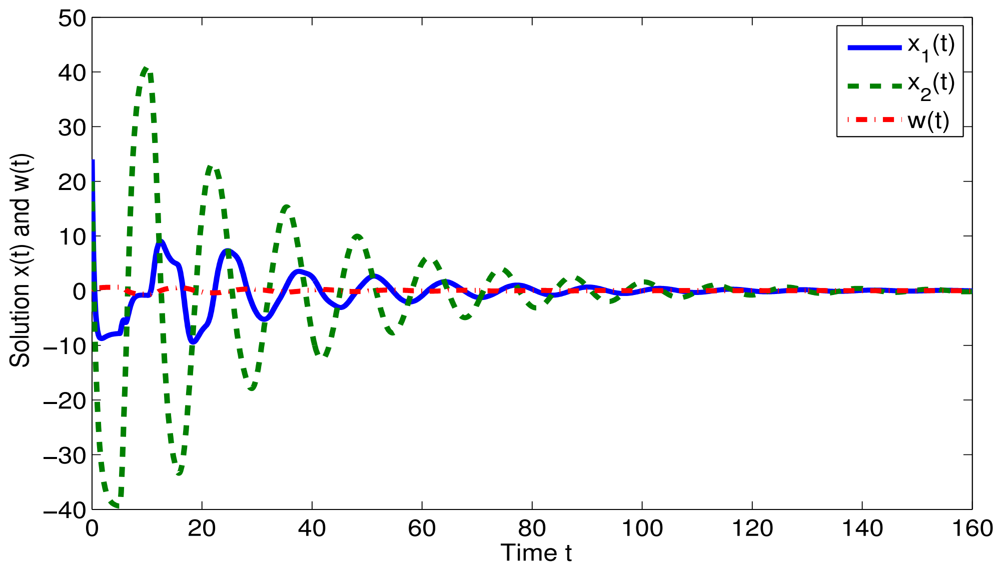

Figure 2. State responses of the closed-loop system and $w(t)$ defined in (5).

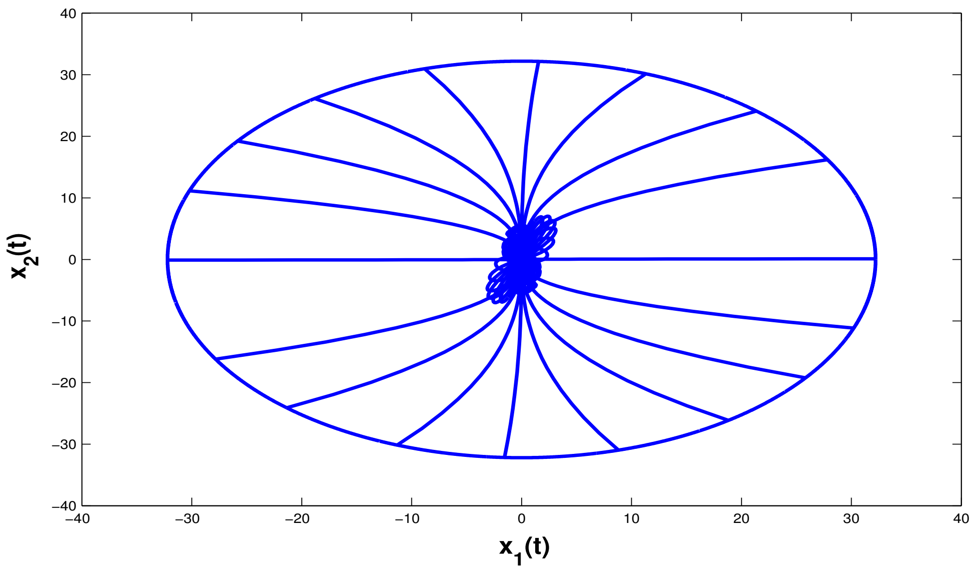

Figure 3. System trajectories.

Table 1. The scalars $\rho_{\max}$ are associated with the set X_ρ for the given d_1 and d_2.

(d_1, d_2)	(1,0.1)	(1,0.5)	(1,1.5)	(1,2)	(1,4)
$\rho_{\max}$(Prob. 1)	161.9218	145.9009	116.4877	97.2339	75.8305
$\rho_{\max}$(Case 1)	161.8783	141.6443	100.1767	83.0704	47.7115
$\rho_{\max}$(Case 2)	159.4388	128.9824	99.8500	93.4002	75.7668
(d_1, d_2)	(2,0.1)	(2,0.5)	(2,1.4)	(2,2.1)	(2,3.5)
$\rho_{\max}$(Prob. 1)	89.1132	87.3734	81.7620	86.3634	47.9163
$\rho_{\max}$(Case 1)	88.9580	86.6829	52.8286	54.5928	—
$\rho_{\max}$(Case 2)	87.9240	77.7870	57.6496	48.0497	35.4435

Table 2. Admissible range of the time delay d_2 to ensure stability of the open-loop system for a given d_1.

d_1	2.3	2.4	2.5	3.0	3.5
d_2([31])	[0.08, 3.57]	[0.22, 3.61]	[0.35, 3.65]	[1.04, 3.77]	[1.88, 3.90]
d_2([32])	[0.07, 3.57]	[0.17, 3.61]	[0.28, 3.65]	[0.80, 3.77]	[1.60, 3.90]
d_2(Corollary 1)	[0, 4.54]	[0, 4.54]	[0, 4.55]	[0, 4.69]	[0, 4.77]

Example 2. [52] Consider the uncertain system (1), where $N = 2$ and

$$A_0 = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix},\ A_1 = \begin{bmatrix} 0.6 & -0.4 \\ 0 & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 0 \\ 0 & -0.5 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ M = I,$$
$$E_0 = \begin{bmatrix} 0.16 & 0 \\ 0 & 0.16 \end{bmatrix},\ E_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0.04 \end{bmatrix},\ E_2 = \begin{bmatrix} 0.04 & 0 \\ 0 & 0 \end{bmatrix},\ E_B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{36}$$

This system can be seen as a linearized model of combustion in a liquid monopropellant rocket motor chambers [53]. For this example, by solving Prob. 2 in this paper with $N = 2$, $\delta = 0.7$, and $e = 10$, we can obtain the scalars $\rho_{\max}$, which are listed in Table 3. Table 3 shows that the scalars $\rho_{\max}$ decrease with the larger d_2. By using the proposed controller ($K = [-0.9926 - 2.9132]$), the state responses of the closed-loop system are plotted in Figure 4, where $\varphi(t) = [0.4\ 0.1]^T$. Figure 4 shows that the closed-loop system (1) is stable, which also shows the effectiveness of the proposed conditions.

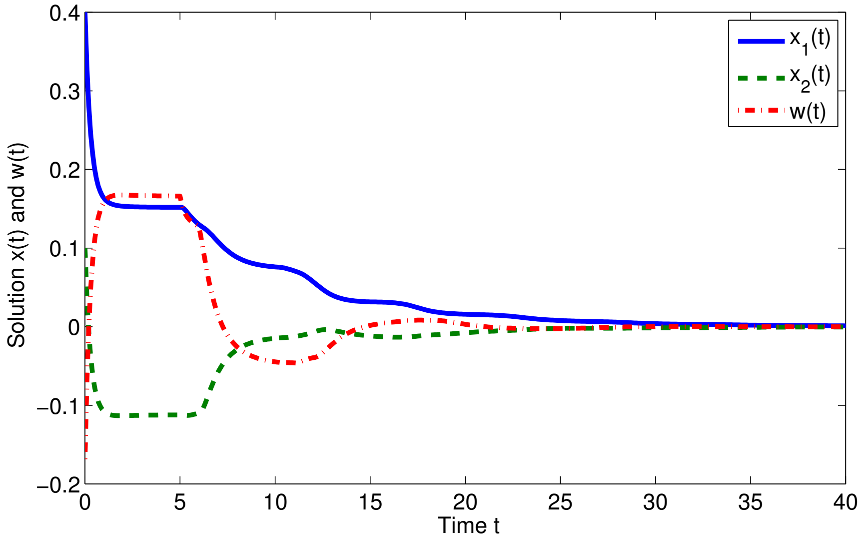

Figure 4. State responses of the closed-loop system and $w(t)$ defined in (5).

Table 3. The scalars $\rho_{\max}$ are associated with the set X_ρ for the given d_1 and d_2.

(d_1, d_2)	(1,0.1)	(1,0.5)	(1,1.5)	(1,2)	(1,4)
$\rho_{\max}$(Prob. 2)	1.1163	1.0439	0.8584	0.7185	0.3630
(d_1, d_2)	(2,0.1)	(2,0.5)	(2,1.4)	(2,2.1)	(2,3.5)
$\rho_{\max}$(Prob. 2)	0.9910	0.9285	0.7103	0.8890	0.5009

Remark 9. From Examples 1 and 2, it is clear that our proposed results can provide a larger estimate of the initial condition set than some existing ones. However, it is worth mentioning that more free-weighting matrices are involved in our conditions, and the computational complexity is increased when solving the optimization problems.

5. Conclusions

In this paper, some new delay-dependent local stabilization criteria have been obtained for multiple time-delay systems with actuator saturation. The saturation nonlinearity is represented as the convex combination of the state feedback, auxiliary distributed-delay feedback, and some cross-terms related to different time delays. Then, by combining the saturation with an augmented L-K functional and some integral inequalities (the Wirtinger integral inequality and the Jensen integral inequality), the stabilization and robust stabilization criteria have been proposed in terms of LMIs. Moreover, the estimation of the domain of attraction has been discussed. A numerical example is provided to show the values of the proposed results. Our proposed results can be readily extended to more general systems such as switched systems and T-S fuzzy systems [54].

In addition, is should be pointed out that our proposed results in this paper are mainly concerned the case with multiple constant delays. For the case with multiple time-varying delays, the corresponding results can also be established, which is our further work.

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Conflict of interest

The authors declare that there are no conflicts of interest.

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