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Research article

Robust synchronization analysis of delayed fractional order neural networks with uncertain parameters

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Abstract: This paper is concerned with the robust synchronization analysis of delayed fractional order neural networks with uncertain parameters (DFNNUPs). Firstly, the DFNNUPs drive system model and response system model are established. Secondly, using multiple matrix quadratic Lyapunov function approach and inequality analysis technique, the robust synchronization conditions are derived in the form of the matrix inequalities. Finally, the correctness of the theoretical results is verified by an example.

Keywords: robust synchronization; delayed fractional order neural networks; uncertain parameters; multiple matrix quadratic Lyapunov function; matrix inequalities
Mathematics Subject Classification: 93A30, 93D09

1. Introduction

Nowadays, synchronization has been widely used in various fields, such as secure communication [1], stochastic network design [2] and information processing [3], especially network secure communication and target control [4]. It is well known that fractional order neural network (FNNs) has always been the research content of scholars. And synchronization is one of the central issues in FNNs, which is an effective tool to control the chaos of FNNs. Thus, many researchers have paid attention to the synchronization of FNNs [5,6].

As we all known that time delay is a common phenomenon, which is sometimes inevitable in the process of building the neural network model. Recently, some related results on the dynamical properties of delayed fractional order neural networks (DFNNs) have been put forward [7,8]. Furthermore, authors proposed the criterion for finite-time synchronization of DFNNs in view of a new fractional order Gronwall inequality with time delay [9]. Moreover, the synchronization analysis [10,11], pinning synchronization [12] and fixed-time synchronization [13] of DFNNs with impulse have been analyzed. In addition, the paper [14] investigated the fixed-time synchronization for semi-Markovian switching complex dynamical networks with hybrid couplings and time-varying delays. Moreover, the synchronization issue of complex-valued DFNNs has been studied [15–17].

It is particularly worth mentioning that owing to being affected by the outside world, the system parameters in the industrial control process are often uncertain. Therefore, uncertain parameters are always considered by scholars when establishing the mathematical model of FNNs. Based on Lyapunov direct method, the paper [18] studied the synchronization for commensurate Riemann-Liouville fractional order memristor-based neural networks with unknown parameters. Furthermore, the literature [19] put forward adaptive synchronization conditions of DFNNUPs in view of the suitable linear feedback controller. The article [20] analyzed the robust synchronization of fractional-order Hopfield neural networks with parameter uncertainties. Besides, several sufficient conditions regarding synchronization of fractional order complex neural networks were also proposed [21].

As mentioned above, delay and uncertain parameters are two phenomena in the fractional order neural networks models. However, researches on fractional-order neural networks containing both phenomena are limited. In order to pursue the more general synchronization conditions of DFNNUPs, this paper studies the robust synchronization conditions of DFNNUPs through LMIs technique and the changes in Lyapunov function construction. The crucial novelty of our contribution lies in following two aspects: (1) The Lyapunov function is in the form of multiple matrix quadratic Lyapunov function, which makes that the robust synchronization conditions has a broader scope. (2) The robust synchronization sufficient conditions are represented in term of matrix inequality. Therefore, the difficulties of this article can be listed as follows: (1) The first difficulty is the scaling of derivatives in the proof process. (2) The second difficulty is the resolve of the matrix inequality to obtain the robust synchronization conditions.

The structure of this paper is outlined as follows: Some definitions on fractional derivative, useful lemmas, the DFNNUPs drive model and the DFNNUPs response model are given in Section 2. The main results of robust synchronization analysis are derived in Section 3. The numerical simulation example and conclusions are respectively presented in Sections 4 and 5.

Notation: Throughout this paper, R denotes the set of real numbers, R^n is the n-dimensional real vector, and $R^{m\times n}$ represents the set of all $m \times n$ real matrices. Moreover, Z^+ is the positive integer. R^+ is the real number. $f(t) \in R^n[0, \infty)$ represents that the f(t) is an n-dimensional real vector defined from $[0, +\infty)$. x^T and A^T stand for the transpose of $x \in R^n$ and any matrix $A \in R^{m\times n}$, respectively. $P > 0$ ($P < 0$) means that P is the positive definite matrix (negative definite matrix). $0_{n\times n}$ represents an $n \times n$ matrix with elements 0. $E_{n\times n}$ denotes the $n \times n$ identity matrix.

2. Preliminaries

Some definitions, lemmas about fractional calculus and assumptions are introduced in this section. Besides, the considered DFNNUPs drive system and response system are also described.

Definition 2.1. *[22] The Riemann-Liouville fractional integral with order $\gamma > 0$ for a continuous function $f(t) \in R^n[0, +\infty)$ is defined as*

$$D_{0,t}^{-\gamma} f(t) = \frac{1}{\Gamma(\gamma)} \int_0^t (t-\mu)^{\gamma-1} f(\mu) d\mu,$$

where $t > 0$, $\Gamma(\cdot)$ is the Gamma function.

Definition 2.2. *[22] The Caputo fractional derivative with order $\gamma > 0$ for a continuous function $f(t) \in R^n[0, +\infty)$ is defined by*

$$D_{0,t}^{\gamma} f(t) = D_{0,t}^{-(n-\gamma)} \frac{d^n}{dt^n} f(t) = \frac{1}{\Gamma(n-\gamma)} \int_0^t (t-\mu)^{n-\gamma-1} f^{(n)}(\mu) d\mu,$$

where $t > 0$, $n - 1 < \gamma < n \in Z^+$. Particularly, when $n = 1$, that is $0 < \gamma < 1$, it has

$${}^{C}D_{0,t}^{\gamma} f(t) = \frac{1}{\Gamma(1-\gamma)} \int_0^t (t-\mu)^{-\gamma} f'(\mu) d\mu.$$

Remark 2.1. *[23] If continuous function $f(t) \in R^n[0, +\infty)$, and $n - 1 < \alpha,\gamma < n \in Z^+$, then,*
(1) $D^{-\alpha} D^{-\gamma} f(t) = D^{-\alpha-\gamma} f(t),\ \alpha, \gamma \geq 0$.
(2) $D^{\alpha} D^{-\gamma} f(t) = D^{\alpha-\gamma} f(t),\ \alpha, \gamma \geq 0$.
Especially, when $\alpha = \gamma$, then, $D^{\alpha} D^{-\gamma} f(t) = f(t)$.
(3) $D^{-\gamma} D^{\gamma} f(t) = f(t) - \sum_{m=0}^{n-1} \frac{f^m}{m!} f^{(m)}(0), \gamma \geq 0$.
In particular, if $0 < \gamma \leq 1$ and $f(t) \in R^1[0, \infty)$, then $D^{-\gamma} D^{\gamma} f(t) = f(t) - f(0)$.
(4) For any real constants k_1 and k_2, it has

$$D^{\gamma}(k_1 f_1(t) + k_2 f_2(t)) = k_1 D^{\gamma} f_1(t) + k_2 D^{\gamma} f_2(t), \gamma \geq 0.$$

Lemma 2.1. *[24] Let $0 < \gamma < 1$ and $f(t) \in R^n[0, +\infty)$, the following inequality is satisfied.*

$$D_{0,t}^{\gamma} \left[\frac{1}{2} f^T(t) M f(t) \right] \leq f^T(t) M D_{0,t}^{\gamma} f(t),$$

where M is the positive definite matrix.

Lemma 2.2. *[25] For any $A \in R^{n\times n}$, $x, y \in R^n$ and real value $\omega > 0$, the following inequality holds.*

$$x^T A y \leq \frac{1}{2\omega} x^T A A^T x + \frac{\omega}{2} y^T y.$$

Lemma 2.3. *[26] Suppose that $A \in R^{m\times n}$, $M \in R^{l\times m}$ and $H(t) \in R^{n\times l}$ are the real matrices. Moreover, $H(t)$ is satisfied with $H^T(t)H(t) \leq E_{l\times l}$. Then, for any real value $\omega > 0$, it has*

$$AH(t)M + M^T H^T(t) A^T \leq \frac{1}{\omega} A A^T + \omega M M^T.$$

Lemma 2.4. *[27] Let $V : [t_0 - \rho, +\infty) \to R^+$ be bounded on $[t_0 - \rho, +\infty)$ and continuous on $[t_0, +\infty)$. If there exist ϕ, ν_h with $h = 1, 2, \ldots, m$ such that*

$$D_{t_0,t}^{\gamma} V(t) \leq -\phi V(t) + \sum_{h=1}^{m} \nu_h \sup_{-\rho_h \leq \omega \leq 0} V(t+\omega), \quad t \geq t_0,$$

where $0 < \gamma < 1$, $\nu_h > 0$, $\phi > \sum_{h=1}^{m} \nu_h$, $\rho = max\{\rho_1, \rho_2, \cdots, \rho_m\}$, then, $\lim_{t\to+\infty} V(t) = 0$.

Consider the following differential equation system as the DFNNUPs drive system:

$$\begin{cases} D^{\gamma}x(t) = -Cx(t) + (A + \triangle A)f(x(t)) + (B + \triangle B)g(x(t-\tau)) + I, & t \in [0, +\infty), \\ x(t) = \psi(t), & t \in [-\tau, 0), \end{cases} \tag{2.1}$$

where $0 < \gamma < 1$, $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in R^n$ stands for the state vector; C=diag$\{c_i\} \in R^{n\times n}$ denotes the self connection weight matrix; $A = (a_{ij})_{n\times n} \in R^{n\times n}$ and $B = (b_{ij})_{n\times n} \in R^{n\times n}$ represent the connection of the jth neuron to the ith neuron at time t and $t - \tau$, respectively. $\triangle A = (\triangle a_{ij}(t))_{n\times n} \in R^n$ and $\triangle B = (\triangle b_{ij}(t))_{n\times n} \in R^n$ are uncertain parameters matrices. Moreover, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T \in R^n$, $g(x(t-\tau)) = (g_1(x_1(t-\tau)), g_2(x_2(t-\tau)), \ldots, g_n(x_n(t-\tau)))^T \in R^n$ are the neuron activation functions; τ represents the delay and $I \in R^n$ is an external input vector.

The corresponding response system is given by

$$\begin{cases} D^{\gamma}y(t) = -Cy(t) + (A + \triangle A)f(y(t)) + (B + \triangle B)g(y(t-\tau)) + I + U(t), & t \in [0, +\infty), \\ y(t) = \varphi(t), & t \in [-\tau, 0), \end{cases} \tag{2.2}$$

where $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T \in R^n$ means the state vector; $U(t)$ is an control input vector; the γ, τ, A, B, $\triangle A$, $\triangle B$, $f(y(t))$ $g(y(t-\tau))$ and I are defined the same as the ones in the system (2.1).

For the neuron activation function f, g and uncertain parameters $\triangle A$, $\triangle B$, the following assumptions are made:

A_1: For any $x, y \in R^n$, there exist $F, G \in R^{n\times n}$, such that

$$\| f(x) - f(y) \| \leq \| F(x-y) \|, \;\; \| g(x) - g(y) \| \leq \| G(x-y) \| .$$

A_2: Uncertain parameters $\triangle A$ and $\triangle B$ are norm bound and content

$$[\triangle A \; \triangle B] = QH(t)[N_A N_B],$$

where $Q \in R^{n\times m}$ is known diagonal constant matrices, $N_A \in R^{l\times n}$ and $N_B \in R^{l\times n}$ are arbitrary constant matrices. Furthermore, $H(t) \in R^{m\times l}$ is an unknown real matrix and Lebesgue norm measurable elements, satisfying $N_A^T H^T(t)H(t)N_A \leq E_{n\times n}$ and $N_B^T H^T(t)H(t)N_B \leq E_{n\times n}$.

Definition 2.3. *[20] The DFNNUPs systems (2.1) and (2.2) are said to achieve robust synchronization if* $\|y(t) - x(t)\| \to 0$, *when* $t \to +\infty$.

3. Main results

The robust synchronization conditions are obtained between the DFNNUPs systems (2.1) and (2.2) under the controller $U(t)$ in this section. The $U(t) = (u_1(t), u_2(t), \cdots, u_n(t))^T$ is designed as

$$U(t) = -K(y(t) - x(t)), \tag{3.1}$$

where $K = diag(k_i) \in R^{n\times n}$.

Theorem 3.1. *Suppose that the assumptions A_1 and A_2 hold, given real constants $\omega > 0$, $\delta > \varepsilon > 0$, if there exist matrices $M \in R^{n\times n}$ (full rank matrix), $P \in R^{n\times n}$ ($P > 0$) and matrix $K = diag(k_i) \in R^{n\times n}$ satisfied*

$$\Phi = \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} < 0, \tag{3.2}$$

where $\Phi_{12} = \Phi_{21} = 0_{n\times n}$, $\Phi_{11} = -M^TPM(C+K)-(C^T+K^T)M^TP^TM+\frac{2}{\omega}F^TF+\omega M^TPMAA^TM^TP^TM+\omega M^TPMBB^TM^TP^TM+2\omega M^TPMQQ^TM^TP^TM+\frac{\delta}{2}M^TPM+\frac{\delta}{2}M^TP^TM$ *and* $\Phi_{22} = \frac{2}{\omega}G^TG-\frac{\varepsilon}{2}M^TPM-\frac{\varepsilon}{2}M^TP^TM$. *Then, the DFNNUPs systems (2.1) and (2.2) achieve robust synchronization under the controller (3.1).*

Proof. Let $e(t) = y(t) - x(t)$, the DFNNUPs error system can be obtained through (2.1) and (2.2),

$$\begin{cases} D^\gamma e(t) = -Ce(t) + (A + \triangle A)[f(y(t)) - f(x(t))] \\ \qquad +(B + \triangle B)[g(y(t-\tau)) - g(x(t-\tau))] + U(t), & t \in [0, +\infty), \\ e(t) = \varphi(t) - \psi(t), & t \in [-\tau, 0). \end{cases} \tag{3.3}$$

Substitute linear controller (3.1) into DFNNUPs error system (3.3), it can derive that

$$\begin{cases} D^\gamma e(t) = -(C+K)e(t) + (A + \triangle A)[f(y(t)) - f(x(t))] \\ \qquad +(B + \triangle B)[g(y(t-\tau)) - g(x(t-\tau))] & t \in [0, +\infty), \\ e(t) = \varphi(t) - \psi(t), & t \in [-\tau, 0). \end{cases} \tag{3.4}$$

Select the following multiple matrix quadratic Lyapunov function $V(t)$:

$$V(t) = e^T(t)M^TPMe(t),$$

where M is an full rank matrix and $P > 0$, namely, $M^TPM > 0$.

Based on Lemma 2.1, it gets

$$D^\gamma V(t) \le 2e^T(t)M^TPMD^\gamma e(t). \tag{3.5}$$

In view of (3.4) and (3.5), it yields

$$\begin{aligned} D^\gamma V(t) \le & 2e^T(t)M^TPM[-(C+K)e(t) + 2(A+\triangle A)[f(y(t)) - f(x(t))] \\ & + 2(B+\triangle B)[g(y(t-\tau)) - g(x(t-\tau))]. \end{aligned}$$

Then,

$$\begin{aligned} & D^\gamma V(t) + \delta V(t) - \varepsilon \sup_{-\tau\le\nu\le 0} V(t+\nu) \\ \le & 2e^T(t)M^TPM\{-(C+K)e(t) + 2(A+\triangle A)[f(y(t)) - f(x(t))] \\ & + 2(B+\triangle B)[g(y(t-\tau)) - g(x(t-\tau))]\} + \delta e^T(t)M^TPMe(t) \\ & - \varepsilon e^T(t-\tau)M^TPMe(t-\tau) \\ = & -2e^T(t)M^TPM(C+K)e(t) + 2e^T(t)M^TPMA[f(y(t)) - f(x(t))] \\ & + 2e^T(t)M^TPM\triangle A[f(y(t)) - f(x(t))] + 2e^T(t)M^TPMB[g(y(t-\tau)) - g(x(t-\tau))] \\ & + 2e^T(t)M^TPM\triangle B[g(y(t-\tau)) - g(x(t-\tau))] + \delta e^T(t)M^TPMe(t) \\ & - \varepsilon e^T(t-\tau)M^TPMe(t-\tau). \end{aligned} \tag{3.6}$$

According to Lemma 2.2 and A_1, it has

$$\begin{aligned} & 2e^T(t)M^TPMA[f(y(t)) - f(x(t))] \\ \le & \frac{1}{\omega}[f(y(t)) - f(x(t))]^T[f(y(t)) - f(x(t))] + \omega e^T(t)M^TPMAA^TM^TP^TMe(t) \\ \le & \frac{1}{\omega}e^T(t)F^TFe(t) + \omega e^T(t)M^TPMAA^TM^TP^TMe(t) \end{aligned} \tag{3.7}$$

and

$$\begin{aligned}&2e^T(t)M^TPMB[g(y(t-\tau))-g(x(t-\tau))]\\ \leq&\frac{1}{\omega}[g(y(t-\tau))-g(x(t-\tau))]^T[g(y(t-\tau))-g(x(t-\tau))]+\omega e^T(t)M^TPMBB^TM^TP^TMe(t)\\ \leq&\frac{1}{\omega}e^T(t-\tau)G^TGe(t-\tau)+\omega e^T(t)M^TPMBB^TM^TP^TMe(t).\end{aligned} \tag{3.8}$$

By virtue of the Lemma 2.3 and A_2, it gets

$$\begin{aligned}&2e^T(t)M^TPM\triangle A[f(y(t))-f(x(t))]\\ =&2e^T(t)M^TPMQH(t)N_A[f(y(t))-f(x(t))]\\ \leq&\frac{1}{\omega}[f(y(t))-f(x(t))]^TN_A^TH^T(t)H(t)N_A[f(y(t))-f(x(t))]+\omega e^T(t)M^TPMQQ^TM^TP^TMe(t)\\ \leq&\frac{1}{\omega}e^T(t)F^TFe(t)+\omega e^T(t)M^TPMQQ^TM^TP^TMe(t)\end{aligned} \tag{3.9}$$

and

$$\begin{aligned}&2e^T(t)M^TPM\triangle B[g(y(t-\tau))-g(x(t-\tau))]\\ =&2e^T(t)M^TPMQH(t)N_B[g(y(t-\tau))-g(x(t-\tau))]\\ \leq&\frac{1}{\omega}[g(y(t-\tau))-g(x(t-\tau))]^TN_B^TH^T(t)H(t)N_B[g(y(t-\tau))-g(x(t-\tau))]\\ &+\omega e^T(t)M^TPMQQ^TM^TP^TMe(t)\\ \leq&\frac{1}{\omega}e^T(t-\tau)G^TGe(t-\tau)+\omega e^T(t)M^TPMQQ^TM^TP^TMe(t).\end{aligned} \tag{3.10}$$

Applying the inequalities (3.7)–(3.10) into (3.6), it can be obtained that

$$\begin{aligned}&D^\gamma V(t)+\delta V(t)-\varepsilon\sup_{-\tau\leq\nu\leq0}V(t+\nu)\\ \leq&-2e^T(t)M^TPM(C+K)e(t)+\frac{2}{\omega}e^T(t)F^TFe(t)+\frac{2}{\omega}e^T(t-\tau)G^TGe(t-\tau)\\ &+\omega e^T(t)M^TPMAA^TM^TP^TMe(t)+\omega e^T(t)M^TPMBB^TM^TP^TMe(t)\\ &+2\omega e^T(t)M^TPMQQ^TM^TP^TMe(t)+\delta e^T(t)M^TPMe(t)\\ &-\varepsilon e^T(t-\tau)M^TPMe(t-\tau),\end{aligned}$$

then,

$$D^\gamma V(t)+\delta V(t)-\varepsilon\sup_{-\tau\leq\nu\leq0}V(t+\nu)\leq(e^T(t)\ \ e^T(t-\tau))\Phi\left(e(t)e(t-\tau)\right),$$

where

$$\Phi=\begin{pmatrix}\Phi_{11}&\Phi_{12}\\ \Phi_{21}&\Phi_{22}\end{pmatrix}\in R^{2n\times2n},$$

with $\Phi_{12}=\Phi_{21}=0_{n\times n}$, $\Phi_{11}=-M^TPM(C+K)-(C^T+K^T)M^TP^TM+\frac{2}{\omega}F^TF+\omega M^TPMAA^TM^TP^TM+\omega M^TPMBB^TM^TP^TM+2\omega M^TPMQQ^TM^TP^TM+\frac{\delta}{2}M^TPM+\frac{\delta}{2}M^TP^TM$ and $\Phi_{22}=\frac{2}{\omega}G^TG-\frac{\varepsilon}{2}M^TPM-\frac{\varepsilon}{2}M^TP^TM$.

In line with Theorem 3.1 and Lemma 2.4, it can deduce that $\lim\limits_{t\to+\infty}V(t)=0$, implying $\lim\limits_{t\to+\infty}e(t)=0$.

Remark 3.1. *Fractional order neural networks model established in this paper contains both time delay and uncertain parameters, which is more in line with the performance in actual problems compared with literature [28,29].*

Remark 3.2. *Aim to the synchronization issue, the Lyapunov function is usually designed as* $V(t) = e^T(t)Pe(t)$ *[25,30]. However, the Lyapunov function in this paper is constructed as* $V(t) = e^T(t)M^TPMe(t)$*, which makes the conditions in Theorem 3.1 have a broader scope.*

Corollary 3.1. *When* $\triangle A = \triangle B = 0_{n\times n}$*, the synchronization conditions in Theorem 3.1 can be modified as*

$$\Phi = \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} \in R^{2n\times 2n}, \tag{3.11}$$

with $\Phi_{12} = \Phi_{21} = 0_{n\times n}$, $\Phi_{11} = -M^TPM(C+K) - (C^T+K^T)M^TP^TM + \frac{2}{\omega}F^TF + \omega M^TPMAA^TM^TP^TM + \omega M^TPMBB^TM^TP^TMP^TM + \frac{\delta}{2}M^TPM + \frac{\delta}{2}M^TP^TM$ *and* $\Phi_{22} = \frac{2}{\omega}G^TG - \frac{\varepsilon}{2}M^TPM - \frac{\varepsilon}{2}M^TP^TM$*. Suppose that the assumptions* A_1 *and* A_2 *are satisfied, if there exist real constants* $\omega > 0$*,* $\delta > \varepsilon > 0$*, matrices* $M \in R^{n\times n}$ *(full rank matrix),* $P \in R^{n\times n}$ *(*$P > 0$*), matrix* $K = diag(k_i) \in R^{n\times n}$ *meet (3.11), then, the two DFNNs systems (2.1) and (2.2) with* $\triangle A = \triangle B = 0_{n\times n}$ *are synchronized under the controller (3.1).*

4. Illustrative example

In order to illustrate the validity of the proposed results, a numerical example is performed in this section.

Example 4.1. The three-state DFNNUPs drive system is designed as

$$D^\gamma x(t) = -Cx(t) + (A + \triangle A)f(x(t)) + (B + \triangle B)g(x(t-\tau)) + I, \tag{4.1}$$

where $x(t) = (x_1(t), x_2(t), x_3(t))^T$, $\tau = 1$ and the activation function is selected as $f_j(x_j(t)) = sin(x_j(t))$, $g_j(x_j(t)) = tanh(x_j(t))$, $j = 1, 2, 3$.

In addition, the matrices C, A and B are defined as

$$C = \begin{pmatrix} 0.90 & 0 & 0 \\ 0 & 0.90 & 0 \\ 0 & 0 & 0.90 \end{pmatrix}, \quad A = \begin{pmatrix} -0.22 & 0.08 & 0.19 \\ 0.28 & -0.21 & 0.12 \\ -0.19 & 0.15 & -0.06 \end{pmatrix}, \quad B = \begin{pmatrix} -0.09 & 0.07 & 0.10 \\ 0.18 & 0.17 & 0.13 \\ -0.25 & -0.09 & -0.08 \end{pmatrix},$$

and $I = (0.1, 0.1, 0.1)^T$. Furthermore, the other parameters Q, $H(t)$, N_A and N_B are given as

$$Q = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{pmatrix}, \quad H(t) = \begin{pmatrix} 1.5cos(t) & 0 & cos(t) \\ 0 & 1.8sin(t) & 0 \\ 0 & 0 & 1.2cos(t) \end{pmatrix},$$

$$N_A = \begin{pmatrix} -0.25 & 0.32 & 0.24 \\ 0.45 & 0.36 & 0.28 \\ 0.39 & 0.18 & -0.22 \end{pmatrix}, \quad N_B = \begin{pmatrix} 0.41 & 0.35 & -0.25 \\ 0.38 & 0.32 & 0.16 \\ 0.09 & 0.27 & 0.15 \end{pmatrix}.$$

The matrices mentioned above satisfied the A_2: $N_A^T H^T(t)H(t)N_A \leq E_{3\times3}$ and $N_B^T H^T(t)H(t)N_B \leq E_{3\times3}$. From $[\triangle A\ \triangle B] = QH(t)[N_A\ N_B]$, it can be calculated that

$$\triangle A = \begin{pmatrix} 0.0075cos(t) & 0.3300cos(t) & 0.0700cos(t) \\ 0.4050sin(t) & 0.3240sin(t) & 0.2520sin(t) \\ 0.2340cos(t) & 0.1080cos(t) & -0.1320cos(t) \end{pmatrix},$$

$$\triangle B = \begin{pmatrix} 0.7561cos(t) & 0.8510cos(t) & 0.5287cos(t) \\ 0.7032sin(t) & 0.8684sin(t) & 0.4770sin(t) \\ 0.2581cos(t) & 0.3180cos(t) & 0.2473cos(t) \end{pmatrix}.$$

The three-state response system is given as follow:

$$D^{\gamma}y(t) = -Cy(t) + (A + \triangle A)f(y(t)) + (B + \triangle B)f(y(t-\tau)) + I + U(t), \tag{4.2}$$

where $y(t) = (y_1(t), y_2(t), y_3(t))^T$; the activation function is described as $f_j(x_j(t)) = sin(x_j(t))$, $g_j(x_j(t)) = tanh(x_j(t))$, $j = 1, 2, 3$. The delay τ, order γ and matrices C, A, $\triangle A$, B, $\triangle B$ are defined as the same with the system (4.1).

Let $\omega = 1.1$ and positive values $\delta = 6.1$, $\varepsilon = 6$ which satisfies $\omega > 0$ and $\delta > \varepsilon > 0$. Given matrices

$$M = \begin{pmatrix} -0.03 & -0.102 & 0.036 \\ 0.039 & -0.141 & 0.240 \\ 0.120 & -0.060 & -0.100 \end{pmatrix},\ P = \begin{pmatrix} 223.56 & -36.288 & 15.552 \\ -36.288 & 173.696 & 15.034 \\ 15.552 & 15.034 & 414.936 \end{pmatrix},$$

where $M^T PM > 0$. Then, solving (3.2) in Theorem 3.1 by MATLAB linear matrix toolbox, it can obtain that the controller gain K is

$$K = \begin{pmatrix} 35.370 & 0 & 0 \\ 0 & 16.308 & 0 \\ 0 & 0 & 13.338 \end{pmatrix}.$$

Set $\gamma = 0.66$. Figure 1 shows that the three-state DFNNUPs drive system (4.1) is not stable. In addition, the robust synchronization of drive system (4.1) and response system (4.2) with the controller (3.1) has been exhibited in Figures 2 and 3.

Remark 4.1. *When $\triangle A = \triangle B = 0_{3\times3}$, the other values in the systems (4.1) and (4.2) content the Corollary 3.1. Given $\gamma = 0.99$, Figure 4 shows the instability of the three-state Given DFNNUPs drive system (4.1). Figure 5 shows the synchronization behaviour of the (4.1) and (4.2) in the same figure, and Figure 6 illustrates the error system state trajectories between the (4.1) and (4.2), which demonstrates the effectiveness of the conditions (3.11).*

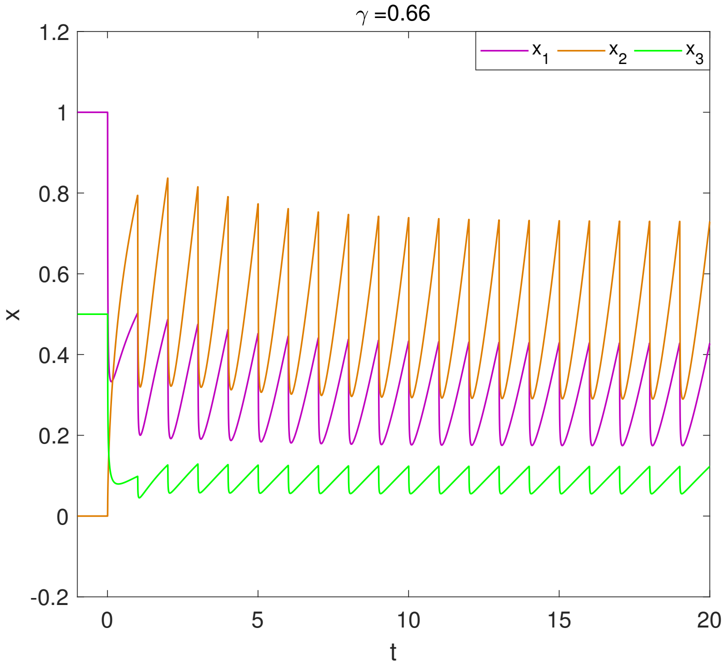

Figure 1. The state trajectory of the drive system (4.1) with initial value $x_0 = (1, 0, 0.5)^T$.

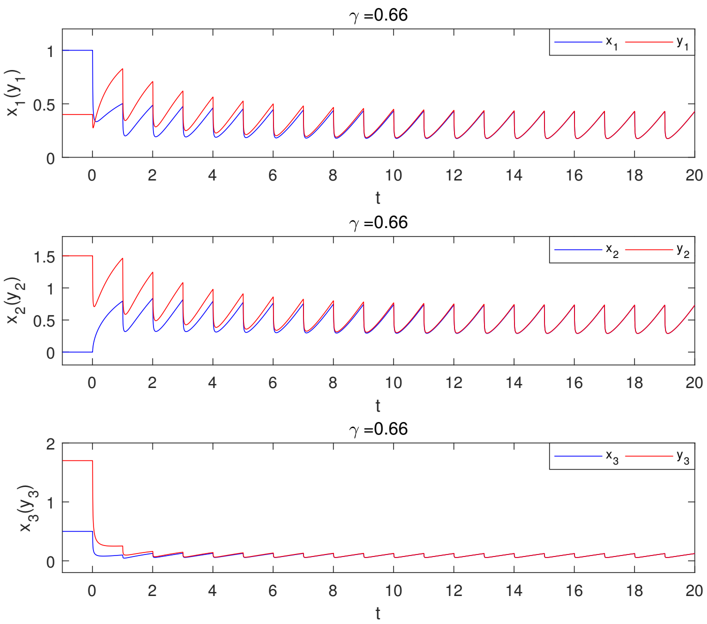

Figure 2. The state trajectories of the systems (4.1) and (4.2) under control with initial values $x_0 = (1, 0, 0.5)^T$ and $y_0 = (0.4, 1.5, 1.7)^T$.

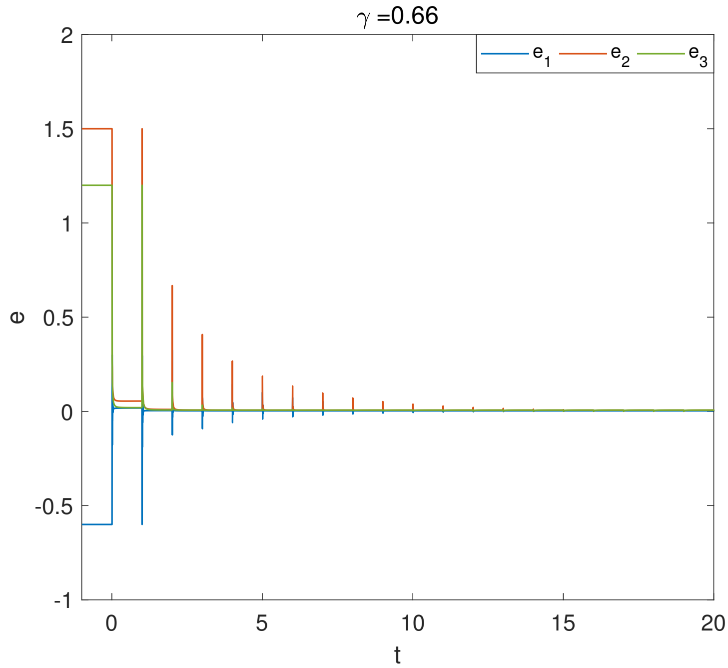

Figure 3. The trajectory of the synchronization error $e(t)$

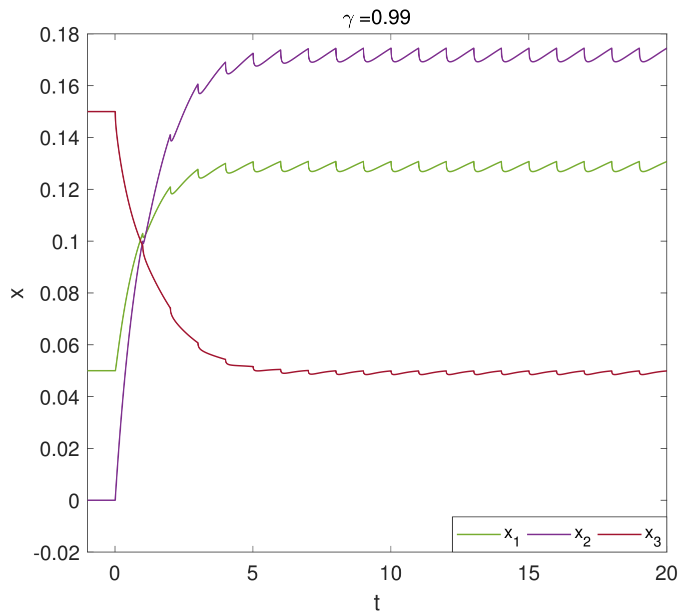

Figure 4. The state trajectory of the drive system (4.1) with $\triangle A = \triangle B = 0_{3\times3}$.

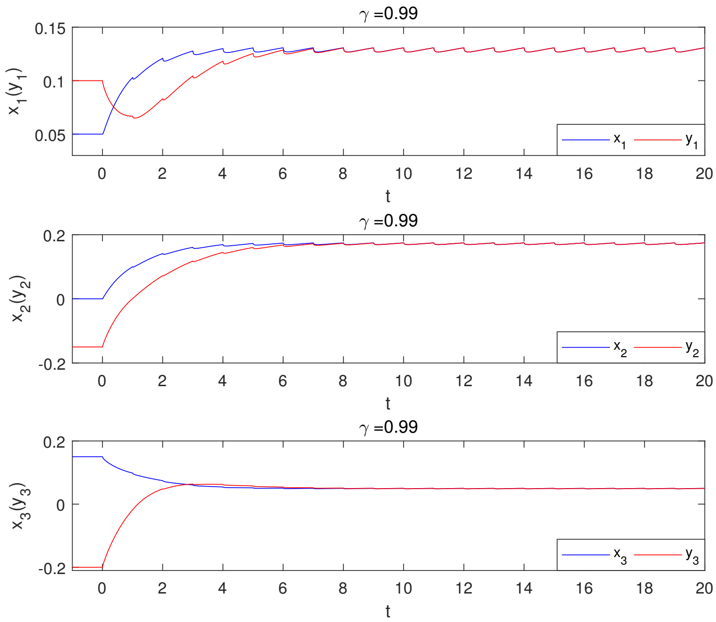

Figure 5. The state trajectories of the system (4.1) and system (4.2) under control with $\triangle A = \triangle B = 0_{3\times3}$.

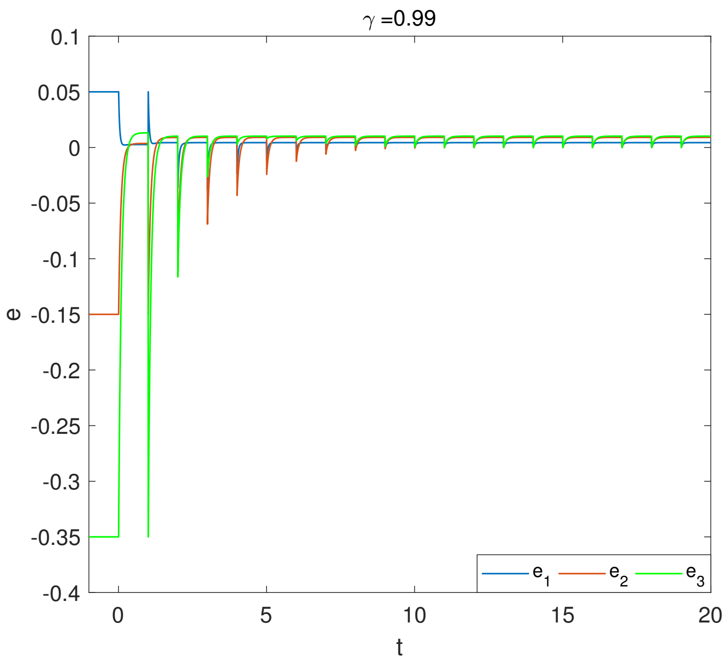

Figure 6. The trajectory of the synchronization error $e(t)$ with $\triangle A = \triangle B = 0_{3\times3}$.

5. Conclusions

In this paper, the robust synchronization analysis of the DFNNUPs is investigated. Due to the multiple matrix quadratic Lyapunov function approach, the sufficient conditions have been obtained in line with matrix inequality. Furthermore, a numerical simulation is presented to demonstrate the validity of the obtained results.

This paper will be applicable to the construction of DFNNUPs model. Moreover, The methods in this paper are suitable for the study of robust synchronization of DFNNUPs, which can be applied to the field of secure communication.

The uncertain parameters of the drive system and response system are the same in this paper. The response system without uncertain parameters or with different uncertain parameters will be included in our future research. Moreover, Because complex signals also exist in applications of neural networks, the further work is to study the robust synchronization of delayed fractional-order complex-valued neural networks with uncertain parameters.

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Conflict of interest

The authors declare no conflicts of interest.

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