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Research article

New stability criteria for semi-Markov jump linear systems with time-varying delays

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Abstract: In this paper, the delay-dependent stochastic stability problem of semi-Markov jump linear systems (S-MJLS) with time-varying delays is investigated. By constructing a Lyapunov-Krasovskii functional (LKF) with two delay-product-type terms, a new sufficient condition on stochastic stability of S-MJLSs is derived in terms of linear matrix inequalities (LMIs). Furthermore, the combination use of a slack condition on Lyapunov matrix and the improved Wirtinger's integral inequality reduces the conservatism of the result. Numerical examples are provided to verify the effectiveness and superiority of the presented results.

Keywords: semi-Markov jump linear systems; time-varying delays; delay-dependent stability; Wirtinger's integral inequality
Mathematics Subject Classification: 93B52, 93C42

1. Introduction

Markov jump linear systems (MJLSs) are a special class of hybrid stochastic systems. Many different types of systems subjects to random abrupt variations can be modeled by MJLSs, such as target tracking systems, mechanical systems and networked control systems [1,2]. In the past decades, the research of MJLSs has received widespread attention and has obtained many significant results, such as [3–6]. In the previous literature, the sojourn times of MJLSs are assumed to be a random variable that follows an exponential distribution, which results in the constant transition rates due to the memoryless property of exponential distribution. However, such an assumption cannot be appropriate for many situations, for example, DNA analysis [7] and fault-tolerant control systems [8]. Different from the MJLSs, semi-Markov jump linear systems (S-MJLSs) allow the sojourn time to follow an non-exponential distribution. S-MJLSs are determined by an embedded Markov chain of semi-Markov processes and probability density functions of the sojourn time. Obviously, due to the relaxed conditions on the probability distributions, the S-MJLS has a wider application domain than the MJLS. Recently, some important results on S-MJLSs have been provided. In [9] and [10], the stochastic stability of S-MJLSs are studied, where in the sojourn time in each state follows the phase-type distributions. For the Weibull distribution of the sojourn time, the stochastic stability and stochastic stabilization of the S-MJLS are discussed in [11]. The problems of controller design of the S-MJLS are investigated in [12–17]. The anti-windup design for stochastic S-MJLSs with saturation nonlinearity and stochastic disturbance is studied in [18]. For semi-Markov discrete-time jump systems, the stabilization problems are addressed in [19–21]. The nonlinear systems with semi-Markov parameters are studied in [22, 23]. The problems of the synchronization of the complex network with semi-Markov process are discussed in [24, 25].

Time delay is universal in practical dynamic systems, such as communication systems, network control systems, process control systems [26–28]. The time delay can degrade the performance of system, and even make the system unstable. In recent years, the stability analysis of time-delay systems has been a hot research topic [29–33]. The existing stability criteria can be classified into two categories: delay-dependent stability criteria and delay-independent stability criteria. Because the former contains more time lag information, it has received more attention [34,35]. Generally speaking, there are two ways to reduce conservatism. One is to construct appropriate Lyapunov functionals, such as delay decomposition approaches [36], augmented Lyapunov functionals [37], convex combination methods [38] and multiple integral functional [39]. The other is to estimate the integral term through appropriate integral inequality, for example, Jensen's inequality [40], Wirting's inequality [41, 42], Reciprocally convex matrix inequality [43, 44], free-matrix-based integral inequality [45, 46], and relaxed integral inequality [47,48]. Some less conservative stability results have been derived by using the above techniques.

Lots of significant results on stochastic stability analysis of MJLSs with time delays have been reported, but the obtained upper bounds of delays are far from the desired values. Compared with MJLSs, S-MJLSs are more powerful in modeling, and the results on analysis and synthesis are relatively few, Which is our primary motivation to write this paper. On the other hand, delay-product-type terms can be added into LKFs which are benefit the final results; Furthermore, In [40, 41, 43], these kinds of integral inequalities have been proved to get better results. It can be expected that some more effective and less conservative results can be derived for delayed S-MJLSs via these approaches, which is the second motivation for this paper.

This paper mainly investigates the stochastic stability of S-MJLSs with time-varying delays. Based on LKF method, we derive some less conservative stability criteria. Numerical examples illustrate the validity and superiority of the results of this paper.The main contribution are listed as follows:

1) An appropriate LKF with two delay-product-type terms is established.

2) Based on the combination use of a slack condition on Lyapunov matrix and improved integral inequalities, a new stability criteria are proposed.

Notation: Throughout this paper, (Ω, F, P) is a probability space, with Ω is the sample space, F is the $\sigma-$algebra of the sample space and P is the probability measure on F; the superscript 'T' and '-1' mean the transpose and the inverse of a matrix; N_+ stands for the set of non-negative integers; R_+ refers to the set of non-negative real numbers; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathbb{E}\{\cdot\}$ refers to the expectation operator with respect to some probability measure P; $Z > 0$ (< 0) means Z is a symmetric positive (negative) definite matrix; Sym$\{Y\}$ stands for $Y + Y^T$; The symbol $*$ in LMIs denotes the symmetric term of the matrix. Identity matrix, of appropriate dimensions, will be denoted by I; $diag(\cdot,\cdot)$ denotes a diagonal matrix; $\Omega(i, j)$ means the element in the i-th row and j-th column of the block matrix Ω.

2. Problem statement

In this section, we first introduce some concepts, notation and terminology related to semi-Markov processes.

i) The stochastic process $\{r_k\}_{k\in N_+}$ takes values in space $S = \{1, 2, \cdots, N\}$, where r_k represents system mode at the kth transition.

ii) The stochastic process $\{t_k\}_{k\in N_+}$ takes values in $\mathbb{R}_+$, where t_k (t_0=0) stands for the time at kth transition and monotonically increases with k.

iii) The stochastic process $\{\theta_k\}_{k\in N_+}$ takes values in $\mathbb{R}_+$, where $\theta_k = t_k - t_{k-1}$ for $\forall k \in N_{\geqslant 1}$ represents the sojourn-time of mode r_{k-1} between the $(k-1)$th transition and kth transition.

In order to introduce the S-MJLS, we consider the following jumping system:

$$\dot{x}(t) = A(r_t)x(t), \quad t_k \leqslant t \leqslant t_{k+1}, \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector, $A(r_t)$ is a matrix function of random jumping process $\{r_t\} \in S$ and suppose that the initial condition $t_0 = 0$ and $r(0)$ is a constant.

Definition 1. *[12] The stochastic process $r_t := r_k, t \in [t_k, t_{k-1})$ is a semi-Markov process, and system* (1) *is a continuous-time S-MJLS if for $i, j \in S$ and $t_0, t_1, \cdots, t_k \geqslant 0$, the following conditions are satisfied:*

i) It hold that the $Pr(r_{k+1} = j, \theta_{k+1} \leq h \mid r_k, \cdots, r_0, t_k, \cdots, t_0) = Pr(r_{k+1} = j, \theta_{k+1} \leq h \mid r_k)$.

ii) The probability $Pr(r_{k+1} = j, \theta_{k+1} \leq h \mid r_k = i)$ is independent on k.

In this paper,we consider the following smei-Markov jump time-delay systems in the space (Ω, F, P) as:

$$\begin{cases} \dot{x}(t) = A(r_t)x(t) + A_d(r_t)x(t - d(t)), \\ x(t) = \phi(t), \quad t \in [-h, 0], \end{cases} \tag{2.2}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector, $\{r_t, t \geqslant 0\}$ is a continuous-time semi-Markov process taking values in a finite space $S = \{1, 2, \cdots, N\}$, $A(r_t)$ and $A_d(r_t)$ are matrix functions of the random jumping process $\{r_t\}$; For notation simplicity, when the system operates in the mode, $A(r_t)$ and $A_d(r_t)$ are respectively denoted by A_i and A_{di}. $\phi(t)$ is a continuous vector-valued initial function defined on the interval $[-h, 0]$. The $d(t)$ is the time-varying delay satisfying:

$$0 \leqslant d(t) \leqslant h, \mu_1 \leqslant \dot{d}(t) \leqslant \mu_2. \tag{2.3}$$

The evolution of the Markov process $\{r_t, t \geqslant 0\}$ is governed by the following probability transitions:

$$\Pr\{r_{t+\theta} = j | r(t) = i\} = \begin{cases} \lambda_{ij}(\theta)\theta + o(\theta), & j \neq i, \\ 1 + \lambda_{ii}(\theta)\theta + o(\theta), & j = i, \end{cases} \tag{2.4}$$

where $\lambda_{ij}(\theta)$ is the transition probability from mode i at the time to j at time $t + \theta$ when $i \neq j$ and $\lambda_{ii}(\theta) = -\sum_{j=1, j\neq i}^{N} \lambda_{ij}(\theta)$; $o(\theta)$ is little-o notation and $\lim_{\theta\to 0} \frac{o(\theta)}{\theta} = 0$.

The following definition and lemmas are needed in the proof of our main results.

Definition 2. *[1] System (2.2) is said to be stochastically stable, if for any initial state (x_0, r_0), the following relation holds:*

$$\lim_{t\to+\infty} \mathbb{E}\left\{\|x(t)\|^2 | x_0, r_0\right\} = 0. \tag{2.5}$$

Lemma 1. *[40] For a positive definite matrix $R \in \mathbb{R}^{n\times n}$, the following inequality holds for all differential function $\omega(s)$ in $[a, b] \to \mathbb{R}^n$:*

$$(b-a)\int_a^b \omega^T(s)R\omega(s)ds \geqslant \int_a^b \omega^T(s)dsR\int_a^b \omega(s)ds \tag{2.6}$$

Lemma 2. *[44] For given vectors β_1 and β_2, scalar α in the interval (0,1), symmetric positive definite matrix $R \in \mathbb{R}^{n\times n}$ and any matrix $X \in \mathbb{R}^{n\times n}$ satisfying $\begin{bmatrix} R & X \\ * & R \end{bmatrix} \geqslant 0$, then the following inequality holds:*

$$\frac{1}{\alpha}\beta_1^T R\beta_1 + \frac{1}{1-\alpha}\beta_2^T R\beta_2 \geqslant \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}^T \begin{bmatrix} R & X \\ * & R \end{bmatrix}\begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \tag{2.7}$$

Lemma 3. *[41] For a given symmetric positive definite matrix $R > 0$, scalars a and b satisfying $a < b$, and any differential function ω in $[a, b] \to \mathbb{R}^n$, the following inequality holds:*

$$\int_a^b \omega^T(s)R\omega(s)ds \geqslant \frac{1}{b-a}\chi_1^T R\chi_1 + \frac{3}{b-a}\chi_2^T R\chi_2 \tag{2.8}$$

where $\chi_1 = \int_a^b \omega(s)ds$ and $\chi_2 = \chi_1 - \frac{2}{b-a}\int_a^b\int_a^s \omega(u)duds$.

3. Improved stability criterion

Before stating the main results, some notations are given. Let

$$\begin{aligned}
&\nu_1(t) = \int_{t-d(t)}^t \frac{x(s)}{d(t)}\mathrm{d}s, \quad \nu_2(t) = \int_{t-h}^{t-d(t)} \frac{x(s)}{h-d(t)}\mathrm{d}s, \quad \nu_3(t) = x(t) - x(t-d(t)),\\
&\nu_4(t) = x(t-d(t)) - x(t-h), \quad \nu_5(t) = x(t) - x(t-d(t)) - 2\nu_1(t),\\
&\nu_6(t) = x(t-d(t)) - x(t-h) - 2\nu_2(t), \quad \Gamma(t) = [x^T(t) \quad x^T(t-d(t))], \xi_4(t) = [x^T(s) \quad \dot{x}^T(s)]^T,\\
&\xi_1(t) = [\Gamma(t) \quad \int_{t-d(t)}^t x^T(s)\mathrm{d}s \quad \int_{t-h}^{t-d(t)} x^T(s)\mathrm{d}s]^T, \quad \xi_2(t) = [\Gamma(t) \quad \nu_1^T(t)]^T, \xi_3(t) = [\Gamma(t) \quad \nu_2^T(t)]^T,\\
&\zeta(t) = [\Gamma(t), x^T(t-h), \nu_1^T(t), \nu_2^T(t), \dot{x}^T(t-d(t))].
\end{aligned}$$

$$\mathscr{P}_i = \begin{bmatrix} P_{11i} & P_{12} & P_{13} & P_{14} \\ * & P_{22} & P_{23} & P_{24} \\ * & * & P_{33} & P_{34} \\ * & * & * & P_{44} \end{bmatrix}, \quad (i \in S).$$

$$\Upsilon = \begin{bmatrix} 4R & \Upsilon_{12} & \Upsilon_{13} & -6R & \Upsilon_{15} & 0 \\ * & \Upsilon_{22} & \Upsilon_{23} & \Upsilon_{24} & \Upsilon_{25} & 0 \\ * & * & 4R & 2\Upsilon_{34} & -6R & 0 \\ * & * & * & 12R & 4X_{22} & 0 \\ * & * & * & * & 12R & 0 \\ * & * & * & * & * & 0 \end{bmatrix}$$

$\Upsilon_{12} = 2R + X_{11} + X_{21} + X_{12} + X_{22}, \quad \Upsilon_{13} = -X_{11} - X_{21} + X_{12} + X_{22}, \quad \Upsilon_{15} = -2X_{12} - 2X_{22},$
$\Upsilon_{22} = 8R - sym\{X_{11} + X_{12} - X_{21} - X_{22}\}, \quad \Upsilon_{23} = 2R + X_{11} - X_{21} - X_{12} + X_{22},$
$\Upsilon_{24} = -6R - 2X_{21}^T - 2X_{22}^T, \quad \Upsilon_{25} = -6R + 2X_{12} - 2X_{22}, \quad \Upsilon_{34} = 2X_{21}^T - 2X_{22}^T.$

In this section, we will present a new stochastic stability criterion in terms of LMIs by using Lyapunov–Krasovskii functional method and Wirtinger-based inequality.

Theorem 1. *Given scalars h, μ_1 and μ_2, the time-varying delay system* (2.2) *is stochastically stable, if there exist symmetric matrices $\mathscr{P}_i \in \mathbb{R}^{4n\times 4n}$, $G \in \mathbb{R}^{3n\times 3n}$, $M \in \mathbb{R}^{3n\times 3n}$, symmetric positive definite matrices $Q \in \mathbb{R}^{2n\times 2n}$, $Z \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{n\times n}$, and any matrices $S \in \mathbb{R}^{n\times n}$, $X \in \mathbb{R}^{2n\times 2n}$, such that the following holds:*

$$\Psi = \begin{bmatrix} \tilde{R} & X \\ * & \tilde{R} \end{bmatrix} \geqslant 0, \quad \tilde{R} = diag\{R, 3R\} \tag{3.1}$$

$$\Xi_1 = \begin{bmatrix} EGE^T & S \\ * & EME^T + Z \end{bmatrix} \geqslant 0, \tag{3.2}$$

$$\Xi_2 = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ * & * & \Omega_{33} & \Omega_{34} \\ * & * & * & \Omega_{44} \end{bmatrix} > 0, \tag{3.3}$$

$$\Phi = \begin{bmatrix} \hat{\Omega}_{11} & \hat{\Omega}_{12} & \hat{\Omega}_{13} & \hat{\Omega}_{14} & \hat{\Omega}_{15} & \hat{\Omega}_{16} \\ * & \hat{\Omega}_{22} & \hat{\Omega}_{23} & \hat{\Omega}_{24} & \hat{\Omega}_{25} & \hat{\Omega}_{26} \\ * & * & \hat{\Omega}_{33} & \hat{\Omega}_{34} & \hat{\Omega}_{35} & \hat{\Omega}_{36} \\ * & * & * & \hat{\Omega}_{44} & \hat{\Omega}_{45} & \hat{\Omega}_{46} \\ * & * & * & * & \hat{\Omega}_{55} & \hat{\Omega}_{56} \\ * & * & * & * & * & \hat{\Omega}_{66} \end{bmatrix} < 0, \tag{3.4}$$

where

$$\Omega_{11} = P_{11i} + d(t)G_{11} + (h - d(t))M_{11}, \quad \Omega_{12} = P_{12} + d(t)G_{12} + (h - d(t))M_{12}, \quad \Omega_{13} = P_{13} + G_{13},$$
$$\Omega_{14} = P_{14} + M_{13}, \quad \Omega_{22} = P_{22} + d(t)G_{22} + (h - d(t))M_{22}, \quad \Omega_{23} = P_{23} + G_{23},$$
$$\Omega_{24} = P_{24} + M_{23}, \quad \Omega_{33} = P_{33} + \frac{G_{33}}{h}, \quad \Omega_{34} = P_{34} + \frac{S}{h}, \quad \Omega_{44} = P_{44} + \frac{M_{33} + Z}{h},$$
$$\hat{\Omega}_{11} = sym\{P_{11i}A_i + P_{13} + d(t)(G_{11} - M_{11})A_i + hM_{11}A_i + G_{13} + Q_{12}A_i\} + A_i^T(Q_{22} + h^2 R)A_i + Q_{11}$$
$$+ \dot{d}(t)(G_{11} - M_{11}) + Z - 4R + \sum_{j=1}^{N} \lambda_{ij}(\theta)\mathscr{P}_{11j},$$
$$\hat{\Omega}_{12} = (1 - \dot{d}(t))(P_{14} - P_{13} - G_{13} + M_{13}) + P_{12}A_{di} + d(t)(G_{11}A_{di} + A_i^T G_{21}^T) + P_{23}^T + G_{23}^T + Q_{11}A_{di}$$
$$+ (h - d(t))(M_{11}A_{di} + A_i^T M_{21}^T) + A_i^T Q_{22}A_{di} + h^2 A_i^T R A_{di} + A_i^T P_{21}^T + \dot{d}(t)(G_{12} - M_{12}) - 2R$$

$$- X_{11} - X_{12} - X_{21} - X_{22},$$

$$\hat{\Omega}_{13} = -P_{14} - M_{13} + X_{11} + X_{21} - X_{12} - X_{22}, \quad \hat{\Omega}_{15} = (h - d(t))(A_i^T P_{41}^T + P_{43}^T + A_i^T M_{31}^T) + 2X_{12} + 2X_{22}$$

$$\hat{\Omega}_{14} = d(t)(A_i^T P_{31}^T + A_i^T G_{31}^T + P_{33}) + G_{33} + 6R, \quad \hat{\Omega}_{16} = (1 - \dot{d}(t))[P_{12} + d(t)G_{12} + (h - d(t))M_{12}],$$

$$\hat{\Omega}_{22} = sym\{P_{21}A_{di} + (1 - \dot{d}(t))(P_{24} - P_{23} + M_{23} - G_{23}) + d(t)G_{21}A_{di} + (h - d(t))M_{21}A_{di}\} + A_{di}^T Q_{22} A_{di}$$

$$- (1 - \dot{d}(t))(Q_{11} - Z) + h^2 A_{di}^T R A_{di} - 8R + \dot{d}(t)(G_{22} - M_{22}) + sym\{X_{11} + X_{12} - X_{21} - X_{22}\},$$

$$\hat{\Omega}_{23} = -P_{24} - M_{23} - 2R - X_{11} + X_{21} + X_{12} - X_{22}, \quad \hat{\Omega}_{66} = -(1 - \dot{d}(t))Q_{22},$$

$$\hat{\Omega}_{24} = d(t)(A_{di}^T P_{31}^T + A_{di}^T G_{31}^T) + 6R + 2X_{21}^T + 2X_{22}^T + (1 - \dot{d}(t))(d(t)P_{34}^T - d(t)P_{33} - G_{33}),$$

$$\hat{\Omega}_{25} = (1 - \dot{d}(t))[(h - d(t))(P_{44} - P_{43}^T) + M_{33}] + 6R + (h - d(t))(A_{di}^T P_{41}^T + A_{di}^T M_{31}^T) - 2X_{12} + 2X_{22},$$

$$\hat{\Omega}_{26} = (1 - \dot{d}(t))[d(t)G_{22} + (h - d(t))M_{22} + P_{22} - Q_{12}], \quad \hat{\Omega}_{34} = -d(t)P_{34}^T - 2X_{21}^T + 2X_{22}^T$$

$$\hat{\Omega}_{33} = -Z - 4R, \quad \hat{\Omega}_{35} = d(t)P_{44} - hP_{44} - M_{33} + 6R, \quad \hat{\Omega}_{44} = -\dot{d}(t)G_{33} - 12R, \quad \hat{\Omega}_{45} = -4X_{22}^T,$$

$$\hat{\Omega}_{46} = d(t)(1 - \dot{d}(t))(G_{32} + P_{32}), \quad \hat{\Omega}_{55} = \dot{d}(t)M_{33} - 12R, \quad \hat{\Omega}_{56} = (1 - \dot{d}(t))(h - d(t))(P_{42} + M_{32}).$$

Proof. Consider the Lyapunov–Krasovskii function given by

$$V(x_t, i) = \sum_{k=1}^{5} V_k(x_t, i) \tag{3.5}$$

where

$$\begin{aligned}
V_1(x_t, i) &= \xi_1^T(t)\mathscr{P}(r_t)\xi_1(t), \\
V_2(x_t, i) &= d(t)\xi_2^T(t)G\xi_2(t) + [h - d(t)]\xi_3^T(t)M\xi_3(t), \\
V_3(x_t, i) &= \int_{t-d(t)}^{t} \xi_4^T(s)Q\xi_4(s)\mathrm{d}s, \\
V_4(x_t, i) &= \int_{t-h}^{t-d(t)} x^T(s)Zx(s)\mathrm{d}s, \\
V_5(x_t, i) &= h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s\mathrm{d}\theta.
\end{aligned}$$

We first show that, for some $\epsilon > 0$, the Lyapunov–Krasovskii functional condition $V(x_t, i) \geqslant \epsilon\|x_t\|^2$ for any initial condition (x_0, r_0).

$$\begin{aligned}
V(x_t, i) = \xi_1^T(t)\mathscr{P}(r_t)\xi_1(t) &+ \begin{bmatrix} x(t) \\ x(t-d(t)) \\ 0 \end{bmatrix}^T [d(t)G + (h - d(t))M] \begin{bmatrix} x(t) \\ x(t-d(t)) \\ 0 \end{bmatrix} \\
&+ 2\begin{bmatrix} x(t) \\ x(t-d(t)) \\ 0 \end{bmatrix}^T G \begin{bmatrix} 0 \\ 0 \\ \varpi_1(t) \end{bmatrix} + 2\begin{bmatrix} x(t) \\ x(t-d(t)) \\ 0 \end{bmatrix}^T M \begin{bmatrix} 0 \\ 0 \\ \varpi_2(t) \end{bmatrix} \\
&+ \frac{[\varpi_1(t)]^T EGE^T[\varpi_1(t)]}{d(t)} + \frac{[\varpi_2(t)]^T EME^T[\varpi_2(t)]}{h - d(t)} + \int_{t-d(t)}^{t} \xi_4^T(s)Q\xi_4(s)\mathrm{d}s
\end{aligned}$$

$$+\int_{t-h}^{t-d(t)} x^T(s)Zx(s)\mathrm{d}s + h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s\mathrm{d}\theta. \tag{3.6}$$

where $\varpi_1(t) = d(t)\nu_1(t)$, $\varpi_2(t) = (h - d(t))\nu_2(t)$, and $E = [0 \quad 0 \quad I]$.

Note that Z is a symmetric positive definite matrix. It follows from Lemma 1 that

$$\int_{t-h}^{t-d(t)} x^T(s)Zx(s)\mathrm{d}s \geqslant \frac{[(h-d(t))\nu_2(t)]^T Z[(h-d(t)) - \nu_2(t)]}{(h-d(t))} \tag{3.7}$$

Further, considering $\Xi_1 > 0$ and applying Lemma 2 yield

$$\frac{[\varpi_1(t)]^T EGE^T[\varpi_1(t)]}{d(t)} + \frac{[\varpi_2(t)]^T EME^T[\varpi_2(t)]}{h-d(t)} \geqslant \begin{bmatrix} \varpi_1(t) \\ \varpi_2(t) \end{bmatrix}^T \frac{\Xi_1}{h} \begin{bmatrix} \varpi_1(t) \\ \varpi_2(t) \end{bmatrix} \tag{3.8}$$

Based on the inequalities (3.7) and (3.8), LKF (3.6) can be written as

$$V(x_t, i) \geqslant \xi_5^T(t)\Xi_2\xi_5(t) + \int_{t-d(t)}^{t} \xi_4^T(s)Q\xi_4(s)\mathrm{d}s + h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s\mathrm{d}\theta. \tag{3.9}$$

where $\xi_5(t) = [x^T(t), x^T(t-d(t)), \omega_1^T(t), \omega_2^T(t)]$. From $\Xi_2 > 0$, $Q > 0$ and $R > 0$, we conclude that there exists a sufficiently small positive number ϵ_1 such that $V(x_t, i) \geqslant \epsilon_1\|x_t\|^2$.

We next show that $\mathscr{L}V(x_t, i) \leqslant -\epsilon_2\|x(t)\|^2$ for a sufficiently small positive number ϵ_2. Let $\mathscr{L}$ be the weak infinitesimal generator of the random process $\{x_t, r_t\}$.

Similar to the procedures of calculating the derivative of the Lyapunov function in [8], we have

$$\mathscr{L}V_1(x_t, i) = \xi_1^T(t)\left(\sum_{j=1}^{N}\lambda_{ij}(\theta)\mathscr{P}_j\right)\xi_1(t) + 2\xi_1^T(t)\mathscr{P}_i\dot{\xi}_1(t) \tag{3.10}$$

The derivative of $V_2(x_t, i)$ is displayed as follows:

$$\begin{aligned}\mathscr{L}V_2(x_t, i) = {}& \dot{d}(t)\xi_2^T(t)G\xi_2(t) - \dot{d}(t)\xi_3^T(t)M\xi_3(t) + 2d(t)\xi_2^T(t)M\begin{bmatrix} \dot{x}(t) \\ (1-\dot{d}(t))\dot{x}(t-d(t)) \\ \frac{x(t)-(1-\dot{d}(t))x(t-d(t))-\dot{d}(t)\nu_1(t)}{d(t)} \end{bmatrix} \\ & + 2[h-d(t)]\xi_3^T(t)M\begin{bmatrix} \dot{x}(t) \\ (1-\dot{d}(t))\dot{x}(t-d(t)) \\ \frac{(1-\dot{d}(t))x(t-d(t))-x(t-h)+\dot{d}(t)\nu_2(t)}{h-d(t)} \end{bmatrix}\end{aligned} \tag{3.11}$$

Calculating the derivative of other terms in $V(x_t, i)$ yields

$$\begin{aligned}\mathscr{L}\sum_{k=3}^{5} V_k(x_t, i) = {}& x^T(t)(Q + A_i^T QA)x(t) + x^T A_i^T QA_d x(t-d(t)) + x^T(t-d(t))(1-\dot{d}(t))Zx(t-d(t)) \\ & + x^T(t-d(t))(A_d^T QA_d - [1-\dot{d}(t)]Q)x(t-d(t)) + x^T(t-d(t))A_d^T QA_i x(t) \\ & - x^T(t-h)Zx(t-h) + h^2\dot{x}^T(t)R\dot{x}(t) - h\int_{t-h}^{t}\dot{x}^T(s)R\dot{x}(s)\mathrm{d}s\end{aligned} \tag{3.12}$$

Note that $\Psi > 0$ holds for any matrix X. Applying Lemma 2 and Lemma 3 to estimate the last term in (3.12) yields

$$\begin{aligned} h\int_{t-h}^{t}\dot{x}^T(s)R\dot{x}(s)\mathrm{d} &\geqslant \frac{h}{d(t)}\begin{bmatrix} \nu_3(t) \\ \nu_5(t) \end{bmatrix}^T \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix} \begin{bmatrix} \nu_3(t) \\ \nu_5(t) \end{bmatrix} + \frac{h}{h-d(t)}\begin{bmatrix} \nu_4(t) \\ \nu_6(t) \end{bmatrix}^T \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix} \begin{bmatrix} \nu_4(t) \\ \nu_6(t) \end{bmatrix} \\ &\geqslant \zeta^T(t)\Upsilon\zeta(t). \end{aligned} \tag{3.13}$$

Combining the inequalities (3.10)-(3.13), we have

$$\mathscr{L}V(x_t, i) \leqslant \zeta^T(t)\Phi\zeta(t) \tag{3.14}$$

Thus, $\Phi < 0$ leads to $\mathscr{L}V(x_t, i) < 0$, which implies there exists a sufficiently small positive number ϵ_2 such that $\mathscr{L}V(x_t, i) \leqslant -\epsilon_2\|x(t)\|^2$ for any initial condition (x_0, r_0). By Dynkin's formula, we further have

$$\mathbb{E}\{V(x_t, i)\} - \{V(x_0, r_0)\} \leqslant -\sigma\mathbb{E}\left\{\int_0^t \|x(s)\|^2\mathrm{d}s|(x_0, r_0)\right\} \tag{3.15}$$

which indicates

$$\mathbb{E}\left\{\int_0^t \|x(s)\|^2\mathrm{d}s|(x_0, r_0)\right\} \leqslant \frac{1}{\sigma}\{V(x_0, r_0)\} < \infty \tag{3.16}$$

The previous inequality means that

$$\lim_{t\to+\infty}\mathbb{E}\{\|x(t)\|^2|(x_0, r_0)\} = 0 \tag{3.17}$$

Thus, system (2.2) is stochastically stable by Definition 2.

Remark 1. *In order to guarantee the Lyapunov–Krasovskii functional $V(x_t, r_t) > 0$, most authors require the Lyapunov matrix $\mathscr{P}_i > 0$ in $V_1(x_t, r_t)$ (see, e.g., [8, 10]). In this paper, we derive a relaxed condition for ensuring the positive definite of LKF, i.e., $\Xi_2 > 0$, $Q > 0$ and $R > 0$. The relaxed condition can reduce the conservatism of the result.*

Remark 2. *As we known, the appropriate augmented Lyapunov functionals is an effective method to reduce the conservatism of the stability conditions (see, e.g., [29, 33]). An augmented LKF with two delay-product-type terms is constructed in this paper, so that the information about the delay is fully considered, which can further reduce the conservatism of the result.*

Remark 3. *It is well known that the integral inequalities have played a key role when dealing with the integral terms (see, e.g., [40, 41, 44]). In this paper, we use Lemma 2 and Lemma 3 to deal with the integral terms in the $\mathscr{L}V_5(x_t, i)$ and use Lemma 1 to deal with $V_4(x_t, i)$, the conservatism of stability conditions is further reduced efficiently.*

When the sojourn-time following an exponential distribution, the transition rate $\lambda_{ij}(\theta)$ will become to a constant λ_{ij}. In such a case, the S-MJLS (2.2) reduces to an MJLS. The stochastic stability criterion for the MJLS is given as follows:

Corollary 1. *Given scalars h, μ_1 and μ_2, the MJLS is stochastically stable, if there exist symmetric matrices $\mathscr{P}_i \in \mathbb{R}^{4n\times 4n}$, $G \in \mathbb{R}^{3n\times 3n}$, $M \in \mathbb{R}^{3n\times 3n}$, symmetric positive definite matrices $Q \in \mathbb{R}^{2n\times 2n}$, $Z \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{n\times n}$, and any matrices $S \in \mathbb{R}^{n\times n}$, $X \in \mathbb{R}^{2n\times 2n}$, such that the following holds:*

$$\Psi = \begin{bmatrix} \tilde{R} & X \\ * & \tilde{R} \end{bmatrix} \geqslant 0, \quad \tilde{R} = diag\{R, 3R\} \tag{3.18}$$

$$\Xi_1 = \begin{bmatrix} EGE^T & S \\ * & EME^T + Z \end{bmatrix} \geqslant 0, \tag{3.19}$$

$$\Xi_2 = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ * & * & \Omega_{33} & \Omega_{34} \\ * & * & * & \Omega_{44} \end{bmatrix} > 0, \tag{3.20}$$

$$\Phi = \begin{bmatrix} \widetilde{\Omega}_{11} & \hat{\Omega}_{12} & \hat{\Omega}_{13} & \hat{\Omega}_{14} & \hat{\Omega}_{15} & \hat{\Omega}_{16} \\ * & \hat{\Omega}_{22} & \hat{\Omega}_{23} & \hat{\Omega}_{24} & \hat{\Omega}_{25} & \hat{\Omega}_{26} \\ * & * & \hat{\Omega}_{33} & \hat{\Omega}_{34} & \hat{\Omega}_{35} & \hat{\Omega}_{36} \\ * & * & * & \hat{\Omega}_{44} & \hat{\Omega}_{45} & \hat{\Omega}_{46} \\ * & * & * & * & \hat{\Omega}_{55} & \hat{\Omega}_{56} \\ * & * & * & * & * & \hat{\Omega}_{66} \end{bmatrix} < 0, \tag{3.21}$$

where

$$\begin{aligned} \widetilde{\Omega}_{11} = {} & sym\{P_{11i}A_i + P_{13} + d(t)(G_{11} - M_{11})A_i + hM_{11}A_i + G_{13} + Q_{12}A_i\} \\ & + Q_{11} + \dot{d}(t)(G_{11} - M_{11}) + A_i^T(Q_{22} + h^2R)A_i + Z - 4R + \sum_{j=1}^{N} \lambda_{ij}\mathscr{P}_{11j}. \end{aligned}$$

and Ω_{ls} $(l, s = 1, 2, 3, 4)$, $\hat{\Omega}_{mn}$ $(m, n = 1, 2, 3, 4, 5, 6)$ are defined as in Theorem 1.

Remark 4. *LMIs (3.3), (3.4), (3.20) and (3.21) contain $d(t)$ and $\dot{d}(t)$. Note that these LMIs are affine with respect to parameter $\dot{d}(t)$. For a given $d(t) = h$, we need to check the feasibility of LMIs with $\dot{d}(t) = \mu_1$ and $\dot{d}(t) = \mu_2$, respectively.*

4. Numerical examples

Example 1. Consider the S-MJLS (2.2) with the following parameters

$$A_1 = \begin{bmatrix} -3.4888 & 0.8057 \\ -0.6451 & -3.2684 \end{bmatrix} A_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 1.3396 & -0.0211 \end{bmatrix}$$
$$A_{d1} = \begin{bmatrix} -0.8620 & -1.2919 \\ -0.6841 & -2.0729 \end{bmatrix} A_{d2} = \begin{bmatrix} -2.8306 & 0.4978 \\ -0.8436 & -1.0115 \end{bmatrix}$$

The sojourn time is assumed to follow the Weibull distribution. Its probability density function is

$$f(h) = \begin{cases} \dfrac{\beta}{\alpha^{\beta}} h^{\beta-1} \exp\left[-\left(\dfrac{h}{\alpha}\right)^{\beta}\right], & h \geq 0 \\ 0, & h < 0 \end{cases}.$$

For different modes, we select the parameters α and β as

$$\begin{cases} \alpha = 1,\ \beta = 1, & \text{for } i = 1 \\ \alpha = 1,\ \beta = 2, & \text{for } i = 2 \end{cases}$$

The embedded Markov chain is assumed to be

$$\mathbf{P} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Based on the definition of transition rate functions, one has

$$\Lambda(h) = [\lambda_{ij}(h)] = \begin{bmatrix} -1 & 1 \\ 2h & -2h \end{bmatrix}$$

Further, by using $\mathbb{E}\{\lambda_{ij(h)}\} = \int_0^{\infty} \lambda_{ij}(h) f(h) \mathrm{d}h$, we can derive the mathematical expectation of the transition rate functions as follows:

$$\bar{\Lambda} = [\bar{\lambda}_{ij}] = \begin{bmatrix} -1 & 1 \\ 1.7725 & -1.7725 \end{bmatrix}$$

In this example, we assume that $\mu_1 = -\mu_2 \leq 0$. Using Theorem 1 of our paper, the admissible upper bounds h for different μ_2 can be found in Table 1. It follows from Table 1 that, with the increase of μ_2, the admissible upper bounds of time delay becomes smaller.

Table 1. Admissible upper bounds h for different μ_2.

μ_2	0.1	0.3	0.5	0.7	0.9
Th. 1	0.7699	0.7312	0.7040	0.6878	0.6749

Take $d(t) = 0.1\sin(t) + 0.6699$, the simulation results are provided in Figure 1 and Figure 2. It can be seen from Figure 2 that the S-MJLS (2.2) is stochastically stable under the maximum allowable delay $h = 0.7699$.

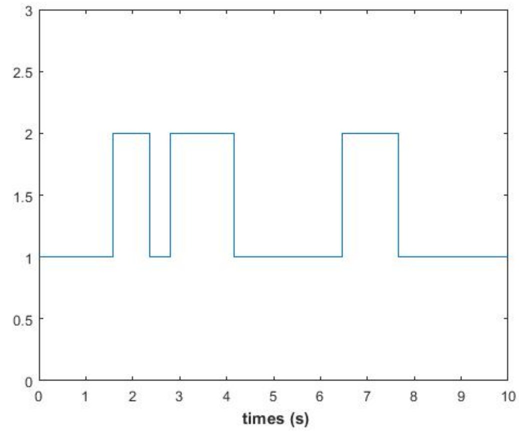

Figure 1. The simulation of system modes.

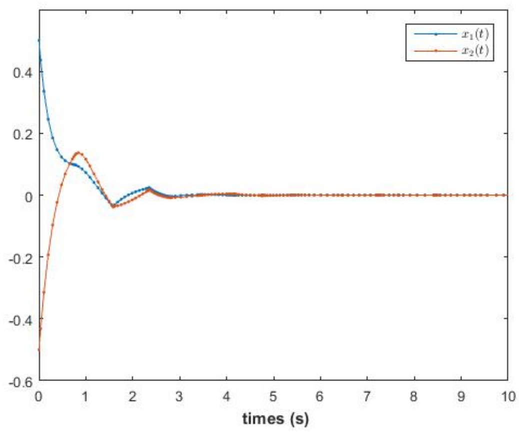

Figure 2. The trajectory of $x(t)$.

Example 2. Consider the MJLS with the following parameters:

$$A_1 = \begin{bmatrix} -2.3 & 0.8 \\ 1.0 & -2.9 \end{bmatrix},\ A_2 = \begin{bmatrix} -1.9 & 0.2 \\ 0.6 & -0.8 \end{bmatrix} A_{d1} = \begin{bmatrix} 0.8 & 1.2 \\ 0.7 & -3.5 \end{bmatrix},\ A_{d2} = \begin{bmatrix} 1.3 & -2.6 \\ 0.5 & -1.4 \end{bmatrix}$$
$$\lambda_{11} = -\lambda_{12}, \quad \lambda_{22} = -\lambda_{21}$$

In this example, we assume $\mu_1 = -\mu_2 = -0.5$ and $\lambda_{22} = -\lambda_{21} = -3$. Using Corollary 1 of our paper, the admissible upper bounds h for different λ_{11} can be found in Table 2.

Table 2. Admissible upper bounds h for different λ_{11}.

λ_{11}	-0.1	-0.5	-0.8	-1
Theorem 2 of [49]	0.500	0.496	0.493	0.492
Theorem 1 of [50]	0.503	0.501	0.499	0.499
Cor. 1	0.796	0.757	0.736	0.724

From this table, it is clear that Corollary 1 proposed in this paper is less conservative than the results in [49, 50].

Take $d(t) = 0.5\sin(t) + 0.296$, the simulation results are provided in Figure 3 and Figure 4. It can be seen from Figure 4 that the MJLS is stochastically stable under the maximum allowable delay $h = 0.796$.

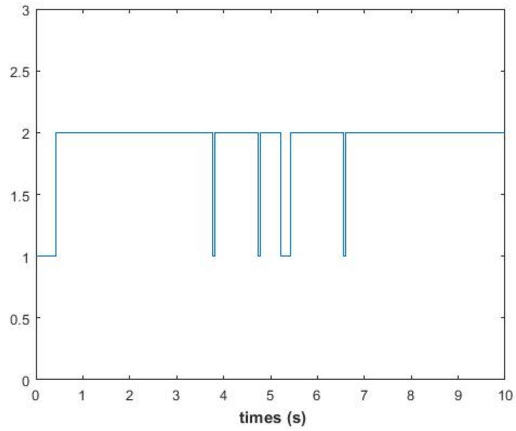

Figure 3. The simulation of system modes.

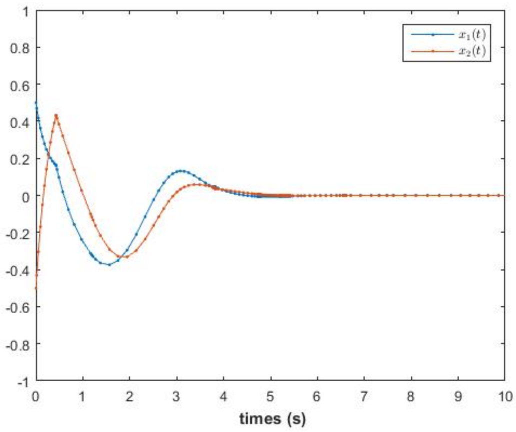

Figure 4. The trajectory of $x(t)$.

5. Conclusions

In this paper, we have provided a new sufficient condition on stochastic stability of S-MJLSs with time-varying delays by construing an augmented LKF with two delay-product-type terms. In order to reduce the conservatism of the result, a slack condition on Lyapunov matrix has been introduced. In addition, the improved Wirtinger's integral inequality are used to deal with the integral term in the time derivative of the LKF. To compare with the existing results, we also provide a sufficient condition on stochastic stability of MJLSs with time-varying delays. Numerical examples are presented to show the superiority of the proposed method.

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Conflict of interest

The authors declare that they have no competing interests concerning the publication of this article.

References

1. X. R. Mao, C. R. Yuan, *Stochastic differential equations with Markovian switching*, Imperial college press, 2006
2. S. Yang, Y. Bo, Robust mixed H_2/H_∞ control of networked control systems with random time delays in both forward and backward communication links, *Automatica,* **47** (2011), 754–760.
3. S. Y. Xu, J. Lam, X. R. Mao, Delay-dependent H_∞ control and filtering for uncertain Markovian jump systems with time-varying delays, *IEEE T. Circuits-I,* **54** (2007), 2070–2077.
4. L. X. Zhang, E. Boukas, Stability and stabilization of Markovian jump linear systems with partly unknown transition probabilities, *Automatica,* **45** (2009), 463–468.
5. Z. G. Wu, J. H. Park, H. S. Su, J. Chu, Delay-dependent passivity for singular Markov jump systems with time-delays, *Commun. Nonlinear Sci.,* **18** (2013), 669–681.
6. J. Cheng, Y. N. Shan, J. D. Cao, J. H. Park, Nonstationary control for T-S fuzzy Markovian switching systems with variable quantization density, *IEEE T. Fuzzy Syst.,* (2020), in press.
7. V. Barbu, N. Limnios, *Semi-Markov chains and hidden semi-Markov models toward applications: their use in reliability and DNA analysis*, Business Media: Springer Science, 2008.
8. J. Huang, Y. Shi, Stochastic stability of semi-Markov jump linear systems: An LMI approach, *IEEE conference on decision and control,* (2011), 4668–4673.
9. Z. T. Hou, J. W. Luo, P. Shi, S. K. Nguang, Stochastic stability of ito differential equations with semi-Markovian jump parameters, *IEEE T. Automat. Contr.,* **51** (2006), 1383–1387.
10. F. B. Li, L. G. Wu, P. Shi, Stochastic stability of semi-Markovian jump systems with mode-dependent delays, *Int. J. Robust Nonlin.,* **24** (2014), 3317–3330.
11. B. P. Jiang, Y. G. Kao, H. R. Karimi, C. C. Gao, Stability and stabilization for singular switching semi-markovian jump systems with generally uncertain transition rates, *IEEE T. Automat. Contr.,* **63** (2018), 3919–3926.
12. Y. L. Wei, J. H. Park, J. Qiu, L. G. Wu, H. Y. Jung, Sliding mode control for semi-Markovian jump systems via output feedback, *Automatica,* **81** (2017), 133–141.
13. W. H. Qi, G. D. Zong, H. R. Karimi, Sliding mode control for nonlinear stochastic singular semi-Markov jump systems, *IEEE T. Automat. Contr.,* **65** (2019), 361–368.

14. W. H. Qi, X. W. Gao, C. K. Ahn, J. D. Cao, J. Cheng, Fuzzy Integral Sliding-Mode Control for Nonlinear Semi-Markovian Switching Systems With Application, *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* (2020), 1–10.

15. W. H. Qi, G. D. Zong, H. R. Karimi, SMC for nonlinear stochastic switching systems with quantization, *IEEE T. Circuits-II,* (2020), 1–1.

16. W. H. Qi, G. D. Zong, W. X. Zheng, Adaptive Event-Triggered SMC for Stochastic Switching Systems With Semi-Markov Process and Application to Boost Converter Circuit Model, *IEEE T. Circuits-I,* **68** (2021), 786–796.

17. B. P. Jiang, H. R. Karimi, Y. G., Kao, C. C. Gao, Adaptive control of nonlinear semi-Markovian jump T-S fuzzy systems with immeasurable premise variables via sliding mode observer, *IEEE T. Cybernetics,* **50** (2020), 810–820.

18. W. H. Qi, J. H. Park, G. D. Zong, J. D. Cao, J. Chen, Anti-windup design for saturated semi-Markovian switching systems with stochastic disturbance, *IEEE T. Circuits-II,* **66** (2019), 1187–1191.

19. L. X. Zhang, Y. Leng, P. Colaneri, Stability and stabilization of discrete-time semi-Markov jump linear systems via semi-Markov kernel approach, *IEEE T. Automat. Contr.,* **61** (2016), 503–508.

20. L. X. Zhang, T. Yang, P. Colaneri, Stability and stabilization of semi-Markov jump linear systems with exponential modulated periodic distribution of sojourn time, *IEEE T. Automat. Contr.,* **62** (2017), 2870–2885.

21. H. Shen, F. Li, S. Y. Xu, V. Sreeram, Slow state variables feedback stabilization for semi-Markov jump systems with singular perturbations, *IEEE T. Automat. Contr.,* **63** (2018), 2709–2714.

22. J. Wang, Y. Wang, H. C. Yan, J. D. Cao, H. Shen, Hybrid Event-Based Leader-Following Consensus of Nonlinear Multiagent Systems With Semi-Markov Jump Parameters, *IEEE Systems Journal,* (2020), 1–12.

23. W. H. Qi, J. H. Park, G. D. Zong, J. D. Cao, Filter for Positive Stochastic Nonlinear Switching Systems With Phase-Type Semi-Markov Parameters and Application, *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* (2021), 1–12.

24. Y. Liu, J. W. Xia, B. Meng, X. N. Song, H. Shen, Extended dissipative synchronization for semi-Markov jump complex dynamic networks via memory sampled-data control scheme, *J. Franklin I.,* **357** (2020), 10900–10920.

25. J. Wang, J. W. Xia, H. Shen, M. P. Xing, J. H. Park, H_∞ Synchronization for Fuzzy Markov Jump Chaotic Systems with Piecewise-Constant Transition Probabilities Subject to PDT Switching Rule, *IEEE T. Fuzzy Syst.,* (2020), 1–1.

26. B. Y. Zhang, J. Lam, S. Y. Xu, Stability analysis of distributed delay neural networks based on relaxed Lyapunov-Krasovskii functionals, *IEEE T. Neur. Net. Lear.,* **26** (2015), 1480–1492.

27. J. Li, Y. He, M. Wu, Improved delay-dependent stability analysis of discrete-time neural networks with time-varying delay, *J. Franklin I.,* **354** (2017), 1922–1936.

28. V. H. Le, H. Trinh, An enhanced stability criterion for time-delay systems via a new bounding technique, *J. Franklin I.,* **352** (2015), 4407–4422.

29. M. Wu, Y. He, J. H. She, G. P. Liu, Delay-dependent criteria for robust stability of time-varying delay systems, *Automatica,* **40** (2004), 1435–1439.
30. H. Y. Shao, New delay-dependent stability criteria for systems with interval delay, *Automatica,* **45** (2009), 744–749.
31. J. Kim, Delay-dependent robust and non-fragile guaranteed cost control for uncertain singular systems with time-varying state and input delays, *Int. J. Control Autom.,* **7** (2009), 357–364.
32. M. Wu, Z. Feng, Y. He, Improved delay-dependent absolute stability of Lure systems with time-delay, *Int. J. Control Autom.,* **7** (2009), 1009–1014.
33. J. W. Wen, L. Peng, S. K. Nguang, Finite-time control for discrete-time Markovian jump systems with deterministic switching and time-delay, *Int. J. Control Autom.,* **12** (2014), 473–485.
34. F. O. Souza, Further improvement in stability criteria for linear systems with interval time-varying delay, *IET Control Theory A.,* **7** (2013), 440–446.
35. P. L. Liu, New delay-derivative-dependent stability criteria for linear systems with interval time-varying delays, *International Journal of Electrical Engineering,* **24** (2017), 47–57.
36. Q. L. Han, A discrete delay decomposition approach to stability of linear retarded and neutral systems, *Automatica,* **45** (2009), 517–524.
37. K. S. Ko, W. I. Lee, P. G. Park, D. K. Sung, Delays-dependent region partitioning approach for stability criterion of linear systems with multiple time-varying delays, *Automatica,* **87** (2018), 389–394.
38. M. Tang, Y. Wang, C. Y. Wen, Improved delay-range-dependent stability criteria for linear systems with interval time-varying delays, *IET Control Theory A.,* **6** (2012), 868–873.
39. M. Fang, J. H. Park, A multiple integral approach to stability of neutral time-delay systems, *Appl. Math. Comput.,* **224** (2013), 714–718.
40. X. L. Zhu, G. H. Yang, Jensen integral inequality approach to stability analysis of continuous-time systems with time-varying delay, *IET Control Theory A.,* **2** (2008), 524–534.
41. A. Seuret, F. Gouaisbaut, Wirtinger-based integral inequality: Application to time-delay systems, *Automatica,* **49** (2013), 2860–2866.
42. M. Park, O. Kwon, J. H. Park, S. M. Lee, E. J. Cha, Stability of time-delay systems via Wirtinger-based double integral inequality, *Automatica,* **55** (2015), 204–208.
43. C. K. Zhang, Y. He, L. Jiang, Q. G. Wang, An extended reciprocally convex matrix inequality for stability analysis of systems with time-varying delay, *Automatica,* **85** (2017), 481–485.
44. P. G. Park, J. W. Ko, C. Jeong, Reciprocally convex approach tostability of systems with time-varying delays, *Automatica,* **47** (2011), 235–238.
45. H. Zeng, Y. He, M. Wu, J. She, Free-Matrix-Based integral inequality for stability analysis of systems with time-varying delay, *IEEE T. Automat. Contr.,* **60** (2015), 2768–2772.
46. Y. L. Zhi, Y. He, C. K. Zhang, M. Wu, New method for stability of systems with time-varying delay via improved free-matrix-based integral inequality, *IFAC-PapersOnLine,* **50** (2017), 1281–1285.
47. C. K. Zhang, Y. He, L. Jiang, M. Wu, H. B. Zeng, Stability analysis of systems with time-varying delay via relaxed integral inequalities, *Syst. Control Lett.,* **92** (2016), 52–61.

48. D. X. Liao, S. M Zhong, J. N. Luo, X. J. Zhang, Y. B. Yu, Q. S. Zhong, Improved delay-dependent stability criteria for networked control system with two additive input delays, *Int. J. Control Autom.*, **17** (2019), 2174–2182.

49. Z. Li, Z. Fei, H. Gao, Stability and stabilisation of Markovian jump systems with time-varying delay: An input-output approach, *IET Control Theory A.*, **6** (2012), 2601–2610.

50. Z. C. Li, Y. L. Xu, Z. Y. Fei, H. Huang, S. Misra, Stability analysis and stabilization of Markovian jump systems with time-varying delay and uncertain transition information, *Int. J. Robust Nonlin.*, **28** (2018), 68–85.