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Research article

Analysis of a stochastic predator-prey system with mixed functional responses and Lévy jumps

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Abstract: In this paper, we consider a stochastic two predator-one prey system consisting of prey, intermediate predator and top predator with Lévy jumps. Here we consider Ratio-dependent function response between intermediate predator and top predator and other function responses are assumed to be linear. Firstly, we prove that the existence and boundedness of pth moment of the positive solution. Then under some assumptions, we establish sufficient criteria for the extinction of the system. The results reveal an important property that the Lévy jumps are unfavorable for the existence of species. Furthermore, we establish sufficient condition for the asymptotically stable in distribution under certain conditions. Finally, some numerical simulations are introduced to demonstrate the theoretical results.

Keywords: mixed functional response; Lévy jumps, extinction; asymptotically stable in distribution
Mathematics Subject Classification: 34K20, 34K57, 34D05, 60K37

1. Introduction

Recently a lot of attention has been paid to the dynamic relation between two predators sharing a common prey species [1,2]. Three species predator-prey models are fundamental for building blocks of large scale ecosystems. In [3–8], the authors considered a widely existing two-predator-prey population model. The three species predator-prey model with intraguild predation involves a prey, an intermediate predator which feeds upon only prey and a top predator (called intraguild predator) which feeds upon both prey and intermediate predator [9, 10]. D. Sen [10] considered Holling type-II function responses between intermediate predator and top predator and other function responses were assumed to be linear which can be denoted by

$$\begin{cases} dx(t) = x(t)\left[a_1 - b_{11}x(t) - b_{12}y(t) - b_{13}z(t)\right]dt, \\ dy(t) = y(t)\left[-a_2 + b_{21}x(t) - b_{22}y(t) - \dfrac{c_1 z(t)}{1+y(t)}\right]dt, \\ dz(t) = z(t)\left[-a_3 + b_{31}x(t) - b_{33}z(t) + \dfrac{c_2 y(t)}{1+y(t)}\right]dt, \end{cases} \tag{1.1}$$

where $x(t)$, $y(t)$ and $z(t)$ represent population size of each species at time t, respectively, $a_1 > 0$ stands for the growth rate of the species $x(t)$, a_2 and a_3 stands for the death rate of species $y(t)$ and $z(t)$, respectively, $b_{jj}(j = 1, 2, 3)$ is the intra-specific competition rate of the species $x(t), y(t)$ and $z(t)$. b_{12} and b_{13} represent the predation or consumption rate of intermediate and top predator feeding upon prey respectively, c_1 is the predation rate of the top predator feeding upon intermediate predator, b_{21} and b_{31} measure prey consumption into reproduction for intermediate predator and top predator respectively and c_2 measures intermediate predator consumption into reproduction for top predator. Parameters a_i, b_{jj} and c_i are positive constants for $i = 1, 2, j = 1, 2, 3$.

In fact, in the real world, most natural phenomena can not be explained by deterministic laws and are always affected by environmental noise, which is an inevitable property of any ecosystem dynamics (see [11–16]). So, modelling population dynamics with white noise and jumps has become an active and fruitful topic in mathematical biology. A large number of literatures show that many researchers introduce Brownian motion into the deterministic model to describe stochastic effects and establish predator-prey stochastic model. May [17] pointed out that due to environmental fluctuations, models involved in the birth rate or other parameters, bearing capacity, competition coefficient showed stochastic fluctuations more or less. X. H. Zhang [18] considered a stochastic Holling II one-predator two-prey system with jumps. Mao [19] pointed out that even a small amount of environmental noise can suppress a potential population explosion. Liu [20] established a random predator-prey model with stage structure in response to predator and Holling II functions, investigated the existence of uniqueness of traversal stable distribution, and obtained sufficient conditions for the extinction of predator populations.

In model (1.1), if the intrinsic growth rate of prey and the death rate of predators are not constants but are subject to environmental noise, then a_j $(j = 1, 2, 3)$ are stochastically perturbed with

$$a_1 \to a_1 + \sigma_1 \dot{B}(t), \qquad -a_l \to -a_l + \sigma_l \dot{B}(t) \quad (l = 2, 3),$$

where σ_1^2, σ_2^2 and σ_3^2 stand for the intensities of the white noise, $B(t)$ denotes the standard Brownian motion which is defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}\}_{t\geq 0}, \mathbb{P})$. Then we obtain the following stochastic three species predator-prey model with white noise and mixed functional responses

$$\begin{cases} dx(t) = x(t)\left[a_1 - b_{11}x(t) - b_{12}y(t) - b_{13}z(t)\right]dt + \sigma_1 x(t)\,dB(t), \\ dy(t) = y(t)\left[-a_2 + b_{21}x(t) - b_{22}y(t) - \dfrac{c_1 z(t)}{1+y(t)}\right]dt + \sigma_2 y(t)\,dB(t), \\ dz(t) = z(t)\left[-a_3 + b_{31}x(t) - b_{33}z(t) + \dfrac{c_2 y(t)}{1+y(t)}\right]dt + \sigma_3 z(t)\,dB(t). \end{cases} \tag{1.2}$$

In mathematical modelling, some sudden environmental disturbances such as earthquake, epidemic diseases can't be described by white noise. These events are so strong that they can break the continuity of the sample path. Therefore, white noise cannot accurately describe these phenomena. In this case, introducing a Lévy jumps might be a reasonable approach. Based on the Brownian motion cases, jump process is introduced as the noise source by authors [21–23]. Bao et al. [21] initially used Lévy jumps to describe these phenomena and proposed stochastic competition model with jumps. Liu [23] investigated some asymptotic properties of a stochastic n-species Gilpin-Ayala competitive model and illustrated that these properties have close relationships with Lévy jumps. Inspired by the above discussion, we establish the following three predator-prey model with mixed responses and Lévy jumps:

$$\begin{cases} dx(t) = x(t)\left[a_1 - b_{11}x(t) - b_{12}y(t) - b_{13}z(t)\right]dt + \sigma_1 x(t)\,dB(t) + x(t^-)\int_{\mathbb{Z}}\gamma_1(u)\tilde{N}(dt,du), \\ dy(t) = y(t)\left[-a_2 + b_{21}x(t) - b_{22}y(t) - \dfrac{c_1 z(t)}{1+y(t)}\right]dt + \sigma_2 y(t)\,dB(t) + y(t^-)\int_{\mathbb{Z}}\gamma_2(u)\tilde{N}(dt,du), \\ dz(t) = z(t)\left[-a_3 + b_{31}x(t) - b_{33}z(t) + \dfrac{c_2 y(t)}{1+y(t)}\right]dt + \sigma_3 z(t)\,dB(t) + z(t^-)\int_{\mathbb{Z}}\gamma_3(u)\tilde{N}(dt,du), \end{cases} \tag{1.3}$$

with initial data

$$X(0) = (x(0), y(0), z(0)) \in \mathbb{R}^3_+,$$

where $x(t^-)$, $y(t^-)$ and $z(t^-)$ represent the left limit of $x(t)$, $y(t)$ and $z(t)$, respectively. Parameters a_j, b_{jj}, c_i and σ_j, $(i = 1, 2, j = 1, 2, 3)$ are all positive constant. N is a poisson counting measure with compensator $\tilde{N}$ and characteristic measure λ on a measurable subset $\mathbb{Z}$ of $(0, \infty)$ with $\lambda(\mathbb{Z}) < \infty$ and $\tilde{N}(dt, du) = N(dt, du) - \lambda(du)dt$. Parameter γ_j is the effect of Lévy noise on the *jth* species. $\gamma_j(u) > 0$ represents the increasing of the species, and $\gamma_j(u) < 0$ represents the decreasing of the species. Therefore, it is reasonable to assume that $1 + \gamma_j(u) > 0$ for $u \in \mathbb{Z}$, $j = 1, 2, 3$.

The organization of this paper is as follows. In Section 3, we obtain the global existence of positive unique solution of system (1.3). In Section 4, we discuss the asymptotical properties of system (1.3). In Section 5, we establish sufficient conditions for the asymptotic stability in distribution of (1.3). Finally, some numerical simulations are provided to illustrate our main results.

2. Preliminaries

Throughout this paper, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathcal{F}_0$ contains all $\mathbb{P} - null$ sets). Let $B(t)(t \geq 0)$ be a scalar standard Brownian motion defined on this probability space. We assume that N and B are independent. For biological reason, we suppose that $1 + \gamma_j(u) > 0$, where $\gamma_j(u) > 0$ means the increasing of the species (e.g., planting) and $-1 < \gamma_j(u) < 0$ means the decreasing of the species (e.g., harvesting and epidemics), $u \in \mathbb{Z}$, $j = 1, 2, 3$.

For our discussion, some technical assumptions are given as follows.

Assumption 1 We assume

$$\int_{\mathbb{Z}}\left\{\left(1+\gamma_j(u)\right)^p - 1 - p\gamma_j(u)\right\}\lambda(du) \leq C_1 < \infty,$$
$$\int_{\mathbb{Z}}\max\left\{|\gamma_j(u)|^2, \left[\ln\left(1+\gamma_j(u)\right)\right]^2\right\}\lambda(du) \leq C_2 < \infty,$$

where $C_i(i = 1, 2)$ is positive constant, $j = 1, 2, 3$.

Assumption 2 $b_{22}-b_{21}>0, b_{33}-b_{31}>0$, which means that the influence of intraspecific competition is greater than the interaction between different species.

For convenience, we cite the following notions:

$$r_1 = a_1 - \frac{\sigma_1^2}{2} - \int_{\mathbb{Z}} [\gamma_1(u) - \ln(1+\gamma_1(u))]\lambda(du),$$
$$r_l = a_l + \frac{\sigma_l^2}{2} + \int_{\mathbb{Z}} [\gamma_l(u) - \ln(1+\gamma_l(u))]\lambda(du), l = 2, 3,$$
$$D_1 = \frac{r_1}{b_{11}}, \qquad D_2 = \frac{b_{21}D_1 - r_2}{b_{22}}, \qquad D_3 = \frac{c_2 + b_{31}D_1 - r_3}{b_{33}},$$
$$A_1 = r_1 - b_{12}D_2 - b_{13}D_3, \quad A_2 = -r_2 + \frac{b_{21}A_1}{b_{11}} - c_1D_3, \quad A_3 = -r_3 + b_{31}\frac{A_1}{b_{11}},$$
$$\tilde{\gamma}_j =: \int_{\mathbb{Z}} \left[\left(1+\gamma_j(u)\right)^p - 1 - p\gamma_j(u)\right]\lambda(du), p \geq 1, j = 1, 2, 3,$$
$$\langle f(t)\rangle = \frac{1}{t}\int_0^t f(s)ds, \text{where } f(t) \text{ is a bounded continuous function.}$$

3. Positive and global solution

In this section, under Assumption 1, we show the solution of system (1.3) is not only positive, but also will not explode to infinity at any finite time. That is, for any positive initial value, (1.3) has unique positive global solution.

Theorem 3.1. *Under Assumption 1, system* (1.3) *has a unique solution* $X(t) = (x(t), y(t), z(t))$ *on* $t \geq 0$ *a.s. for any initial value* $X(0) = (x(0), y(0), z(0)) \in \mathbb{R}_+^3$.

Proof. Since the coefficients of (1.3) are locally Lipschitz continuous, then for any initial value $X(0) = (x(0), y(0), z(0)) \in \mathbb{R}_+^3$, there is a unique local solution $X(t) = (x(t), y(t), z(t))$ on $t \in [0, \tau_e)$ a.s., where τ_e is the explosion time. To show this solution is global, we need to show that $\tau_e = \infty$ a.s. Let $k_0 > 0$ be sufficiently large such that $(x(0), y(0), z(0)) \in (\frac{1}{k_0}, k_0)$. For each integer $k \geq k_0$, define the stopping time

$$\tau_k = \inf\left\{t \in [0, \tau_e) : x(t) \notin \left(\frac{1}{k}, k\right) \ or\ y(t) \notin \left(\frac{1}{k}, k\right) \ or\ z(t) \notin \left(\frac{1}{k}, k\right)\right\},$$

where $\inf \emptyset = \infty$ (as usual $\emptyset$ denotes the empty set). Clearly, τ_k is increasing as $k \to \infty$. Set $\tau_\infty = \lim_{k\to\infty} \tau_k$, whence $\tau_\infty \leq \tau_e$ a.s. If we can show that $\tau_\infty = \infty$ a.s., then $\tau_e = \infty$ and therefore $(x(t), y(t), z(t)) \in \mathbb{R}_+^3$ a.s. In other words, to complete the proof we need to show that $\tau_\infty = \infty$ a.s. If this statement is false, there is a pair of constant $T > 0$ and $\epsilon \in (0, 1)$ such that $\mathbb{P}\{\tau_\infty \leq T\} > \epsilon$. Hence there is an integer $k_1 \geq k_0$ such that $\mathbb{P}\{\tau_k \leq T\} > \epsilon$ for all $k \geq k_1$.

We write $x(t) = x, y(t) = y, z(t) = z$ and define a C^2-function $V : \mathbb{R}_+^3 \to \mathbb{R}$ by

$$V(x, y, z) = [x - 1 - \ln(x)] - \alpha[y - 1 - \ln(y)] - \varrho[z - 1 - \ln(z)],$$

where $\alpha = \frac{b_{12}}{b_{21}}$ and $\varrho = \frac{b_{13}}{b_{31}}$ are both positive constants. The nonnegativity of this function can be seen from

$$\varphi - 1 - \ln(\varphi) \geq 0, \varphi \geq 0.$$

By the generalized Itô's formula, we yield

$$
\begin{aligned}
LV(x,y,z) = &(x-1)\left[a_1 - b_{11}x - b_{12}y - b_{13}z\right] + \int_{\mathbb{Z}} [\gamma_1(u) - \ln(1+\gamma_1(u))]\lambda(du) \\
&+ \alpha(y-1)\left[-a_2 + b_{21}x - b_{22}y - \frac{c_1 z}{1+y}\right] + \alpha\int_{\mathbb{Z}} [\gamma_2(u) - \ln(1+\gamma_2(u))]\lambda(du) \\
&+ \varrho(z-1)\left[-a_3 + b_{31}x - b_{33}z + \frac{c_2 y}{1+y}\right] + \varrho\int_{\mathbb{Z}} [\gamma_3(u) - \ln(1+\gamma_3(u))]\lambda(du) \\
&+ \frac{\sigma_1^2 + \alpha\sigma_2^2 + \varrho\sigma_3^2}{2} \\
= &a_1 x - b_{11}x^2 - a_1 + b_{11}x + b_{12}y + b_{13}z + \int_{\mathbb{Z}} [\gamma_1(u) - \ln(1+\gamma_1(u))]\lambda(du) \\
&- \alpha a_2 y - \alpha b_{22}y^2 - \frac{\alpha c_1 zy}{1+y} + \alpha a_2 - \alpha b_{21}x + \alpha b_{22}y + \frac{\alpha c_1 z}{1+y} + \alpha\int_{\mathbb{Z}} [\gamma_2(u) - \ln(1+\gamma_2(u))]\lambda(du) \\
&- \varrho a_3 z - \varrho b_{33}z^2 + \frac{\varrho c_2 zy}{1+y} + \varrho a_3 - \varrho b_{31}x + \varrho b_{33}z - \frac{\varrho c_2 y}{1+y} + \varrho\int_{\mathbb{Z}} [\gamma_3(u) - \ln(1+\gamma_3(u))]\lambda(du) \\
&+ \frac{\sigma_1^2 + \alpha\sigma_2^2 + \varrho\sigma_3^2}{2} \\
\le &a_1 x - b_{11}x^2 + b_{11}x + b_{12}y + b_{13}z + \int_{\mathbb{Z}} [\gamma_1(u) - \ln(1+\gamma_1(u))]\lambda(du) \\
&- \alpha b_{22}y^2 + \alpha a_2 + \alpha b_{22}y + \alpha c_1 z + \alpha\int_{\mathbb{Z}} [\gamma_2(u) - \ln(1+\gamma_2(u))]\lambda(du) \\
&- \varrho b_{33}z^2 + \varrho c_2 z + \varrho a_3 + \varrho b_{33}z + \varrho\int_{\mathbb{Z}} [\gamma_3(u) - \ln(1+\gamma_3(u))]\lambda(du) \\
&+ \frac{\sigma_1^2 + \alpha\sigma_2^2 + \varrho\sigma_3^2}{2} \\
=: &\hat{\mathcal{G}}(x,y,z).
\end{aligned}
$$

It is easy to see that there is a constant G such that $\hat{\mathcal{G}}(x,y,z) \le G$, that is $LV(x,y,z) \le G$. We have

$$
\begin{aligned}
dV(x,y,t) \le &Gdt + (x-1)\sigma_1 dB(t) + \int_{\mathbb{Z}} [\gamma_1(u)x - \ln(1+\gamma_1(u))]\tilde{N}(dt,du) \\
&+ \alpha(y-1)\sigma_2 dB(t) + \alpha\int_{\mathbb{Z}} [\gamma_2(u)y - \ln(1+\gamma_2(u))]\tilde{N}(dt,du) \\
&+ \varrho(z-1)\sigma_3 dB(t) + \varrho\int_{\mathbb{Z}} [\gamma_3(u)z - \ln(1+\gamma_3(u))]\tilde{N}(dt,du).
\end{aligned} \tag{3.1}
$$

Integrating both sides of (3.1) from 0 to $\tau_k \wedge T = \min\{\tau_k, T\}$, and then taking expectations, yield

$$
\begin{aligned}
\mathbb{E}V(x(\tau_k \wedge T), y(\tau_k \wedge T), z(\tau_k \wedge T)) \le &V(x(0), y(0), z(0)) + G\mathbb{E}(\tau_k \wedge T) \\
\le &V(x(0), y(0), z(0)) + GT.
\end{aligned}
$$

For $k \geq k_1$, let $\Omega_k = \tau_k \leq T$. According to $\mathbb{P}(\tau_k \leq T) \geq \epsilon$, then $\mathbb{P}(\Omega_k) \geq \epsilon$, and we obtain

$$\begin{aligned} V(x(0), y(0), z(0)) + GT \geq & \mathbb{E}\left[I_{\Omega_k} V(x(\tau_k \wedge T)), y(\tau_k \wedge T), z(\tau_k \wedge T)\right] \\ \geq & \epsilon \left\{ \left[(k-1-\ln k) \wedge \left(\frac{1}{k} - 1 - \ln \frac{1}{k}\right) \right] \wedge \alpha \left[(k-1-\ln k) \wedge \left(\frac{1}{k} - 1 - \ln \frac{1}{k}\right) \right] \right. \\ & \left. \wedge \varrho \left[(k-1-\ln k) \wedge \left(\frac{1}{k} - 1 - \ln \frac{1}{k}\right) \right] \right\}, \end{aligned}$$

where I_{Ω_k} is the indicator function of Ω_k. Letting $k \to \infty$ leads to the following contradiction

$$\infty > V(x(0), y(0), z(0)) + GT = \infty.$$

Then $\tau_\infty = \infty$ a.s., which means $\tau_e = \infty$ a.s. This completes the proof.

Lemma 3.1. *([24]) For any $\beta, \varpi \in \mathbb{R}$ and any $s, r, \varepsilon > 0$, we have the variation of the Young's inequality*

$$|\beta|^s |\varpi|^r \leq |\beta|^{s+r} + \frac{r}{s+r} \left[\frac{s}{\varepsilon(s+r)}\right]^{\frac{s}{r}} |\varpi|^{s+r}.$$

Lemma 3.2. *Let $X(t) = (x(t), y(t), z(t))$ be a solution of system* (1.3). *Under Assumptions 2, for any initial value $X(0) = (x(0), y(0), z(0)) \in \mathbb{R}^3_+$, there exists $K_j(p) > 0$, $j = 1, 2, 3$ such that*

$$\begin{cases} \limsup_{t\to\infty} \mathbb{E}(x(t)) \leq K_1(p) \\ \limsup_{t\to\infty} \mathbb{E}(y(t)) \leq K_2(p) \\ \limsup_{t\to\infty} \mathbb{E}(z(t)) \leq K_3(p) \end{cases}, \ for\ any\ p \geq 1. \tag{3.2}$$

That is, the p-th moment of the positive solution to (1.3) *is upper bounded.*

Proof. By the Itô's formula, we have

$$\begin{aligned} d(x^p(t)) = & x^p(t) \left\{ p \left[a_1 - b_{11}x(t) - b_{12}y(t) - b_{13}z(t) + \frac{1}{2}(p-1)\sigma_1^2 \right] + \tilde{\gamma}_1 \right\} dt \\ & + p x^p(t) \sigma_1 dB(t) + x^p(t) \int_{\mathbb{Z}} \left[(1+\gamma_1(u))^p - 1 \right] \tilde{N}(dt, du). \end{aligned} \tag{3.3}$$

Integrating two sides of (3.3) and taking expectations leads to

$$\mathbb{E}(x^p(t)) = x^p(0) + \int_0^t \mathbb{E}\left(x^p(s) \left\{ p \left[a_1 - b_{11}x(s) - b_{12}y(s) - b_{13}z(s) + \frac{1}{2}(p-1)\sigma_1^2 \right] + \tilde{\gamma}_1 \right\} \right) ds.$$

Therefore, we have

$$\begin{aligned} \frac{d\mathbb{E}(x^p(t))}{dt} = & \mathbb{E}\left(x^p(t) \left\{ p \left[a_1 - b_{11}x(t) - b_{12}y(t) - b_{13}z(t) + \frac{1}{2}(p-1)\sigma_1^2 \right] + \tilde{\gamma}_1 \right\} \right) \\ \leq & p a_1 \mathbb{E}(x^p(t)) - p b_{11} \mathbb{E}\left(x^{p+1}(t) \right) + \frac{1}{2}(p-1)\sigma_1^2 \mathbb{E}(x^p(t)) + \tilde{\gamma}_1 \mathbb{E}(x^p(t)) \\ \leq & p \mathbb{E}(x^p(t)) \left\{ \left[a_1 + \frac{1}{2}(p-1)\sigma_1^2 + \frac{\tilde{\gamma}_1}{p} \right] - b_{11} \mathbb{E}[(x^p(t))]^{\frac{1}{p}} \right\}. \end{aligned}$$

Let $\hat{X}(t) = \mathbb{E}(x^p(t))$, then

$$\frac{d\hat{X}(t)}{dt} \leq p\hat{X}(t)\left[a_1 + \frac{1}{2}(p-1)\sigma_1^2 + \frac{\tilde{\gamma}_1}{p} - b_{11}\hat{X}^{1/p}(t)\right].$$

Let $x(0) < \frac{a_1+\frac{1}{2}(p-1)\sigma_1^2+\frac{\tilde{\gamma}_1}{p}}{b_{11}}$, $p \geq 1$, then

$$0 < b_{11}\hat{X}^{1/p}(0) = b_{11}x(0) < a_1 + \frac{1}{2}(p-1)\sigma_1^2 + \frac{\tilde{\gamma}_1}{p}.$$

By the standard comparison argument, we have

$$[\mathbb{E}(x^p(t))]^{1/p} = \hat{X}^{1/p}(t) \leq \frac{a_1 + \frac{1}{2}(p-1)\sigma_1^2 + \frac{\tilde{\gamma}_1}{p}}{b_{11}}.$$

Thus,

$$\limsup_{t\to\infty} \mathbb{E}(x^p(t)) \leq \left(\frac{a_1 + \frac{1}{2}(p-1)\sigma_1^2 + \frac{\tilde{\gamma}_1}{p}}{b_{11}}\right)^p := K_1(p).$$

Next, we prove the boundedness of $y(t)$. Making use of the Itô's formula to $e^t y^p$, we have

$$\begin{aligned} d(e^t y^p(t)) =& e^t y^p(t)\left\{p\left[\frac{1}{p} - a_2 + b_{21}x(t) - b_{22}y(t) - \frac{c_1 z(t)}{1+y(t)} + \frac{1}{2}(p-1)\sigma_2^2\right] + \tilde{\gamma}_2\right\}dt \\ &+ pe^t y^p(t)\sigma_2 dB(t) + e^t y^p(t)\int_{\mathbb{Z}}[(1+\gamma_2(u))^p - 1]\tilde{N}(dt,du). \end{aligned}$$

Integrating the both sides from 0 to t and taking expectations yields

$$\begin{aligned} \mathbb{E}(e^t y^p(t)) &= y^p(0) + \int_0^t \mathbb{E}(e^s y^p(s))\left\{p\left[\frac{1}{p} - a_2 + b_{21}x(s) - b_{22}y(s) - \frac{c_1 z(s)}{1+y(s)} + \frac{1}{2}(p-1)\sigma_2^2\right] + \tilde{\gamma}_2\right\}ds \\ &\leq y^p(0) + p\mathbb{E}\int_0^t e^s y^p(s)\left(\left[\frac{1+\tilde{\gamma}_2}{p} + \frac{1}{2}(p-1)\sigma_2^2\right] + b_{21}x(s) - b_{22}y(s)\right)ds \\ &\leq y^p(0) + p\mathbb{E}\int_0^t e^s y^p(s)\left(\left[\frac{1+\tilde{\gamma}_2}{p} + \frac{1}{2}(p-1)\sigma_2^2\right] - b_{22}y(s)\right)ds + p\mathbb{E}\int_0^t b_{21}e^s x(s)y^p(s)ds. \end{aligned}$$

Using Lemma 3.1, we have

$$\begin{aligned} \mathbb{E}(e^t y(t)^p) \leq& y^p(0) + p\mathbb{E}\int_0^t e^s y^p(s)\left(\left[\frac{1+\tilde{\gamma}_2}{p} + \frac{1}{2}(p-1)\sigma_2^2\right] - (b_{22}-b_{21})y(s)\right)ds \\ &+ \left[\frac{p}{1+p}\right]^{1+p}\mathbb{E}\int_0^t b_{21}e^s x^{p+1}(s)ds. \end{aligned}$$

Let $\Gamma(y) = y^p\left(\left[\frac{1+\tilde{\gamma}_2}{p} + \frac{1}{2}(p-1)\sigma_2^2\right] - (b_{22}-b_{21})y\right)$. To attain the maximum value of $\Gamma(y)$, we obtain $y = \frac{p\left[\frac{1+\tilde{\gamma}_2}{p} + \frac{1}{2}(p-1)\sigma_2^2\right]}{(1+p)(b_{22}-b_{21})} > 0$. So the maximum value of $\Gamma(y)$ is given by

$$\Gamma_{\max} = \left(\frac{p}{(b_{22}-b_{21})}\right)^p\left(\frac{\left[\frac{1+\tilde{\gamma}_2}{p} + \frac{1}{2}(p-1)\sigma_2^2\right]}{1+p}\right)^{1+p}.$$

Hence, we obtain that

$$\begin{aligned}\mathbb{E}\left(e^t y^p(t)\right) \leq & y^p(0) + p\mathbb{E}\int_0^t e^s \Gamma_{\max} ds + \left[\frac{p}{1+p}\right]^{1+p} \mathbb{E}\int_0^t e^s b_{21} x^{p+1}(s) ds. \\ \leq & y^p(0) + p\left(\frac{p}{(b_{22}-b_{21})}\right)^p \left(\frac{\left[\frac{1+\tilde{\gamma}_2}{p} + \frac{1}{2}(p-1)\sigma_2^2\right]}{1+p}\right)^{1+p} (e^t - 1) \\ & + \left[\frac{p}{1+p}\right]^{1+p} (b_{21} K_1(p+1))(e^t - 1).\end{aligned}$$

One can observe that for $t = 0$, $\mathbb{E}(y^p(t)) \leq y^p(0)$ and when $t \to \infty$,

$$\limsup_{t\to\infty} \mathbb{E}\left(y^p(t)\right) \leq \left[\frac{p}{1+p}\right]^{p+1} \left(\frac{\left[\frac{1+\tilde{\gamma}_2}{p} + \frac{1}{2}(p-1)\sigma_2^2\right]^{p+1}}{(b_{22}-b_{21})^p} + b_{21}\left(K_1(p+1)\right)\right) := K_2(p).$$

Finally, we use the same method to prove the boundedness of $z(t)$. Obviously,

$$\begin{aligned}d\left(e^t z^p(t)\right) = & e^t z^p(t)\left\{p\left[\frac{1}{p} - a_3 + b_{31}x(t) - b_{33}z(t) - \frac{c_2 y(t)}{1+y(t)} + \frac{1}{2}(p-1)\sigma_3^2\right] + \tilde{\gamma}_3\right\} dt \\ & + p e^t z^p(t)\sigma_3 dB(t) + e^t z^p(t)\int_{\mathbb{Z}} \left[(1+\gamma_3(u))^p - 1\right] \tilde{N}(dt, du).\end{aligned} \tag{3.4}$$

Integrating two sides of (3.4) and taking expectations leads to

$$\begin{aligned}\mathbb{E}\left(e^t z^p(t)\right) = & z^p(0) + \int_0^t \mathbb{E}\left(e^s z^p(s)\right)\left\{p\left[\frac{1}{p} - a_3 + b_{31}x(s) - b_{33}z(s) + \frac{c_2 y(s)}{1+y(s)} + \frac{1}{2}(p-1)\sigma_3^2\right] + \tilde{\gamma}_3\right\} ds \\ \leq & z^p(0) + p\mathbb{E}\int_0^t \left(e^s z^p(s)\right)\left(\left[\frac{1+\tilde{\gamma}_3}{p} + c_2 + \frac{1}{2}(p-1)\sigma_3^2\right] + b_{31}x(s) - b_{33}z(s)\right) ds \\ \leq & z^p(0) + p\mathbb{E}\int_0^t \left(e^s z^p(s)\right)\left\{\left[\frac{1+\tilde{\gamma}_3}{p} + c_2 + \frac{1}{2}(p-1)\sigma_3^2\right] - b_{33}z(s)\right\} ds \\ & + b_{31}\mathbb{E}\int_0^t e^s x(s) z^p(s) ds.\end{aligned}$$

As a result,

$$\limsup_{t\to\infty} \mathbb{E}\left(z^p(t)\right) \leq \left[\frac{p}{p+1}\right]^{p+1} \left(\frac{\left[\frac{1+\tilde{\gamma}_3}{p} + c_2 + \frac{1}{2}(p-1)\sigma_3^2\right]^{p+1}}{(b_{33}-b_{31})^p} + b_{31}K_1(p+1)\right) := K_3(p).$$

Under Assumption 2, it is clear that $K_j(p) > 0 (j = 1, 2, 3)$. Therefore, the p-th moment of the positive solution to (1.3) is upper bounded. The proof is completed.

By Lemma 3.2, together with the Chebyshev inequality, we can obtain the following result.

Corollary 3.1. *Under Assumption 2, the solution of* (1.3) *is stochastically ultimate bounded.*

4. Asymptotical property

In this section, we investigate the asymptotical property of (1.3).

Theorem 4.1. *Under Assumption 1, the solution* $X(t) = (x(t), y(t), z(t))$ *of system* (1.3) *with any positive initial value has the property that*

$$\begin{aligned} \limsup_{t\to\infty} \frac{\ln x(t)}{t} &\le r_1, \ a.s, \\ \limsup_{t\to\infty} \frac{\ln y(t)}{t} &\le -r_2, \ a.s, \\ \limsup_{t\to\infty} \frac{\ln z(t)}{t} &\le -r_3, \ a.s. \end{aligned} \tag{4.1}$$

Particularly, if $r_1 < 0$*, then* $X(t) = (x(t), y(t), z(t))$ *will go to extinction.*

Proof. By Theorem 3.1, the solution $X(t) = (x(t), y(t), z(t))$ with initial value $X(0) = (x(0), y(0), z(0)) \in \mathbb{R}^3_+$ remains in $\mathbb{R}_+$ with probability one. By the generalized Itô's formula, we derive from (1.3) that

$$\frac{\ln x(t)}{t} = r_1 - b_{11}\langle x(t)\rangle - b_{12}\langle y(t)\rangle - b_{13}\langle z(t)\rangle + \frac{\ln x(0)}{t} + t^{-1}\sum_{i=1}^{2} N_{1i}(t), \tag{4.2}$$

$$\frac{\ln y(t)}{t} = -r_2 + b_{21}\langle x(t)\rangle - b_{22}\langle y(t)\rangle - c_1\left\langle \frac{z(t)}{1+y(t)}\right\rangle + \frac{\ln y(0)}{t} + t^{-1}\sum_{i=1}^{2} N_{2i}(t), \tag{4.3}$$

$$\frac{\ln z(t)}{t} = -r_3 + b_{31}\langle x(t)\rangle - b_{33}\langle z(t)\rangle - c_2\left\langle \frac{y(t)}{1+y(t)}\right\rangle + \frac{\ln z(0)}{t} + t^{-1}\sum_{i=1}^{2} N_{3i}(t), \tag{4.4}$$

where, for $j = 1, 2, 3$,

$$\begin{cases} N_{j1}(t) &= \int_0^t \sigma_j B(s), \\ N_{j2}(t) &= \int_0^t \int_{\mathbb{Z}} \ln\left[1 + \gamma_j(u)\right] \tilde{N}(ds, du), \end{cases}$$

are local martingale with the quadratic variations $\left\langle N_{ji}(t)\right\rangle := \left\langle N_{ji}(t), N_{ji}(t)\right\rangle, i = 1, 2$ [11], that is

$$\begin{cases} \left\langle N_{j1}(t)\right\rangle &= \int_0^t \sigma_j^2 ds \le \max(\sigma_j^2)t, \\ \left\langle N_{j2}(t)\right\rangle &= \int_0^t \int_{\mathbb{Z}} \ln\left[1 + \gamma_j(u)\right]^2 \lambda(du)\, ds \\ &\le \max\left\{\left[\ln\left(1 + \gamma_j(u)\right)\right]^2\right\} \lambda(\mathbb{Z})\, t. \end{cases}$$

By Lemma 3.1 in [21] and the strong law of large numbers, we obtain

$$\lim_{t\to+\infty} t^{-1} N_{ij}(t) = 0, \ a.s., \ i = 1, 2, \ j = 1, 2, 3. \tag{4.5}$$

From (4.2) and (4.5), we have $\limsup_{t\to\infty} \frac{\ln x(t)}{t} \le r_1$ *a.s.* If $r_1 < 0$, then

$$\lim_{t\to\infty} x(t) = 0, \ a.s.$$

Applying L'Hospital's rule, it follows that

$$\lim_{t\to\infty} \langle x(t)\rangle = 0, \ a.s. \tag{4.6}$$

Further, by combining (4.3), (4.5) and (4.6), we can get $\limsup_{t\to\infty} \frac{\ln y(t)}{t} \le -r_2 + b_{21} \lim_{t\to\infty} \langle x(t)\rangle = -r_2 < 0$ $a.s.$ So

$$\lim_{t\to\infty} y(t) = 0, \ a.s.$$

Similarly, we have $\lim_{t\to\infty} \langle y(t)\rangle = 0$ $a.s.$ Noting $\left\langle \frac{y(t)}{1+y(t)} \right\rangle \le \langle y(t)\rangle$, and from the positivity of $y(t)$, we have

$$\lim_{t\to\infty} \left\langle \frac{y(t)}{1+y(t)} \right\rangle = 0, \ a.s. \tag{4.7}$$

Combining (4.4), (4.5) and (4.7), we get $\limsup_{t\to\infty} \frac{\ln z(t)}{t} \le -r_3 + b_{31} \lim_{t\to\infty} \langle x(t)\rangle + c_2 \lim_{t\to\infty} \left\langle \frac{y(t)}{1+y(t)} \right\rangle = -r_3 < 0$ $a.s.$ Thus,

$$\lim_{t\to\infty} z(t) = 0, \ a.s.$$

The proof is therefore completed.

Theorem 4.2. *If $A_j > 0, D_j > 0$ $(j = 1, 2, 3)$, then for any given initial value $X(0) = (x(0), y(0), z(0)) \in \mathbb{R}^3_+$, the solution $X(t) = (x(t), y(t), z(t))$ to system* (1.3) *has the following property*

$$\begin{aligned} \frac{A_1}{b_{11}} &\le \liminf_{t\to\infty} \langle x(t)\rangle \le \limsup_{t\to\infty} \langle x(t)\rangle \le D_1, \\ \frac{A_2}{b_{22}} &\le \liminf_{t\to\infty} \langle y(t)\rangle \le \limsup_{t\to\infty} \langle y(t)\rangle \le D_2, \\ \frac{A_3}{b_{33}} &\le \liminf_{t\to\infty} \langle z(t)\rangle \le \limsup_{t\to\infty} \langle z(t)\rangle \le D_3. \end{aligned} \tag{4.8}$$

That is, all the populations in system (1.3) *are persistent in mean.*

Proof. It follows from (4.2) and (4.5) that

$$\begin{aligned} \ln(x(t)) &= \ln(x(0)) + r_1 t - b_{11} \int_0^t x(s)ds - b_{12} \int_0^t y(s)ds - b_{13} \int_0^t z(s)ds + \sum_{i=1}^{2} N_{1i}(t) \\ &\le \ln(x(0)) + r_1 t - b_{11} \int_0^t x(s)ds + \sum_{i=1}^{2} N_{1i}(t). \end{aligned}$$

From Lemma 2 in [25], we obtain

$$\limsup_{t\to\infty} \langle x(t)\rangle \le \frac{r_1}{b_{11}} =: D_1. \tag{4.9}$$

From (4.3)–(4.5) and (4.9), we get

$$\begin{aligned}\ln(y(t)) &= \ln(y(0)) - r_2 t + b_{21}\int_0^t x(s)ds - b_{22}\int_0^t y(s)ds - c_1\int_0^t \frac{z(s)}{1+y(t)}ds + \sum_{i=1}^{2} N_{2i}(t)\\ &\le \ln(y(0)) + (b_{21}D_1 - r_2)t - b_{22}\int_0^t y(s)ds + \sum_{i=1}^{2} N_{2i}(t),\\ \ln(z(t)) &= \ln(z(0)) - r_3 t + b_{31}\int_0^t x(s)ds - b_{33}\int_0^t z(s)ds + c_2\int_0^t \frac{y(s)}{1+y(t)}ds + \sum_{i=1}^{2} N_{2i}(t)\\ &\le \ln(z(0)) + (c_2 + b_{31}D_1 - r_3)t - b_{33}\int_0^t z(s)ds + \sum_{i=1}^{2} N_{3i}(t).\end{aligned}$$

Thus, from Lemma 2 in [25], we have

$$\limsup_{t\to\infty}\langle y(t)\rangle \le \frac{b_{21}D_1 - r_2}{b_{22}} =: D_2, \tag{4.10}$$

$$\limsup_{t\to\infty}\langle z(t)\rangle \le \frac{c_2 + b_{31}D_1 - r_3}{b_{33}} =: D_3. \tag{4.11}$$

From Theorem 4.1, we have $\limsup_{t\to\infty}\frac{\ln x(t)}{t} = 0$. By combining (4.2), (4.5), (4.10) and (4.11), then

$$\begin{aligned}b_{11}\liminf_{t\to\infty}\langle x(t)\rangle &= \liminf_{t\to\infty}\left\{r_1 - b_{12}\langle y(t)\rangle - b_{13}\langle z(t)\rangle - \frac{\ln(x(0))}{t} - \frac{\ln(x(t))}{t} + \sum_{i=1}^{2} N_{1i}(t)\right\}\\ &\ge r_1 - b_{12}\limsup_{t\to\infty}\langle y(t)\rangle - b_{13}\limsup_{t\to\infty}\langle z(t)\rangle - \frac{\ln(x(0))}{t} - \limsup_{t\to\infty}\frac{\ln(x(t))}{t} + \sum_{i=1}^{2} N_{1i}(t)\\ &\ge r_1 - b_{12}\limsup_{t\to\infty}\langle y(t)\rangle - b_{13}\limsup_{t\to\infty}\langle z(t)\rangle\\ &\ge r_1 - b_{12}D_2 - b_{13}D_3\\ &=: A_1.\end{aligned}$$

Similarly, by Theorem 4.1, noting $\lim_{t\to\infty}\left\langle\frac{y(t)}{1+y(t)}\right\rangle = 0$ *a.s.*, we have

$$\begin{aligned}b_{22}\liminf_{t\to\infty}\langle y(t)\rangle &= \liminf_{t\to\infty}\left\{-r_2 + b_{21}\langle x(t)\rangle - c_1\left\langle\frac{z(t)}{1+y(t)}\right\rangle + \frac{\ln y(0)}{t} - \frac{\ln y(t)}{t} + \sum_{i=1}^{2} N_{2i}\right\}\\ &\ge -r_2 + b_{21}\liminf_{t\to\infty}\langle x(t)\rangle - c_1\limsup_{t\to\infty}\langle z(t)\rangle - \limsup_{t\to\infty}\frac{\ln y(t)}{t} + \sum_{i=1}^{2} N_{2i}\\ &\ge -r_2 + b_{21}\liminf_{t\to\infty}\langle x(t)\rangle - c_1\limsup_{t\to\infty}\langle z(t)\rangle\\ &\ge -r_2 + \frac{b_{21}A_1}{b_{11}} - c_1 D_3\\ &=: A_2.\end{aligned}$$

and

$$\begin{aligned}
b_{33}\liminf_{t\to\infty}\langle z(t)\rangle &= \liminf_{t\to\infty}\left\{-r_3+b_{31}\langle x(t)\rangle+c_2\left\langle\frac{y(t)}{1+y(t)}\right\rangle+\frac{\ln z(0)}{t}-\frac{\ln z(t)}{t}+\sum_{i=1}^{2}N_{3i}\right\}\\
&\geq -r_3+b_{31}\liminf_{t\to\infty}\langle x(t)\rangle+c_2\liminf_{t\to\infty}\left\langle\frac{y(t)}{1+y(t)}\right\rangle-\limsup_{t\to\infty}\frac{\ln z(t)}{t}\\
&\geq -r_3+b_{31}\frac{A_1}{b_{11}}\\
&=: A_3.
\end{aligned}$$

Therefore, under above conditions, all the populations of (1.3) are persistent in mean. This completes the proof.

Remark 4.1. *Compared with the literature [26], the influence of Lévy jumps to system* (1.3) *is considered in this paper, while it is ignored in [26]. For the case without jump-diffusion coefficient, our main results are consistent with those in [26]. Therefore, the results of literature [26] are generalized in this paper.*

5. Asymptotic stability in distribution

In [27], for all parameter values, deterministic system (1.1) has a trivial equilibrium point $E_0 = (0,0,0)$ and an axial equilibrium point $E_1 = (\frac{a_1}{b_{11}},0,0)$. In addition, when $b_{31}a_1 - b_{11}a_3 > 0$ and $b_{21}a_1 - b_{11}a_2 > 0$, respectively, deterministic system (1.1) has two boundary equilibria $E_2 = (\frac{b_{33}a_1+b_{13}a_3}{b_{11}b_{33}+b_{13}b_{31}}, 0, \frac{b_{31}a_1-b_{11}a_3}{b_{11}b_{33}+b_{13}b_{31}})$ and $E_3 = (\frac{b_{22}a_1+b_{12}a_2}{b_{11}b_{22}+b_{12}b_{21}}, \frac{b_{21}a_1-b_{11}a_2}{b_{11}b_{22}+b_{12}b_{21}}, 0)$. However, in the real world, population systems are often affected by environmental noise. So, when considering the introduction of Lévy jumps into population models, we are also interested in knowing how jumps affect the long-term dynamic behavior of species. That is what happens to the statistical characteristics of the species' long-term dynamic behavior. Thus, invariant distribution plays an important role in many actual ecosystems. In this section, our work aims to find sufficient conditions to obtain that the probability density function is asymptotically stable for the system (1.3) perturbed by Lévy noise. First we give the following definition and lemmas.

Definition 5.1. *Let $X_1(t) = (x_1(t), y_1(t), z_1(t))$ be a positive solution of* (1.3) *with initial value $X_1(0) = (x_1(0), y_1(0), z_1(0)) \in \mathbb{R}^3_+$. $X_1(t)$ is said to be globally asymptotically stable in expectation if for any other solution $X_2(t) = (x_2(t), y_2(t), z_2(t))$ of* (1.3)*, we have*

$$\mathbb{P}\left\{\lim_{t\to+\infty}\mathbb{E}\left(|X_1(t)| - X_2(t)|\right) = 0\right\} = 1,$$

where $\mathbb{E}$ denotes the expectation of some stochastic variable.

Lemma 5.1. *([28, 29]) Let $X(t)$ be an n-dimensional stochastic process on $t \geq 0$. Suppose that there exist positive constant α, β, ξ such that*

$$\mathbb{E}|X(t) - X(s)|^{\alpha} \leq \xi|t-s|^{1+\beta},\ 0 \leq s, t < \infty.$$

Then there exists continuous modification $\tilde{X}(t)$ of $X(t)$, and almost every sample path of $\tilde{X}(t)$ is local but uniformly Höder continuous with exponent $\kappa < \frac{\alpha}{\beta}$. In other words, the continuous modification $\tilde{X}(t)$ of $X(t)$ has the property that for every $\kappa \in (0, \frac{\alpha}{\beta})$,

$$\mathbb{P}\left\{\zeta : \sup_{0<|t-s|<f(\zeta),0\leq s,t<\infty} \frac{|\tilde{X}(t,\zeta) - \tilde{X}(s,\zeta)}{|t-s|^{\kappa}} \leq \frac{2}{1-2^{-\kappa}}\right\} = 1.$$

Lemma 5.2. *Assume that Assumptions 1 and 2 hold. Let $X(t) = (x(t), y(t), z(t))$ be a solution of (1.3) on $t \geq 0$ with initial data $X(0) = (x(0), y(0), z(0)) \in \mathbb{R}^3_+$, then almost every sample path of $X(t)$ is uniformly continuous on $t \geq 0$.*

Proof. We rewrite $x(t) - x(0)$ as the following integral form:

$$\begin{aligned} x(t) - x(0) = &\int_0^t x(t)[a_1 - b_{11}x(t) - b_{13}y(t) - b_{13}z(t)]ds + \int_0^t x(t)\sigma_1 dB(s) \\ &+ \int_0^t \int_{\mathbb{Z}} x(t)\gamma_1(u)\tilde{N}(ds, du). \end{aligned}$$

Let $f_1 = x(t)[a_1 - b_{11}x(t) - b_{13}y(t) - b_{13}z(t)], g_1 = x(t)\sigma_1, h_1 = x(t)\gamma_1(u)$. By Lemma 3.2, there is a positive constant $K_1(p)$ such that $\mathbb{E}(x^p(t)) \leq K_1(p)$ on $t \geq 0$. Then we can derive that

$$\begin{aligned} \mathbb{E}(|f_1|^p) =&\mathbb{E}(x^p(s)|a_1 - b_{11}x(s) - b_{12}y(s) - b_{13}z(s)|^p) \\ \leq&\frac{1}{2}\mathbb{E}\left(x^{2p}(s)\right) + \frac{1}{2}\mathbb{E}\left[(a_1 - b_{11}x(s) - b_{12}y(s) - b_{13}z(s))^{2p}\right] \\ \leq&\frac{1}{2}\mathbb{E}\left(x^{2p}(s)\right) + \frac{1}{2}\mathbb{E}\left[(a_1 - b_{11}x(s))^{2p}\right] \\ \leq&\frac{1}{2}\mathbb{E}\left(x^{2p}(s)\right) + \frac{1}{2}(n+1)^{2p-1}\left[a_1^{2p} + b_{11}^{2p}\mathbb{E}\left(x^{2p}(s)\right)\right] \\ =:&Q_1(p) \end{aligned} \tag{5.1}$$

and

$$\mathbb{E}(|g_1|^p) = \mathbb{E}\left(x^p(t)\sigma_1^p\right) = \sigma_1^p\mathbb{E}(x^p(t)) \leq \sigma_1^p K_1(p) =: Q_2(p). \tag{5.2}$$

We assume $p > 2$. For $0 \leq s < t < \infty$, using the moment inequality (cf. Friedman [30]) on (5.1) leads to

$$\mathbb{E}|\int_s^t g_1 dB(v)|^p \leq \left[\frac{p(p-1)}{2}\right]^{p/2}(t-s)^{(p-2)/2}\int_s^t \mathbb{E}|g_1|^p dB(v). \tag{5.3}$$

Under Assumption 2, with Kunita's first inequality (see Theorem 4.4.23, [31]), we have

$$\begin{aligned} \mathbb{E}\left[|\int_s^t \int_{\mathbb{Z}} h_1\tilde{N}(dv, du)|^p\right] \leq&2^{p-1}\left\{\mathbb{E}\left[\int_s^t \int_{\mathbb{Z}} |x(s)\gamma_1(u)|^2\lambda(du)dv\right]^{\frac{p}{2}} + \mathbb{E}\left[\int_s^t \int_{\mathbb{Z}} |x(s)\gamma_1(u)|^p\lambda(du)dv\right]\right\} \\ \leq&2^{p-1}\left\{(t-s)^{\frac{p}{2}}C_1^{\frac{p}{2}}K_1(p) + (t-s)C_1^{\frac{p}{2}}K_1(p)\right\}. \end{aligned} \tag{5.4}$$

Let $0 < s < T < \infty, t - s \leq 1, 1/p + 1/q = 1$, then from (5.1)–(5.4), we obtain

$$\begin{aligned}
\mathbb{E}|x(t) - x(s)|^p \leq& 2^{p-1}\mathbb{E}\left(\int_s^t |f_1|dv\right)^p + 2^{p-1}\mathbb{E}\left(\int_0^t |g_1|dB(v)\right)^p + 2^{p-1}\mathbb{E}\left(|\int_s^t \int_{\mathbb{Z}} h_1\tilde{N}(dv, du)|^p\right) \\
\leq& 2^{p-1}\left(\int_s^t 1^q dv\right)^{\frac{p}{q}} \mathbb{E}\left(\int_s^t |f_1|^p dv\right) + 2^{p-1}\left[\frac{p(p-1)}{2}\right]^{p/2} (t-s)^{(p-2)/2} \int_s^t \mathbb{E}|g_1|^p dB(v) \\
&+ 2^{p-1}\left\{2^{p-1}(t-s)^{\frac{p}{2}} C_1^{\frac{p}{2}} K_1(p) + 2^{p-1}(t-s) C_1^{\frac{p}{2}} K_1(p)\right\} \\
=& 2^{p-1}(t-s)^{(p-1)+1} Q_1(p) + 2^{p-1}\left[\frac{p(p-1)}{2}\right]^{p/2} (t-s)^{\frac{(p-2)}{2}+1} Q_2(p) \\
&+ 2^{p-1}\left\{2^{p-1}(t-s)^{\frac{p}{2}} C_1^{\frac{p}{2}} K_1(p) + 2^{p-1}(t-s) C_1^{\frac{p}{2}} K_1(p)\right\} \\
\leq& 2^{p-1}(t-s)^{\frac{p}{2}}\left\{(t-s)^{\frac{p}{2}} Q_1(p) + \left[\frac{p(p-1)}{2}\right]^{p/2} Q_2(p) + 2^{p-1}C_1^{\frac{p}{2}} K_1(p)\right. \\
&\left. + 2^{p-1}(t-s)^{\frac{p}{2}} C_1^{\frac{p}{2}} K_1(p)\right\} \\
\leq& 2^{p-1}(t-s)^{\frac{p}{2}} Q(p),
\end{aligned}$$

where $Q(p) = (t-s)^{\frac{p}{2}} Q_1(p) + \left[\frac{p(p-1)}{2}\right]^{p/2} Q_2(p) + 2^{p-1}C_1^{\frac{p}{2}} K_1(p) + 2^{p-1}(t-s)^{\frac{p}{2}} C_1^{\frac{p}{2}} K_1(p) < \infty$. We see from Lemma 5.1 that almost every sample path of $x(t)$ is locally but uniformly *Höder* continuous with exponent κ for every $\kappa \in (0, \frac{p-2}{2p})$. Therefore almost every sample path of $x(t)$ is uniformly continuous on $t \geq 0$. Similarly, we have

$$\begin{aligned}
y(t) - y(0) =& \int_0^t y(t)\left[-a_2 + b_{21}x(t) - b_{22}y(t) - \frac{c_1 z(t)}{1 + y(t)}\right]ds + \int_0^t y(t)\sigma_2 dB(s) \\
&+ \int_0^t \int_{\mathbb{Z}} y(t)\gamma_2(u)\tilde{N}(ds, du),
\end{aligned}$$

and

$$\begin{aligned}
z(t) - z(0) =& \int_0^t z(t)\left[-a_3 + b_{31}x(t) - b_{33}z(t) + \frac{c_2 y(t)}{1 + y(t)}\right]ds + \int_0^t z(t)\sigma_3 dB(s) \\
&+ \int_0^t \int_{\mathbb{Z}} z(t)\gamma_3(u)\tilde{N}(ds, du).
\end{aligned}$$

Let

$$f_2 = y(t)\left[-a_2 + b_{21}x(t) - b_{22}y(t) - \frac{c_1 z(t)}{1 + y(t)}\right], \quad f_3 = z(t)\left[-a_3 + b_{31}x(t) - b_{33}z(t) + \frac{c_2 y(t)}{1 + y(t)}\right],$$

$$g_2 = y(t)\sigma_2, \quad g_3 = z(t)\sigma_3, \quad h_2 = y(t)\gamma_2(u), \quad h_3 = z(t)\gamma_3(u),$$

then,

$$
\begin{aligned}
\mathbb{E}\left(|f_2|^p\right) =& \mathbb{E}\left(y^p(s)\,|-a_2+b_{21}x(s)-b_{22}y(s)-\frac{c_1z(s)}{1+y(s)}|^p\right)\\
\le& \frac{1}{2}\mathbb{E}\left(y^{2p}(s)\right)+\frac{1}{2}\mathbb{E}\left(\left(-a_2+b_{21}x(s)-b_{22}y(s)-\frac{c_1z(s)}{1+y(s)}\right)^{2p}\right)\\
\le& \frac{1}{2}\mathbb{E}\left(y^{2p}(s)\right)+\frac{1}{2}(n+1)^{2p-1}\,\mathbb{E}\left[(-a_2-b_{22}y(s))^{2p}+\left(b_{21}x(s)-\frac{c_1z(s)}{1+y(s)}\right)^{2p}\right]\\
\le& \frac{1}{2}\mathbb{E}\left(y^{2p}(s)\right)+\frac{1}{2}(n+1)^{4p-2}\left[a_2^{2p}+b_{22}^{2p}\mathbb{E}\left(y^{2p}(s)\right)\right]\\
&+\frac{1}{2}(n+1)^{4p-2}\left[b_{21}^{2p}\mathbb{E}\left(x^{2p}(s)\right)+c_1^{2p}\mathbb{E}\left(z^{2p}(s)\right)\right]\\
\le& \frac{1}{2}K_2(2p)+\frac{1}{2}(n+1)^{4p-2}\left[a_2^{2p}+b_{22}^{2p}K_2(2p)\right]+\frac{1}{2}(n+1)^{4p-2}\left[b_{21}^{2p}K_1(2p)+c_1^{2p}K_3(2p)\right]\\
=:& W_1(p)
\end{aligned}
$$

and

$$
\begin{aligned}
\mathbb{E}\left(|f_3|^p\right) =& \mathbb{E}\left(z^p(s)\,|-a_3+b_{31}x(s)-b_{33}z(s)+\frac{c_2y(s)}{1+y(s)}|^p\right)\\
\le& \frac{1}{2}\mathbb{E}\left(z^{2p}(s)\right)+\frac{1}{2}(n+1)^{4p-2}\left(a_3^{2p}+b_{33}^{2p}\mathbb{E}\left(z^{2p}(s)\right)\right)\\
&+\frac{1}{2}(n+1)^{4p-2}\left(c_2^{2p}+b_{31}^{2p}\mathbb{E}\left(x^{2p}(s)\right)\right)\\
\le& \frac{1}{2}K_3(2p)+\frac{1}{2}(n+1)^{4p-2}\left[a_3^{2p}+c_2^{2p}+b_{33}^{2p}K_3(2p)+b_{31}^{2p}K_1(2p)\right]\\
=:& F_1(p).
\end{aligned}
$$

A similar discussion to $\mathbb{E}|y(t)-y(s)|^p$ and $\mathbb{E}|z(t)-z(s)|^p$, we can conclude that almost every sample path of $y(t)$ and $z(t)$ is uniformly continuous on $t \ge 0$. This completes the proof.

Lemma 5.3. *Let $f(t)$ be a nonnegative function defined on $[0,\infty)$ such that $f(t)$ is integrable on $[0,\infty)$ and is uniformly continuous on $[0,\infty)$, then $\lim_{t\to+\infty} f(t) = 0$.*

For later proof, we give the following technical assumption.

Assumption 3

$$
\begin{cases}
b_{11}+b_{21}-b_{31}>0,\\
-b_{12}+b_{22}+c_1M_2+c_2<0,\\
-b_{13}-b_{33}+c_1M_1<0,
\end{cases}
$$

where

$$
M_1 =: \left[\frac{3}{4}\right]^{4/3}\left(\frac{\left[\frac{1+\tilde{\gamma}_2}{3}+\sigma_2^2\right]^4}{(b_{22}-b_{21})^3}+b_{21}\left(\frac{a_1+\frac{3\sigma_1^2}{2}+\frac{\tilde{\gamma}_1}{4}}{b_{11}}\right)^4\right)^{1/3},
$$

$$
M_2 =: \left[\frac{3}{4}\right]^{4/3}\left(\frac{\left[\frac{1+\tilde{\gamma}_3}{3}+c_2+\sigma_3^2\right]^4}{(b_{33}-b_{31})^3}+b_{31}\left(\frac{a_1+\frac{3\sigma_1^2}{2}+\frac{\tilde{\gamma}_1}{4}}{b_{11}}\right)^4\right)^{1/3}.
$$

Lemma 5.4. *If Assumption 2 and 3 hold, then system* (1.3) *is globally asymptotically stable in expectation.*

Proof. Let $X_1(t) = (x_1(t), y_1(t), z_1(t))$ and $X_2(t) = (x_2(t), y_2(t), z_2(t))$ be any two solutions of system (1.3) with positive initial data. Consider a Lyapunov function $V(t)$ defined by

$$V(t) = |\ln x_1(t) - \ln x_2(t)| + |\ln y_1(t) - \ln y_2(t)| + |\ln z_1(t) - \ln z_2(t)|, t \geq 0. \tag{5.5}$$

Making use of the Itô's formula with jumps, one can deduce that

$$\begin{aligned}
&d^+V(t)\\
=&\operatorname{sgn}(x_1(t) - x_2(t))\{-b_{11}(x_1(t) - x_2(t)) - b_{12}(y_1(t) - y_2(t)) - b_{13}(z_1(t) - z_2(t))\}dt\\
&+\operatorname{sgn}(y_1(t) - y_2(t))\left\{-b_{12}(x_1(t) - x_2(t)) + b_{22}(y_1(t) - y_2(t)) - c_1\left(\frac{z_1(t)}{1+y_1(t)} - \frac{z_2(t)}{1+y_2(t)}\right)\right\}dt\\
&+\operatorname{sgn}(z_1(t) - z_2(t))\left\{b_{31}(x_1(t) - x_2(t)) - b_{33}(z_1(t) - z_2(t)) + c_2\left(\frac{y_1(t)}{1+y_1(t)} - \frac{y_2(t)}{1+y_2(t)}\right)\right\}dt.
\end{aligned}$$

Integrating from 0 to t and taking expectations yields

$$\begin{aligned}
&\mathbb{E}(V(t)) - \mathbb{E}(V(0))\\
=&\mathbb{E}\int_0^t \{\operatorname{sgn}(x_1(t) - x_2(t))[-b_{11}(x_1(t) - x_2(t)) - b_{12}(y_1(t) - y_2(t)) - b_{13}(z_1(t) - z_2(t))]\\
&+\operatorname{sgn}(y_1(t) - y_2(t))\left[-b_{21}(x_1(t) - x_2(t)) + b_{22}(y_1(t) - y_2(t)) - c_1\left(\frac{z_1(t)}{1+y_1(t)} - \frac{z_2(t)}{1+y_2(t)}\right)\right]\\
&+\operatorname{sgn}(z_1(t) - z_2(t))\left[b_{13}(x_1(t) - x_2(t)) - b_{33}(z_1(t) - z_2(t)) + c_2\left(\frac{y_1(t)}{1+y_1(t)} - \frac{y_2(t)}{1+y_2(t)}\right)\right]\Bigg\}ds.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{d\mathbb{E}(V(t))}{dt} \leq& -b_{11}\mathbb{E}(|x_1(t) - x_2(t)|) - b_{12}\mathbb{E}(|y_1(t) - y_2(t)|) - b_{13}\mathbb{E}(|z_1(t) - z_2(t)|)\\
&- b_{21}\mathbb{E}(|x_1(t) - x_2(t)|) + b_{22}\mathbb{E}(|y_1(t) - y_2(t)|) + c_1\mathbb{E}\left(|\frac{z_1(t)}{1+y_1(t)} - \frac{z_2(t)}{1+y_2(t)}|\right)\\
&+ b_{31}\mathbb{E}(|x_1(t) - x_2(t)|) - b_{33}\mathbb{E}(|z_1(t) - z_2(t)|) + c_2\mathbb{E}\left(|\frac{y_1(t)}{1+y_1(t)} - \frac{y_2(t)}{1+y_2(t)}|\right)\\
\leq& -(b_{11} + b_{21} - b_{31})\mathbb{E}(|x_1(t) - x_2(t)|) - (b_{12} - b_{22})\mathbb{E}(|y_1(t) - y_2(t)|)\\
&- (b_{13} + b_{33})\mathbb{E}(|z_1(t) - z_2(t)|) + c_1\mathbb{E}(|z_1(t) - z_2(t)|) + c_1\mathbb{E}(|z_1(t)y_2(t) - z_2(t)y_1(t)|)\\
&+ c_2\mathbb{E}(|y_1(t) - y_2(t)|)\\
\leq& -(b_{11} + b_{21} - b_{31})\mathbb{E}(|x_1(t) - x_2(t)|)\\
&+ (-b_{12} + b_{22} + c_1\mathbb{E}(z_1(t)) + c_2)\mathbb{E}(|y_1(t) - y_2(t)|)\\
&+ (-b_{13} - b_{33} + c_1\mathbb{E}(y(t)))\mathbb{E}(|z_1(t) - z_2(t)|)\\
\leq& -(b_{11} + b_{21} - b_{31})\mathbb{E}(|x_1(t) - x_2(t)|)\\
&+ \left(-b_{12} + b_{22} + c_1\mathbb{E}\left(z_1^3(t)\right)^{1/3} + c_2\right)\mathbb{E}(|y_1(t) - y_2(t)|)\\
&+ \left(-b_{13} - b_{33} + c_1\mathbb{E}\left(y_1^3(t)\right)^{1/3}\right)\mathbb{E}(|z_1(t) - z_2(t)|).
\end{aligned}$$

By Lemma 3.2,

$$\mathbb{E}\left(y^3(t)\right)^{1/3} \le \left[\frac{3}{4}\right]^{4/3}\left(\frac{\left[\frac{1+\tilde{\gamma}_2}{3}+\sigma_2^2\right]^4}{(b_{22}-b_{21})^3}+b_{21}\left(\frac{a_1+\frac{3\sigma_1^2}{2}+\frac{\tilde{\gamma}_1}{4}}{b_{11}}\right)^4\right)^{1/3} := M_1,$$

$$\mathbb{E}\left(z^3(t)\right)^{1/3} \le \left[\frac{3}{4}\right]^{4/3}\left(\frac{\left[\frac{1+\tilde{\gamma}_3}{3}+c_2+\sigma_3^2\right]^4}{(b_{33}-b_{31})^3}+b_{31}\left(\frac{a_1+\frac{3\sigma_1^2}{2}+\frac{\tilde{\gamma}_1}{4}}{b_{11}}\right)^4\right)^{1/3} := M_2.$$

Therefore

$$\begin{aligned}\frac{d\mathbb{E}(V(t))}{dt} \le &-(b_{11}+b_{21}-b_{31})\,\mathbb{E}(|x_1(t)-x_2(t)|)\\ &+(-b_{12}+b_{22}+c_1M_2+c_2)\,\mathbb{E}(|y_1(t)-y_2(t)|)\\ &+(-b_{13}-b_{33}+c_1M_1)\,\mathbb{E}(|z_1(t)-z_2(t)|).\end{aligned}$$

By Assumption 3, then

$$\begin{aligned}\mathbb{E}(V(t)) \le &V(0)-(b_{11}+b_{21}-b_{31})\int_0^t \mathbb{E}(|x_1(t)-x_2(t)|)ds\\ &-(b_{12}-b_{22}-c_1M_2-c_2)\int_0^t \mathbb{E}(|y_1(t)-y_2(t)|)ds\\ &-(b_{13}+b_{33}-c_1M_1)\int_0^t \mathbb{E}(|z_1(t)-z_2(t)|)ds\\ <&\infty.\end{aligned}$$

It then follow from $V(t) \ge 0$ that $\mathbb{E}|x_1(t)-x_2(t)| \in L^1[0,\infty)$, $\mathbb{E}|y_1(t)-y_2(t)| \in L^1[0,\infty)$ and $\mathbb{E}|z_1(t)-z_2(t)| \in L^1[0,\infty)$. Therefore,

$$\begin{aligned}&\mathbb{E}(|(x_1(t),y_1(t),z_1(t))-(x_2(t),y_2(t),z_2(t))|)\\ =&\mathbb{E}\left\{\left[|x_1(t)-x_2(t)|^2+|y_1(t)-y_2(t)|^2+|z_1(t)-z_2(t)|^2\right]^{1/2}\right\}\\ \le&\mathbb{E}(|x_1(t)-x_2(t)|)+\mathbb{E}(|y_1(t)-y_2(t)|)+\mathbb{E}(|z_1(t)-z_2(t)|)\\ \in&L^1[0,\infty).\end{aligned}$$

Further, we can easily see from Lemma 5.2 that $|x_1(t)-x_2(t)|$, $|y_1(t)-y_2(t)|$ and $|z_1(t)-z_2(t)|$ are uniformly continuous with respect to t. So by Lemma 5.3 we easily obtain

$$\lim_{t\to+\infty}\mathbb{E}(|(x_1(t),y_1(t),z_1(t))-(x_2(t),y_2(t),z_2(t))|)=0 \text{ for almost all } \zeta\in\Omega. \tag{5.6}$$

This completes the proof.

Remark 5.1. *From Lemma 5.4, it shows that the jump-diffusion coefficient γ_j has influence on the globally asymptotically stable of system* (1.3) *in expectation. In other words, when Lévy noise intensity is too high then the system is not globally asymptotically stable in expectation.*

Theorem 5.1. *Under the conditions of Lemma 5.4, system* (1.3) *is asymptotically stable in distribution. That is, there exists a unique probability measure $\mu(\cdot)$ such that for any initial value $X(0)=(x(0),y(0),z(0))\in\mathbb{R}_+^3$, the transition probability $p(t,X(0),\cdot)$ of $\tilde{X}(t)$ weakly converges to $\mu(\cdot)$ as $t\to\infty$.*

Proof. Let $p(t, X(0), dY)$ denote the transition probability of the event $X(t; X(0)) \in \mathcal{B}$, where $\mathcal{B}$ is a Borel measurable set of $\mathbb{R}_+^3$. Let $\mathcal{P}\left(\mathbb{R}_+^3\right)$ denote all probability measures on $\mathbb{R}_+^3$. For any $\mathcal{P}_1, \mathcal{P}_2 \in \mathcal{P}\left(\mathbb{R}_+^3\right)$, we define metric $d_{\mathcal{K}}$ as follow:

$$d_{\mathcal{K}}(\mathcal{P}_1, \mathcal{P}_2) = \sup_{g\in\mathcal{K}} \left| \int_{\mathbb{R}_+^3} g(X)\mathcal{P}_1(dX) - \int_{\mathbb{R}_+^3} g(X)\mathcal{P}_2(dX) \right|,$$

where $\mathcal{K} = \left\{g : \mathbb{R}_+^3 \to \mathbb{R} : |g(X) - g(Y)| \le \|X - Y\|, |g(\cdot) \le 1\right\}$. First, we prove $p((t, X(0), dY) : t \ge 0)$ is cauchy in the space $\mathcal{P}\left(\mathbb{R}_+^3\right)$ with metric $d_{\mathcal{K}}$. According to Lemma 3.2 and Chebyshev inequality, $p((t, X(0), dY) : t \ge 0)$ is tight. For any $g \in \mathcal{K}$ and $t, s > 0$, we have

$$\begin{aligned}
&|\mathbb{E}g(X(X(0); t+s)) - \mathbb{E}g(X(X(0); t))| \\
&= |\mathbb{E}\left[\mathbb{E}(g(X(X(0); t+s))|\mathcal{F}_s)\right] - \mathbb{E}g(X(X(0); t))| \\
&= \left| \int_{\mathbb{R}_+^3} \mathbb{E}g\left(X\left(\tilde{X}(0); t\right)\right) p\left(s, X(0), d\tilde{X}(0)\right) - \mathbb{E}g(X(X(0); t)) \right| \\
&\le \int_{\mathbb{R}_+^3} \left|\mathbb{E}g\left(X\left(\tilde{X}(0); t\right)\right) - \mathbb{E}g(X(X(0); t))\right| p\left(s, X(0), d\tilde{X}(0)\right).
\end{aligned}$$

It follows from (5.6) that there is a constant $T \ge 0$ such that

$$\sup_{g\in\mathcal{K}} \left|\mathbb{E}g\left(X\left(\tilde{X}(0); t\right)\right) - \mathbb{E}g(X(X(0); t))\right| \le \varepsilon, \forall t \ge T.$$

Thanks to the arbitrariness of g, we have

$$\sup_{\mathcal{K}} |\mathbb{E}g(X(0); t+s) - \mathbb{E}g(X(0); t)| \le \varepsilon_1, \ \forall t \ge T, s > 0. \tag{5.7}$$

(5.7) is equivalent to

$$d_{\mathcal{K}}(p(t+s, X(0), \cdot), p(t, X(0), \cdot)) \le \varepsilon, \ \forall t \ge T, s > 0.$$

Therefore, the transition probability $p((t, X(0), \cdot) : t \ge 0)$ of the solution of system (1.3) is cauchy in the space $\mathcal{P}\left(\mathbb{R}_+^3\right)$ with metric $d_{\mathcal{K}}$. So there is a unique $\mu(\cdot)$ such that

$$\lim_{t\to\infty} d_{\mathcal{K}}(\mathcal{P}(t, 0, \cdot), \mu(\cdot)) = 0. \tag{5.8}$$

Then for any fix $X(0) \in \mathbb{R}_+^3$, combining with (5.7) and (5.8), we have

$$\lim_{t\to\infty} d_{\mathcal{K}}(\mathcal{P}(X(0), t, \cdot), \mu(\cdot)) \le \lim_{t\to\infty} \left[d_{\mathcal{K}}(\mathcal{P}(0, t, \cdot), \mu(\cdot)) + d_{\mathcal{K}}(\mathcal{P}(0, t, \cdot), \mathcal{P}(0, t, \cdot))\right].$$

That is,

$$\lim_{t\to\infty} d_{\mathcal{K}}(\mathcal{P}(X(0), t, \cdot \times \cdot), \mu(\cdot \times \cdot)) = 0.$$

The proof is completed.

Remark 5.2. *According to the proof of Theorem 5.1, we can use the method presented in this paper to discuss the stability of stochastic systems driven by discrete time noises without Lévy jumps.*

6. Numerical simulations

In the section, we give some examples to demonstrate our main results by using the Milstein method [32]. We always choose $a_1 = 0.7, a_2 = 0.02, a_3 = 0.025, b_{11} = 0.5, b_{21} = 0.2, b_{31} = 0.06, b_{12} = 0.6, b_{22} = 0.4, b_{33} = 0.4, b_{13} = 0.12, c_1 = 0.065, c_2 = 0.05, \mathbb{Z} = (0, +\infty), \lambda(\mathbb{Z}) = 1$ with initial value $(x(0), y(0), z(0)) = (0.8, 0.35, 0.15)$.

First, we illustrate the effect of white noise on population dynamics. Let $\sigma_1 = \sigma_2 = \sigma_3 = \gamma_1 = \gamma_2 = \gamma_2 = 0$, then (1.3) is a deterministic system. The dynamics of deterministic case is showed in Figure 1.

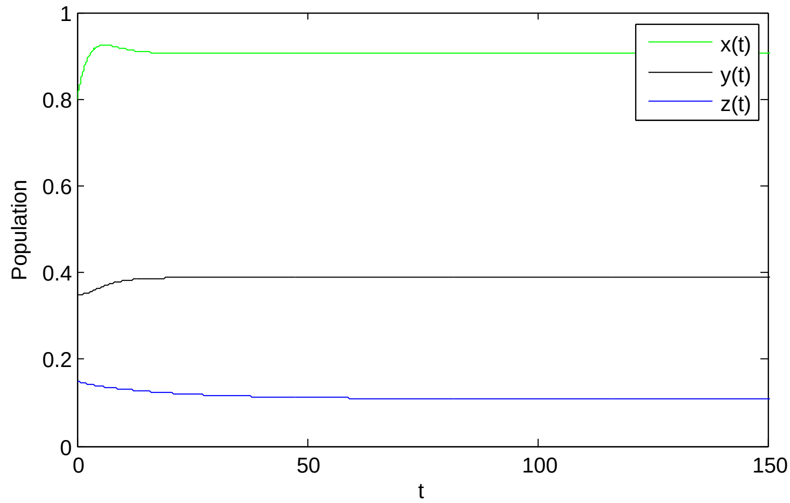


Figure 1. Dynamical behaviors of the deterministic case of (1.3) with $\Delta t = 0.01$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

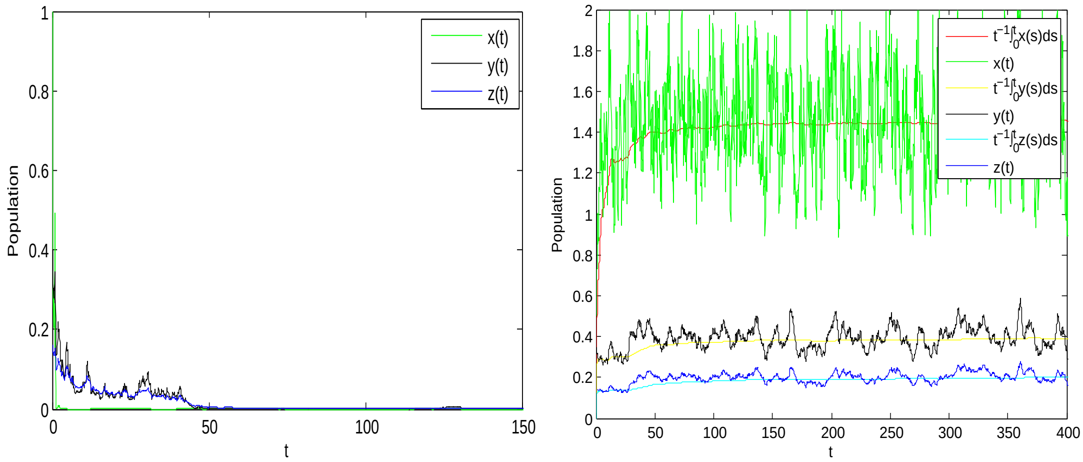
For the stochastic case, we choose $\sigma_1 = 1.2, \sigma_2 = 0.6, \sigma_3 = 0.2, \gamma_1 = \gamma_2 = \gamma_2 = 0.48$. After a simple calculation, we have $r_1 = -0.1080 < 0$. By Theorem 4.1, system (1.3) is extinctive, see Figure 2(a). If $\sigma_1 = 0.2, \sigma_2 = 0.05, \sigma_3 = 0.04, \gamma_1 = \gamma_2 = \gamma_3 = 0.3$, then

$$\begin{aligned}
&r_1 = a_1 - \frac{\sigma_1^2}{2} - \int_{\mathbb{Z}} [\gamma_1(u) - \ln(1 + \gamma_1(u))]\lambda(du) = 0.6424 > 0,\\
&b_{22} - b_{21} = 0.2 > 0, b_{33} - b_{31} = 0.34 > 0,\\
&b_{11} + b_{21} - b_{31} = 0.6400 > 0,\\
&-b_{12} - b_{22} + c_1 M_2 + c_2 = -0.1117 < 0,\\
&-b_{13} - b_{33} + c_1 M_1 = -0.3962 < 0,\\
&A_1 = 0.3262 > 0, A_2 = 0.1917 > 0, A_3 = 0.0148 > 0,\\
&D_1 = 1.2847 > 0, D_2 = 0.4951 > 0, D_3 = 0.1591 > 0,
\end{aligned}$$

which means all conditions of Theorem 4.2 hold. Further, by computation we have

$$\begin{aligned}
0.6524 \le \liminf_{t\to\infty} \langle x(t)\rangle \le \limsup_{t\to\infty} \langle x(t)\rangle \le 1.2847,\\
0.4793 \le \liminf_{t\to\infty} \langle y(t)\rangle \le \limsup_{t\to\infty} \langle y(t)\rangle \le 0.4951,\\
0.0371 \le \liminf_{t\to\infty} \langle z(t)\rangle \le \limsup_{t\to\infty} \langle z(t)\rangle \le 0.1591.
\end{aligned}$$

That is, (1.3) is persistence in mean, see Figure 2(b). The distributions of all species may see Figure 3.



(a) (b)

Figure 2. Dynamics of system (1.3). (a) is with $\sigma_1 = 1.4, \sigma_2 = 0.6, \sigma_3 = 0.2, \gamma_1 = \gamma_2 = \gamma_3 = 0.78$. (b) is with $\sigma_1 = 0.2, \sigma_2 = 0.05, \sigma_3 = 0.04, \gamma_1 = \gamma_2 = \gamma_3 = 0.3$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

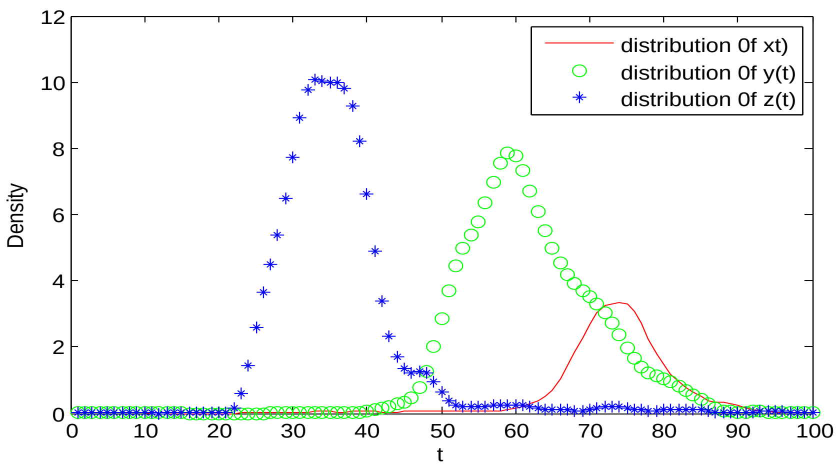


Figure 3. Distribution of $x(t), y(t), z(t)$ for the case of persistence in mean.

Next, we demonstrate the effect of Lévy jumps on population dynamics. Let $\sigma_1 = \sigma_2 = \sigma_3 = 0.2$, $\gamma_1 = \gamma_2 = 0.25$, $\gamma_3 = 0.05$. We can check that

$$\begin{aligned}
&r_1 = a_1 - \frac{\sigma_1^2}{2} - \int_{\mathbb{Z}} [\gamma_1(u) - \ln(1+\gamma_1(u))]\lambda(du) = 0.6531 > 0,\\
&\int_{\mathbb{Z}} \left\{\left(1+\gamma_j(u)\right)^p - 1 - p\gamma_j(u)\right\}\lambda(du) \le 0.0076, p = 3, j = 1, 2.3,\\
&\int_{\mathbb{Z}} \max\left\{|\gamma_j(u)|^2, \left[\ln\left(1+\gamma_j(u)\right)\right]^2\right\}\lambda(du) \le 0.0024, j = 1, 2, 3,\\
&b_{22} - b_{21} = 0.2 > 0, b_{33} - b_{31} = 0.34 > 0,\\
&b_{11} + b_{21} - b_{31} = 0.6400 > 0,\\
&-b_{12} - b_{22} + c_1 M_2 + c_2 = -0.1243 < 0,\\
&-b_{13} - b_{33} + c_1 M_1 = -0.3943 < 0,\\
&A_1 = 0.3369 > 0, A_2 = 0.1893 > 0, A_3 = 0.0346 > 0,\\
&D_1 = 1.3063 > 0, D_2 = 0.4860 > 0, D_3 = 0.2054 > 0.
\end{aligned}$$

Thus, Assumptions 1–3 are satisfied and $A_j > 0, D_j > 0 (j = 1, 2, 3)$. By Theorem 4.2, we have

$$0.6738 \le \liminf_{t\to\infty} \langle x(t)\rangle \le \limsup_{t\to\infty} \langle x(t)\rangle \le 1.3063,$$
$$0.4733 \le \liminf_{t\to\infty} \langle y(t)\rangle \le \limsup_{t\to\infty} \langle y(t)\rangle \le 0.4860,$$
$$0.0866 \le \liminf_{t\to\infty} \langle z(t)\rangle \le \limsup_{t\to\infty} \langle z(t)\rangle \le 0.2054,$$

which may see Figure 4 (a).

If $\sigma_1 = \sigma_2 = \sigma_3 = 0.2$, $\gamma_1 = 1.68, \gamma_2 = 0.45$, $\gamma_3 = 0.06$, then $r_1 = -0.0142 < 0$. Theorem 4.1 implies that $x(t), y(t)$ and $z(t)$ are all extinct a.s.(Figure 4(b)). Comparing Figure 4(a) with Figure 4(b), we can find that the Lévy jumps may suppress the survival of the species.

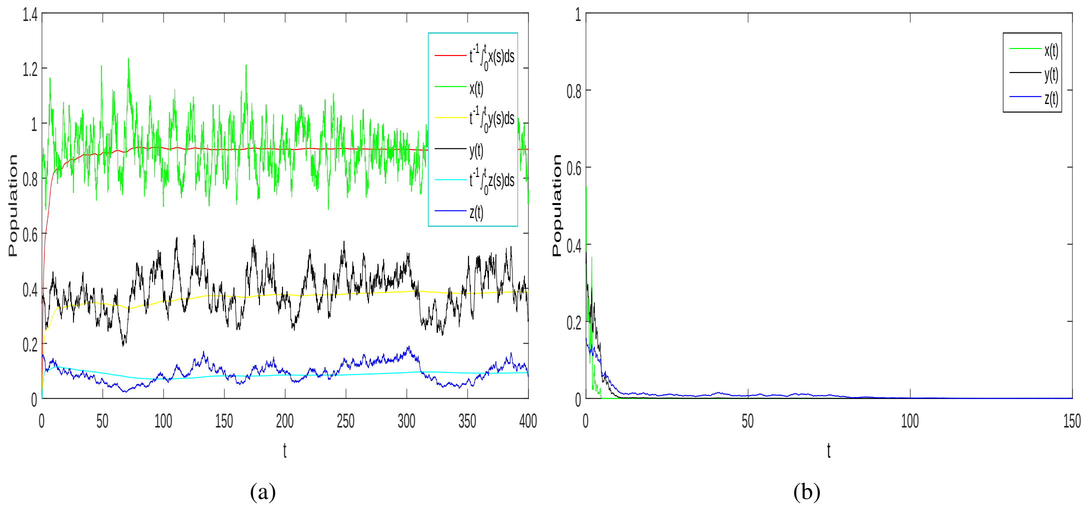


Figure 4. Dynamics of system (1.3) with $\sigma_1 = \sigma_2 = \sigma_3 = 0.2$. (a) is with $\gamma_1 = \gamma_2 = 0.25, \gamma_3 = 0.05$. (b) is with $\gamma_1 = 1.68, \gamma_2 = 0.45, \gamma_3 = 0.06$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

Remark 6.1. *For the same set of parameter values, we note that in the absence of environmental noise the deterministic system is uniformly persistent (see Figure 1). In stochastic system, relatively smaller intensity of white noise and Lévy jumps can maintain the survival of species, while the species will be extinctive with larger intensity of white noise and Lévy jumps (see Figure 4). Therefore, whether the species of the stochastic system are persistence in mean or not depends on the intensity of the noise.*

7. Conclusions

In this paper, we discuss a stochastic two predator-one prey system with Lévy jumps and mixed functional responses,which contains ratio-dependent type (between intermediate predator and top predator) and linear functional responses. Firstly, we discuss the existence and the pth moment-boundedness of positive solution. Then under Assumption 2, we establish sufficient criteria for the extinction of system (1.3). The result reveals an important property of the Lévy jumps: They are unfavorable for the existence of species. Furthermore, we establish sufficient condition for asymptotically stable in distribution for system (1.3) under certain conditions. Finally, some numerical simulations are introduced to demonstrate the theoretical results.

Theorem 4.1 shows that the intensity of white noise and Lévy jumps can make prey extinction. This means that unpredictable events in nature are so severe and intense that they can dramatically change population size in a short period of time. The extinction of prey will lead to the extinction of intermediate and top predators. Theorem 4.2 shows that if the intensities γ_1, γ_2 and γ_3 are small, then all population will persist in mean. Theorem 5.1 shows the existence of a unique invariant probability measure for system (1.3).

Some interesting topics deserve further investigations. As done in [16, 33], one can introduce time delays in stochastic system (1.3). Moreover, one can study deterministic system (1.1) with other perturbations, such as Markovian switching (see [34–38]) or second-order stochastic perturbation (see [39]). We leave these investigations for future work.

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Conflict of interest

The authors declare no conflict of interest.

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