Mathematics

Research article

Amply soft set and its topologies: AS and PAS topologies

Orhan Göçür*

Department of Statistics and Computer Sciences, Faculty of Science and Literature, Bilecik Seyh Edebali University, 11000 Bilecik-Turkey

* Correspondence: Email: orhan.gocur@bilecik.edu.tr; Tel: +902282141538.

Abstract: Do the topologies of each dimension have to be same of any space? I show that this is not necessary with amply soft topology produced by classical topologies. For example, an amply soft topology produced by classical topologies may have got any indiscrete topologies, discrete topologies or any topological spaces in each different dimension. The amply soft topology allows to write elements of different classical topologies in its each parameter sets. The classical topologies may be finite, infinite, countable or uncountable. This situation removes the boundary in soft topology and cause it to spread over larger areas. Amply soft topology produced by classical topologies is a special case of an amply soft topology. For this, I define a new soft topology it is called as an amply soft topology. I introduce amply soft open sets, amply soft closed sets, interior and closure of an amply soft set and subspace of any amply soft topological space. I gave parametric separation axioms which are different from T_i separation axioms. T_i questions the relationship between the elements of space itself while P_i questions the strength of the connection between their parameters. An amply soft topology is built on amply soft sets. Amply soft sets use any kind of universal parameter set or initial universe (such as finite or infinite, countable or uncountable). Also, subset, superset, equality, empty set, whole set on amply soft sets are defined. And operations such as union, intersection, difference of two amply soft sets and complement of an amply soft set are given. Then three different amply soft point such as amply soft whole point, amply soft point and monad point are defined. And also I give examples related taking a universal set as uncountable.

1. Introduction

Greene [1] mentioned in his book "The Elegance of the Universe" that the 11th dimension appears when the Heterotic-E matching constant is greater than 1 but not less than 1. In this case, it may be thought that the 11th dimension is a different topology. I think this may be a discrete topology. So, is a topology possible in which different topologies of different dimensions can exist? In this context, for a topological space in which different topologies can be written in different parameters, it should be looked at whether a set is created in which different sets can be written in different parameters. So, let us first analyze Molodtsov's soft set [2] which makes it possible to write different sets with different parameters.

Molodtsov [2] defined a soft set and gave some properties about it. For this, he thought that there are many uncertainties to solve complicated problems such as in sociology, economics, engineering, medical science, environment problems, statistics, etc. There is no deal to solve them successfully. However, there are some theories such as vague sets theory [3], fuzzy sets [4], probability, intuitionistic fuzzy sets [5], rough sets [6], interval mathematics [7], etc., but these studies have their own complexities. Also, you can see the latest study examples based on them [8–13].

Maji et al. [14] defined soft subset, soft superset, soft equality and they gave some operators such as intersection, union of two soft sets and complement of a soft set. They presented some properties about them. But some properties of them are false. So, Yang [15] gave counterexample about some of them. Also, Ali et al. [16] gave counter example about the others. Then they redefined operations of soft sets. Then the other researchers pointed out these false about soft operators and they gave some new definitions about soft operators of soft sets [17–25]. These are so valuable studies.

Up to now, there are many studies on the soft sets and their operations, also, taking universal set of parameters E is finite and countable because of the definition of the soft set. There are many soft sets and their operators defined. As first, in the soft set defined by Molodtsov [2] and developed by Maji et al. [14] the union of any soft set and its complement need not be a universal set. So, this situation makes a lot of deficiencies like a problem about complement of any soft set. To overcome this problem, researchers see two different ways. One way is using fixed parameter set in soft sets like Shabir and Naz's work [26]. They defined a lot of concept about a soft set and its topology. They used same parameter sets in their soft sets. This situation limits the soft topology. As a result of the fixed parameter, a soft point could be defined with fixed parameter. As a result of this, all soft sets have same values in all their parameters [27–30]. Some researchers tried to overcome this situation by defining another soft point [17,24,27,31]. You can see the latest study examples based on Shabir and Naz's work [32–38].

The other way is to redefine soft set and its operations. As first, Çağman and Enginoğlu [19] redefined soft set and its operations. This study is so valuable. But anyone could not take uncountable or infinite universal parameter set in practice. Because it is not pointed out what will we do with parameter sets between two soft sets exactly while using soft operators. You can see the latest study examples based on Çağman and Enginoglu's work [32,36–40]. Çağman et al. [41] defined a soft topology. They use different parameter sets in their soft sets. But all of them are finite because of the definition of soft operations. Then, Zhu and Wen [25] redefined a soft set and gave operations of it. Also, they pointed out what will we do with parameters set between two soft sets while using soft operators. But it is so complicated and as a result, cause same problems. You can see the latest study examples based on Zhu and Wen's work [42–44].

A similar situation drew attention of Fatimah et al. in [45]. They said that in their study all soft and hybrid soft sets used so far binary operation (either 0 or 1) or else real numbers between 0 and 1.

So they defined a new soft set and it is called an N-Soft set. They used n parameter in their study, $n \in \mathbb{N}$, is natural numbers. Riaz et al. defined an N-Soft topology in [42]. Therefore, we can say that they use set of parameters E as infinite and countable. They use initial universe X as finite and countable. But in fact, in real life or in space they do not have to be finite and countable.

In order to overcome all the problems mentioned about above and to define a more comprehensive and a more generalized soft set, I defined an amply soft set. I named this soft set as an amply soft set, together with its operations, in order to eliminate the complexity by selecting the ones that are suitable for a certain purpose among the previously defined soft sets and the operations between them and redefining otherwise. Amply soft sets use any kind of universal parameter set and initial universe (such as finite or infinite, countable or uncountable). Also, I introduced subset, superset, equality, empty set, and whole set about amply soft sets. And I gave operations such as union, intersection, difference of two amply soft sets and complement of an amply soft set. Then three different amply soft point such as amply soft whole point, amply soft point and monad point are defined. Also examples related taking universal set as uncountable are given.

I defined a new soft topology, and it is called as a PAS topology. The PAS topology allows to write different elements of classical topologies in its each parameter sets. The classical topologies may be finite, infinite, countable or uncountable. This situation removes all of the boundaries in a soft topology and cause it to spread over larger areas. A PAS topology is a special case of an AS topology. For this purpose, I defined a new soft topology, and it is called as an amply soft topology or briefly an AS topology. I introduce AS open sets, AS closed sets, interior and closure of an AS set and subspace of any AS topological space. I gave parametric separation axioms which are different from T_i separation axioms. T_i questions the relationship between the elements of space itself while P_i questions the strength of the connection between their parameters.

2. Materials and methods

These terminologies are used hereafter in the paper: X denotes an initial universe, E denotes a universal set of parameters; A, B, C are subsets of E.

Definition 1 *Let* $P(X)$ *denote the power set of* X. *If* $F: E \rightarrow P(X)$ *is a mapping given by*

$$F(e) = \begin{cases} F(e), & \forall e \in A; \\ \emptyset, & \forall e \in E - A, \end{cases}$$

then F with A is called as an amply soft set over X and it is denoted by $F * A$. We can say an AS set instead of an amply soft set for briefness.

Example 1 *Let* $X = \{x_1, x_2, x_3, x_4\}$ *be a universal set,* $E = \{e_1, e_2, e_3, e_4, e_5\}$ *a set of parameters and* $A = \{e_1, e_2, e_4, e_5\}$ *a subset of* E. *Let* $F: A \rightarrow P(X)$ *be the mapping given by* $F(e_1) = \{x_1, x_2\}, F(e_2) = \{x_2, x_4\}, F(e_4) = \{x_3\}, F(e_5) = \{x_3\}$. *Then we can show it looks like the following:*

$$F * A = \{\{x_1\}_{\{e_1\}}, \{x_2\}_{\{e_1, e_2\}}, \{x_3\}_{\{e_4, e_5\}}, \{x_4\}_{\{e_2\}}\}.$$

Definition 2 *Let* $F * A$ *and* $G * B$ *be two amply soft sets over* X. $F * A$ *is subset of* $G * B$, *denoted by* $F * A \stackrel{\sim}{\subseteq} G * B$, *if* $F(e) \subseteq G(e)$, *for all* $e \in A$.

Definition 3 *Let* $F * A$ *and* $G * B$ *be two amply soft sets over* X. $F * A$ *is superset of* $G * B$, *denoted by*

$$F * A \stackrel{\sim}{\supseteq} G * B, \text{ if } F(e) \supseteq G(e), \text{ for all } e \in B.$$

Definition 4 *Let $F * A$ and $G * B$ be two amply soft sets over X. If $F * A$ is subset of $G * B$ and $G * B$ is subset of $F * A$ also, then $F * A$ and $G * B$ are said to be an equal and denoted by $F * A \cong G * B$.*

Definition 5 *An amply soft set $F * A$ over X is said to be an empty amply soft set denoted by $\widetilde{\emptyset}$ if $F(e) = \emptyset$ for all $e \in E$.*

Definition 6 *An amply soft set $F * E$ over X is said to be an absolute amply soft set denoted by $\widetilde{X}$ if for all $e \in E$, $F(e) = X$.*

Definition 7 *The union of two amply soft sets of $F * A$ and $G * B$ over a common universe X is the amply soft set $H * C$, where $C = A \cup B$ and for all $e \in E$,*

$$H(e) = \begin{cases} F(e), & \forall e \in A - B, \\ G(e), & \forall e \in B - A, \\ F(e) \cup G(e), & \forall e \in A \cap B, \\ \emptyset, & \forall e \in E - C. \end{cases}$$

We can write $F * A \,\widetilde{\cup}\, G * B \cong F \cup G * A \cup B \cong H * C$.

Definition 8 *The intersection $H * C$ of two amply soft sets $F * A$ and $G * B$ over a common universe X denoted by $F * A \,\widetilde{\cap}\, G * B$ is defined as $C = A \cap B$ and for all $e \in E$,*

$$H(e) = \begin{cases} F(e) \cap G(e), & \forall e \in C, \\ \emptyset, & \forall e \in E - C. \end{cases}$$

We can write $F * A \,\widetilde{\cap}\, G * B \cong F \cap G * A \cap B \cong H * C$.

Definition 9 *The difference $H * A$ of two amply soft sets $F * A$ and $G * B$ over X is denoted by $F * A \widetilde{\backslash} G * B$ and it is defined as*

$$H(e) = \begin{cases} F(e), & \forall e \in A - B, \\ F(e) \backslash G(e), & \forall e \in A \cap B, \\ \emptyset, & \forall e \in E - A. \end{cases}$$

We can write $F * A \widetilde{\backslash} G * B \cong F \backslash G * A \cong H * A$.

Definition 10 *Let $F * A$ be an amply soft set over $\widetilde{X}$. The complement of an amply soft set $F * A$ over X is denoted by $(F * A)^{\widetilde{r}} \cong F' * E$ where $F' : E \to P(X)$ a mapping is defined as $F'(e) = X - F(e)$ for all $e \in E$.*

Note that, $F' : E \to P(X)$ is a mapping given by

$$F'(e) = \begin{cases} X - F(e), & \forall e \in A, \\ X, & \forall e \in E - A. \end{cases}$$

Example 2 *Let $X = \{h_1, h_2, h_3, h_4, h_5\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$ and its subsets $A = \{e_1, e_2, e_3\}$, $B = \{e_1, e_3, e_4\}$, $C = \{e_1, e_4\}$ and $F * A, G * B, H * C$ are amply soft sets over X defined as follows respectively,*

$$F * A \cong \{\{h_1, h_2\}_{\{e_1\}}, \{h_2, h_3, h_4\}_{\{e_2\}}, \{h_1, h_2, h_5\}_{\{e_3\}}\},$$

$$G * B \cong \{\{h_1, h_4\}_{\{e_1\}}, \{h_1, h_2, h_4\}_{\{e_3\}}, \{h_3, h_5\}_{\{e_4\}}\},$$

$$H * C \cong \{\{h_1, h_2\}_{\{e_1\}}, \{h_1, h_2, h_3\}_{\{e_4\}}\}.$$

Then,

1. $F * A \,\widetilde{\cup}\, G * B \cong F \cup G * A \cup B$
$\cong \{\{h_1, h_2, h_4\}_{\{e_1\}}, \{h_2, h_3, h_4\}_{\{e_2\}}, \{h_1, h_2, h_4, h_5\}_{\{e_3\}}, \{h_3, h_5\}_{\{e_4\}}\}.$

2. $F*A \tilde{\cap} G*B \cong F \cap G * A \cap B \cong \{\{h_1\}_{\{e_1\}}, \{h_1, h_2\}_{\{e_3\}}\}$.

3. $F*A \tilde{\setminus} G*B \cong F\setminus G * A \cong \{\{h_2\}_{\{e_1\}}, \{h_2, h_3, h_4\}_{\{e_2\}}, \{h_5\}_{\{e_3\}}\}$.

4. $G*B \tilde{\setminus} F*A \cong G\setminus F * B \cong \{\{h_4\}_{\{e_1, e_3\}}, \{h_3, h_5\}_{\{e_4\}}\}$.

5. $(F*A)^{\tilde{\prime}} \cong F' * E \cong \{\{h_3, h_4, h_5\}_{\{e_1\}}, \{h_1, h_5\}_{\{e_2\}}, \{h_3, h_4\}_{\{e_3\}}, X_{\{e_4, e_5\}}\}$.

6. $6. (G*B)^{\tilde{\prime}} \cong G' * E \cong \{\{h_2, h_3, h_5\}_{\{e_1\}}, \{h_3, h_5\}_{\{e_3\}}, \{h_1, h_2, h_4\}_{\{e_4\}}, X_{\{e_2, e_5\}}\}$.

Proposition 1 *Let $F*A, G*B$ and $H*C$ be amply soft sets over X; $A, B, C \subseteq E$. Then the followings are hold.*

1. $F*A \tilde{\cup} F*A \cong F*A$,
2. $F*A \tilde{\cap} F*A \cong F*A$,
3. $F*A \tilde{\cup} \tilde{\emptyset} \cong F*A$,
4. $F*A \tilde{\cap} \tilde{\emptyset} \cong \tilde{\emptyset}$,
5. $F*A \tilde{\cup} \tilde{X} \cong \tilde{X}$,
6. $F*A \tilde{\cap} \tilde{X} \cong F*A$,
7. $F*A \tilde{\cup} G*B \cong G*B \tilde{\cup} F*A$,
8. $F*A \tilde{\cap} G*B \cong G*B \tilde{\cap} F*A$,
9. $(F*A \tilde{\cup} G*B) \tilde{\cup} H*C \cong F*A \tilde{\cup} (G*B \tilde{\cup} H*C)$,
10. $(F*A \tilde{\cap} G*B) \tilde{\cap} H*C \cong F*A \tilde{\cap} (G*B \tilde{\cap} H*C)$,
11. $F*A \tilde{\cap} (G*B \tilde{\cup} H*C) \cong (F*A \tilde{\cap} G*B) \tilde{\cup} (F*A \tilde{\cap} H*C)$,
12. $F*A \tilde{\cup} (G*B \tilde{\cap} H*C) \cong (F*A \tilde{\cup} G*B) \tilde{\cap} (F*A \tilde{\cup} H*C)$.
13. $((F*A)^{\tilde{\prime}})^{\tilde{\prime}} \cong F*A$
14. $(F*A \tilde{\cup} G*B)^{\tilde{\prime}} \cong (F*A)^{\tilde{\prime}} \tilde{\cap} (G*B)^{\tilde{\prime}}$
15. $(F*A \tilde{\cap} G*B)^{\tilde{\prime}} \cong (F*A)^{\tilde{\prime}} \tilde{\cup} (G*B)^{\tilde{\prime}}$

Proof. Let $F*A$ and $G*B$ be amply soft sets over X; $A, B \subseteq E$.

1. $F*A$ defined as from the *Definition 1*,

$$F(e) = \begin{cases} F(e), & \forall e \in A, \\ \emptyset, & \forall e \in E - A. \end{cases}$$

So

$$F(e) \cup F(e) = \begin{cases} F(e), & \forall e \in A \\ \emptyset, & \forall e \in E - A \end{cases} \cup \begin{cases} F(e), & \forall e \in A \\ \emptyset, & \forall e \in E - A \end{cases} \overset{A \cup A = A}{\cong} \begin{cases} F(e) \cup F(e), & \forall e \in A \\ \emptyset \cup \emptyset, & \forall e \in E - A \end{cases}$$
$$= \begin{cases} F(e), & \forall e \in A \\ \emptyset, & \forall e \in E - A \end{cases} = F(e).$$

Then,

$$F*A \tilde{\cup} F*A \cong F*A$$

2. It can be shown like above.

3. $F*A$ defined as from the *Definition 1*,

$$F(e) = \begin{cases} F(e), & \forall e \in A, \\ \emptyset, & \forall e \in E - A. \end{cases}$$

And $\tilde{\emptyset}$ defined as from the *Definition 5*, $F(e) = \emptyset$ for all $e \in E$.
So

$$\begin{cases} F(e), & \forall e \in A \\ \emptyset, & \forall e \in E - A \end{cases} \cup \emptyset = \begin{cases} F(e) \cup \emptyset, & \forall e \in A \\ \emptyset \cup \emptyset, & \forall e \in E - A \end{cases} = \begin{cases} F(e), & \forall e \in A \\ \emptyset, & \forall e \in E - A \end{cases} = F(e).$$

Then,

$$F * A \,\widetilde{\cup}\, \widetilde{\emptyset} \cong F * A$$

4. It can be shown like above.

5. $F * A$ defined as from the *Definition 1*,

$$F(e) = \begin{cases} F(e), & \forall e \in A, \\ \emptyset, & \forall e \in E - A. \end{cases}$$

And $\tilde{X}$ defined as from the *Definition 6*, $F(e) = X$ for all $e \in E$.
So

$$\begin{cases} F(e), & \forall e \in A \\ \emptyset, & \forall e \in E - A \end{cases} \cup X = \begin{cases} F(e) \cup X, & \forall e \in A \\ \emptyset \cup X, & \forall e \in E - A \end{cases} = \begin{cases} X, & \forall e \in A \\ X, & \forall e \in E - A \end{cases} = X.$$

Then,

$$F * A \,\widetilde{\cup}\, \tilde{X} \cong \tilde{X}$$

6. $F * A$ defined as from the *Definition 1*,

$$F(e) = \begin{cases} F(e), & \forall e \in A; \\ \emptyset, & \forall e \in E - A, \end{cases}$$

And $\tilde{X}$ defined as from the *Definition 6*, $F(e) = X$ for all $e \in E$.
So

$$\begin{cases} F(e), & \forall e \in A \\ \emptyset, & \forall e \in E - A \end{cases} \cap X = \begin{cases} F(e) \cap X, & \forall e \in A \\ \emptyset \cap X, & \forall e \in E - A \end{cases} = \begin{cases} F(e), & \forall e \in A \\ \emptyset, & \forall e \in E - A \end{cases} = F(e), \textit{ for all } e \in E.$$

Then,

$$F * A \,\widetilde{\cap}\, \tilde{X} \cong F * A$$

7. $F * A \,\widetilde{\cup}\, G * B = F \cup G * A \cup B$. Then from the *Definition 7*, for all $e \in E$

$$F \cup G(e) = \begin{cases} F(e), & \forall e \in A - B \\ G(e), & \forall e \in B - A \\ F(e) \cup G(e), & \forall e \in A \cap B \\ \emptyset, & \forall e \in E - (A \cap B) \end{cases} = \begin{cases} G(e), & \forall e \in B - A \\ F(e), & \forall e \in A - B \\ G(e) \cup F(e), & \forall e \in B \cap A \\ \emptyset, & \forall e \in E - (B \cap A) \end{cases} = G \cup F(e).$$

So any one can see that clearly, $F \cup G(e) = G \cup F(e)$ for all $e \in E$.
Then,

$$F * A \,\widetilde{\cup}\, G * B \cong G * B \,\widetilde{\cup}\, F * A.$$

8. $F * A \,\widetilde{\cap}\, G * B \cong F \cap G * A \cap B$. Then from the *Definition 8*, for all $e \in E$

$$F \cap G(e) = \begin{cases} F(e) \cap G(e), & \forall e \in A \cap B \\ \emptyset, & \forall e \in E - (A \cap B) \end{cases} = \begin{cases} G(e) \cap F(e), & \forall e \in B \cap A \\ \emptyset, & \forall e \in E - (B \cap A) \end{cases} = G \cap F(e)$$

So any one can see that clearly, $F \cap G(e) = G \cap F(e)$ for all $e \in E$.
Then,

$$F * A \widetilde{\cap} G * B \cong G * B \widetilde{\cap} F * A.$$

The others can be shown like aboves.

Let $F * A$ and $G * B$ be amply soft sets over $X, A, B \subset E$. $(F * A)^{\widetilde{\prime}}$ is defined as

$$F'(e) = \begin{cases} \mathbf{X} - \mathbf{F(e)}, & \forall \mathbf{e} \in \mathbf{A}, \\ \mathbf{X}, & \forall \mathbf{e} \in \mathbf{E} - \mathbf{A}, \end{cases}$$

from the *Definition 10.* Then $((F * A)^{\widetilde{\prime}})^{\widetilde{\prime}} \cong (F' * E)^{\widetilde{\prime}} = (F')' * E$ defined as

$$(F')'(e) = \begin{cases} \mathbf{X} - (\mathbf{X} - \mathbf{F(e)}), & \forall \mathbf{e} \in \mathbf{A} \\ \mathbf{X} - \mathbf{X}, & \forall \mathbf{e} \in \mathbf{E} - \mathbf{A} \end{cases} = \begin{cases} F(e), & \forall e \in A \\ \emptyset, & \forall e \in E - A \end{cases} = F(e).$$

Then, it means from the *Definition 1*,

$$((\boldsymbol{F} * \boldsymbol{A})^{\widetilde{\prime}})^{\widetilde{\prime}} \cong \boldsymbol{F} * \boldsymbol{A}.$$

The others can be shown like aboves. □

Example 3 Let us consider the amply soft sets $F * A, G * B, H * C$ over X on the *Example 2*.

1. Let us see if $(F * A \cup G * B) \cup H * C = F * A \cup (G * B \cup H * C)$.

$$\Longrightarrow):$$
$$F * A \widetilde{\cup} G * B \cong F \cup G * A \cup B \cong \{\{h_1, h_2, h_4\}_{\{e_1\}}, \{h_2, h_3, h_4\}_{\{e_2\}}, \{h_1, h_2, h_4, h_5\}_{\{e_3\}}, \{h_3, h_5\}_{\{e_4\}}\}.$$

Then,

$$(F \cup G * A \cup B) \widetilde{\cup} H * C \cong (F \cup G) \cup H * (A \cup B)$$
$$\cup C \cong \{\{h_1, h_2, h_4\}_{\{e_1\}}, \{h_2, h_3, h_4\}_{\{e_2\}}, \{h_1, h_2, h_4, h_5\}_{\{e_3\}}, \{h_1, h_2, h_3, h_5\}_{\{e_4\}}\}.$$

$$\Longleftarrow): \quad G * B \widetilde{\cup} H * C \cong G \cup H * B \cup C \cong \{\{h_1, h_2, h_4\}_{\{e_1, e_3\}}, \{h_1, h_2, h_3, h_5\}_{\{e_4\}}\}.$$

Then,

$$F * A \widetilde{\cup} (G \cup H * B \cup C) \cong F \cup (G \cup H) * A \cup (B$$
$$\cup C) \cong \{\{h_1, h_2, h_4\}_{\{e_1\}}, \{h_2, h_3, h_4\}_{\{e_2\}}, \{h_1, h_2, h_4, h_5\}_{\{e_3\}}, \{h_1, h_2, h_3, h_5\}_{\{e_4\}}\}.$$

So ,

$$(F * A \widetilde{\cup} G * B) \widetilde{\cup} H * C \cong F * A \widetilde{\cup} (G * B \widetilde{\cup} H * C).$$

2. Let us see if $(F * A) \widetilde{\cap} G * B) \widetilde{\cap} H * C \cong F * A \widetilde{\cap} (G * B \widetilde{\cap} H * C)$.

$$\Longrightarrow): F * A \widetilde{\cap} G * B \cong F \cap G * A \cap B \cong \{\{h_1\}_{\{e_1\}}, \{h_1, h_2\}_{\{e_3\}}\}.$$

Then,

$$(F \cap G * A \cap B) \widetilde{\cap} H * C \cong \{\{h_1\}_{\{e_1\}}\}$$

$$\Longleftarrow) : G * B \widetilde{\cap} H * C \cong G \cap H * C \cong \{\{h_1\}_{\{e_1\}}, \{h_3\}_{\{e_4\}}\}.$$

Then,

$$F * A \widetilde{\cap} (G \cap H * B \cap C) \cong F \cap (G \cap H) * A \cap (B \cap C) \cong \{\{h_1\}_{\{e_1\}}\}.$$

So,

$$(F * A \tilde{\cap} G * B) \tilde{\cap} H * C \cong F * A \tilde{\cap} (G * B \tilde{\cap} H * C).$$

3. Let us see if $F * A \tilde{\cap} (G * B \tilde{\cup} H * C) \cong (F * A \tilde{\cap} G * B) \tilde{\cup} (F * A \tilde{\cap} H * C)$.

$\Longrightarrow$): $G * B \tilde{\cup} H * C \cong G \cup H * B \cup C = \{\{h_1, h_2, h_4\}_{\{e_1\}}, \{h_1, h_2, h_4\}_{\{e_3\}}, \{h_1, h_2, h_3, h_5\}_{\{e_4\}}\}$.

Then,

$$F * A \tilde{\cap} (G \cup H * B \cup C) \cong F \cap (G \cup H) * A \cap (B \cup C) \cong \{\{h_1, h_2\}_{\{e_1, e_3\}}\}.$$

$\Longleftarrow$): $F * A \tilde{\cap} G * B \cong F \cap G * A \cap B \cong \{\{h_1\}_{\{e_1\}}, \{h_1, h_2\}_{\{e_3\}}\}$.

Then,

$$F * A \tilde{\cap} H * C \cong F \cap H * A \cap C \cong \{\{h_1, h_2\}_{\{e_1\}}\}.$$

$(F * A \tilde{\cap} G * B) \tilde{\cup} (F * A \tilde{\cap} H * C) \cong (F \cap G) \cup (F \cap H) * (A \cup B) \cap (A \cap C) = \{\{h_1, h_2\}_{\{e_1, e_3\}}\}.$

So,

$$F * A \tilde{\cap} (G * B \tilde{\cup} H * C) \cong (F * A \tilde{\cap} G * B) \tilde{\cup} (F * A \tilde{\cap} H * C).$$

4. Let us see if $F * A \tilde{\cup} (G * B \tilde{\cap} H * C) \cong (F * A \tilde{\cup} G * B) \tilde{\cap} (F * A \tilde{\cup} H * C)$.

$\Longrightarrow$): $G * B \tilde{\cap} H * C \cong G \cap H * B \cap C = \{\{h_1\}_{\{e_1\}}, \{h_3\}_{\{e_4\}}\}$.

Then,

$$F * A \tilde{\cup} (G \cap H * B \cap C) \cong F \cup (G \cap H) * A \cup (B$$
$$\cap C) \cong \{\{h_1, h_2\}_{\{e_1\}}, \{h_2, h_3, h_4\}_{\{e_2\}}, \{h_1, h_2, h_5\}_{\{e_3\}}, \{h_3\}_{\{e_4\}}\}.$$

$\Longleftarrow$):
$F * A \tilde{\cup} G * B \cong F \cup G * A \cup B \cong \{\{h_1, h_2, h_4\}_{\{e_1\}}, \{h_2, h_3, h_4\}_{\{e_2\}}, \{h_1, h_2, h_4, h_5\}_{\{e_3\}}, \{h_3, h_5\}_{\{e_4\}}\}.$

$F * A \tilde{\cup} H * C \cong F \cup H * A \cup C \cong \{\{h_1, h_2\}_{\{e_1\}}, \{h_2, h_3, h_4\}_{\{e_2\}}, \{h_1, h_2, h_5\}_{\{e_3\}}, \{h_1, h_2, h_3\}_{\{e_4\}}\}.$

$F \cup G * A \cup B \tilde{\cap} F \cup H * A \cup C \cong (F \cup G) \cap (F \cup H) * (A \cup B) \cap (A$
$\cup C) \cong \{\{h_1, h_2\}_{\{e_1\}}, \{h_2, h_3, h_4\}_{\{e_2\}}, \{h_1, h_2, h_5\}_{\{e_3\}}, \{h_3\}_{\{e_4\}}\}.$

So,

$$F * A \tilde{\cup} (G * B \tilde{\cap} H * C) \cong (F * A \tilde{\cup} G * B) \tilde{\cap} (F * A \tilde{\cup} H * C).$$

5. Let us see if $((F * A)^{\tilde{r}})^{\tilde{r}} \cong F * A$.
$(F * A)^{\tilde{r}}$ is showed in *matter 5* on the *Example 2*. Then,

$$((F * A)^{\tilde{r}})^{\tilde{r}} \cong (F' * E)^{\tilde{r}} = (F')' * E \cong \{\{h_1, h_2\}_{\{e_1\}}, \{h_2, h_3, h_4\}_{\{e_2\}}, \{h_1, h_2, h_5\}_{\{e_3\}}, \emptyset_{\{e_4, e_5\}}\}$$

and it is equal

$$\{\{h_1, h_2\}_{\{e_1\}}, \{h_2, h_3, h_4\}_{\{e_2\}}, \{h_1, h_2, h_5\}_{\{e_3\}}\} \cong F * A.$$

Then

$$((F * A)^{\tilde{r}})^{\tilde{r}} \cong F * A.$$

6. Let us see if $(F * A \, \widetilde{\cup} \, G * B)^{\widetilde{r}} \cong (F * A)^{\widetilde{r}} \, \widetilde{\cap} \, (G * B)^{\widetilde{r}}$.

$F * A \, \widetilde{\cup} \, G * B \cong F \cup G * A \cup B$ is showed in *matter 1* on the *Example 2*. Then,

$$(F * A \, \widetilde{\cup} \, G * B)^{\widetilde{r}} \cong (F \cup G)' * E \cong \{\{h_3, h_5\}_{\{e_1\}}, \{h_1, h_5\}_{\{e_2\}}, \{h_3\}_{\{e_3\}}, \{h_1, h_2, h_4\}_{\{e_4\}}, X_{\{e_5\}}\}.$$

$(F * A)^{\widetilde{r}}$ is showed as in *matter 5* on the *Example 2* and $(G * B)^{\widetilde{r}}$ is showed as in *matter 6* on the *Example 2*. Then,

$$(F * A)^{\widetilde{r}} \, \widetilde{\cap} \, (G * B)^{\widetilde{r}} \cong F' * E \, \widetilde{\cap} \, G' * E \cong F' \cap G'$$
$$* E \cong \{\{h_3, h_5\}_{\{e_1\}}, \{h_1, h_5\}_{\{e_2\}}, \{h_3\}_{\{e_3\}}, \{h_1, h_2, h_4\}_{\{e_4\}}, X_{\{e_5\}}\}.$$

So,

$$(F * A \, \widetilde{\cup} \, G * B)^{\widetilde{r}} \cong (F * A)^{\widetilde{r}} \, \widetilde{\cap} \, (G * B)^{\widetilde{r}}.$$

7. *Let us see if* $(F * A \, \widetilde{\cap} \, G * B)^{\widetilde{r}} \cong (F * A)^{\widetilde{r}} \, \widetilde{\cup} \, (G * B)^{\widetilde{r}}$.

$F * A \, \widetilde{\cap} \, G * B$ is showed as in *matter 2* on the *Example 2*. Then,

$$(F * A \, \widetilde{\cap} \, G * B)^{\widetilde{r}} \cong (F \cap G * A \cap B)^{\widetilde{r}} \cong (F \cap G)' * E \cong \{\{h_2, h_3, h_4, h_5\}_{\{e_1\}}, \{h_3, h_4, h_5\}_{\{e_3\}}, X_{\{e_2, e_4, e_5\}}\}$$

$(F * A)^{\widetilde{r}}$ is showed as in *matter 5* on the *Example 2*. And $(G * B)^{\widetilde{r}}$ is showed as in *matter 6* on the *Example 2*. Then,

$$(F * A)^{\widetilde{r}} \, \widetilde{\cup} \, (G * B)^{\widetilde{r}} \cong F' * E \, \widetilde{\cup} \, G' * E \cong F' \cup G' * E \cong \{\{h_2, h_3, h_4, h_5\}_{\{e_1\}}, \{h_3, h_4, h_5\}_{\{e_3\}}, X_{\{e_2, e_4, e_5\}}\}.$$

So,

$$(F * A \, \widetilde{\cap} \, G * B)^{\widetilde{r}} \cong (F * A)^{\widetilde{r}} \, \widetilde{\cup} \, (G * B)^{\widetilde{r}}.$$

The following example, we take E as an uncountable parameter.

Example 4 Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$ be set of walnut types, E be a parameter set of resistance in percent against to walnut illness; F and G, denotes different types of walnut illnesses, be mappings from E to $P(X)$; $E = [0,100] \subset \mathbb{R}$ and $A = (60,90]$, $B = (80,100] \subset E$, $\mathbb{R}$ is set of real numbers.

And let $F * A$ be an amply soft set for walnut types resistancing in percent against to a type of walnut illness, is defined by

$$F * (60,90] = \{\{x_1, x_3, x_7, x_8, x_9, x_{10}\}_{(60,70]}, \{x_1, x_7, x_9, x_{10}\}_{(70,80]}, \{x_1, x_9, x_{10}\}_{(80,90]}\}$$

and

$G * B$ be an amply soft set for walnut types resistancing in percent against to another type of walnut illness, is defined by

$$G * (75,95] = \{\{x_1, x_2, x_3, x_{10}\}_{(75,85]}, \{x_1, x_2\}_{(85,95]}\}.$$

Then we can find the followings:

1. $F * (60,90] \, \widetilde{\cup} \, G * (75,95] \cong F \cup G *$
$(60,95] \cong \{\{x_1, x_3, x_7, x_8, x_9, x_{10}\}_{(60,70]}, \{x_1, x_7, x_9, x_{10}\}_{(70,75]},$
$\{x_1, x_2, x_3, x_7, x_9, x_{10}\}_{(75,80]}, \{x_1, x_2, x_3, x_9, x_{10}\}_{(80,85]}, \{x_1, x_2, x_9, x_{10}\}_{(85,90]}, \{x_1, x_2\}_{(90,95]}\}.$

2. $F * (60,90] \, \widetilde{\cap} \, G * (75,95] \cong F \cap G * (75,90] = \{\{x_1, x_{10}\}_{(75,85]}, \{x_1\}_{(85,90]}\}.$

3. $F * (60,90] \widetilde{\backslash} G * (75,95] \cong F \backslash G *$
$(60,90] \cong \{\{x_1, x_3, x_7, x_8, x_9, x_{10}\}_{(60,70]}, \{x_1, x_7, x_9, x_{10}\}_{(70,75]}, \{x_7, x_9\}_{(75,80]}, \{x_9\}_{(80,85]}, \{x_9, x_{10}\}_{(85,90]}\}$.

4. $G * (75,95] \widetilde{\backslash} F * (60,90] \cong G \backslash F * (75,95] \cong \{\{x_2, x_3\}_{(75,85]}, \{x_2\}_{(85,90]}, \{x_1, x_2\}_{(90,95]}\}$.

5. $(F * (60,90])^{\widetilde{\prime}} \cong F' *$
$[0,100] \cong \{\{x_2, x_4, x_5, x_6\}_{(60,70]}, \{x_2, x_3, x_4, x_5, x_6, x_8\}_{(70,80]}, \{x_2, x_3, x_4, x_5, x_6, x_7, x_8\}_{(80,90]}, X_{[0,60] \cup (95,100]}\}$.

6. $(G * (75,95])^{\widetilde{\prime}} \cong G' *$
$[0,100] \cong \{\{x_4, x_5, x_6, x_7, x_8, x_9\}_{(75,85]}, \{x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}_{(85,95]}, X_{[0,75] \cup (95,100]}\}$.

The following example, we take E and X as an uncountable.

Example 5 Let $X = \mathbb{R}, E = (0,10] \subset \mathbb{R}, F * (2,5]$ and $G * (3,8]$ be amply soft sets, defined as follows respectively, over X, $\mathbb{R}$ is a set of real numbers.

$$F * (2,5] \cong \{[10,15]_{(2,4]}, \{13\}_{(4,5]}\}$$

$$G * (3,8] \cong \{[7,12]_{(3,7]}, (0,100)_{(7,8]}\}$$

Then we can find the following:

1. $F * (2,5] \widetilde{\cup} G * (3,8] \cong F \cup G * (2,8]$
$\cong \{[10,15]_{(2,3]}, [7,15]_{(3,4]}, [7,12] \cup \{13\}_{(4,5]}, [7,12]_{(5,7]}, (0,100)_{(7,8]}\}$

2. $F * (2,5] \widetilde{\cap} G * (3,8] \cong F \cap G * (3,5] \cong \{[10,12]_{(3,4]}\}$

3. $F * (2,5] \widetilde{\backslash} G * (3,8] \cong F \backslash G * (2,5] \cong \{[10,15]_{(2,3]}, (12,15]_{(3,4]}, \{13\}_{(4,5]}\}$

4. $G * (3,8] \widetilde{\backslash} F * (2,5] \cong G \backslash F * (3,8] \cong \{[7,10)_{(3,4]}, [7,12]_{(4,7]}, (0,100)_{(7,8]}\}$

5. $(F * (2,5])^{\widetilde{\prime}} \cong F' * (0,10] \cong \{\mathbb{R} - [10,15]_{(2,4]}, \mathbb{R} - \{13\}_{(4,5]}, \mathbb{R}_{(0,10]-(2,5]}\}$

6. $(G * (3,8])^{\widetilde{\prime}} \cong G' * (0,10] \cong \{\mathbb{R} - [7,12]_{(3,7]}, \mathbb{R} - (0,100)_{(7,8]}, \mathbb{R}_{(0,10]-(3,8]}\}$

Definition 11 Let $F * E$ be an amply soft set over X and $x \in X$. If $F * E$ defined as $F(e) = \{x\}$ for all $e \in E$, then $F * E$ is called as an amply soft whole point and it is denoted by $\tilde{x}$.

Definition 12 Let $F * A$ be an amply soft set over X and $x \in X$. If $F * A$ defined as $F(e) = \{x\}$ for all $e \in A$, then $F * A$ is called as an amply soft point and it is denoted by x.

Definition 13 Let $\{a\} \subset E$ and let $F * \{a\}$ be an amply soft set over $X, x \in X$. If $F * \{a\}$ defined as $F(a) = \{x\}$, then $F * \{a\}$ is called a monad point and it is denoted by x_a.

Definition 14 Let $F * E$ be an amply soft set over X and $x \in X$. We say that $\tilde{x} \widetilde{\in} F * E$ read as amply soft whole point $\tilde{x}$ belongs to the amply soft set $F * E$ if $x \in F(e)$ for all $e \in E$.

Definition 15 Let $F * E$ be an amply soft set over X and $x \in X$. We say that $\tilde{x} \widetilde{\notin} F * E$ if $x \notin F(e)$ for some $e \in E$.

Definition 16 Let $F * A$ be an amply soft set over $X, x \in X$. We say that $x \widetilde{\in} F * A$ read as amply soft point x belongs to the amply soft set $F * A$ if $x \in F(e)$ for all $e \in A$.

Definition 17 Let $F * A$ be an amply soft set over X and $x \in X$. We say that $x \widetilde{\notin} F * A$ if $x \notin F(e)$ for some $e \in A$.

Definition 18 *Let $a \in A$ and $F * A$ be an amply soft set over X, $x \in X$. We say that $x_a \widetilde{\in} F * A$ read as monad point x belongs to the amply soft set $F * A$ if $x \in F(a)$.*

Definition 19 *Let $a \in E$ and $F * \{a\}$ be an amply soft set over X, $x \in X$. We say that $x_a \widetilde{\notin} F * \{a\}$ if $x \notin F(a)$.*

Proposition 2 Let $F * A$ and $G * B$ be amply soft sets over $X, x \in X$. If $x \widetilde{\in} F * A$ and $x \widetilde{\in} G * B$ then $x \widetilde{\in} F * A \widetilde{\cup} G * B$.

Proof. It is obvious from the *Definition 7* and the *Definition 16*. □

Proposition 3 Let $F * A$ and $G * B$ be amply soft sets over $X, x \in X$. If $x \widetilde{\in} F * A, x \widetilde{\in} G * B$ and $F * A \widetilde{\cap} G * B \widetilde{\neq} \widetilde{\emptyset}$ then $x \widetilde{\in} F * A \widetilde{\cap} G * B$.

Proof.   It is obvious from the *Definition 8* and the *Definition 16*.   □

Proposition 4 Let $F * A$ and $G * B$ be amply soft sets over $X, x \in X, x \in F * A$ and $x \widetilde{\in} G * B$. If $A \cup B = E$, then $\tilde{x} \widetilde{\in} F * A \widetilde{\cup} G * B$.

Proof.   It is obvious from the *Proposition 2* and the *Definition 14*.   □

Example 6 Let us consider the *Example 2*.

We can see that clearly, $h_1 \widetilde{\notin} F * A, h_1 \widetilde{\in} H * C, h_2 \widetilde{\in} F * A, h_2 \widetilde{\in} H * C$

1. $F * A \widetilde{\cap} H * C \cong F \cap H * A \cap C = F \cap H * \{e_1\} = \{\{h_1, h_2\}_{\{e_1\}}\}$.

So we can see that clearly, $h_1 \widetilde{\in} F * A \widetilde{\cap} H * C, h_2 \widetilde{\in} F * A \widetilde{\cap} H * C$.

2. $F * A \widetilde{\cup} H * C \cong F \cup H * A \cup C \cong F \cup H *$ $\{e_1, e_2, e_3, e_4\} \cong \{\{h_1, h_2\}_{\{e_1\}}, \{h_2, h_3, h_4\}_{\{e_2\}}, \{h_1, h_2, h_5\}_{\{e_3\}}, \{h_1, h_2, h_3\}_{\{e_4\}}\}$.

So we can see that clearly, $h_1 \widetilde{\notin} F * A \widetilde{\cup} H * C$ and $h_2 \widetilde{\in} F * A \widetilde{\cup} H * C$.

Proposition 5 Let $F * A$ be an amply soft set over X and $a \in A$. If monad points $x_a \widetilde{\in} F * A$ for all $a \in A$ then $x \widetilde{\in} F * A$.

Proof.   It is obvious from the *Definition 18* and the *Definition 12*.   □

Proposition 6 Let $F * E$ be an amply soft set over X. If monad points $x_a \widetilde{\in} F * E$ for all $a \in E$ then $\tilde{x} \widetilde{\in} F * E$.

Proof.   It is obvious from the *Definition 18* and the *Definition 11*.   □

Example 7 Let us consider the *Example 2*.

Monad points $h_{2e_1}, h_{2e_2}, h_{2e_3} \widetilde{\in} F * A$. Then we can see that clearly, $h_2 \widetilde{\in} F * A$.

3.   AS and PAS topologies

Definition 20 *Let $\tilde{\tau}$ be the collection of amply soft sets over X, then $\tilde{\tau}$ is said to be an amply soft topology (or briefly AS topology) on $\tilde{X}$ if,*
1. $\widetilde{\emptyset}, \widetilde{X}$ belong to $\widetilde{\tau}$
2.   The union of any number of amply soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$
3.   The intersection of any two amply soft sets in $\tilde{\tau}$ belongs to $\widetilde{\tau}$.

The triplet $(\widetilde{X}, \widetilde{\tau}, E)$ is called as an amply soft topological space over $\widetilde{X}$.

We will use AS topological space $\widetilde{X}$ instead of amply soft topological space $(\widetilde{X}, \widetilde{\tau}, E)$ for shortly.

Definition 21 *Let $(\tilde{X}, \tilde{\tau}, E)$ be an AS topological space, then the members of $\tilde{\tau}$ are said to be AS open sets in an AS topological space $\tilde{X}$.*

Definition 22 *Let $(\tilde{X}, \tilde{\tau}, E)$ be an AS topological space. An amply soft set $F * A$ over $\tilde{X}$ is said to be an AS closed set in an AS topological space $\tilde{X}$, if its complement $(F * A)^{\widetilde{\imath}}$ belongs to $\tilde{\tau}$.*

Proposition 7 *Let $(\tilde{X}, \tilde{\tau}, E)$ be an AS topological space. Then*
1. $\widetilde{\emptyset}, \widetilde{X}$ are AS closed sets in AS topological space $\widetilde{X}$,
2.   The intersection of any number of AS closed sets is an AS closed set in an AS topological space $\widetilde{X}$,
3.   The union of any two AS closed sets is an AS closed set in an AS topological space $\widetilde{X}$.

Proposition 8 *Let $(\widetilde{X}, \tilde{\tau}, E)$ be an AS topological space. Then the collection $\tau_e = \{F(e) | F * E \widetilde{\in} \tilde{\tau}\}$ for each $e \in E$, defines topologies on X.*

Proof.   Let $(\widetilde{X}, \widetilde{\tau}, E)$ be an AS topological space.
1.   Because $(\widetilde{X}, \widetilde{\tau}, E)$ is an AS topological space; $\widetilde{\emptyset}, \widetilde{X} \widetilde{\in} \widetilde{\tau}$ from the *Definition 20*.

a. Since $\widetilde{X} \widetilde{\in} \widetilde{\tau}$, there exist an AS open set $F * E$ such that $F(e) = X$ for all $e \in E$ from the *Definition 6*. And we can see that clearly, $X \in \tau_e$ for all $e \in E$.

b. Since $\widetilde{\emptyset} \widetilde{\in} \widetilde{\tau}$, there exist an AS open set $G * E$ such that $G(e) = \emptyset$ for all $e \in E$ from the *Definition 5*. And we can see that clearly, $\emptyset \in \tau_e$ for all $e \in E$.

2. Because $(\widetilde{X}, \widetilde{\tau}, E)$ is an AS topological space, the union of any number of amply soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$ from the *Definition 20*. Then from the *Definition 7*, the union of any number of sets in τ_e belongs to τ_e for all $e \in E$.

3. Because $(\widetilde{X}, \widetilde{\tau}, E)$ is an AS topological space, the intersection of any two amply soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$ from the *Definition 20*. Then from the *Definition 8*, the intersection of any two sets in τ_e belongs to τ_e for all $e \in E$.

So $\tau_e = \{F(e) | (F, E) \widetilde{\in} \widetilde{\tau}\}$ for each $e \in E$, defines topologies on X. □

Example 8 Let $X = \{x, y, z\}$ be a universal set, $E = \{e_1, e_2, e_3\}$ a parameter set and

$$\widetilde{\tau} =$$
$$\{\widetilde{\emptyset}, \widetilde{X}, \{\{x,y\}_{\{e_1\}}, \{y\}_{\{e_2\}}\}, \{\{y,z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}, \{\{x,z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}, \{\{y\}_{\{e_2\}}\}, \{\{x\}_{\{e_1\}}\}, \{\{x,y\}_{\{e_1\}}, \{y,z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}$$
,
$$\{X_{\{e_1\}}, \{y\}_{\{e_2,e_3\}}\}, \{\{x,z\}_{\{e_1\}}, \{y,z\}_{\{e_2,e_3\}}\}, \{\{x\}_{\{e_1\}}, \{y,z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}, \{X_{\{e_1\}}, \{y,z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}, \{X_{\{e_1\}}, \{y,z\}_{\{e_2,e_3\}}\},$$
$$\{\{x,z\}_{\{e_1\}}, \{y\}_{\{e_2,e_3\}}\}, \{\{x\}_{\{e_1\}}, \{y\}_{\{e_2\}}\}$$

such that every members an amply soft set over X.

We can see that clearly $\widetilde{\tau}$ is an AS topology from the *Definition 20*. And so, we can see that

$$\tau_{e_1} = \{\emptyset, X, \{x,y\}, \{x,z\}, \{x\}\},$$

$$\tau_{e_2} = \{\emptyset, X, \{y\}, \{y,z\}\}$$

and

$$\tau_{e_3} = \{\emptyset, X, \{y\}, \{z\}, \{y,z\}\}$$

are topologies on X.

Also,

Let $F * E \cong \{\{z\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, X_{\{e_3\}}\}$ and $G * E \cong \{X_{\{e_1\}}, \{x\}_{\{e_2\}}, \{x,y\}_{\{e_3\}}\}$ be amply soft sets over X. We can see that clearly these amply soft sets are AS closed sets on $\widetilde{\tau}$.

1. We want to learn whether the union of these AS closed sets is an AS closed set.

$F * E \widetilde{\cup} G * E \cong F \cup G * E = \{X_{\{e_1,e_3\}}, \{x,z\}_{\{e_2\}}\}$. And its complement is $(F \cup G * E)^{\widetilde{\prime}} \cong (F \cup G)' * E \cong \{\{y\}_{\{e_2\}}\}$. It is an AS open set that we can see it clearly in $\widetilde{\tau}$. From the *Definition 22*, the union of $F * E$ and $G * E$ is an AS closed set.

2. We want to learn whether the intersection of these AS closed sets is an AS closed set.

$F * E \widetilde{\cap} G * E \cong F \cap G * E \cong \{\{z\}_{\{e_1\}}, \{x\}_{\{e_2\}}, \{x,y\}_{\{e_3\}}\}$. And its complement is

$(F \cap G * E)^{\widetilde{\prime}} \cong (F \cap G)^{\widetilde{\prime}} * E \cong \{\{x,y\}_{\{e_1\}}, \{y,z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}$.

It is an AS open set that we can see it clearly in $\widetilde{\tau}$. From the *Definition 22*, the intersection of $F * E$ and $G * E$ is an AS closed set.

Definition 23 Let $(\widetilde{X}, \widetilde{\tau_1}, E)$ and $(\widetilde{X}, \widetilde{\tau_2}, E)$ be two AS topological space on $\widetilde{X}$. Then the followings are hold.

1. If $\widetilde{\tau_2} \widetilde{\subseteq} \widetilde{\tau_1}$ then $\widetilde{\tau_1}$ is finer then $\widetilde{\tau_2}$,
2. If $\widetilde{\tau_2} \widetilde{\subseteq} \widetilde{\tau_1}$ then $\widetilde{\tau_2}$ is coarser then $\widetilde{\tau_1}$.

Definition 24 *Let $(\widetilde{X}, \tilde{\tau}, E)$ be an AS topological space and $\tilde{\beta} \cong \tilde{\tau}$. If every element of $\tilde{\tau}$ can be written as any union of elements of $\tilde{\beta}$, then $\tilde{\beta}$ is called as an AS basis for the AS topology $\tilde{\tau}$. Then we can say that each element of $\tilde{\beta}$ is an AS basis element.*

Example 9 Let us consider the AS topology on the *Example 8.* Then

$$\widetilde{\beta} = \{\widetilde{\emptyset}, \widetilde{X}, \{\{x,y\}_{\{e_1\}}, \{y\}_{\{e_2\}}\}, \{\{y,z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}, \{\{x,z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}, \{\{y\}_{\{e_2\}}\}, \{\{x\}_{\{e_1\}}\}\}$$

is an AS basis for the AS topology $\tilde{\tau}$.

Definition 25 *Let $n \in \mathbb{N}$, $e_n \in E$. And let (X, τ_n) be topological spaces over same initial universe X. And let f be mapping from τ_n to τ_{e_n} for all $e \in E$. Then $\tilde{\beta} \cong \{\tau_{e_n}\}$ is an AS basis for an AS topology $\tilde{\tau}$. We can say that it is called as an AS topology produced by classical topologies and for shortly a PAS topology.*

Example 10 Let $X = \{x, y, z\}$, $E = \{e_1, e_2, e_3\}$ and $(X, \tau_1), (X, \tau_2), (X, \tau_3)$ are topological spaces over X such that $\tau_1 = \{\emptyset, X, \{x,y\}, \{x,z\}, \{x\}\}$, $\tau_2 = \{\emptyset, X, \{y\}, \{y,z\}\}$, $\tau_3 = \{\emptyset, X, \{y\}, \{z\}, \{y,z\}\}$.

Let $n \in \{1, 2, 3\}$, f is mapping from τ_n to τ_{e_n} for all $e \in E$. We take $\widetilde{\beta} \cong \{\tau_{e_1}, \tau_{e_2}, \tau_{e_3}\}$ with helping f, for produce AS topology over X.

$$\widetilde{\beta} \cong \{\emptyset_{\{e_1\}}, X_{\{e_1\}}, \{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_1\}}, \{x\}_{\{e_1\}}, \emptyset_{\{e_2\}}, X_{\{e_2\}}, \{y\}_{\{e_2\}}, \{y,z\}_{\{e_2\}}, \emptyset_{\{e_3\}}, X_{\{e_3\}}, \{y\}_{\{e_3\}}, \{z\}_{\{e_3\}}, \{y,z\}_{\{e_3\}}\}.$$

Then, we can define PAS topology $\tilde{\tau}$ over $\widetilde{X}$. For this we have to take any union of element of $\widetilde{\beta}$ from the *Definition 24.* So;

$$\tilde{\tau} =$$
$$\{\widetilde{\emptyset}, \widetilde{X}, \{X_{\{e_1\}}\}, \{\{x,y\}_{\{e_1\}}\}, \{\{x,z\}_{\{e_1\}}\}, \{\{x\}_{\{e_1\}}\}, \{X_{\{e_2\}}\}, \{\{y\}_{\{e_2\}}\}, \{\{y,z\}_{\{e_2\}}\},$$
$$\{X_{\{e_3\}}\}, \{\{y\}_{\{e_3\}}\}, \{\{z\}_{\{e_3\}}\}, \{\{y,z\}_{\{e_3\}}\},$$
$$\{X_{\{e_1,e_2\}}\}, \{X_{\{e_1\}}, \{y\}_{\{e_2\}}\}, \{X_{\{e_1\}}, \{y,z\}_{\{e_2\}}\}, \{\{x,y\}_{\{e_1\}}, X_{\{e_2\}}\},$$
$$\{\{x,y\}_{\{e_1\}}, \{y\}_{\{e_2\}}\}, \{\{x,y\}_{\{e_1\}}, \{y,z\}_{\{e_2\}}\}, \{\{x,z\}_{\{e_1\}}, X_{\{e_2\}}\}, \{\{x,z\}_{\{e_1\}}, \{y\}_{\{e_2\}}\},$$
$$\{\{x,z\}_{\{e_1\}}, \{y,z\}_{\{e_2\}}\}, \{\{x\}_{\{e_1\}}, X_{\{e_2\}}\}, \{\{x\}_{\{e_1\}}, \{y\}_{\{e_2\}}\}, \{\{x\}_{\{e_1\}}, \{y,z\}_{\{e_2\}}\},$$
$$\{X_{\{e_2,e_3\}}\}, \{X_{\{e_2\}}, \{y\}_{\{e_3\}}\}, \{X_{\{e_2\}}, \{z\}_{\{e_3\}}\}, \{X_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}, \{\{y\}_{\{e_2,e_3\}}\},$$
$$\{\{y\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}, \{\{y\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}, \{\{y,z\}_{\{e_2\}}, X_{\{e_3\}}\}, \{\{y,z\}_{\{e_2\}}, \{y\}_{\{e_3\}}\},$$
$$\{\{y,z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}, \{\{y,z\}_{\{e_2,e_3\}}\}, \{X_{\{e_1,e_3\}}\}, \{X_{\{e_1\}}, \{y\}_{\{e_3\}}\}, \{X_{\{e_1\}}, \{z\}_{\{e_3\}}\},$$
$$\{X_{\{e_1\}}, \{y,z\}_{\{e_3\}}\}, \{\{x,y\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}, \{\{x,y\}_{\{e_1\}}, \{z\}_{\{e_3\}}\}, \{\{x,y\}_{\{e_1\}}, \{y,z\}_{\{e_3\}}\},$$
$$\{\{x,z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}, \{\{x,z\}_{\{e_1\}}, \{z\}_{\{e_3\}}\}, \{\{x,z\}_{\{e_1\}}, \{y,z\}_{\{e_3\}}\}, \{\{x\}_{\{e_1\}}, \{y\}_{\{e_3\}}\},$$
$$\{\{x\}_{\{e_1\}}, \{z\}_{\{e_3\}}\}, \{\{x\}_{\{e_1\}}, \{y,z\}_{\{e_3\}}\}, \{X_{\{e_1,e_2\}}, \{y\}_{\{e_3\}}\}, \{X_{\{e_1,e_2\}}, \{z\}_{\{e_3\}}\},$$
$$\{X_{\{e_1,e_2\}}, \{y,z\}_{\{e_3\}}\}, \{X_{\{e_1\}}, \{y\}_{\{e_2,e_3\}}\}, \{X_{\{e_1\}}, \{y\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}, \{X_{\{e_1\}}, \{y\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\},$$
$$\{X_{\{e_1\}}, \{y,z\}_{\{e_2\}}, \{y\}_{\{e_3\}}\}, \{X_{\{e_1\}}, \{y,z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}, \{X_{\{e_1\}}, \{y,z\}_{\{e_2,e_3\}}\},$$
$$\{\{x,y\}_{\{e_1\}}, X_{\{e_2\}}, \{y\}_{\{e_3\}}\}, \{\{x,y\}_{\{e_1\}}, X_{\{e_2\}}, \{z\}_{\{e_3\}}\},$$
$$\{\{x,y\}_{\{e_1\}}, X_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}, \{\{x,y\}_{\{e_1\}}, \{y\}_{\{e_2,e_3\}}\}, \{\{x,y\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{z\}_{\{e_3\}}\},$$
$$\{\{x,y\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}, \{\{x,y\}_{\{e_1\}}, \{y,z\}_{\{e_2\}}, \{y\}_{\{e_3\}}\}, \{\{x,y\}_{\{e_1\}}, \{y,z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\},$$
$$\{\{x,y\}_{\{e_1\}}, \{y,z\}_{\{e_2,e_3\}}\}, \{\{x,z\}_{\{e_1\}}, X_{\{e_2\}}, \{y\}_{\{e_3\}}\}, \{\{x,z\}_{\{e_1\}}, X_{\{e_2\}}, \{z\}_{\{e_3\}}\}, \{\{x,z\}_{\{e_1\}}, X_{\{e_2\}},$$
$$\{y,z\}_{\{e_3\}}\}, \{\{x,z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{y\}_{\{e_3\}}\}, \{\{x,z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{z\}_{\{e_3\}}\},$$
$$\{\{x,z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}, \{\{x,z\}_{\{e_1\}}, \{y,z\}_{\{e_2\}}, \{y\}_{\{e_3\}}\}, \{\{x,z\}_{\{e_1\}}, \{y,z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\},$$
$$\{\{x,z\}_{\{e_1\}}, \{y,z\}_{\{e_2,e_3\}}\}, \{\{x\}_{\{e_1\}}, X_{\{e_2\}}, \{y\}_{\{e_3\}}\}, \{\{x\}_{\{e_1\}}, X_{\{e_2\}}, \{z\}_{\{e_3\}}\},$$
$$\{\{x\}_{\{e_1\}}, X_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}, \{\{x\}_{\{e_1\}}, \{y\}_{\{e_2,e_3\}}\}, \{\{x\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{z\}_{\{e_3\}}\},$$
$$\{\{x\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}, \{\{x\}_{\{e_1\}}, \{y,z\}_{\{e_2\}}, \{y\}_{\{e_3\}}\}, \{\{x\}_{\{e_1\}}, \{y,z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\},$$
$$\{\{x\}_{\{e_1\}}, \{y,z\}_{\{e_2,e_3\}}\}\}.$$

Hence, we establish PAS topological space $(\widetilde{X}, \tilde{\tau}, E)$ over $\widetilde{X}$ from the AS basis $\widetilde{\beta} \cong \{\tau_{e_1}, \tau_{e_2}, \tau_{e_3}\}$.

Proposition 9 Any PAS topological space $(\widetilde{X}, \widetilde{\tau_1}, E)$ is finer then the AS topological space $(\widetilde{X}, \widetilde{\tau_2}, E)$ over same universe $\widetilde{X}$.

Example 11 Let us consider the PAS topology on the *Example 10* and consider the AS topology $\tilde{\tau}$ over same universe on *Example 8*. We can see that clearly, any element of the AS topology is subset of the PAS topology. So we can say that PAS topological space $(\widetilde{X}, \widetilde{\tau_1}, E)$ is finer then AS topological space $(\widetilde{X}, \widetilde{\tau_2}, E)$ over same universe $\widetilde{X}$.

Proposition 10 Let $n \in \mathbb{N}, e_n \in E, (X, \tau_n)$ be topological spaces over same initial universe X, β_n be basis for each τ_n and let f_1 be mapping from β_n to β_{e_n} for all $e \in E, \widetilde{\beta_1} \cong \{\beta_{e_n}\}$ be an AS basis for an AS topology $\widetilde{\tau_1}$. And let $\widetilde{\beta_2} \cong \{\tau_{e_n}\}$ be an AS basis for an AS topology $\widetilde{\tau_2}$ such that f_2 be mapping from τ_n to τ_{e_n} for all $e \in E$. Then PAS topological space $(\widetilde{X}, \widetilde{\tau_1}, E)$ is finer then PAS topological space $(\widetilde{X}, \widetilde{\tau_2}, E)$.

Definition 26 Let $(\widetilde{X}, \tilde{\tau}, E)$ be an AS topological space and $F * A$ be an AS set over $\widetilde{X}$. Then the collection,

$$\widetilde{\tau_{F*A}} \cong \{F * A \,\widetilde{\cap}\, (G * B)_i : (G * B)_i \,\widetilde{\in}\, \tilde{\tau}, i \in I\}$$

is called a subspace AS topology on $F * A$. Hence, $(F * A, \widetilde{\tau_{F*A}}, A)$ is called as an AS topological subspace of $(\widetilde{X}, \tilde{\tau}, E)$.

Definition 27 Let (X, τ, E) be an AS topological space and $F * A$ be an AS set over $\widetilde{X}$. Then the interior of $F * A$ denoted by $(F * A)^{\tilde{\delta}}$ is defined as a union of all AS open subsets of $F * A$.

Note that this, $(F * A)^{\tilde{\delta}}$ is the biggest AS open set that is contained by $F * A$.

Example 12 Let us consider the AS topology $(\widetilde{X}, \tilde{\tau}, E)$ on the *Example 8*. And $F * A \cong \{\{x, y\}_{\{e_1\}}, \{z\}_{\{e_2\}}\}$ be an AS set over X such that $A = \{e_1, e_2\} \subset E$. Then,

$$(F * A)^{\tilde{\delta}} = \{\{x\}_{\{e_1\}}\} \,\widetilde{\cup}\, \widetilde{\emptyset} \cong \{\{x\}_{\{e_1\}}\}.$$

Example 13 Let us consider the PAS topology $(X, \tilde{\tau}, E)$ on the *Example 10* and $F * A \cong \{\{x, y\}_{\{e_1\}}, \{z\}_{\{e_2\}}\}$ be an AS set over X such that $A = \{e_1, e_2\} \subset E$. Then,

$$(F * A)^{\tilde{\delta}} \cong \{\{x, y\}_{\{e_1\}}\} \,\widetilde{\cup}\, \widetilde{\emptyset} \cong \{\{x, y\}_{\{e_1\}}\}.$$

Proposition 11 Let $F * A$ be an AS set on X and let τ_1 be a PAS topological space and τ_2 be an AS topological space on X. Then, $(F * A)^{\tilde{\delta}}$ on $\widetilde{\tau_2}$ is contained by $(F * A)^{\tilde{\delta}}$ on $\widetilde{\tau_1}$.

Proof. It is obvious from the *Proposition 9* and the *Definition 27*. □

Definition 28 Let (X, τ, E) be an AS topological space and $F * A$ be an AS set over $\widetilde{X}$. Then the closure of $F * A$ denoted by $\overline{F * A}$ is defined as the intersection of all AS closed supersets of $F * A$.

Note that this, $\overline{F * A}$ is the smallest AS closed set is containing $F * A$.

Example 14 Let us consider the AS topology $(\widetilde{X}, \tilde{\tau}, E)$ on the *Example 8* and $F * A \cong \{\{x, y\}_{\{e_1\}}, \{z\}_{\{e_2\}}\}$ be an AS set over $\widetilde{X}$ such that $A = \{e_1, e_2\} \subset E$. Then,

$$F * A \cong \widetilde{X} \,\widetilde{\cap}\, \{X_{\{e_1, e_3\}}, \{x, z\}_{\{e_2\}}\} \cong \{X_{\{e_1, e_3\}}, \{x, z\}_{\{e_2\}}\}.$$

Example 15 Let us consider the PAS topology $(\widetilde{X}, \tilde{\tau}, E)$ on the *Example 10* and $F * A \cong \{\{x, y\}_{\{e_1\}}, \{z\}_{\{e_2\}}\}$ be an AS set over X such that $A = \{e_1, e_2\} \subset E$. Then,

$$F * A \cong \widetilde{X} \,\widetilde{\cap}\, \{X_{\{e_1, e_3\}}, \{x, z\}_{\{e_2\}}\} \,\widetilde{\cap}\, \{X_{\{e_1, e_2\}}\} \,\widetilde{\cap}\, \{X_{\{e_1, e_2\}}, \{z\}_{\{e_3\}}\} \,\widetilde{\cap}\, \{X_{\{e_1, e_2\}}, \{y\}_{\{e_3\}}\} \,\widetilde{\cap}\,$$
$$\{X_{\{e_1, e_2\}}, \{y, z\}_{\{e_3\}}\} \,\widetilde{\cap}\, \{X_{\{e_1\}}, \{x, z\}_{\{e_2, e_3\}}\} \,\widetilde{\cap}\, \{X_{\{e_1\}}, \{x, z\}_{\{e_2\}}, \{x, y\}_{\{e_3\}}\} \,\widetilde{\cap}\, \{X_{\{e_1\}}, \{x, z\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$$
$$\cong \{X_{\{e_1\}}, \{x, z\}_{\{e_2\}}\}.$$

Proposition 12 Let $F * A$ be an AS set, $\widetilde{\tau_1}$ be a PAS topological space and $\widetilde{\tau_2}$ be an AS topological space on $\widetilde{X}$. Then, $\overline{F * A}$ on $\widetilde{\tau_2}$ is containing $\overline{F * A}$ on $\widetilde{\tau_1}$.

Proof. It is obvious from the *Proposition 9* and the *Definition 28*. □

Note that these separation axioms defined as parametric separations in below are different from T_i separation axioms. T_i questions the relationship between the elements of space itself while P_i questions the strength of the connection between their parameters.

Definition 29 Let $(\widetilde{X}, \widetilde{\tau}, E)$ be an AS topological space and $a, b \in E$ such that $a \neq b$, if there exist $x \in X$ and. AS open sets $F * A$ and $G * B$ such that $x_a \widetilde{\in} F * A$ and $x_b \widetilde{\notin} F * A$; or $x_b \widetilde{\in} G * B$ and $x_a \widetilde{\notin} G * B$, then $(\widetilde{X}, \widetilde{\tau}, E)$ is called as a P_0 space.

Example 16 Let us consider the AS topology $(\widetilde{X}, \widetilde{\tau}, E)$ on the *Example 8*.

For $e_1, e_2 \in E$, there exist $x \in X$ and AS open set $\{\{x, y\}_{\{e_1\}}, \{y\}_{\{e_2\}}\}$ such that $x_{e_1} \widetilde{\in} \{\{x, y\}_{\{e_1\}}, \{y\}_{\{e_2\}}\}$ and $x_{e_2} \widetilde{\notin} \{\{x, y\}_{\{e_1\}}, \{y\}_{\{e_2\}}\}$.

For $e_1, e_3 \in E$, there exist $y \in X$ and AS open set $\{\{x, z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}$ such that $y_{e_3} \widetilde{\in} \{\{x, z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}$ and $y_{e_1} \widetilde{\notin} \{\{x, z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}$.

For $e_2, e_3 \in E$, there exist $y \in X$ and AS open set $\{\{y, z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}$ such that $y_{e_2} \widetilde{\in} \{\{y, z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}$ and $y_{e_3} \widetilde{\notin} \{\{y, z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}$.

So $(\widetilde{X}, \widetilde{\tau}, E)$ is a P_0 space from the *Definition 29*. □

Definition 30 Let $(\widetilde{X}, \widetilde{\tau}, E)$ be an AS topological space and $a, b \in E$ such that $a \neq b$. If there exist $x \in X$ and AS open sets $F * A$ and $G * B$ such that $x_a \widetilde{\in} F * A$ and $x_b \widetilde{\notin} F * A$; and $x_b \widetilde{\in} G * B$ and $x_a \widetilde{\notin} G * B$, then $(\widetilde{X}, \widetilde{\tau}, E)$ is called as a P_1 space.

Example 17 Let us consider the AS topology $(\widetilde{X}, \widetilde{\tau}, E)$ on the *Example 8*.

For $e_1, e_2 \in E$ there exist $z \in X$ and AS open sets $\{\{x, z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}$ and $\{\{y, z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}$ such that $z_{e_1} \widetilde{\in} \{\{x, z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}$ and $z_{e_2} \widetilde{\notin} \{\{x, z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}$; and $z_{e_2} \widetilde{\in} \{\{y, z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}$ and $z_{e_1} \widetilde{\notin} \{\{y, z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}$.

For $e_1, e_3 \in E$ there exist $y \in X$ and AS open sets $\{\{x, z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}$ and $\{\{x, y\}_{\{e_1\}}, \{y\}_{\{e_2\}}\}$ such that $y_{e_3} \widetilde{\in} \{\{x, z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}$ and $y_{e_1} \widetilde{\notin} \{\{x, z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}$; and $y_{e_1} \widetilde{\in} \{\{x, y\}_{\{e_1\}}, \{y\}_{\{e_2\}}\}$ and $y_{e_3} \widetilde{\notin} \{\{x, y\}_{\{e_1\}}, \{y\}_{\{e_2\}}\}$.

For $e_2, e_3 \in E$ there exist $y \in X$ and AS open sets $\{\{y, z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}$ and $\{\{x, z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}$ such that $y_{e_2} \widetilde{\in} \{\{y, z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}$ and $y_{e_3} \widetilde{\notin} \{\{y, z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}$ and $y_{e_3} \widetilde{\in} \{\{x, z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}$ and $y_{e_2} \widetilde{\notin} \{\{x, z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}$.

So $(\widetilde{X}, \widetilde{\tau}, E)$ is a P_1 space from the *Definition 30*.

Theorem 1 Let $(\widetilde{X}, \widetilde{\tau}, E)$ be a P_1 space, then it is also a P_0 space.

Proof. It is obvious from the *Definition 30* and the *Definition 29*. □

Definition 31 Let $(\widetilde{X}, \widetilde{\tau}, E)$ be an AS topological space and $a, b \in E$ such that $a \neq b$. If there exist $x \in X$ and AS open sets $F * A$ and $G * B$ such that $x_a \widetilde{\in} F * A$, $x_b \widetilde{\in} G * B$ and $F * A \widetilde{\cap} G * B \cong \widetilde{\emptyset}$, then $(\widetilde{X}, \widetilde{\tau}, E)$ is called as a P_2 space.

Example 18 Let us consider the AS topology $(\widetilde{X}, \widetilde{\tau}, E)$ on the *Example 8*.

For $e_1, e_2 \in E$, there exist $z \in X$ and AS open sets $\{\{x, z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}$ and $\{\{y, z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}$ such that $z_{e_1} \widetilde{\in} \{\{x, z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\}, z_{e_2} \widetilde{\in} \{\{y, z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}$ and

$$\{\{x, z\}_{\{e_1\}}, \{y\}_{\{e_3\}}\} \widetilde{\cap} \{\{y, z\}_{\{e_2\}}, \{z\}_{\{e_3\}}\}\} \cong \widetilde{\emptyset},$$

But for $e_1, e_3 \in E$, there aren't any exist $x \in X$ and AS open sets $F * A$ and $G * B$ such that $x_{e_1} \widetilde{\in} F * A$, $x_{e_3} \widetilde{\in} G * B$ and $F * A \widetilde{\cap} G * B \cong \widetilde{\emptyset}$.

So (X, τ, E) is not a P_2 space.

Theorem 2 *Let* $(\tilde{X}, \tilde{\tau}, E)$ *be a* P_2 *space, then it is also a* P_1 *space.*

Proof. Let $(\tilde{X}, \tilde{\tau}, E)$ be a P_2 space and $\mathbf{a}, \mathbf{b} \in \mathbf{E}$ such that $\mathbf{a} \neq \mathbf{b}$. Then there exist $\mathbf{x} \in \mathbf{X}$ and AS open sets $\mathbf{F} * \mathbf{A}$ and $\mathbf{G} * \mathbf{B}$ such that $\mathbf{x_a} \widetilde{\in} \mathbf{F} * \mathbf{A}, \mathbf{x_b} \in \mathbf{G} * \mathbf{B}$ and $\mathbf{F} * \mathbf{A} \widetilde{\cap} \mathbf{G} * \mathbf{B} = \widetilde{\emptyset}$ from the *Definition 31*. Because of $\mathbf{F} * \mathbf{A} \widetilde{\cap} \mathbf{G} * \mathbf{B} \cong \widetilde{\emptyset}$, there exist AS open sets $\mathbf{F} * \mathbf{A}$ and $\mathbf{G} * \mathbf{B}$ clearly that $\mathbf{x_a} \widetilde{\in} \mathbf{F} * \mathbf{A}, \mathbf{x_b} \widetilde{\notin} \mathbf{F} * \mathbf{A}$ and $\mathbf{x_b} \widetilde{\in} \mathbf{G} * \mathbf{B}, \mathbf{x_a} \widetilde{\notin} \mathbf{G} * \mathbf{B}$. This is true for all $\mathbf{x} \in \mathbf{X}$ and then, it is a P_1 space from the *Definition 30*. □

Theorem 3 *Any PAS topological space* $(\tilde{X}, \tilde{\tau}, E)$ *is a* P_2 *space.*

Proof. Let $(\tilde{X}, \tilde{\tau}, E)$ be a PAS topological space, $n \in \mathbb{N}$, $e_n \in E$. There exist topological spaces (X, τ_n) over same initial universe X. And f be mapping from τ_n *to* τ_{e_n} for all e$\in E$; $\tilde{\beta} = \{\tau_{e_n}\}$ be an AS basis for a PAS topology $\tilde{\tau}$ from the *Definition 25*. For $e_1, e_2 \in E$ such that $e_1 \neq e_2$. Therefore, there exist AS open sets $\{X_{\{e_1\}}\}$ and $\{X_{\{e_2\}}\}$ such that $x_{e_1} \widetilde{\in} \{X_{\{e_1\}}\}, x_{e_2} \widetilde{\in} \{X_{\{e_2\}}\}$ and $\{X_{\{e_1\}}\} \widetilde{\cap} \{X_{\{e_2\}}\} \cong \widetilde{\emptyset}$. This is true for all $x \in X$ and then, (X, τ, E) is a P_2 space from the *Definition 31*. □

Example **19** Let us consider the PAS topology $(\tilde{X}, \tilde{\tau}, E)$ on the *Example 10*. It is clearly seen that $(\tilde{X}, \tilde{\tau}, E)$ is a P_2 space from the *Definition 31*.

Example **20** Let $e_1, e_2 \in E, x \in X$ and (X, τ_{e_1}) and (X, τ_{e_2}) be indiscrete topological spaces over same universe X. And let $(\tilde{X}, \tilde{\tau}, E)$ be their PAS topology. So; $\tilde{\tau} = \{\widetilde{\emptyset}, \tilde{X}, \{X_{\{e_1\}}\}, \{X_{\{e_2\}}\}\}$ is an AS topological space over $\tilde{X}$.

Therefore there exist AS open sets $\{X_{\{e_1\}}\}$ and $\{X_{\{e_2\}}\}$ such that $x_{\mathbf{e_1}} \widetilde{\in} \{X_{\{e_1\}}\}$, $x_{\mathbf{e_2}} \widetilde{\in} \{X_{\{e_2\}}\}$ and $\{X_{\{e_1\}}\} \widetilde{\cap} \{X_{\{e_2\}}\} \cong \widetilde{\emptyset}$. So $(\tilde{X}, \tilde{\tau}, E)$ is a P_2 space from the *Definition 31*.

Definition 32 *Let* $(\tilde{X}, \tilde{\tau}, E)$ *be an AS topological space,* $H * C$ *be an AS closed set,* $a \in E$, $x \in X$ *such that* $x_a \widetilde{\notin} H * C$. *If there exist* $x \in X$ *and AS open sets* $F * A$ *and* $G * B$ *such that* $x_a \widetilde{\in} F * A$, $H * C \widetilde{\subseteq} G * B$ *and* $F * A \widetilde{\cap} G * B \cong \widetilde{\emptyset}$ *then* $(\tilde{X}, \tilde{\tau}, E)$ *is called as a Halime space.*

Definition 33 *Let* $(\tilde{X}, \tilde{\tau}, E)$ *be an AS topological space. Then it is said to be a* P_3 *space if it is both a Halime space and a* P_1 *space.*

Example **21** Let us consider the AS topology $(\tilde{X}, \tilde{\tau}, E)$ on the *Example 8*. Let us see if it is Halime space.

For $e_1 \in E$, choose x_{e_1}, there exist AS closed sets $H * C$ such as $\{\{z\}_{\{e_1\}}, \{x, z\}_{\{e_2\}}, X_{\{e_3\}}\}$, $\{\{y\}_{\{e_1\}}, X_{\{e_2\}}, \{x, z\}_{\{e_3\}}\}$, $\{\{y, z\}_{\{e_1\}}, X_{\{e_2, e_3\}}\}$, $\{\{z\}_{\{e_1\}}, \{x\}_{\{e_2\}}, \{x, y\}_{\{e_3\}}\}$ such that $x_{e_1} \notin H * C$. And AS open set of super set of each $H * C$ is only $\tilde{X}$. So there is not any exist AS open set $F * A$ such that $x_{e_1} \widetilde{\in} F * A$ and $F * A \widetilde{\cap} \tilde{X} \cong \widetilde{\emptyset}$.

This situation is similarly for y_{e_1} and z_{e_1}. So (X, τ, E) is not a Halime space.

Example 22 *Let* $X = \{x, y, z\}$ *be a universal set,* $E = \{e_1, e_2, e_3\}$ *be a parameter set,* $\tilde{\tau} = \{\widetilde{\emptyset}, \tilde{X}, \{\{x, y\}_{\{e_1\}}, \{x, z\}_{\{e_2\}}, \{y, z\}_{\{e_3\}}\}, \{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}\}$ *such that every members an amply soft set* $E \to P(X)$. *Then* $(\tilde{X}, \tilde{\tau}, E)$ *is an AS topological space. Let us see if it is a* P_3 *space.*

Firstly, let us see if it is a Halime space.

For $e_1 \in E$, choose $x_{\mathbf{e_1}}$ and for an AS closed set $\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$ such that $x_{\mathbf{e_1}} \widetilde{\notin} \{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$. There exists an AS open set $\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$ such that $\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\} \widetilde{\subseteq} \{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$. And there exists an AS open set $\{\{x, y\}_{\{e_1\}}, \{x, z\}_{\{e_2\}}, \{y, z\}_{\{e_3\}}\}$ such that $x_{\mathbf{e_1}} \widetilde{\in} \{\{x, y\}_{\{e_1\}}, \{x, z\}_{\{e_2\}}, \{y, z\}_{\{e_3\}}\}$ and $\{\{x, y\}_{\{e_1\}}, \{x, z\}_{\{e_2\}}, \{y, z\}_{\{e_3\}}\} \widetilde{\cap} \{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\} \cong \widetilde{\emptyset}$.

For $e_2 \in E$, choose $x_{\mathbf{e_2}}$ and for an AS closed set $\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$ such that $x_{\mathbf{e_2}} \widetilde{\notin} \{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$. There exists an AS open set $\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$ such that $\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\} \widetilde{\subseteq} \{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$. And there exists an AS open set

$\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}$ such   that $x_{e_2} \widetilde{\in} \{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}$ and $\{\{x,y\}_{\{e^1\}}, \{x,z\}_{\{e^2\}}, \{y,z\}_{\{e^3\}}\} \widetilde{\cap} \{\{z\}_{\{e^1\}}, \{y\}_{\{e^2\}}, \{x\}_{\{e^3\}}\} \cong \widetilde{\emptyset}$.

For $e_3 \in E$, choose z_{e_3} and for an AS closed set $\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$ such that $z_{e_3} \widetilde{\notin} \{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$. There exists an AS open set $\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$ such that $\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\} \widetilde{\subseteq} \{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$. And there exists an AS open set $\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}$ such   that $z_{e_3} \widetilde{\in} \{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}$ and $\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\} \widetilde{\cap} \{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\} \cong \widetilde{\emptyset}$,

So $(\widetilde{X}, \widetilde{\tau}, E)$ is a Halime space from the *Definition 32*.

Finally, let us see if it is a P_1 space.

For $e_1, e_2 \in E$, there exist $y \in X$ and AS open sets $\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}$ and $\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$ such   that $y_{e_1} \widetilde{\in} \{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}$ and $y_{e_2} \widetilde{\notin} \{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}$;   and $y_{e_2} \in \{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$ and $y_{e_1} \widetilde{\notin} \{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$.

For $e_1, e_3 \in E$, there exist $x \in X$ and AS open sets $\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}$ and $\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$ such   that $x_{e_1} \widetilde{\in} \{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}$ and $x_{e_3} \widetilde{\notin} \{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}$;   and $x_{e_3} \widetilde{\in} \{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$ and $x_{e_1} \widetilde{\notin} \{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$.

For $e_2, e_3 \in E$, there exist $y \in X$ and AS open sets $\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}$ and $\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$ such   that $y_{e_3} \widetilde{\in} \{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}$ and $y_{e_2} \widetilde{\notin} \{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}$;   and $y_{e_2} \widetilde{\in} \{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$ and $y_{e_3} \notin \{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$.

Therefore, $(\widetilde{X}, \widetilde{\tau}, E)$ is a P_1 space from the Definition 19. $(\widetilde{X}, \widetilde{\tau}, E)$ is a P_3 space because it is both a Halime space and a P_1 space.

Theorem 4 Let $(\widetilde{X}, \widetilde{\tau}, E)$ be a P_3 space, then it is also a P_2 space.

Proof. Let $(\widetilde{X}, \widetilde{\tau}, E)$ be a P_3 space. Then it is both a P_1 and a Halime space from the *Definition 33*.

Because $(\widetilde{X}, \widetilde{\tau}, E)$ is a Halime space, for an AS closed set $H * C$ and $a \in E, x \in X$ such that $x_a \widetilde{\notin} H * C$, there exist AS open sets $F * A$ and $G * B$ such that $x_a \widetilde{\in} F * A, H * C \widetilde{\subseteq} G * B$ and $F * A \widetilde{\cap} G * B \cong \widetilde{\emptyset}$. So from last equality it is seen that $x_a \widetilde{\in} F * A, x_a \widetilde{\notin} G * B$. Because $(\widetilde{X}, \widetilde{\tau}, E)$ is also a P_1 space and $G * B$ is an AS open set, there exist $x_b \widetilde{\in} G * B$ such that $b \in E$. And now on, We can see that clearly, there exist AS open sets $F * A$ and $G * B$ such that $x_a \widetilde{\in} F * A, x_b \widetilde{\in} G * B$ and $F * A \widetilde{\cap} G * B \cong \widetilde{\emptyset}$ so (X, τ, E) is a P_2 space, from the *Definition 31*.   □

Definition 34 Let $(\widetilde{X}, \widetilde{\tau}, E)$ be an AS topological space, $H * C$ and $K * D$ be AS closed sets such that $H * C \widetilde{\cap} K * D = \widetilde{\emptyset}$. If there exist AS open sets $F * A$ and $G * B$ such that $H * C \widetilde{\subseteq} F * A$, $K * D \widetilde{\subseteq} G * B$ and $F * A \widetilde{\cap} G * B \cong \widetilde{\emptyset}$ then $(\widetilde{X}, \widetilde{\tau}, E)$ is called as an Orhan space.

Definition 35 Let $(\widetilde{X}, \widetilde{\tau}, E)$ be an AS topological space. Then it is said to be a P_4 space if it is both an Orhan space and a P_1 space.

Example 23 Let us consider the AS topology $(\widetilde{X}, \widetilde{\tau}, E)$ on the Example 22. Let us see if it is a P_4 space.

$\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\}$ and $\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\}$ are disjoint AS closed sets on it. And we know that these are also disjoint AS open sets. So $(\widetilde{X}, \widetilde{\tau}, E)$ is an Orhan Space. And also $(\widetilde{X}, \widetilde{\tau}, E)$ is a P_1 space from the *Example 22, so* $(\widetilde{X}, \widetilde{\tau}, E)$ is a P_4 space from the *Definition 35*.

Example 24 Let us consider the PAS topology on the Example 20. Let us see if it is a P_4 space.

Firstly, let's see if it is an Orhan space. There exist AS closed sets $\{X_{\{e_1\}}\}$ and $\{X_{\{e_2\}}\}$ such that $\{X_{\{e_1\}}\} \, \widetilde{\cap} \, \{X_{\{e_2\}}\} \cong \widetilde{\emptyset}$. And we know that $\{X_{\{e_1\}}\}$ and $\{X_{\{e_2\}}\}$ are also AS open sets such that $\{X_{\{e_1\}}\} \, \widetilde{\subseteq} \, \{X_{\{e_1\}}\}, \{X_{\{e_2\}}\} \, \widetilde{\subseteq} \, \{X_{\{e_2\}}\}$ and $\{X_{\{e_1\}}\} \, \widetilde{\cap} \, \{X_{\{e_2\}}\} \cong \widetilde{\emptyset}$. So $(\widetilde{X}, \widetilde{\tau}, E)$ is an Orhan space.

Finally, let's see if it is a P_1 space.

There exist AS open sets $\{X_{\{e_1\}}\}$ and $\{X_{\{e_2\}}\}$ such that $x_{e_1} \widetilde{\in} \{X_{\{e_1\}}\}$, $x_{e_2} \widetilde{\notin} \{X_{\{e_1\}}\}$ and $x_{e_2} \widetilde{\in} \{X_{\{e_2\}}\}$, $x_{e_1} \widetilde{\notin} \{X_{\{e_2\}}\}$. So $(\widetilde{X}, \widetilde{\tau}, E)$ is a P_1 space. Because $(\widetilde{X}, \widetilde{\tau}, E)$ is both an Orhan space and a P_1 space, it is a P_4 space from the *Definition 35*.

Proposition 13 *Any PAS topological space* $(\widetilde{X}, \widetilde{\tau}, E)$ *may not be a* P_4 *space.*

Example 25 *Let* $\mathbb{R}$ *be real numbers,* $E = \{e_1, e_2\}$ *be a universal parameter set,* $(\mathbb{R}, \tau_1)$ *a discrete topological space and* $(\mathbb{R}, \tau_2)$ *a finite complement topological space and from the* Definition 25, *their PAS topology over* $\widetilde{\mathbb{R}}$ $(\widetilde{\mathbb{R}}, \widetilde{\tau}, E)$.

Now let us see if $\widetilde{\mathbb{R}}$ is a P_4 space.

Firstly, we can say that the PAS topology is a P_1 space because of the *Theorem 3* and the *Theorem 2*. Let us see it,

For e_1, e_2; there exist $2 \in \mathbb{R}$ and $\{\{2\}_{\{e_1\}}, (\mathbb{R} - \{2, 3, 5\})_{\{e_2\}}\}$, $\{\{3\}_{\{e_1\}}, (\mathbb{R} - \{3, 5\})_{\{e_2\}}\} \, \widetilde{\in} \, \widetilde{\tau}$ such that $2_{e_1} \widetilde{\in} \{\{2\}_{\{e_1\}}, (\mathbb{R} - \{2, 3, 5\})_{\{e_2\}}\}$ and $2_{e_2} \widetilde{\notin} \{\{2\}_{\{e_1\}}, (\mathbb{R} - \{2, 3, 5\})_{\{e_2\}}\}$; $2_{e_2} \widetilde{\in} \{\{3\}_{\{e_1\}}, (\mathbb{R} - \{3,5\})_{\{e_2\}}\}$ and $2_{\{e_1\}} \widetilde{\notin} \{\{3\}_{\{e_1\}}, (\mathbb{R} - \{3,5\})_{\{e_2\}}\}$. Therefore, it is clearly seen that $(\mathbb{R}, \tau, E)$ is a P_1 space from the *Definition 30*.

Finally, let us see if $\widetilde{\mathbb{R}}$ is an Orhan space. Let $U, V \widetilde{\subseteq} \mathbb{R}$ be finite sets. Then we can choose $\{U_{\{e_2\}}\}$ and $\{V_{\{e_2\}}\}$ AS closed sets such that $\{U_{\{e_2\}}\} \, \widetilde{\cap} \, \{V_{\{e_2\}}\} \cong \widetilde{\emptyset}$ But for $A, B \subseteq E$, we can't find any AS open sets $F * A$ and $G * B$ such that $\{U_{\{e_2\}}\} \widetilde{\subseteq} F * A$, $\{V_{\{e_2\}}\} \widetilde{\subseteq} G * B$ and $F * A \, \widetilde{\cap} \, G * B \cong \widetilde{\emptyset}$. For this purpose, suppose that, there exist $\{(R - U)_{\{e_2\}}\}$, $\{(R - V)_{\{e_2\}}\}$ be AS open sets such that $\{V_{\{e_2\}}\} \widetilde{\subseteq} \{(R - U)_{\{e_2\}}\}$, $\{U_{\{e_2\}}\} \widetilde{\subseteq} \{(R - V)_{\{e_2\}}\}$ and $\{(R - U)_{\{e_2\}}\} \, \widetilde{\cap} \, \{(R - V)_{\{e_2\}}\} = \emptyset$. Therefore, we can obtain $U \cup V = \mathbb{R}$. This result contradicts the finite selection of U and V. Hence, $(\widetilde{\mathbb{R}}, \widetilde{\tau}, E)$ is not an Orhan space and so it is not a P_4 space.

Theorem 5 *Let* $(\widetilde{X}, \widetilde{\tau}, E)$ *be a* P_4 *space, then it is also a* P_3 *space.*

Proof. Let $(\widetilde{X}, \widetilde{\tau}, E)$ be a P_4 space. Then it is both a P_1 and an Orhan space from the *Definition 35*. Because $(\widetilde{X}, \widetilde{\tau}, E)$ is an Orhan space, for AS closed sets $H * C$ and $K * D$ such that $H * C \, \widetilde{\cap} \, K * D \cong \widetilde{\emptyset}$, there exist AS open sets $F * A$ and $G * B$ such that $H * C \widetilde{\subseteq} F * A, K * D \widetilde{\subseteq} G * B$ and $F * A \cap G * B = \emptyset$. For this,

1. Let AS closed set $H * C = \emptyset$, then for $x \in X, a \in E; x_a \widetilde{\notin} H * C$ and for $x_a \widetilde{\in} G * B$, there exist $F * A$ and $G * B$ such that $H * C \widetilde{\subseteq} F * A, K * D \widetilde{\subseteq} G * B$ and $F * A \, \widetilde{\cap} \, G * B \cong \widetilde{\emptyset}$, so (X, τ, E) is a Halime space from the *Definition 32*.

2. Let AS closed set $H * C \not\cong \widetilde{\emptyset}$, then there exist $x_a \widetilde{\in} H * C$ and then there exist AS closed set $K * D$ such that $x_a \widetilde{\notin} K * D$ because $H * C \, \widetilde{\cap} \, K * D \cong \widetilde{\emptyset}$. Then $(\widetilde{X}, \widetilde{\tau}, E)$ is an Orhan space, so there exist AS open sets $F * A$ and $G * B$ such that $H * C \widetilde{\subseteq} F * A$, $K * D \widetilde{\subseteq} G * B$ and $F * A \, \widetilde{\cap} \, G * B \cong \widetilde{\emptyset}$. Therefore $(\widetilde{X}, \widetilde{\tau}, E)$ is also a Halime space. And since $(\widetilde{X}, \widetilde{\tau}, E)$ is both a Halime and a P_1 space, it is a P_3 space from the *Definition 33*. □

Corolary 1 *Any AS topological space* $(\widetilde{X}, \widetilde{\tau}, E)$ *is a* P_4 *space* $\Rightarrow$ P_3 *space* $\Rightarrow$ P_2 *space* $\Rightarrow$ P_1 *space* $\Rightarrow$ P_0 *space.*

Proof. It is clearly seen that from the *Theorem 5*, the *Theorem 4*, the *Theorem 2* and the *Theorem 1* respectively. □

4. Conclusions

I defined an amply soft set. Amply soft sets use any kind of universal parameter set and initial universe (such as finite or infinite, countable or uncountable). Also, I introduced subset, superset, equality, empty set, and whole set about amply soft sets. And I gave operations such as union, intersection, difference of two amply soft sets and complement of an amply soft set. Then three different amply soft point such as amply soft whole point, amply soft point, and monad point were defined. Also examples related taking universal set as uncountable were given.

I defined a new soft topology, and it is called as a PAS topology. The PAS topology allows to write different elements of classical topologies in its each parameter sets. The classical topologies may be finite, infinite, countable or uncountable. This situation removes all of the boundaries in a soft topology and cause it to spread over larger areas. A PAS topology is a special case of an AS topology. For this purpose, I defined a new soft topology, and it is called as an amply soft topology or briefly an AS topology. I introduce AS open sets, AS closed sets, interior and closure of an AS set and subspace of any AS topological space. I gave parametric separation axioms which are different from T_i separation axioms. T_i questions the relationship between the elements of space itself while P_i questions the strength of the connection between their parameters.

In our next study, we will touch on the concept of a monad metrizable space [46] which we took the preliminary step in this study. A space concept in monad metrizable space will gain a new perspective. I will compute distance in real space between different topologies or metric spaces in different dimensions. First of all, I will define passing points between different topologies. And I will define monad metric.

Conflict of interest

The author declares that there is no conflict of interest regarding the publication of this paper.

References

1. B. Greene, Elegant universe superstrings, hidden dimensions, and the quest for the ultimate Theory, In: *The Glitters of M Theory*, 4 Eds., TÜBİTAK: Ankara, Turkey, 2011.
2. D. Molodtsov, Soft set theory first results, *Comput. Math. Appl.,* **37** (1999), 19–31.
3. W. L. Gau, D. J. Buehrer, Vague sets, *IEEE Trans. Syst. Man. Cybernet.,* **23** (1993), 610–614.
4. L. A. Zadeh, Fuzzy sets, *Inf. Control,* **8** (1965), 338–353.
5. K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets Syst.*, **20** (1986), 87–96.
6. Z. Pawlak, Rough sets, *Int. J. Comp. Inf. Sci.,* **11** (1982), 341–356.
7. M. B. Gorzalzany, A method of inference in approximate reasoning based on interval valued fuzzy sets, *Fuzzy Sets Syst.,* **21** (1987), 1–17.
8. Q. T. Bui, B. Vo, H. N. Do, N. Q. V. Hung, V. Snasel, F-mapper: A fuzzy mapper clustering algorithm*, Knowl.-Based Syst.,* **189** (2020), 105107.
9. Q. T. Bui, B. Vo, V. Snasel, W. Pedrycz, M. Y. Chen, SFCM: A fuzzy clustering algorithm of extracting the shape information of data, *IEEE Trans. Fuzzy Syst.,* **29** (2021), 75–89.

10. S. G. Quek, G. Selvachandran, F. Smarandache, J. Vimala, S. H. Le, Q. T. Bui, V. C. Gerogiannis, Entropy measures for plithogenic sets and applications in multi-attribute decision making, *Mathematics,* **8** (2020), 965.

11. J. Zhan, J. C. R. Alcantud, A novel type of soft rough covering and its application to multicriteria group decision making, *Artif. Intell. Rev.,* **52** (2019), 2381–2410.

12. K. Zhang, J. M. Zhang, W. Z. Wu, J. C. A. Alcantud, Fuzzy β-covering based (*I,T*)-fuzzy rough set models and applications to multi-attribute decision-making, *Comput. Indus. Eng.,* **128** (2019), 605–621.

13. J. C. R. Alcantud, V. Torra, Decomposition theorems and extension principles for hesitant fuzzy sets, *Inf. Fusion,* **41** (2018), 48–56.

14. P. K. Maji, R. Biswas, R. Roy, Soft set theory, *Comput. Math. Appl.*, **45** (2003), 555–562.

15. C. F. Yang, A note on "Soft Set Theory" [Comput. Math. Appl. 45 (4–5) (2003), 555–562], *Comput. Math. Appl.,* **56** (2008), 1899–1900.

16. M. I. Ali, F. Feng, X. Liu, W. K. Min, M. Shabir, On some new operations in soft set theory, *Comput. Math. Appl.,* **57** (2009), 1547–1553.

17. A. Aygünoglu, H. Aygün, Some notes on soft topological spaces, *Neural Comp. Appl.*, **21** (2012), 113–119.

18. N. Çağman, Contributions to the theory of soft sets, *J. New Results Sci.,* **4** (2014), 33–41.

19. N. Çağman, S. Enginoğlu, Soft set theory and uni-int decision making, *Eur. J. Oper. Res.,* **207** (2010), 848–855.

20. F. Feng, Y. B. Jun, X. Z. Zhao, Soft semirings, *Comput. Math. Appl.*, **56** (2008), 2621–2628.

21. T. J. Neog, D. M. A. Sut, New approach to the theory of soft sets, *Int. J. Comput. Appl.*, **32** (2011), 1–6.

22. D. Pei, D. Miao, From soft sets to information systems, *IEEE International Conference on Granular Computing*, 2005.

23. A. Sezgın, A. O. Atagün, On operations of soft sets, *Comput. Math. Appl.,* **61** (2011), 1457–1467.

24. G. Şenel, A comparative research on the definition of soft point, *Int. J. Comput. Appl.,* **163** (2017), 1–4.

25. P. Zhu, Q. Wen, Operations on soft sets revisited, *J. Appl. Math.*, **2013** (2013), 693–731.

26. M. Shabir, M. Naz, On soft topological spaces, *Comput. Math. Appl.*, **61** (2011), 1786–1799.

27. S. Das, S. K. Samanta, Soft real sets, soft real numbers and their properties, *J. Fuzzy Math.*, **20** (2012), 551–576.

28. S. Das, S. K. Samanta; Soft metric, *Ann. Fuzzy Math. Inform*, **6** (2013), 77–94.

29. O. Göçür, Soft single point space and soft metrizable, *Ann. Fuzzy Math. Inform,* **13** (2017), 499–507.

30. O. Göçür, Some new results on soft n-T4 spaces, *Igdır Univ. J. Inst. Sci. Technol.,* **9** (2019), 1066–1072.

31. I. Zorlutuna, M. Akdağ, W. K. Min, S. Atmaca, Remarks on soft topological space, *Ann. Fuzzy Math. Inform*, **3** (2012), 171–185.

32. A. Fadel, N. Hassan, Separation axioms of bipolar soft topological space, *J. Phys.: Conf. Ser.*, **1212** (2019), 012017.

33. L. Fu, X. Shi, Path connectedness over soft rough topological space, *J. Adv. Math. Comput. Sci.*, **31** (2019), 1–10.

34. H. Kamacı, A. O. Atagün, E. Aygün, Difference operations of soft matrices with applications in decision making, *Punjab Univ. J. Math.*, **51** (2019), 1–21.

35. S. A. Khandait, R. Bhardwaj, C. Singh, Fixed point result with soft cone metric space with examples, *Math. Theory Model.*, **9** (2019), 62–79.

36. M. Riaz, N. Çağman, I. Zareef, M. Aslam, N-soft topology and its applications to multi-criteria group decision making, *J. Intell. Fuzzy Syst.*, **36** (2019), 6521–6536.

37. M. Riaz, B. Davvaz, A. Firdous, A. Fakhar, Novel concepts of soft rough set topology with applications, *J. Intell. Fuzzy Syst.*, **36** (2019), 3579–3590.

38. M. Riaz, S. T. Tehrim, Certain Properties of bipolar fuzzy soft topology via q-neighborhood, *Punjab Univ. J. Math.*, **51** (2019), 113–131.

39. G. Murtaza, M. Abbas, M. I. Ali, Fixed points of interval valued neutrosophic soft mappings, *Fac. Sci. Math. Univ. Nis Serb.*, **33** (2019), 463–474.

40. S. T. Tehrim, M. Riaz, A novel extension of TOPSIS to MCGDM with bipolar neutrosophic soft topology, *J. Intell. Fuzzy Syst.*, **37** (2019), 5531–5549.

41. N. Çağman, S. Karataş, S. Enginoğlu, Soft topology, *Comput. Math. Appl.*, **62** (2011), 351–358.

42. H. Kamacı, Selectivity analysis of parameters in soft set and its effect on decision making, *H. Int. J. Mach. Learn. & Cyber*., **11** (2020), 313–324.

43. H. Kamacı, Similarity measure for soft matrices and its applications, *J. Intell. Fuzzy Syst.*, **36** (2019), 3061–3072.

44. F. Karaaslan, I. Deli, Soft neutrosophic classical sets and their applications in decision-making, *Palest. J. Math.*, **9** (2020), 312–326.

45. F. Fatimah, D. Rosadi, R. B. F. Hakim, N-soft sets and decision making algorithms, *Soft Comput.*, **22** (2018), 3829–3842.

46. O. Göçür, Monad metrizable space, *Mathematics,* **8** (2020), 1891.