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Research article

Improved results on mixed passive and H_∞ performance for uncertain neural networks with mixed interval time-varying delays via feedback control

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Abstract: This paper studies the mixed passive and H_∞ performance for uncertain neural networks with interval discrete and distributed time-varying delays via feedback control. The interval discrete and distributed time-varying delay functions are not assumed to be differentiable. The improved criteria of exponential stability with a mixed passive and H_∞ performance are obtained for the uncertain neural networks by constructing a Lyapunov-Krasovskii functional (LKF) comprising single, double, triple, and quadruple integral terms and using a feedback controller. Furthermore, integral inequalities and convex combination technique are applied to achieve the less conservative results for a special case of neural networks. By using the Matlab LMI toolbox, the derived new exponential stability with a mixed passive and H_∞ performance criteria is performed in terms of linear matrix inequalities (LMIs) that cover H_∞, and passive performance by setting parameters in the general performance index. Numerical examples are shown to demonstrate the benefits and effectiveness of the derived theoretical results. The method given in this paper is less conservative and more general than the others.

Keywords: uncertain neural networks; mixed passive and H_∞ performance; exponential stability; mixed interval time-varying delays; feedback control
Mathematics Subject Classification: 34D20 , 34H15

1. Introduction

During the past few decades, many researchers have studied neural networks because of their applications in many fields such as parallel computation, fault diagnosis, image processing, optimization problems, industrial automation, and so on [1–5]. To acquire the above applications, we need to first analyze the theoretical stability for the equilibrium point of neural networks. Further, the important factor affecting system analysis is time delay. It is well known that time delay is a normal phenomenon that appears in neural networks since the neural networks consist of a large number of neurons that connect and communicate with each other into a diversity of axon sizes and lengths. Moreover, the existence of time delay causing poor control performance, divergence, oscillation, and instability to the system [6]. Stability analysis of neural networks with constant, discrete, and distributed time-varying delays has received considerable attentions [7–9]. For example, [7], the delay-dependent criterion for exponential stability analysis of neural networks with time-varying delays satisfying $0 \leq \eta(t) \leq \eta, \dot{\eta}(t) \leq \mu$ is obtained. In [8], the problem of dissipativity analysis for neural networks with time-varying delays is investigated. However, practically time delay can occur in an irregular fashion such as sometimes the time-varying delays are not differentiable. So, it inspires us to study neural networks without the restriction on the derivative of time-varying delays.

On the other side, since external perturbation, uncertain or slowly varying parameters, an accurate mathematical model does not get easy. Data tends to be uncertain in many applications [10–12]. Therefore, it is important to guarantee that the model is stable with respect to the uncertainties. Also, uncertainty in neural networks cannot be avoided. Consequently the problem of robust stability analysis for uncertain neural networks has many studied. For example, Subramanian et al. [13] investigated the robust stabilization of uncertain neural networks with two additive time-varying delays based on Wirtinger-based double integral inequality. In [14], Zeng et al. studied the robust passivity analysis of uncertain neural networks with discrete and distributed delays by constructing an augmented Lyapunov functional and combining a new integral inequality with the reciprocally convex approach.

It is well known that passivity is a special case and a general theory of dissipativeness and it performs an influential part in the designing of linear and nonlinear systems. It is widely applied in many areas such as sliding mode control [15], fuzzy control [16], network control [17], and signal processing [18]. The main property of passivity is that can keep the system internally stable. Recently, the passivity problem has been studied in [14, 19–22]. In addition, the H_∞ theory is very important due to the H_∞ control design that exposes the control problem as a mathematical optimization problem to find the controller solution. The H_∞ approaches are used in control theory to synthesize controllers achieving stabilization with guaranteed performance [23, 24]. The problem of mixed H_∞ and passivity analysis was first studied in [25, 26]. It has received a lot of attention from many researchers. For example, the mixed passive and H_∞ synchronization problems of complex dynamical networks have been analyzed in [27, 28]. And, the combined H_∞ and passivity state estimation of memristive neural networks was studied in [29]. Nevertheless, a mixed passive and H_∞ analysis problem for uncertain neural networks with interval discrete and distributed time-varying delays has been few considered which is our motivation.

Inspired by above discussions, the problem of mixed passive and H_∞ performance for uncertain neural networks with interval discrete and distributed time-varying delays via feedback control is studied. The main contributions of this paper are three aspects.

• In this work, the system consists of the interval discrete and distributed time-varying delays such that does not necessitate being differentiable functions, which mean that a fast interval discrete and distributed time-varying delays is approved. The lower bound of the delays does not restrict to be 0, the activation functions are different, and the output is general.

• By using the Lyapunov-Krasovskii stability theory, the new results of the exponential stability with a mixed passive and H_∞ performance for the uncertain neural networks are obtained. Based on the weighting parameter, the results are more general such that H_∞ performance or passive performance for the uncertain neural networks are included.

• Different from the methods in [30–32], the Lyapunov-Krasovskii functional comprising single, double, triple, and quadruple integral terms and integral inequalities are employed. Convex combination idea and zero equation are used. The method used in this paper reveals less conservative results when comparing with existing results [30–32].

This paper is formed in five sections as follows. In Section 2, network model and preliminaries are provided. Section 3 shows exponential stability analysis with a mixed passive and H_∞ performance of the uncertain neural network system, and the stability analysis of a special case neural network. Numerical examples are given in Section 4 and conclusions are addressed in Section 5.

2. Network model and preliminaries

Notations

Throughout this paper, $\mathbb{R}$ and $\mathbb{R}^n$ represent the set of real numbers and the n-dimensional Euclidean spaces, respectively. $M > (\geq)0$ means that the symmetric matrix M is positive (semi-positive) definite. $M < (\leq)0$ denotes that the symmetric matrix M is negative (semi-negative) definite. M^T and M^{-1} denote the transpose and the inverse of matrix M, respectively. $\lambda_{\max}(M)$ and $\lambda_{\min}(M)$ denote the maximum eigenvalue and the minimum eigenvalue of matrix M, respectively. The symbol $*$ represents the symmetric block in a symmetric matrix. I is the identity matrix with appropriate dimensions. e_i represents the unit column vector having one element on its *ith* row and zeros elsewhere. $C([a_1, a_2], \mathbb{R}^n)$ denotes the set of continuous functions mapping the interval $[a_1, a_2]$ to $\mathbb{R}^n$. $\mathcal{L}_2[0,\infty)$ represents the space of functions $\zeta : \mathbb{R}^+ \to \mathbb{R}^n$ with the norm $\|\zeta\|_{\mathcal{L}_2} = \left[\int_0^\infty |\zeta(\theta)|^2\, d\theta\right]^{\frac{1}{2}}$. For $\vartheta \in \mathbb{R}^n$, the norm of ϑ, denoted by $\|\vartheta\|$, is defined by $\|\vartheta\| = \left[\sum_{i=1}^{n} |\vartheta_i|^2\right]^{1/2}$; $\|\vartheta(t+\nu)\|_{cl} = \max\{\sup_{-\max\{\sigma_2,\delta_2\}\leq\nu\leq 0} \|\vartheta(t+\nu)\|^2, \sup_{-\max\{\sigma_2,\delta_2\}\leq\nu\leq 0} \|\dot{\vartheta}(t+\nu)\|^2\}$.

We consider the uncertain neural network model with interval discrete and distributed time-varying delays of the form

$$
\begin{aligned}
\dot{x}(t) = &-(A + \Delta A(t))x(t) + (B + \Delta B(t))f(x(t)) + (C + \Delta C(t))k(x(t - \sigma(t))) \\
&+ (D + \Delta D(t))\int_{t-\delta_2(t)}^{t-\delta_1(t)} h(x(s))\, ds + E\omega(t) + \mathcal{U}(t), \\
z(t) = &C_1 x(t) + C_2 x(t - \sigma(t)) + C_3 \int_{t-\delta_2(t)}^{t-\delta_1(t)} h(x(s))\, ds + C_4\omega(t), \\
x(t) = &\phi(t), \quad t \in [-\varrho, 0],
\end{aligned}
\tag{2.1}
$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the neuron state vector, $f(x(t)), k(x(t)), h(x(t)) \in \mathbb{R}^n$ are the neuron activation functions, $z(t) \in \mathbb{R}^n$ is the output vector, $\omega(t) \in \mathbb{R}^n$ is the input vector such that $\omega(t) \in \mathcal{L}_2[0,\infty)$, $\mathcal{U}(t) \in \mathbb{R}^n$ is the control input, $A = \mathrm{diag}\{a_1, a_2, \ldots, a_n\} > 0$, B is the connection weight matrix, C is the discretely delayed connection weight matrix, D is the distributively delayed connection weight matrix, E, C_1, C_2, C_3, C_4 are given constant matrices, $\phi(t) \in C[[-\varrho, 0], \mathbb{R}^n]$ is the initial function. $\sigma(t)$ is the interval discrete time-varying delay that satisfies $0 \le \sigma_1 \le \sigma(t) \le \sigma_2$ where $\sigma_1, \sigma_2 \in \mathbb{R}$. $\delta_i(t)$ $(i = 1, 2)$ is the interval distributed time-varying delay that satisfies $0 \le \delta_1 \le \delta_1(t) \le \delta_2(t) \le \delta_2$ where $\delta_1, \delta_2 \in \mathbb{R}$. $\varrho = \max\{\sigma_2, \delta_2\}$ is known real constant, the time-varying uncertainties matrices $\Delta A(t), \Delta B(t), \Delta C(t)$, and $\Delta D(t)$ are given by

$$\begin{aligned}\Delta A(t) =& J_1 \mathbb{S}_1(t)\Sigma_1, \quad \Delta B(t) = J_2 \mathbb{S}_2(t)\Sigma_2,\\ \Delta C(t) =& J_3 \mathbb{S}_3(t)\Sigma_3, \quad \Delta D(t) = J_4 \mathbb{S}_4(t)\Sigma_4,\end{aligned}$$

and $J_1, J_2, J_3, J_4, \Sigma_1, \Sigma_2, \Sigma_3$ and Σ_4 are known constant matrices with appropriate dimensions, $\mathbb{S}_1(t), \mathbb{S}_2(t), \mathbb{S}_3(t), \mathbb{S}_4(t)$ are unknown uncertain matrices satisfying

$$\mathbb{S}_1^T(t)\mathbb{S}_1(t) \le I, \quad \mathbb{S}_2^T(t)\mathbb{S}_2(t) \le I, \quad \mathbb{S}_3^T(t)\mathbb{S}_3(t) \le I, \quad \mathbb{S}_4^T(t)\mathbb{S}_4(t) \le I.$$

The neuron activation functions $f(x(t)), k(x(t))$ and $h(x(t))$ satisfy the following conditions:

(A1) f is continuous and satisfies

$$F_i^- \le \frac{f_i(\alpha_1) - f_i(\alpha_2)}{\alpha_1 - \alpha_2} \le F_i^+$$

for all $\alpha_1 \ne \alpha_2$, and $F_i^-, F_i^+ \in \mathbb{R}$, $f_i(0) = 0$.

(A2) k is continuous and satisfies

$$K_i^- \le \frac{k_i(\alpha_1) - k_i(\alpha_2)}{\alpha_1 - \alpha_2} \le K_i^+$$

for all $\alpha_1 \ne \alpha_2$, and $K_i^-, K_i^+ \in \mathbb{R}$, $k_i(0) = 0$.

(A3) h is continuous and satisfies

$$H_i^- \le \frac{h_i(\alpha_1) - h_i(\alpha_2)}{\alpha_1 - \alpha_2} \le H_i^+$$

for all $\alpha_1 \ne \alpha_2$, and $H_i^-, H_i^+ \in \mathbb{R}$, $h_i(0) = 0$.

The state feedback is considered with

$$\mathcal{U}(t) = Kx(t).$$

Substitute $\mathcal{U}(t) = Kx(t)$ into (2.1), we gain

$$\begin{aligned}\dot{x}(t) =& (K - A - \Delta A(t))x(t) + (B + \Delta B(t))f(x(t)) + (C + \Delta C(t))\\ &\times k(x(t - \sigma(t))) + (D + \Delta D(t))\int_{t-\delta_2(t)}^{t-\delta_1(t)} h(x(s))\,ds + E\omega(t),\\ z(t) =& C_1 x(t) + C_2 x(t - \sigma(t)) + C_3 \int_{t-\delta_2(t)}^{t-\delta_1(t)} h(x(s))\,ds + C_4\omega(t),\\ x(t) =& \phi(t), \quad t \in [-\varrho, 0].\end{aligned} \tag{2.2}$$

Definition 2.1. *[28] The uncertain NNs (2.2) with $\omega(t) = 0$ is exponentially stable, if there exist constants $b_1 > 0$ and $b_2 > 0$ such that*

$$\|x(t)\|^2 \le b_1 e^{-b_2 t}\|x(\nu)\|_{cl}.$$

Definition 2.2. *[28] For a given scalar $\upsilon \in [0,1]$, the uncertain NNs (2.2) is exponentially stable and meets a predefined passive and H_∞ performance index γ, if the following conditions can be ensured simultaneously:*

(1) the uncertain NNs (2.2) is exponentially stable in sense of Definition 2.1.
(2) under zero initial condition, there exists a scalar $\gamma > 0$ such that the following inequality is satisfied:

$$\int_0^{\mathcal{T}_p} \left[-\upsilon z^T(t)z(t) + 2(1-\upsilon)\gamma z^T(t)\omega(t)\right] dt \geq -\gamma^2 \int_0^{\mathcal{T}_p} \left[\omega^T(t)\omega(t)\right] dt, \tag{2.3}$$

for any $\mathcal{T}_p \geq 0$ and any non-zero $\omega(t) \in \mathcal{L}_2[0,\infty)$.

Remark 1. *The condition (2.3) includes passive performance index and H_∞ performance index. If $\upsilon = 1$, the condition (2.3) reduces to the H_∞ performance index; and if $\upsilon = 0$, the condition (2.3) reduces to the passive performance index; when υ takes the value in $(0,1)$, then the condition (2.3) becomes to the mixed passive and H_∞ performance index.*

Lemma 2.3. *[33, 34] Suppose $0 \leq \eta_1 < \eta_2$ and $x(t) \in \mathbb{R}^n$, for any matrix $M > 0$ the following inequalities hold:*

$$\begin{aligned}
-(\eta_2-\eta_1)\int_{t-\eta_2}^{t-\eta_1} x^T(s)Mx(s)\,ds &\leq -\int_{t-\eta_2}^{t-\eta_1} x^T(s)\,dsM\int_{t-\eta_2}^{t-\eta_1} x(s)\,ds,\\
-\frac{(\eta_2^2-\eta_1^2)}{2}\int_{-\eta_2}^{-\eta_1}\int_{t+\beta}^{t} x^T(s)Mx(s)\,ds\,d\beta &\leq -\int_{-\eta_2}^{-\eta_1}\int_{t+\beta}^{t} x^T(s)\,ds\,d\beta\\
&\quad\times M\int_{-\eta_2}^{-\eta_1}\int_{t+\beta}^{t} x(s)\,ds\,d\beta,\\
-\frac{\eta_2^3}{6}\int_{-\eta_2}^{0}\int_{\beta}^{0}\int_{t+\lambda}^{t} x^T(s)Mx(s)\,ds\,d\lambda\,d\beta &\leq -\int_{-\eta_2}^{0}\int_{\beta}^{0}\int_{t+\lambda}^{t} x^T(s)\,ds\,d\lambda\,d\beta\\
&\quad\times M\int_{-\eta_2}^{0}\int_{\beta}^{0}\int_{t+\lambda}^{t} x(s)\,ds\,d\lambda\,d\beta.
\end{aligned}$$

Lemma 2.4. *[35] For a matrix $M > 0$, a differentiable function $\{x(\alpha)|\alpha \in [a_1, a_2]\}$, the following inequality holds:*

$$\begin{aligned}
&\int_{a_1}^{a_2} \dot{x}^T(\alpha)M\dot{x}(\alpha)\,d\alpha \geq \frac{1}{a_2-a_1}\left[x(a_2)-x(a_1)\right]^T M\left[x(a_2)-x(a_1)\right] + \frac{3}{a_2-a_1}\\
&\times\left[x(a_2)+x(a_1)-\frac{2}{a_2-a_1}\int_{a_1}^{a_2} x(\alpha)\,d\alpha\right]^T M\left[x(a_2)+x(a_1)-\frac{2}{a_2-a_1}\int_{a_1}^{a_2} x(\alpha)\,d\alpha\right].
\end{aligned}$$

Lemma 2.5. *[36] For given matrices P, Q and R with $R^TR \leq I$ and a scalar $\alpha > 0$, the following inequality holds:*

$$PRQ + (PRQ)^T \leq \alpha PP^T + \alpha^{-1}Q^TQ.$$

Lemma 2.6. *[37] Let P, Q, R be given matrices such that $R > 0$, then*

$$\begin{bmatrix} Q & P^T \\ P & -R \end{bmatrix} < 0 \Leftrightarrow Q + P^TR^{-1}P < 0.$$

3. Main results

3.1. Mixed passive and H_∞ analysis for neural networks

In this section, we will firstly find the sufficient conditions which guarantee the neural networks without parameter uncertainties to be exponentially stable with a mixed passive and H_∞ performance. That is we consider the following model

$$
\begin{aligned}
\dot{x}(t) &= (K-A)x(t) + Bf(x(t)) + Ck(x(t-\sigma(t))) + D\int_{t-\delta_2(t)}^{t-\delta_1(t)} h(x(s))\,ds \\
&\quad +E\omega(t), \\
z(t) &= C_1x(t) + C_2x(t-\sigma(t)) + C_3\int_{t-\delta_2(t)}^{t-\delta_1(t)} h(x(s))\,ds + C_4\omega(t), \\
x(t) &= \phi(t), \quad t \in [-\varrho, 0].
\end{aligned} \tag{3.1}
$$

In this paper, we define the denotations as follows

$$
\begin{aligned}
&\tilde{F}_i = \max\{|F_i^-|, |F_i^+|\}, \quad \tilde{K}_i = \max\{|K_i^-|, |K_i^+|\}, \quad \tilde{H}_i = \max\{|H_i^-|, |H_i^+|\}, \\
&F_1 = \mathrm{diag}\{F_1^-F_1^+, F_2^-F_2^+, \ldots, F_n^-F_n^+\}, \\
&F_2 = \mathrm{diag}\left\{\frac{F_1^- + F_1^+}{2}, \frac{F_2^- + F_2^+}{2}, \ldots, \frac{F_n^- + F_n^+}{2}\right\}, \\
&K_1 = \mathrm{diag}\{K_1^-K_1^+, K_2^-K_2^+, \ldots, K_n^-K_n^+\}, \\
&K_2 = \mathrm{diag}\left\{\frac{K_1^- + K_1^+}{2}, \frac{K_2^- + K_2^+}{2}, \ldots, \frac{K_n^- + K_n^+}{2}\right\}, \\
&H_1 = \mathrm{diag}\{H_1^-H_1^+, H_2^-H_2^+, \ldots, H_n^-H_n^+\}, \\
&H_2 = \mathrm{diag}\left\{\frac{H_1^- + H_1^+}{2}, \frac{H_2^- + H_2^+}{2}, \ldots, \frac{H_n^- + H_n^+}{2}\right\},
\end{aligned}
$$

$$
\begin{aligned}
\xi^T(t) = \Big[&x^T(t), \dot{x}^T(t), x^T(t-\sigma_1), x^T(t-\sigma_2), x^T(t-\sigma(t)), f^T(x(t)), \\
&k^T(x(t-\sigma(t))), h^T(x(t)), \frac{1}{\sigma_1}\int_{t-\sigma_1}^{t} x^T(s)\,ds, \frac{1}{\sigma_2}\int_{t-\sigma_2}^{t} x^T(s)\,ds, \\
&\frac{1}{\sigma(t)-\sigma_1}\int_{t-\sigma(t)}^{t-\sigma_1} x^T(s)\,ds, \frac{1}{\sigma_2-\sigma(t)}\int_{t-\sigma_2}^{t-\sigma(t)} x^T(s)\,ds, \int_{t-\delta_2(t)}^{t-\delta_1(t)} h^T(x(s))\,ds, \\
&\int_{-\sigma(t)}^{-\sigma_1}\int_{t+\beta}^{t} x^T(s)\,ds\,d\beta, \int_{-\sigma_2}^{-\sigma(t)}\int_{t+\beta}^{t} x^T(s)\,ds\,d\beta, \omega^T(t)\Big].
\end{aligned}
$$

Theorem 3.1. *For given scalars $\sigma_1, \sigma_2, \delta_1, \delta_2, \beta_1, \beta_2, \gamma > 0$, and $\upsilon \in [0, 1]$, if there exist eleven $n \times n$ matrices $P > 0, Q_1 > 0, Q_2 > 0, R_1 > 0, R_2 > 0, U > 0, L > 0, X_1 > 0, X_2 > 0, N > 0, Z$ and three $n \times n$ positive diagonal matrices $Y_1 > 0, Y_2 > 0, Y_3 > 0$ such that the following LMIs hold:*

$$\Theta + \Theta_1 < 0, \tag{3.2}$$

$$\Theta + \Theta_2 < 0, \tag{3.3}$$

wherein,

$$\Theta_1 = -\, e_{15}X_1e_{15}^T,$$
$$\Theta_2 = -\, e_{14}X_1e_{14}^T,$$
$$\Theta = \begin{bmatrix} \Theta(1,1) & \Theta(1,2) \\ * & \Theta(2,2) \end{bmatrix},$$

with:

$$\Theta(1,1) = \begin{bmatrix} \theta_{1,1} & \theta_{1,2} & -2R_1 & -2R_2 & \upsilon C_1^TC_2 & \theta_{1,6} & \beta_1N^TC & H_2Y_3 \\ * & \theta_{2,2} & 0 & 0 & 0 & \beta_2N^TB & \beta_2N^TC & 0 \\ * & * & \theta_{3,3} & 0 & -2U & 0 & 0 & 0 \\ * & * & * & \theta_{4,4} & -2U & 0 & 0 & 0 \\ * & * & * & * & \theta_{5,5} & 0 & K_2Y_2 & 0 \\ * & * & * & * & * & -Y_1 & 0 & 0 \\ * & * & * & * & * & * & -Y_2 & 0 \\ * & * & * & * & * & * & * & \theta_{8,8} \end{bmatrix},$$

$$\Theta(1,2) = \begin{bmatrix} 6R_1 & 6R_2 & 0 & 0 & \theta_{1,13} & \frac{\sigma_2^2-\sigma_1^2}{2}X_2 & \frac{\sigma_2^2-\sigma_1^2}{2}X_2 & \theta_{1,16} \\ 0 & 0 & 0 & 0 & \beta_2N^TD & 0 & 0 & \beta_2N^TE \\ 6R_1 & 0 & 6U & 0 & 0 & 0 & 0 & 0 \\ 0 & 6R_2 & 0 & 6U & 0 & 0 & 0 & 0 \\ 0 & 0 & 6U & 6U & \upsilon C_2^TC_3 & 0 & 0 & \theta_{5,16} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Theta(2,2) = \begin{bmatrix} -12R_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -12R_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -12U & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -12U & 0 & 0 & 0 & 0 \\ * & * & * & * & \theta_{13,13} & 0 & 0 & \theta_{13,16} \\ * & * & * & * & * & \theta_{14,14} & -X_2 & 0 \\ * & * & * & * & * & * & \theta_{15,15} & 0 \\ * & * & * & * & * & * & * & \theta_{16,16} \end{bmatrix},$$

in which:

$$\begin{aligned} \theta_{1,1} &= Q_1 + Q_2 - 4R_1 - 4R_2 + \upsilon C_1^TC_1 - F_1Y_1 - H_1Y_3 + 2\beta_1Z - 2\beta_1N^TA \\ &\quad + \frac{(\sigma_2^2-\sigma_1^2)^2}{4}X_1 - \frac{(\sigma_2^2-\sigma_1^2)^2}{4}X_2, \\ \theta_{1,2} &= P - \beta_1N^T + \beta_2Z^T - \beta_2N^TA, \quad \theta_{1,6} = F_2Y_1 + \beta_1N^TB, \\ \theta_{1,13} &= \upsilon C_1^TC_3 + \beta_1N^TD, \quad \theta_{1,16} = \upsilon C_1^TC_4 - (1-\upsilon)\gamma C_1^T + \beta_1N^TE, \end{aligned}$$

$$\begin{aligned}
\theta_{2,2} &= \sigma_1^2 R_1 + \sigma_2^2 R_2 + (\sigma_2-\sigma_1)^2 U - 2\beta_2 N^T + \frac{(\sigma_2^3-\sigma_1^3)^2}{36} X_2, \\
\theta_{3,3} &= -Q_1 - 4R_1 - 4U, \quad \theta_{4,4} = -Q_2 - 4R_2 - 4U, \\
\theta_{5,5} &= -8U - K_1 Y_2 + \upsilon C_2^T C_2, \quad \theta_{5,16} = \upsilon C_2^T C_4 - (1-\upsilon)\gamma C_2^T, \\
\theta_{8,8} &= (\delta_2-\delta_1)^2 L - Y_3, \quad \theta_{13,13} = -L + \upsilon C_3^T C_3, \\
\theta_{13,16} &= \upsilon C_3^T C_4 - (1-\upsilon)\gamma C_3^T, \quad \theta_{14,14} = -X_1 - X_2, \\
\theta_{15,15} &= -X_1 - X_2, \quad \theta_{16,16} = \upsilon C_4^T C_4 - 2(1-\upsilon)\gamma C_4^T - \gamma^2 I,
\end{aligned}$$

then, the NNs (3.1) is exponentially stable with a mixed passive and H_∞ performance. Moreover, the controller is in the form

$$K = N^{-1} Z.$$

Proof. Consider the model (3.1) with the following Lyapunov-Krasovskii functional

$$V(x(t),t) = \sum_{i=1}^{9} V_i(x(t),t),$$

where

$$\begin{aligned}
V_1(x(t),t) &= x^T(t) P x(t), \\
V_2(x(t),t) &= \int_{t-\sigma_1}^{t} x^T(s) Q_1 x(s)\, ds, \\
V_3(x(t),t) &= \int_{t-\sigma_2}^{t} x^T(s) Q_2 x(s)\, ds, \\
V_4(x(t),t) &= \sigma_1 \int_{-\sigma_1}^{0} \int_{t+s}^{t} \dot{x}^T(\tau) R_1 \dot{x}(\tau)\, d\tau\, ds, \\
V_5(x(t),t) &= \sigma_2 \int_{-\sigma_2}^{0} \int_{t+s}^{t} \dot{x}^T(\tau) R_2 \dot{x}(\tau)\, d\tau\, ds, \\
V_6(x(t),t) &= (\sigma_2-\sigma_1) \int_{-\sigma_2}^{-\sigma_1} \int_{t+s}^{t} \dot{x}^T(\tau) U \dot{x}(\tau)\, d\tau\, ds, \\
V_7(x(t),t) &= (\delta_2-\delta_1) \int_{-\delta_2}^{-\delta_1} \int_{t+s}^{t} h^T(x(\tau)) L h(x(\tau))\, d\tau\, ds, \\
V_8(x(t),t) &= \frac{(\sigma_2^2-\sigma_1^2)}{2} \int_{-\sigma_2}^{-\sigma_1} \int_{\beta}^{0} \int_{t+\lambda}^{t} x^T(s) X_1 x(s)\, ds\, d\lambda\, d\beta, \\
V_9(x(t),t) &= \frac{(\sigma_2^3-\sigma_1^3)}{6} \int_{-\sigma_2}^{-\sigma_1} \int_{\beta}^{0} \int_{\lambda}^{0} \int_{t+\varphi}^{t} \dot{x}^T(s) X_2 \dot{x}(s)\, ds\, d\varphi\, d\lambda\, d\beta.
\end{aligned} \tag{3.4}$$

We find time derivatives of $V_i(x(t),t), i = 1, 2, \ldots, 9$, along the trajectories of (3.1), we achieve

$$\dot{V}_1(x(t),t) = x^T(t) P \dot{x}(t) + \dot{x}^T(t) P x(t), \tag{3.5}$$

$$\dot{V}_2(x(t),t) = x^T(t) Q_1 x(t) - x^T(t-\sigma_1) Q_1 x(t-\sigma_1), \tag{3.6}$$

$$\dot{V}_3(x(t),t) = x^T(t) Q_2 x(t) - x^T(t-\sigma_2) Q_2 x(t-\sigma_2), \tag{3.7}$$

$$
\begin{aligned}
\dot{V}_4(x(t),t) &= \sigma_1 \int_{-\sigma_1}^{0} \left[\dot{x}^T(t)R_1\dot{x}(t) - \dot{x}^T(t+s)R_1\dot{x}(t+s)\right] ds \\
&= \sigma_1^2\dot{x}^T(t)R_1\dot{x}(t) - \sigma_1 \int_{t-\sigma_1}^{t} \dot{x}^T(\alpha)R_1\dot{x}(\alpha)\, d\alpha, \qquad (3.8)\\
\dot{V}_5(x(t),t) &= \sigma_2 \int_{-\sigma_2}^{0} \left[\dot{x}^T(t)R_2\dot{x}(t) - \dot{x}^T(t+s)R_2\dot{x}(t+s)\right] ds \\
&= \sigma_2^2\dot{x}^T(t)R_2\dot{x}(t) - \sigma_2 \int_{t-\sigma_2}^{t} \dot{x}^T(\alpha)R_2\dot{x}(\alpha)\, d\alpha, \qquad (3.9)\\
\dot{V}_6(x(t),t) &= (\sigma_2-\sigma_1) \int_{-\sigma_2}^{-\sigma_1} \left[\dot{x}^T(t)U\dot{x}(t) - \dot{x}^T(t+s)U\dot{x}(t+s)\right] ds \\
&= (\sigma_2-\sigma_1)^2\dot{x}^T(t)U\dot{x}(t) - (\sigma_2-\sigma_1) \int_{t-\sigma_2}^{t-\sigma_1} \dot{x}^T(\alpha)U\dot{x}(\alpha)\, d\alpha, \qquad (3.10)\\
\dot{V}_7(x(t),t) &= (\delta_2-\delta_1) \int_{-\delta_2}^{-\delta_1} \left[h^T(x(t))Lh(x(t)) - h^T(x(t+s))Lh(x(t+s))\right] ds \\
&= (\delta_2-\delta_1)^2 h^T(x(t))Lh(x(t)) - (\delta_2-\delta_1) \int_{t-\delta_2}^{t-\delta_1} h^T(x(\alpha))Lh(x(\alpha))\, d\alpha \\
&\le (\delta_2-\delta_1)^2 h^T(x(t))Lh(x(t)) \\
&\quad - (\delta_2(t)-\delta_1(t)) \int_{t-\delta_2(t)}^{t-\delta_1(t)} h^T(x(\alpha))Lh(x(\alpha))\, d\alpha, \qquad (3.11)\\
\dot{V}_8(x(t),t) &= \frac{(\sigma_2^2-\sigma_1^2)}{2} \int_{-\sigma_2}^{-\sigma_1}\int_{\beta}^{0} \left[x^T(t)X_1x(t) - x^T(t+\lambda)X_1x(t+\lambda)\right] d\lambda\, d\beta \\
&= \frac{(\sigma_2^2-\sigma_1^2)^2}{4} x^T(t)X_1x(t) - \frac{(\sigma_2^2-\sigma_1^2)}{2} \int_{-\sigma_2}^{-\sigma_1}\int_{t+\beta}^{t} x^T(s)X_1x(s)\, ds\, d\beta, \qquad (3.12)\\
\dot{V}_9(x(t),t) &= \frac{(\sigma_2^3-\sigma_1^3)}{6} \\
&\quad \times \int_{-\sigma_2}^{-\sigma_1}\int_{\beta}^{0}\int_{\lambda}^{0} \left[\dot{x}^T(t)X_2\dot{x}(t) - \dot{x}^T(t+\varphi)X_2\dot{x}(t+\varphi)\right] d\varphi\, d\lambda\, d\beta \\
&= \frac{(\sigma_2^3-\sigma_1^3)^2}{36}\dot{x}^T(t)X_2\dot{x}(t) \\
&\quad - \frac{(\sigma_2^3-\sigma_1^3)}{6} \int_{-\sigma_2}^{-\sigma_1}\int_{\beta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)X_2\dot{x}(s)\, ds\, d\lambda\, d\beta. \qquad (3.13)
\end{aligned}
$$

Utilizing Lemma 2.4., the following inequalities are easily obtained:

$$
\begin{aligned}
-\sigma_1 \int_{t-\sigma_1}^{t} \dot{x}^T(\alpha)R_1\dot{x}(\alpha)\, d\alpha \le & -[x(t)-x(t-\sigma_1)]^T R_1 [x(t)-x(t-\sigma_1)] \\
& -3\left[x(t)+x(t-\sigma_1) - \frac{2}{\sigma_1}\int_{t-\sigma_1}^{t} x(\alpha)\, d\alpha\right]^T \\
& \times R_1 \left[x(t)+x(t-\sigma_1) - \frac{2}{\sigma_1}\int_{t-\sigma_1}^{t} x(\alpha)\, d\alpha\right], \qquad (3.14)
\end{aligned}
$$

$$
\begin{aligned}
-\sigma_2 \int_{t-\sigma_2}^{t} \dot{x}^T(\alpha) R_2 \dot{x}(\alpha)\, d\alpha \leq & -[x(t)-x(t-\sigma_2)]^T R_2 [x(t)-x(t-\sigma_2)] \\
& -3\left[x(t)+x(t-\sigma_2)-\frac{2}{\sigma_2}\int_{t-\sigma_2}^{t} x(\alpha)\, d\alpha\right]^T \\
& \times R_2\left[x(t)+x(t-\sigma_2)-\frac{2}{\sigma_2}\int_{t-\sigma_2}^{t} x(\alpha)\, d\alpha\right],
\end{aligned} \tag{3.15}
$$

$$
\begin{aligned}
&-(\sigma_2-\sigma_1)\int_{t-\sigma_2}^{t-\sigma_1} \dot{x}^T(\alpha) U \dot{x}(\alpha)\, d\alpha \\
&\leq -[x(t-\sigma(t))-x(t-\sigma_2)]^T U [x(t-\sigma(t))-x(t-\sigma_2)] \\
&\quad -3\left[x(t-\sigma(t))+x(t-\sigma_2)-\frac{2}{\sigma_2-\sigma(t)}\int_{t-\sigma_2}^{t-\sigma(t)} x(\alpha)\, d\alpha\right]^T \\
&\quad \times U\left[x(t-\sigma(t))+x(t-\sigma_2)-\frac{2}{\sigma_2-\sigma(t)}\int_{t-\sigma_2}^{t-\sigma(t)} x(\alpha)\, d\alpha\right] \\
&\quad -[x(t-\sigma_1)-x(t-\sigma(t))]^T U [x(t-\sigma_1)-x(t-\sigma(t))] \\
&\quad -3\left[x(t-\sigma_1)+x(t-\sigma(t))-\frac{2}{\sigma(t)-\sigma_1}\int_{t-\sigma(t)}^{t-\sigma_1} x(\alpha)\, d\alpha\right]^T \\
&\quad \times U\left[x(t-\sigma_1)+x(t-\sigma(t))-\frac{2}{\sigma(t)-\sigma_1}\int_{t-\sigma(t)}^{t-\sigma_1} x(\alpha)\, d\alpha\right].
\end{aligned} \tag{3.16}
$$

By utilizing Lemma 2.3, we achieve the following inequalities

$$
\begin{aligned}
&-(\delta_2(t)-\delta_1(t))\int_{t-\delta_2(t)}^{t-\delta_1(t)} h^T(x(\alpha)) L h(x(\alpha))\, d\alpha \\
&\quad \leq -\int_{t-\delta_2(t)}^{t-\delta_1(t)} h^T(x(\alpha))\, d\alpha L \int_{t-\delta_2(t)}^{t-\delta_1(t)} h(x(\alpha))\, d\alpha,
\end{aligned} \tag{3.17}
$$

$$
\begin{aligned}
&-\frac{(\sigma_2^2-\sigma_1^2)}{2}\int_{-\sigma_2}^{-\sigma_1}\int_{t+\beta}^{t} x^T(s) X_1 x(s)\, ds\, d\beta \\
&\leq -\int_{-\sigma_2}^{-\sigma(t)}\int_{t+\beta}^{t} x^T(s)\, ds\, d\beta X_1 \int_{-\sigma_2}^{-\sigma(t)}\int_{t+\beta}^{t} x(s)\, ds\, d\beta \\
&\quad -\varepsilon\int_{-\sigma_2}^{-\sigma(t)}\int_{t+\beta}^{t} x^T(s)\, ds\, d\beta X_1 \int_{-\sigma_2}^{-\sigma(t)}\int_{t+\beta}^{t} x(s)\, ds\, d\beta \\
&\quad -(1-\varepsilon)\int_{-\sigma(t)}^{-\sigma_1}\int_{t+\beta}^{t} x^T(s)\, ds\, d\beta X_1 \int_{-\sigma(t)}^{-\sigma_1}\int_{t+\beta}^{t} x(s)\, ds\, d\beta \\
&\quad -\int_{-\sigma(t)}^{-\sigma_1}\int_{t+\beta}^{t} x^T(s)\, ds\, d\beta X_1 \int_{-\sigma(t)}^{-\sigma_1}\int_{t+\beta}^{t} x(s)\, ds\, d\beta,
\end{aligned} \tag{3.18}
$$

where $\varepsilon = \dfrac{\sigma^2(t) - \sigma_1^2}{\sigma_2^2 - \sigma_1^2}$.

$$
\begin{aligned}
&-\frac{(\sigma_2^3 - \sigma_1^3)}{6} \int_{-\sigma_2}^{-\sigma_1} \int_{\beta}^{0} \int_{t+\lambda}^{t} \dot{x}^T(s) X_2 \dot{x}(s)\, ds\, d\lambda\, d\beta \\
&\leq -\left[\frac{\sigma_2^2 - \sigma_1^2}{2} x^T(t) - \int_{-\sigma_2}^{-\sigma(t)} \int_{t+\beta}^{t} x^T(s)\, ds\, d\beta - \int_{-\sigma(t)}^{-\sigma_1} \int_{t+\beta}^{t} x^T(s)\, ds\, d\beta\right] \\
&\quad \times X_2 \left[\frac{\sigma_2^2 - \sigma_1^2}{2} x(t) - \int_{-\sigma_2}^{-\sigma(t)} \int_{t+\beta}^{t} x(s)\, ds\, d\beta - \int_{-\sigma(t)}^{-\sigma_1} \int_{t+\beta}^{t} x(s)\, ds\, d\beta\right].
\end{aligned} \tag{3.19}
$$

It follows from **(A1)** that $\left[f_i(x_i(t)) - F_i^- x_i(t)\right]\left[f_i(x_i(t)) - F_i^+ x_i(t)\right] \leq 0$ for every $i = 1, 2, \ldots, n$, which are equivalent to

$$
\begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} F_i^- F_i^+ e_i e_i^T & -\dfrac{F_i^- + F_i^+}{2} e_i e_i^T \\ -\dfrac{F_i^- + F_i^+}{2} e_i e_i^T & e_i e_i^T \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \leq 0,
$$

for every $i = 1, 2, \ldots, n$.
Define $Y_1 = \text{diag}\{y_1, y_2, \ldots, y_n\} > 0$, then

$$
\sum_{i=1}^{n} y_i \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} F_i^- F_i^+ e_i e_i^T & -\dfrac{F_i^- + F_i^+}{2} e_i e_i^T \\ -\dfrac{F_i^- + F_i^+}{2} e_i e_i^T & e_i e_i^T \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \leq 0,
$$

which is equivalent to

$$
\begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} -F_1 Y_1 & F_2 Y_1 \\ F_2 Y_1 & -Y_1 \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \geq 0. \tag{3.20}
$$

Similarly, from **(A2)**, **(A3)** define $Y_2 = \text{diag}\{\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_n\} > 0$,
$Y_3 = \text{diag}\{\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_n\} > 0$ we have

$$
\begin{bmatrix} x(t - \sigma(t)) \\ k(x(t - \sigma(t))) \end{bmatrix}^T \begin{bmatrix} -K_1 Y_2 & K_2 Y_2 \\ K_2 Y_2 & -Y_2 \end{bmatrix} \begin{bmatrix} x(t - \sigma(t)) \\ k(x(t - \sigma(t))) \end{bmatrix} \geq 0, \tag{3.21}
$$

$$
\begin{bmatrix} x(t) \\ h(x(t)) \end{bmatrix}^T \begin{bmatrix} -H_1 Y_3 & H_2 Y_3 \\ H_2 Y_3 & -Y_3 \end{bmatrix} \begin{bmatrix} x(t) \\ h(x(t)) \end{bmatrix} \geq 0. \tag{3.22}
$$

We have zero equation as follows

$$
\begin{aligned}
0 =& 2\left[x^T(t)\beta_1 N^T + \dot{x}^T(t)\beta_2 N^T\right]\Big[- \dot{x}(t) + (N^{-1}Z - A)x(t) + Bf(x(t)) \\
&+ Ck(x(t - \sigma(t))) + D\int_{t-\delta_2(t)}^{t-\delta_1(t)} h(x(s))\, ds + E\omega(t)\Big].
\end{aligned}
$$

Adding above zero equation to $\dot{V}(x(t), t)$, we obtain the following inequality from (2.3), (3.5)–(3.22)

$$\begin{aligned}\dot{V}(x(t), t) + \upsilon z^T(t)z(t) - 2(1-\upsilon)\gamma z^T(t)\omega(t) - \gamma^2\omega^T(t)\omega(t) \\ \leq \xi^T(t)\left(\varepsilon\Theta^{(1)} + (1-\varepsilon)\Theta^{(2)}\right)\xi(t),\end{aligned} \tag{3.23}$$

where, $\Theta^{(i)} = \Theta + \Theta_i$ $(i = 1, 2)$ with Θ and Θ_i are defined in (3.2), (3.3).
Since $0 \leq \varepsilon \leq 1$, the term $\varepsilon\Theta^{(1)} + (1-\varepsilon)\Theta^{(2)}$ is a convex combination of $\Theta^{(1)}$ and $\Theta^{(2)}$. The combinations are negative definite only if

$$\Theta^{(1)} < 0, \tag{3.24}$$

$$\Theta^{(2)} < 0. \tag{3.25}$$

So, (3.24) and (3.25) are equivalent to (3.2) and (3.3), respectively.
Hence, we obtain

$$\dot{V}(x(t), t) + \upsilon z^T(t)z(t) - 2(1-\upsilon)\gamma z^T(t)\omega(t) - \gamma^2\omega^T(t)\omega(t) < 0. \tag{3.26}$$

Under the zero initial condition, for any $\mathcal{T}_p$ we find that

$$\begin{aligned}&\int_0^{\mathcal{T}_p} \upsilon z^T(t)z(t) - 2(1-\upsilon)\gamma z^T(t)\omega(t) - \gamma^2\omega^T(t)\omega(t)\, dt \\ &\quad \leq \int_0^{\mathcal{T}_p} \dot{V}(x(t), t) + \upsilon z^T(t)z(t) - 2(1-\upsilon)\gamma z^T(t)\omega(t) - \gamma^2\omega^T(t)\omega(t)\, dt < 0,\end{aligned}$$

that is

$$\int_0^{\mathcal{T}_p} \upsilon z^T(t)z(t) - 2(1-\upsilon)\gamma z^T(t)\omega(t)\, dt \leq \gamma^2 \int_0^{\mathcal{T}_p} \omega^T(t)\omega(t)\, dt.$$

In this case, the condition (2.3) is guaranteed for any non-zero $\omega(t) \in \mathcal{L}_2[0, \infty)$. If $\omega(t) = 0$, in sense of equation (3.26), there exists a scalar $\upsilon_1 > 0$ such that

$$\dot{V}(x(t), t) < -\upsilon_1 x^T(t)x(t). \tag{3.27}$$

By the definitions of $V_i(x(t), t)$, it is easy to derive the following inequalities:

$$\begin{aligned}V_1(x(t), t) &\leq \lambda_{\max}(P)\|x(t)\|^2, \\ V_4(x(t), t) &\leq \sigma_1^2 \int_{t-\sigma_1}^t \dot{x}^T(\alpha)R_1\dot{x}(\alpha)\, d\alpha, \\ V_5(x(t), t) &\leq \sigma_2^2 \int_{t-\sigma_2}^t \dot{x}^T(\alpha)R_2\dot{x}(\alpha)\, d\alpha, \\ V_6(x(t), t) &\leq (\sigma_2 - \sigma_1)^2 \int_{t-\sigma_2}^t \dot{x}^T(\tau)U\dot{x}(\tau)\, d\tau, \\ V_7(x(t), t) &\leq (\delta_2 - \delta_1)^2 \int_{t-\delta_2}^t h^T(x(\tau))Lh(x(\tau))\, d\tau,\end{aligned} \tag{3.28}$$

$$V_8(x(t),t) \leq \frac{(\sigma_2^2-\sigma_1^2)^2}{4}\int_{t-\sigma_2}^{t} x^T(s)X_1x(s)\,ds,$$
$$V_9(x(t),t) \leq \frac{(\sigma_2^3-\sigma_1^3)^2}{36}\int_{t-\sigma_2}^{t} \dot{x}^T(s)X_2\dot{x}(s)\,ds.$$

We are now ready to deal with the exponential stability of (3.1). Consider the Lyapunov–Krasovskii functional $e^{2ct}V(x(t),t)$, where c is a constant. Using (3.27), (3.28), we have

$$\begin{aligned}
&\frac{d}{dt}e^{2ct}V(x(t),t) = e^{2ct}\dot{V}(x(t),t) + 2ce^{2ct}V(x(t),t)\\
&<e^{2ct}\Bigg[-\upsilon_1 + 2c\Bigg(\lambda_{\max}(P) + \sigma_1\lambda_{\max}(Q_1) + \sigma_2\lambda_{\max}(Q_2) + \sigma_1^3\lambda_{\max}(R_1)\\
&\quad + \sigma_2^3\lambda_{\max}(R_2) + \sigma_2(\sigma_2-\sigma_1)^2\lambda_{\max}(U)\\
&\quad + \delta_2(\delta_2-\delta_1)^2\lambda_{\max}(L)\max_{i\in\{1,2,\dots,n\}}(\tilde{H}_i^2) + \frac{\sigma_2(\sigma_2^2-\sigma_1^2)^2}{4}\lambda_{\max}(X_1)\\
&\quad + \frac{\sigma_2(\sigma_2^3-\sigma_1^3)^2}{36}\lambda_{\max}(X_2)\Bigg)\Bigg]\|x(t+\nu)\|_{cl}.
\end{aligned} \tag{3.29}$$

Let

$$\begin{aligned}
\mu_1 =&\lambda_{\max}(P) + \sigma_1\lambda_{\max}(Q_1) + \sigma_2\lambda_{\max}(Q_2) + \sigma_1^3\lambda_{\max}(R_1) + \sigma_2^3\lambda_{\max}(R_2)\\
&+ \sigma_2(\sigma_2-\sigma_1)^2\lambda_{\max}(U) + \delta_2(\delta_2-\delta_1)^2\lambda_{\max}(L)\max_{i\in\{1,2,\dots,n\}}(\tilde{H}_i^2)\\
&+ \frac{\sigma_2(\sigma_2^2-\sigma_1^2)^2}{4}\lambda_{\max}(X_1) + \frac{\sigma_2(\sigma_2^3-\sigma_1^3)^2}{36}\lambda_{\max}(X_2).
\end{aligned}$$

Now, we take c to be a constant satisfying $c \leq \dfrac{\upsilon_1}{2\mu_1}$, and then achieve from (3.29) that

$$\frac{d}{dt}e^{2ct}V(x(t),t) \leq 0, \tag{3.30}$$

which, together with (3.4) and (3.28), imply that

$$\begin{aligned}
&e^{2ct}V(x(t),t) \leq V(x(0),0) = \sum_{i=1}^{9} V_i(x(0),0)\\
&\leq\Bigg[\lambda_{\max}(P)\|x(0)\|^2 + \int_{-\sigma_1}^{0} x^T(s)Q_1x(s)\,ds + \int_{-\sigma_2}^{0} x^T(s)Q_2x(s)\,ds\\
&\quad + \sigma_1^2\int_{-\sigma_1}^{0}\dot{x}^T(\tau)R_1\dot{x}(\tau)\,d\tau + \sigma_2^2\int_{-\sigma_2}^{0}\dot{x}^T(\tau)R_2\dot{x}(\tau)\,d\tau\\
&\quad + (\sigma_2-\sigma_1)^2\int_{-\sigma_2}^{0}\dot{x}^T(\tau)U\dot{x}(\tau)\,d\tau + (\delta_2-\delta_1)^2\int_{-\delta_2}^{0} h^T(x(\tau))Lh(x(\tau))\,d\tau\\
&\quad + \frac{(\sigma_2^2-\sigma_1^2)^2}{4}\int_{-\sigma_2}^{0} x^T(s)X_1x(s)\,ds + \frac{(\sigma_2^3-\sigma_1^3)^2}{36}\int_{-\sigma_2}^{0}\dot{x}^T(s)X_2\dot{x}(s)\,ds\Bigg]
\end{aligned}$$

$$\leq \mu_0 \|x(\nu)\|_{cl},$$

where

$$\begin{aligned}\mu_0 =& \lambda_{\max}(P) + \sigma_1\lambda_{\max}(Q_1) + \sigma_2\lambda_{\max}(Q_2) + \sigma_1^3\lambda_{\max}(R_1) + \sigma_2^3\lambda_{\max}(R_2)\\ &+ \sigma_2(\sigma_2-\sigma_1)^2\lambda_{\max}(U) + \delta_2(\delta_2-\delta_1)^2\lambda_{\max}(L)\max_{i\in\{1,2,\ldots,n\}}(\tilde{H}_i^2)\\ &+ \frac{\sigma_2(\sigma_2^2-\sigma_1^2)^2}{4}\lambda_{\max}(X_1) + \frac{\sigma_2(\sigma_2^3-\sigma_1^3)^2}{36}\lambda_{\max}(X_2),\end{aligned}$$

and therefore

$$V(x(t),t) \leq \mu_0 e^{-2ct}\|x(\nu)\|_{cl}.$$

Noticing $\lambda_{\min}(P)\|x(t)\|^2 \leq V(x(t),t)$, we obtain

$$\|x(t)\|^2 \leq \frac{\mu_0}{\lambda_{\min}(P)} e^{-2ct}\|x(\nu)\|_{cl}. \tag{3.31}$$

Letting $b_1 = \dfrac{\mu_0}{\lambda_{\min}(P)}$ and $b_2 = 2c$, we can rewrite (3.31) as

$$\|x(t)\|^2 \leq b_1 e^{-b_2 t}\|x(\nu)\|_{cl}.$$

Hence, the NNs (3.1) is exponentially stable with a mixed passive and H_∞ performance index γ. The proof is completed. □

3.2. Mixed passive and H_∞ analysis for uncertain neural networks

In the second part, the criteria of exponential stability with a mixed passive and H_∞ performance for the uncertain neural networks are obtained by using similar proof of Theorem 3.1 together with Lemma 2.5, 2.6.

Theorem 3.2. *For given scalars $\sigma_1, \sigma_2, \delta_1, \delta_2, \beta_1, \beta_2, \gamma > 0$, and $\upsilon \in [0,1]$, if there exist eleven $n\times n$ matrices $P > 0, Q_1 > 0, Q_2 > 0, R_1 > 0, R_2 > 0, U > 0, L > 0, X_1 > 0, X_2 > 0, N > 0, Z$, positive diagonal matrices $Y_1 > 0, Y_2 > 0, Y_3 > 0$ and eight positive constants $\alpha_i > 0$ $(i = 1,2,\ldots,8)$ such that the following LMIs hold:*

$$\Psi + \Theta_1 < 0, \tag{3.32}$$

$$\Psi + \Theta_2 < 0, \tag{3.33}$$

wherein,

$$\begin{aligned}\Theta_1 =& -e_{15}X_1e_{15}^T,\\ \Theta_2 =& -e_{14}X_1e_{14}^T,\\ \bar{\Theta} =& \begin{bmatrix}\bar{\Theta}(1,1) & \Theta(1,2)\\ * & \bar{\Theta}(2,2)\end{bmatrix},\end{aligned}$$

$$\Psi = \begin{bmatrix} \bar{\Theta} & N^TJ_1 & N^TJ_1 & N^TJ_2 & N^TJ_2 & N^TJ_3 & N^TJ_3 & N^TJ_4 & N^TJ_4 \\ * & -\alpha_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\alpha_2 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\alpha_3 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\alpha_4 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\alpha_5 I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\alpha_6 I & 0 & 0 \\ * & * & * & * & * & * & * & -\alpha_7 I & 0 \\ * & * & * & * & * & * & * & * & -\alpha_8 I \end{bmatrix},$$

with: $\Theta(1,2)$ *is defined in Theorem 3.1,*

$$\bar{\Theta}(1,1) = \begin{bmatrix} \bar{\theta}_{1,1} & \theta_{1,2} & -2R_1 & -2R_2 & \upsilon C_1^T C_2 & \theta_{1,6} & \beta_1 N^T C & H_2 Y_3 \\ * & \bar{\theta}_{2,2} & 0 & 0 & 0 & \beta_2 N^T B & \beta_2 N^T C & 0 \\ * & * & \theta_{3,3} & 0 & -2U & 0 & 0 & 0 \\ * & * & * & \theta_{4,4} & -2U & 0 & 0 & 0 \\ * & * & * & * & \theta_{5,5} & 0 & K_2 Y_2 & 0 \\ * & * & * & * & * & \bar{\theta}_{6,6} & 0 & 0 \\ * & * & * & * & * & * & \bar{\theta}_{7,7} & 0 \\ * & * & * & * & * & * & * & \theta_{8,8} \end{bmatrix},$$

$$\bar{\Theta}(2,2) = \begin{bmatrix} -12R_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -12R_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -12U & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -12U & 0 & 0 & 0 & 0 \\ * & * & * & * & \bar{\theta}_{13,13} & 0 & 0 & \theta_{13,16} \\ * & * & * & * & * & \theta_{14,14} & -X_2 & 0 \\ * & * & * & * & * & * & \theta_{15,15} & 0 \\ * & * & * & * & * & * & * & \theta_{16,16} \end{bmatrix},$$

in which:

$$\begin{aligned} \bar{\theta}_{1,1} =& Q_1 + Q_2 - 4R_1 - 4R_2 + \upsilon C_1^T C_1 - F_1 Y_1 - H_1 Y_3 + 2\beta_1 Z - 2\beta_1 N^T A \\ &+ \frac{(\sigma_2^2 - \sigma_1^2)^2}{4} X_1 - \frac{(\sigma_2^2 - \sigma_1^2)^2}{4} X_2 + \alpha_1 \beta_1^2 \Sigma_1^T \Sigma_1, \\ \bar{\theta}_{2,2} =& \sigma_1^2 R_1 + \sigma_2^2 R_2 + (\sigma_2 - \sigma_1)^2 U - 2\beta_2 N^T + \frac{(\sigma_2^3 - \sigma_1^3)^2}{36} X_2 + \alpha_2 \beta_2^2 \Sigma_1^T \Sigma_1, \\ \bar{\theta}_{6,6} =& - Y_1 + \alpha_3 \beta_1^2 \Sigma_2^T \Sigma_2 + \alpha_4 \beta_2^2 \Sigma_2^T \Sigma_2, \\ \bar{\theta}_{7,7} =& - Y_2 + \alpha_5 \beta_1^2 \Sigma_3^T \Sigma_3 + \alpha_6 \beta_2^2 \Sigma_3^T \Sigma_3, \\ \bar{\theta}_{13,13} =& - L + \upsilon C_3^T C_3 + \alpha_7 \beta_1^2 \Sigma_4^T \Sigma_4 + \alpha_8 \beta_2^2 \Sigma_4^T \Sigma_4, \end{aligned}$$

then, the uncertain NNs (2.2) is exponentially stable with a mixed passive and H_∞ *performance index* γ.

Proof. We use the same Lyapunov-Krasovskii functional in Theorem 3.1, such that matrices A, B, C, D are replaced by $A + J_1\mathbb{S}_1(t)\Sigma_1$, $B + J_2\mathbb{S}_2(t)\Sigma_2$, $C + J_3\mathbb{S}_3(t)\Sigma_3$, $D + J_4\mathbb{S}_4(t)\Sigma_4$, respectively. Then applying Lemma 2.5, we get

$$
\begin{aligned}
& x^T(t)(-2\beta_1 N^T \Delta A(t))x(t) \\
&\quad \leq \alpha_1 x^T(t)\beta_1\Sigma_1^T\Sigma_1\beta_1 x(t) + \alpha_1^{-1}x^T(t)N^T J_1 J_1^T N x(t), \\
& x^T(t)(-\beta_2 N^T \Delta A(t))\dot{x}(t) + \dot{x}^T(t)(-\beta_2\Delta A^T(t)N)x(t) \\
&\quad \leq \alpha_2 \dot{x}^T(t)\beta_2\Sigma_1^T\Sigma_1\beta_2\dot{x}(t) + \alpha_2^{-1}x^T(t)N^T J_1 J_1^T N x(t), \\
& x^T(t)\beta_1 N^T\Delta B(t) f(x(t)) + f^T(x(t))\beta_1\Delta B^T(t)N x(t) \\
&\quad \leq \alpha_3 f^T(x(t))\beta_1\Sigma_2^T\Sigma_2\beta_1 f(x(t)) + \alpha_3^{-1}x^T(t)N^T J_2 J_2^T N x(t), \\
& \dot{x}^T(t)\beta_2 N^T\Delta B(t) f(x(t)) + f^T(x(t))\beta_2\Delta B^T(t)N\dot{x}(t) \\
&\quad \leq \alpha_4 f^T(x(t))\beta_2\Sigma_2^T\Sigma_2\beta_2 f(x(t)) + \alpha_4^{-1}\dot{x}^T(t)N^T J_2 J_2^T N\dot{x}(t), \\
& x^T(t)\beta_1 N^T\Delta C(t)k(x(t-\sigma(t))) + k^T(x(t-\sigma(t)))\beta_1\Delta C^T(t)N x(t) \\
&\quad \leq \alpha_5 k^T(x(t-\sigma(t)))\beta_1\Sigma_3^T\Sigma_3\beta_1 k(x(t-\sigma(t))) + \alpha_5^{-1}x^T(t)N^T J_3 J_3^T N x(t), \\
& \dot{x}^T(t)\beta_2 N^T\Delta C(t)k(x(t-\sigma(t))) + k^T(x(t-\sigma(t)))\beta_2\Delta C^T(t)N\dot{x}(t) \\
&\quad \leq \alpha_6 k^T(x(t-\sigma(t)))\beta_2\Sigma_3^T\Sigma_3\beta_2 k(x(t-\sigma(t))) + \alpha_6^{-1}\dot{x}^T(t)N^T J_3 J_3^T N\dot{x}(t), \\
& x^T(t)\beta_1 N^T\Delta D(t)\int_{t-\delta_2(t)}^{t-\delta_1(t)} h(x(s))\,ds + \int_{t-\delta_2(t)}^{t-\delta_1(t)} h^T(x(s))\,ds\beta_1\Delta D^T(t)N x(t) \\
&\quad \leq \alpha_7 \int_{t-\delta_2(t)}^{t-\delta_1(t)} h^T(x(s))\,ds\beta_1\Sigma_4^T\Sigma_4\beta_1\int_{t-\delta_2(t)}^{t-\delta_1(t)} h(x(s))\,ds \\
&\qquad + \alpha_7^{-1}x^T(t)N^T J_4 J_4^T N x(t), \\
& \dot{x}^T(t)\beta_2 N^T\Delta D(t)\int_{t-\delta_2(t)}^{t-\delta_1(t)} h(x(s))\,ds + \int_{t-\delta_2(t)}^{t-\delta_1(t)} h^T(x(s))\,ds\beta_2\Delta D^T(t)N\dot{x}(t) \\
&\quad \leq \alpha_8 \int_{t-\delta_2(t)}^{t-\delta_1(t)} h^T(x(s))\,ds\beta_2\Sigma_4^T\Sigma_4\beta_2\int_{t-\delta_2(t)}^{t-\delta_1(t)} h(x(s))\,ds \\
&\qquad + \alpha_8^{-1}\dot{x}^T(t)N^T J_4 J_4^T N\dot{x}(t).
\end{aligned}
$$

Then applying the similar proof of Theorem 3.1 and Lemma 2.6, we have

$$
\begin{aligned}
& \dot{V}(t) + \upsilon z^T(t)z(t) - 2(1-\upsilon)\gamma z^T(t)\omega(t) - \gamma^2\omega^T(t)\omega(t) \\
&\quad \leq \xi^T(t)\left(\varepsilon\Psi^{(1)} + (1-\varepsilon)\Psi^{(2)}\right)\xi(t),
\end{aligned}
$$

where, $\Psi^{(i)} = \Psi + \Theta_i$ $(i = 1, 2)$ with Ψ and Θ_i are defined in (3.32), (3.33).
Since $0 \leq \varepsilon \leq 1$, the term $\varepsilon\Psi^{(1)} + (1-\varepsilon)\Psi^{(2)}$ is a convex combination of $\Psi^{(1)}$ and $\Psi^{(2)}$. The combinations are negative definite only if

$$\Psi^{(1)} < 0, \tag{3.34}$$

$$\Psi^{(2)} < 0. \tag{3.35}$$

Therefore, (3.34) and (3.35) are equivalent to (3.32) and (3.33), respectively. This completes the proof. □

In the third part, we will investigate the stability of a special model of the neural networks, in order to compare the maximum delay with existing results.

Remark 2. *We consider the following neural network model as a special case of the system (2.1)*

$$\dot{x}(t) = -Ax(t) + Bf(x(t)) + Ck(x(t - \sigma(t))). \tag{3.36}$$

Corollary 3.3. *For given scalars $\sigma_1, \sigma_2, \beta_1$ and β_2, if there exist nine $n \times n$ matrices $P > 0, Q_1 > 0, Q_2 > 0, R_1 > 0, R_2 > 0, U > 0, X_1 > 0, X_2 > 0, N > 0$ and two $n \times n$ positive diagonal matrices $Y_1 > 0, Y_2 > 0$ such that the following LMIs hold:*

$$\Pi + \Pi_1 < 0, \tag{3.37}$$
$$\Pi + \Pi_2 < 0, \tag{3.38}$$

where

$$\begin{aligned}
\Pi_1 &= -\, e_{13} X_1 e_{13}^T, \\
\Pi_2 &= -\, e_{12} X_1 e_{12}^T, \\
\Pi &= \left[\theta'_{(i,j)}\right]_{13\times 13},
\end{aligned}$$

with $(\theta'_{(i,j)})^T = \theta'_{(j,i)}$,

$$\begin{aligned}
\theta'_{(1,1)} &= Q_1 + Q_2 - 4R_1 - 4R_2 - F_1 Y_1 - 2\beta_1 N^T A + \tfrac{(\sigma_2^2-\sigma_1^2)^2}{4} X_1 - \tfrac{(\sigma_2^2-\sigma_1^2)^2}{4} X_2, \\
\theta'_{(1,2)} &= P - \beta_1 N^T - \beta_2 N^T A, \quad \theta'_{(1,3)} = -2R_1, \quad \theta'_{(1,4)} = -2R_2, \quad \theta'_{(1,6)} = F_2 Y_1 + \beta_1 N^T B, \\
\theta'_{(1,7)} &= \beta_1 N^T C, \quad \theta'_{(1,8)} = 6R_1, \quad \theta'_{(1,9)} = 6R_2, \quad \theta'_{(1,12)} = \tfrac{\sigma_2^2-\sigma_1^2}{2} X_2, \quad \theta'_{(1,13)} = \tfrac{\sigma_2^2-\sigma_1^2}{2} X_2, \\
\theta'_{(2,2)} &= \sigma_1^2 R_1 + \sigma_2^2 R_2 + (\sigma_2 - \sigma_1)^2 U - 2\beta_2 N^T + \tfrac{(\sigma_2^3-\sigma_1^3)^2}{36} X_2, \quad \theta'_{(2,6)} = \beta_2 N^T B, \\
\theta'_{(2,7)} &= \beta_2 N^T C, \quad \theta'_{(3,3)} = -Q_1 - 4R_1 - 4U, \quad \theta'_{(3,5)} = -2U, \quad \theta'_{(3,8)} = 6R_1, \quad \theta'_{(3,10)} = 6U, \\
\theta'_{(4,4)} &= -Q_2 - 4R_2 - 4U, \quad \theta'_{(4,5)} = -2U, \quad \theta'_{(4,9)} = 6R_2, \quad \theta'_{(4,11)} = 6U, \quad \theta'_{(5,5)} = -8U - K_1 Y_2, \\
\theta'_{(5,7)} &= K_2 Y_2, \quad \theta'_{(5,10)} = 6U, \quad \theta'_{(5,11)} = 6U, \quad \theta'_{(6,6)} = -Y_1, \quad \theta'_{(7,7)} = -Y_2, \\
\theta'_{(8,8)} &= -12R_1, \quad \theta'_{(9,9)} = -12R_2, \quad \theta'_{(10,10)} = -12U, \\
\theta'_{(11,11)} &= -12U, \quad \theta'_{(12,12)} = -X_1 - X_2, \quad \theta'_{(12,13)} = -X_2, \quad \theta'_{(13,13)} = -X_1 - X_2,
\end{aligned}$$

another terms are 0,
then, the NNs (3.36) is exponentially stable.

Proof. We choose the following Lyapunov–Krasovskii functional candidate for the system (3.36) as

$$V(x(t), t) = \sum_{i=1}^{8} V_i(x(t), t),$$

where

$$\begin{aligned}
V_1(x(t), t) &= x^T(t) P x(t), \\
V_2(x(t), t) &= \int_{t-\sigma_1}^{t} x^T(s) Q_1 x(s)\, ds, \\
V_3(x(t), t) &= \int_{t-\sigma_2}^{t} x^T(s) Q_2 x(s)\, ds,
\end{aligned}$$

$$V_4(x(t),t) = \sigma_1 \int_{-\sigma_1}^{0} \int_{t+s}^{t} \dot{x}^T(\tau) R_1 \dot{x}(\tau)\, d\tau\, ds,$$
$$V_5(x(t),t) = \sigma_2 \int_{-\sigma_2}^{0} \int_{t+s}^{t} \dot{x}^T(\tau) R_2 \dot{x}(\tau)\, d\tau\, ds,$$
$$V_6(x(t),t) = (\sigma_2 - \sigma_1) \int_{-\sigma_2}^{-\sigma_1} \int_{t+s}^{t} \dot{x}^T(\tau) U \dot{x}(\tau)\, d\tau\, ds,$$
$$V_7(x(t),t) = \frac{(\sigma_2^2 - \sigma_1^2)}{2} \int_{-\sigma_2}^{-\sigma_1} \int_{\beta}^{0} \int_{t+\lambda}^{t} x^T(s) X_1 x(s)\, ds\, d\lambda\, d\beta,$$
$$V_8(x(t),t) = \frac{(\sigma_2^3 - \sigma_1^3)}{6} \int_{-\sigma_2}^{-\sigma_1} \int_{\beta}^{0} \int_{\lambda}^{0} \int_{t+\varphi}^{t} \dot{x}^T(s) X_2 \dot{x}(s)\, ds\, d\varphi\, d\lambda\, d\beta.$$

By applying similar proof in Theorem 3.1, the system (3.36) is exponentially stable. □

Remark 3. *Recently, the robust passivity problem of uncertain neural networks with interval discrete and distributed time-varying delays has been studied in [14]. Also, robust reliable H_∞ control problem of uncertain neural networks with mixed time delays has been discussed in [23]. However, the problem of mixed passive and H_∞ for uncertain neural networks with interval discrete and distributed time-varying delays has not been investigated yet. The results in this paper provide the sufficient conditions to assure that the uncertain neural network is exponentially stable with mixed passive and H_∞ index γ. The conditions are obtained by constructing a Lyapunov-Krasovskii functional consisting novel integral terms.*

Remark 4. *It is well known that time delay is a normal phenomenon that appears in neural networks since the neural networks consist of a large number of neurons that connect with each other into a diversity of axon sizes and lengths. Practically time delay can occur in an irregular fashion such as sometimes the time-varying delays are not differentiable. So, in this work, the interval discrete and distributed time-varying delays do not necessitate being differentiable functions.*

Remark 5. *It is well known that the H_∞ theory is very important in the control problem. Besides, the H_∞ approaches are used in control theory to synthesize controllers achieving stabilization with an H_∞ norm bound limited to disturbance reduction. The passivity theory is widely used in system synthesis and analysis, as the system with passivity performance can effectively reduce the impact of noise. In fact, the passivity system does not produce energy by itself, but it will use the system's energy. The main property of passivity is that can keep the system internally stable. By the above mentioned, the obtained results are based on mixed passivity and H_∞ problem for uncertain neural networks with mixed time-varying delays. In comparison between the design of mixed H_∞/passive performance and a single H_∞ or passive controller, the control problem under mixed H_∞/passive performance consideration is more general than a single H_∞ or passive controller for example, a simple actual mixed H_∞ and passive performance index is employed in handling with the event-triggered reliable control issue for the fuzzy Markov jump systems (FMJSs), which can achieve the H_∞ or passive event-triggered reliable control problem for FMJSs by turning some fixed parameters. Hence, this paper are more general and convenient than the existing individual passive and H_∞ problem.*

Remark 6. *In this work, the Lyapunov-Krasovskii functional consisting single, double, triple, and quadruple integral terms, which full of the information of the delays $\sigma_1, \sigma_2, \delta_1, \delta_2$, and a state variable*

$x(t)$. Furthermore, more information on activation functions has taken fully into the stability and performance analysis that is $F_i^- \le \dfrac{f_i(x_i(t))}{x_i(t)} \le F_i^+$, $K_i^- \le \dfrac{k_i(x_i(t-\sigma(t)))}{x_i(t-\sigma(t))} \le K_i^+$, and $H_i^- \le \dfrac{h_i(x_i(t))}{x_i(t)} \le H_i^+$ are addressed in the calculation. Hence, the construction and the technique for computation of the Lyapunov-Krasovskii functional are the main key to improve results of this work. In the proof of Theorems 3.1, 3.2, and Corollary 3.3, integral inequalities and convex combination technique are used to bound the derivative of Lyapunov-Krasovskii functional, which provide tighter than the inequalities in [30–32, 38]. All of these lead to the improved results in our work as we can see the compared results with some exiting works in numerical examples. However, the complex computation of the Lyapunov-Krasovskii functional leads to the LMI derived in this work which contains many information of the system. It is feasible for NNs with large number of neurons which can be solved by using the Matlab LMI toolbox. Hence, for further work, it is interesting for researchers to improve the technique for a simple Lyapunov-Krasovskii functional and also achieve better results.

4. Numerical examples

In this section, we provided four numerical examples which are illustrated the effectiveness of the proposed results. Moreover, two numerical examples show less conservative results than others.

Example 4.1. *We consider the neural networks (3.36) with matrix parameters in [30]:*

$$\begin{aligned} A &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1.5 \end{bmatrix}, \quad C = \begin{bmatrix} -2 & 0.5 \\ 0.5 & -2 \end{bmatrix}, \\ F_1 &= K_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \text{and} \quad F_2 = K_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}. \end{aligned}$$

By taking parameters $\beta_1 = \beta_2 = 1$ and solving Example 4.1 using LMIs in Corollary 3.3, we obtain maximum allowable values of σ_2 for different σ_1 without the upper bound of differentiable delay (μ) as shown in Table 1. Table 1 shows that the results derived in this paper are less conservative than the results in [30].

Table 1. The maximum allowable values of σ_2 for different values of σ_1 and μ.

Methods	σ_1	$\mu = 0.8$	$\mu = 0.9$	Unknown μ
[30]	$\sigma_1 = 0.5$	0.8262	0.8215	-
Corollary 3.3		-	-	0.9976
[30]	$\sigma_1 = 0.75$	0.9669	0.9625	-
Corollary 3.3		-	-	1.1233
[30]	$\sigma_1 = 1$	1.1152	1.1108	-
Corollary 3.3		-	-	1.2710

Example 4.2. *We consider the neural networks (3.36) with matrix parameters in [31, 32, 38]:*

$$A = \begin{bmatrix} 1.5 & 0 \\ 0 & 0.7 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0503 & 0.0454 \\ 0.0987 & 0.2075 \end{bmatrix}, \quad C = \begin{bmatrix} 0.2381 & 0.9320 \\ 0.0388 & 0.5062 \end{bmatrix},$$

$$F_1 = K_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad and \quad F_2 = K_2 = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.4 \end{bmatrix}.$$

By taking parameters $\beta_1 = \beta_2 = 1$ *and solving Example 4.2 using LMIs in Corollary 3.3, we get maximum allowable values of* σ_2 *for* $\sigma_1 = 0$ *without the upper bound of differentiable delay* (μ) *as shown in Table 2. Table 2 illustrates that the results obtained in this paper are less conservative than the results in [31, 32, 38].*

Table 2. The maximum allowable values of σ_2 for $\sigma_1 = 0$ and different values of μ.

Methods	$\mu = 0.5$	$\mu = 0.55$	Unknown μ
[38]	3.0594	2.9814	-
[31]	3.3377	3.2350	-
[32]	3.4600	3.4100	-
Corollary 3.3	-	-	3.5814

Example 4.3. *We consider the neural networks (3.1) with* $\sigma_1 = 0.5, \sigma_2 = 1.75, \delta_1 = 0.2, \delta_2 = 1.0, \upsilon = 0.1, \beta_1 = 0.9, \beta_2 = 0.2,$

$$\begin{aligned}
A &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.2 & -0.1 \\ -0.5 & 0.1 \end{bmatrix}, \quad C = \begin{bmatrix} -0.5 & 0 \\ -0.3 & -0.2 \end{bmatrix}, \\
D &= \begin{bmatrix} 0.15 & 0.1 \\ 0 & -0.3 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\
C_2 &= C_3 = C_4 = 0.1I, \quad F_1 = K_1 = H_1 = -0.4I, \\
F_2 &= K_2 = H_2 = 0.4I, \quad h_i(x_i) = \tanh(x_i), \textit{and} \\
f_i(x_i) &= k_i(x_i) = 0.2(\mid x_i + 1 \mid - \mid x_i - 1 \mid).
\end{aligned}$$

LMIs of (3.2), (3.3) in Theorem 3.1 are solved, we obtain

$$\begin{aligned}
P &= \begin{bmatrix} 3.6577 & -0.2200 \\ -0.2200 & 3.7479 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 4.0338 & 0.0460 \\ 0.0460 & 3.9284 \end{bmatrix}, \\
Q_2 &= \begin{bmatrix} 4.1684 & 0.0580 \\ 0.0580 & 4.0614 \end{bmatrix}, \quad R_1 = \begin{bmatrix} 0.1741 & -0.0295 \\ -0.0295 & 0.1987 \end{bmatrix}, \\
R_2 &= \begin{bmatrix} 0.0292 & -0.0121 \\ -0.0121 & 0.0394 \end{bmatrix}, \quad U = \begin{bmatrix} 0.2121 & -0.0178 \\ -0.0178 & 0.2033 \end{bmatrix}, \\
L &= \begin{bmatrix} 3.6430 & 0.0277 \\ 0.0277 & 3.7766 \end{bmatrix}, \quad X_1 = \begin{bmatrix} 2.8416 & 0.0197 \\ 0.0197 & 2.8239 \end{bmatrix}, \\
X_2 &= \begin{bmatrix} 0.0479 & -0.0193 \\ -0.0193 & 0.0642 \end{bmatrix}, \quad N = \begin{bmatrix} 2.3363 & -0.2470 \\ -0.2432 & 2.3840 \end{bmatrix}, \\
Z &= \begin{bmatrix} -13.6905 & -0.9328 \\ -0.5358 & -12.9525 \end{bmatrix}, \quad Y_1 = \begin{bmatrix} 3.9748 & 0 \\ 0 & 3.9748 \end{bmatrix}, \\
Y_2 &= \begin{bmatrix} 0.4324 & 0 \\ 0 & 0.4324 \end{bmatrix}, \quad Y_3 = \begin{bmatrix} 4.6765 & 0 \\ 0 & 4.6765 \end{bmatrix}.
\end{aligned}$$

The state feedback control is obtained by

$$\mathcal{U}(t) = N^{-1}Zx(t) = \begin{bmatrix} -5.9479 & -0.9843 \\ -0.8316 & -5.5335 \end{bmatrix} x(t), \quad t \geq 0.$$

The maximum allowable values of σ_2 for different values of σ_1 are shown in Table 3. Furthermore, we want to find the relation among the scalars σ_2, υ, and γ. For three different values of υ, we set $\upsilon = 0$, $\upsilon = 0.5$, and $\upsilon = 1$, respectively, which means the passivity case, passivity and H_∞ case, and H_∞ case are studied, respectively. Moreover, we choose the values of σ_2 from $\sigma_2 = 0.5$ to $\sigma_2 = 2$ and other parameters are fixed by $\sigma_1 = 0.2$, $\delta_1 = 0.2$, $\delta_2 = 0.8$, $\beta_1 = 0.9$, $\beta_2 = 0.2$. By applying Theorem 3.1 and Matlab LMI toolbox to solve LMIs (3.2) and (3.3), we have the relation among the parameters σ_2, υ, and γ, which is presented in Table 4. Figure 1 shows the response solution $x(t)$ in Example 4.3 where $\omega(t) = 0$ and the initial condition $\phi(t) = [-0.1 \quad 0.1]^T$. Figure 2 shows the response solution $x(t)$ in Example 4.3 where $\omega(t)$ is Gaussian noise with mean 0 and variance 1 and the initial condition $\phi(t) = [-0.1 \quad 0.1]^T$.

The numerical simulations are accomplished using the explicit Runge-Kutta-like method (dde45), extrapolation and interpolation by spline of the third order.

Table 3. The maximum allowable values of σ_2 for different values of σ_1 in Example 4.3.

Method	$\sigma_1 = 0$	$\sigma_1 = 0.5$	$\sigma_1 = 1$	$\sigma_1 = 2$	$\sigma_1 = 3$
Theorem 3.1	2.1176	2.3865	2.5354	3.3564	4.1253

Table 4. The minimum allowable values of γ for mixed passive and H_∞ analysis with different values of σ_2 and υ in Example 4.3.

γ_{min}	$\sigma_2 = 0.5$	$\sigma_2 = 1$	$\sigma_2 = 1.5$	$\sigma_2 = 2$
$\upsilon = 0$	0.5672	0.6835	0.8135	0.9465
$\upsilon = 0.5$	0.7752	0.9683	1.1035	1.2156
$\upsilon = 1$	1.2331	1.4452	1.6862	1.7965

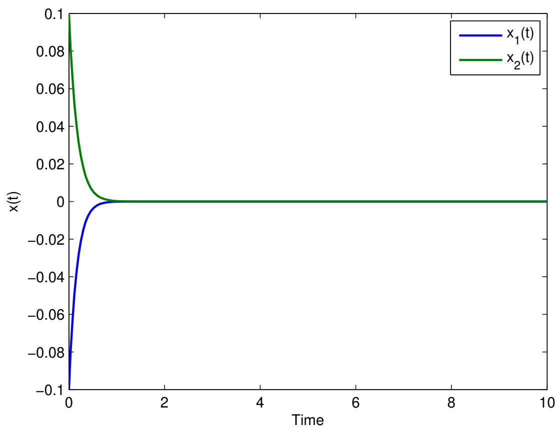

Figure 1. The trajectories of $x_1(t)$ and $x_2(t)$ with $\omega(t) = 0$ in Example 4.3.

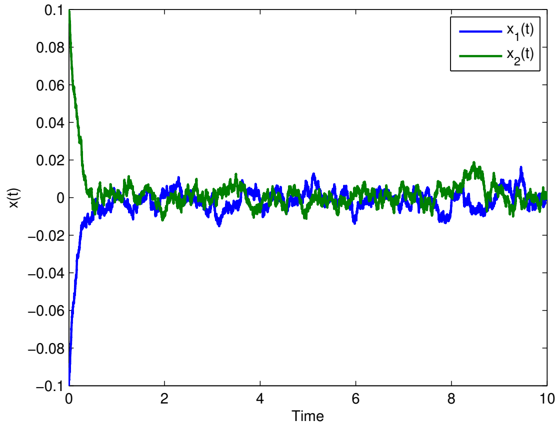

Figure 2. The trajectories of $x_1(t)$ and $x_2(t)$ with Gaussian noise in Example 4.3.

Example 4.4. *We consider the uncertain neural networks (2.2) with* $\sigma_1 = 0.7, \sigma_2 = 1.5, \delta_1 = 0.2, \delta_2 = 1, \upsilon = 0.1, \beta_1 = 0.9, \beta_2 = 0.2,$

$$
\begin{aligned}
A &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.2 & -0.1 \\ -0.5 & 0.1 \end{bmatrix}, \quad C = \begin{bmatrix} -0.5 & 0 \\ -0.3 & -0.2 \end{bmatrix}, \\
D &= \begin{bmatrix} 0.15 & 0.1 \\ 0 & -0.3 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\
C_2 &= C_3 = C_4 = 0.1I, \quad F_1 = K_1 = H_1 = -0.4I, \\
F_2 &= K_2 = H_2 = 0.4I, \quad J_1 = J_2 = J_3 = J_4 = 0.2I, \\
\Sigma_1 &= \Sigma_2 = \Sigma_3 = \Sigma_4 = I, \quad h_i(x_i) = \tanh(x_i), \textit{and} \\
f_i(x_i) &= k_i(x_i) = 0.2(|\ x_i + 1\ | - |\ x_i - 1\ |).
\end{aligned}
$$

LMIs of (3.32), (3.33) in Theorem 3.2 are solved, we obtain

$$
\begin{aligned}
P &= \begin{bmatrix} 2.8510 & -0.1478 \\ -0.1478 & 2.7756 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 2.3154 & 0.1464 \\ 0.1464 & 2.0641 \end{bmatrix}, \\
Q_2 &= \begin{bmatrix} 2.3534 & 0.1402 \\ 0.1402 & 2.1063 \end{bmatrix}, \quad R_1 = \begin{bmatrix} 0.0523 & -0.0200 \\ -0.0200 & 0.0680 \end{bmatrix}, \\
R_2 &= \begin{bmatrix} 0.0131 & -0.0077 \\ -0.0077 & 0.0192 \end{bmatrix}, U = \begin{bmatrix} 0.2104 & -0.0062 \\ -0.0062 & 0.1748 \end{bmatrix}, \\
L &= \begin{bmatrix} 2.2685 & 0.0108 \\ 0.0108 & 2.3401 \end{bmatrix}, \quad X_1 = \begin{bmatrix} 1.3348 & 0.0125 \\ 0.0125 & 1.3274 \end{bmatrix}, \\
X_2 &= \begin{bmatrix} 0.0224 & -0.0103 \\ -0.0103 & 0.0305 \end{bmatrix}, \quad N = \begin{bmatrix} 1.6053 & -0.0435 \\ -0.0978 & 1.4801 \end{bmatrix}, \\
Z &= \begin{bmatrix} -10.2677 & -0.5664 \\ -0.5137 & -9.2901 \end{bmatrix}, \quad Y_1 = \begin{bmatrix} 3.0132 & 0 \\ 0 & 3.0132 \end{bmatrix}, \\
Y_2 &= \begin{bmatrix} 0.4054 & 0 \\ 0 & 0.4054 \end{bmatrix}, \quad Y_3 = \begin{bmatrix} 3.4945 & 0 \\ 0 & 3.4945 \end{bmatrix},
\end{aligned}
$$

$$\begin{aligned}\alpha_1 &= 1.9270, \quad \alpha_2 = 0.6465, \quad \alpha_3 = 1.2926, \quad \alpha_4 = 1.9011,\\ \alpha_5 &= 0.0754, \quad \alpha_6 = 0.6677, \quad \alpha_7 = 1.2734, \quad \alpha_8 = 1.8989.\end{aligned}$$

The state feedback control is obtained by

$$\mathcal{U}(t) = N^{-1}Zx(t) = \begin{bmatrix} -6.4170 & -0.5239 \\ -0.7712 & -6.3115 \end{bmatrix} x(t), \quad t \geq 0.$$

The maximum allowable values of σ_2 for different values of σ_1 are shown in Table 5. Furthermore, we want to find the relation among the scalars σ_2, υ, and γ. For three different values of υ, we set $\upsilon = 0$, $\upsilon = 0.5$, and $\upsilon = 1$, respectively, which means the passivity case, passivity and H_∞ case, and H_∞ case are considered, respectively. Moreover, we choose the values of σ_2 from $\sigma_2 = 0.5$ to $\sigma_2 = 2$ and other parameters are fixed by $\sigma_1 = 0.2$, $\delta_1 = 0.2$, $\delta_2 = 0.8$, $\beta_1 = 0.9$, $\beta_2 = 0.2$. By applying Theorem 3.2 and Matlab LMI toolbox to solve LMIs (3.32) and (3.33), we have the relation among the parameters σ_2, υ, and γ, which is presented in Table 6. Figure 3 shows the response solution $x(t)$ in Example 4.4 where $\omega(t) = 0$ and the initial condition $\phi(t) = [-0.1 \quad 0.1]^T$. Figure 4 shows the response solution $x(t)$ in Example 4.4 where $\omega(t)$ is Gaussian noise with mean 0 and variance 1 and the initial condition $\phi(t) = [-0.1 \quad 0.1]^T$.

Table 5. The maximum allowable values of σ_2 for different values of σ_1 in Example 4.4.

Method	$\sigma_1 = 0$	$\sigma_1 = 0.5$	$\sigma_1 = 1$	$\sigma_1 = 2$	$\sigma_1 = 3$
Theorem 3.2.	1.8308	2.2056	2.4233	3.1232	3.8142

Table 6. The minimum allowable values of γ for mixed passive and H_∞ analysis with different values of σ_2 and υ in Example 4.4.

γ_{min}	$\sigma_2 = 0.5$	$\sigma_2 = 1$	$\sigma_2 = 1.5$	$\sigma_2 = 2$
$\upsilon = 0$	0.6354	0.7534	0.8756	0.9869
$\upsilon = 0.5$	0.8965	1.0231	1.2231	1.4365
$\upsilon = 1$	1.6352	1.7563	1.8641	1.9634

Remark 7. *In this work, we choose $\sigma_1, \sigma_2, \delta_1, \delta_2, \beta_1, \beta_2, \gamma$ are real numbers that satisfy $0 \leq \sigma_1 \leq \sigma(t) \leq \sigma_2$, $0 \leq \delta_1 \leq \delta_1(t) \leq \delta_2(t) \leq \delta_2$, and $\gamma > 0$. In practice, the designing of these parameters can occur in an appropriate range. Furthermore, the suitable values of $\sigma_1, \sigma_2, \delta_1, \delta_2, \beta_1, \beta_2$ lead to the smallest γ for the mixed passive and H_∞ analysis.*

Remark 8. *The stability criteria of Theorem 3.1 in the form LMIs (3.2) and (3.3) can be easily to examine by using LMI toolbox in MATLAB [39]. The improved stability criteria by using the Lyapunov-Krasovskii functional is based on LMIs and the dimension of the LMIs depends on the number of the neurons in neural networks. Thus, the computational burden problem goes up. This problem is the issue in studying needs of LMI optimization in applied mathematics and the optimization research. Hence, in the further, new techniques should be considered to reduce the conservativeness caused by the time-delays such as the delay-fractioning approach and so on.*

Remark 9. *In the future work, it is very challenging to apply some lemmas or Lyapunov-Krasovskii functional used in this paper to apply into the quaternion-valued case to get improved stability conditions.*

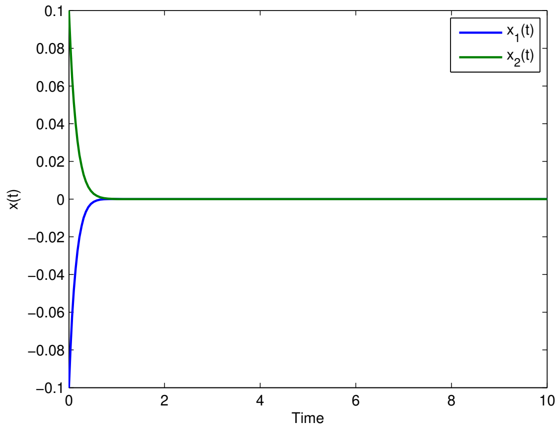

Figure 3. The trajectories of $x_1(t)$ and $x_2(t)$ with $\omega(t) = 0$ in Example 4.4.

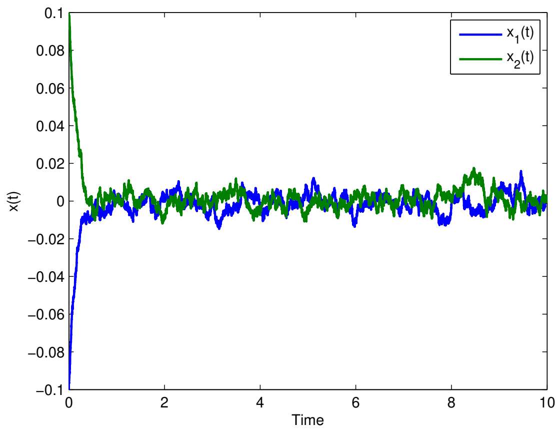

Figure 4. The trajectories of $x_1(t)$ and $x_2(t)$ with Gaussian noise in Example 4.4.

5. Conclusion

The problem of mixed passive and H_∞ analysis for uncertain neural networks with the state feedback control is investigated in this paper. We obtain the new sufficient conditions to guarantee exponential stability with mixed passive and H_∞ performance for the uncertain neural networks by using a Lyapunov-Krasovskii functional consisting single, double, triple, and quadruple integral terms with a feedback controller. Furthermore, integral inequalities and convex combination technique are applied to achieve the less conservative results for a special case of neural networks with interval discrete time-varying delays. The new criteria are in terms of linear matrix inequalities (LMIs) that cover H_∞, and passive performance by setting parameters in the general performance index. Finally, numerical examples have been given to show the effectiveness of the proposed results and improve over some existing results in the literature. In the future work, the derived results and methods in this paper are expected to be applied to other systems such as fuzzy control systems, complex dynamical networks, quaternion-valued neural networks and so on [16, 40, 41].

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Conflict of interest

The authors declare no conflict of interest.

References

1. L. O. Chua, L. Yang, Cellular neural networks: applications, *IEEE Trans. Circuits Syst.*, **35** (1988), 1273–1290.
2. M. A. Cohen, S. Grossberg, Absolute stability of global pattern formation and parallel memory storage by competitive neural networks, *IEEE Trans. Syst. Man Cybern.*, **13** (1983), 815–826.
3. S. Haykin, *Neural Networks*, New Jersey: Englewood Cliffs, 1994.
4. H. Wang, Y. Yu, G. Wen, Stability analysis of fractional-order Hopfield neural networks with time delays, *Neural Networks*, **55** (2014), 98–109.
5. H. Zhang, Z. Wang, New delay-dependent criterion for the stability of recurrent neural networks with time-varying delay, *Sci. China Series F*, **52** (2009), 942–948.
6. L. Wang, X. Zou, Harmless delays in Cohen-Grossberg neural networks, *Phys. D*, **170** (2002), 162–173.
7. A. Farnam, R. M. Esfanjani, A. Ahmadi, Delay-dependent criterion for exponential stability analysis of neural networks with time-varying delays, *IFAC-PapersOnLine*, **49** (2016), 130–135.
8. H. B. Zeng, Y. He, P. Shi, M. Wu, S. P. Xiao, Dissipativity analysis of neural networks with time-varying delays, *Neurocomputing*, **168** (2015), 741–746.
9. Y. Liu, Z. Wang, X. Liu, Global exponential stability of generalized recurrent neural networks with discrete and distributed delays, *Neural Networks*, **19** (2006), 667–675.
10. H. P. Kriegel, M. Pfeifle, Density-based clustering of uncertain data, In: *Proceedings of the ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, 2005, 672–677.
11. G. Cormode, A. McGregor, Approximation algorithms for clustering uncertain data, In: *Proceedings of the ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Database Systems*, 2008, 191–199.
12. C. C. Aggarwal, P. Yu, A framework for clustering uncertain data streams, In: *Proceedings-International Conference on Data Engineering*, 2008, 150–159.

13. K. Subramanian, P. Muthukumar, S. Lakshmanan, Robust stabilization of uncertain neural networks with additive time-varying delays, *IFAC-PapersOnLine*, **49** (2016), 154–159.

14. H. B. Zeng, J. H. Park, H. Shen, Robust passivity analysis of neural networks with discrete and distributed delays, *Neurocomputing*, **149** (2015), 1092–1097.

15. L. Wu, W. X. Zheng, Passivity-based sliding mode control of uncertain singular time-delay systems, *Automatica*, **45** (2009), 2120–2127.

16. G. Calcev, R. Gorez, M. D. Neyer, Passivity approach to fuzzy control systems, *Automatica*, **34** (1998), 339–344.

17. H. Gao, T. Chen, T. Chai, Passivity and passification for networked control systems, *SIAM J. Control Optim.*, **46** (2007), 1299–1322.

18. L. Xie, M. Fu, H. Li, Passivity analysis and passification for uncertain signal processing systems, *IEEE Trans. Signal Process.*, **46** (1998), 2394–2403.

19. H. Li, J. Lam, K. C. Cheung, Passivity criteria for continuous-time neural networks with mixed time-varying delays, *Appl. Math. Comput.*, **218** (2012), 11062–11074.

20. M. V. Thuan, H. Trinh, L. V. Hien, New inequality-based approach to passivity analysis of neural networks with interval time-varying delay, *Neurocomputing*, **194** (2016), 301–307.

21. S. Xu, W. X. Zheng, Y. Zou, Passivity analysis of neural networks with time-varying delays, *IEEE T. Circuits II*, **56** (2009), 325–329.

22. N. Yotha, T. Botmart, K. Mukdasai, W. Weera, Improved delay-dependent approach to passivity analysis for uncertain neural networks with discrete interval and distributed time-varying delays, *Vietnam J. Math.*, **45** (2017), 721–736.

23. Y. Du, X. Liu, S. Zhong, Robust reliable H_∞ control for neural networks with mixed time delays, *Chaos Soliton. Fract.*, **91** (2016), 1–8.

24. M. Syed Ali, R. Saravanakumar, S. Arik, Novel H_∞ state estimation of static neural networks with interval time-varying delays via augmented Lyapunov-Krasovskii functional, *Neurocomputing*, **171** (2016), 949–954.

25. K. Mathiyalagan, J. H. Park, R. Sakthivel, S. M. Anthoni, Robust mixed H_∞ and passive filtering for networked markov jump systems with impulses, *Signal Process.*, **101** (2014), 162–173.

26. M. Meisami-Azad, J. Mohammadpour, K. M. Grigoriadis, Dissipative analysis and control of state-space symmetric systems, *Automatica*, **45** (2009), 1574–1579.

27. L. Su, H. Shen, Mixed H_∞/passive synchronization for complex dynamical networks with sampled-data control, *Appl. Math. Comput.*, **259** (2015), 931–942.

28. J. Wang, L. Su, H. Shen, Z. G. Wu, J. H. Park, Mixed H_∞/passive sampled-data synchronization control of complex dynamical networks with distributed coupling delay, *J. Franklin Inst.*, **354** (2017), 1302–1320.

29. R. Sakthivel, R. Anbuvithya, K. Mathiyalagan, P. Prakash, Combined H_∞ and passivity state estimation of memristive neural networks with random gain fluctuations, *Neurocomputing*, **168** (2015), 1111–1120.

30. J. Qiu, H. Yang, J. Zhang, Z. Gao, New robust stability criteria for uncertain neural networks with interval time-varying delays, *Chaos Soliton. Fract.*, **39** (2009), 579–585.

31. J. Sun, G. P. Liu, J. Chen, D. Rees, Improved stability criteria for neural networks with time-varying delay, *Phys. Lett. A*, **373** (2009), 342–348.

32. J. Tian, X. Xie, New asymptotic stability criteria for neural networks with time-varying delay, *Phys. Lett. A*, **374** (2010), 938–943.

33. A. Farnam, R. M. Esfanjani, Improved linear matrix inequality approach to stability analysis of linear systems with interval time-varying delays, *J. Comput. Appl. Math.*, **294** (2016), 49–56.

34. J. An, Z. Li, X. Wang, A novel approach to delay-fractional dependent stability criterion for linear systems with interval delay, *ISA Trans.*, **53** (2014), 210–219.

35. A. Seuret, F. Gouaisbaut, Jensen's and Wirtinger's inequalities for time-delay systems, In: *Proceedings of the 11th IFAC Workshop on Time-Delay Systems*, 2013, 343–348.

36. Z. Wang, Y. Liu, K. Fraser, X. Liu, Stochastic stability of uncertain Hopfield neural networks with discrete and distributed delays, *Phys. Lett. A*, **354** (2006), 288–297.

37. B. Boyd, L. E. Ghoui, E. Feron, V. Balakrishnan, *Linear matrix inequalities in system and control theory*, Philadephia: SIAM, 1994.

38. Y. He, G. P. Liu, D. Rees, M. Wu, Stability analysis for neural networks with time-varying interval delay, *IEEE Trans. Neural Netw.*, **18** (2007), 1850–1854.

39. G. Balas, R. Chaing, A. Packard, M. Safovov, *Robust control toolbox user's guide*, Natick: The MathWorks, 2010.

40. P. Niamsup, T. Botmart, W. Weera, Modified function projective synchronization of complex dynamical networks with mixed time-varying and asymmetric coupling delays via new hybrid pinning adaptive control, *Adv. Differ. Equ.*, **2017** (2017), 1–31.

41. H. Shu, Q. Song, Y. Liu, Z. Zhao, F. E. Alsaadi, Global μ-stability of quaternion-valued neural networks with non-differentiable time-varying delays, *Neurocomputing*, **247** (2017), 202–212.

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