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Research article

Some differential identities of MA-semirings with involution

Liaqat Ali[1], Yaqoub Ahmed Khan[2], A. A. Mousa[3,4], S. Abdel-Khalek[3] and Ghulam Farid[5,*]

[1] Department of Mathematics, Govt. MAO College Lahore, Pakistan
[2] Department of Mathematics, GIC Lahore, Pakistan
[3] Department of Mathematics and Statistics, College of Science, Taif University, P.O. Box 11099, Taif 21944, Saudi Arabia
[4] Department of Basic Engineering Science, Faculty of Engineering, Menofia University, Shebin El-Kom 32511, Egypt
[5] Department of Mathematics, COMSATS University Islamabad, Attock Campus, Pakistan

* **Correspondence:** Email: faridphdsms@hotmail.com; Tel: +923334426360.

Abstract: In this paper, we discuss some differential identities of MA-semirings with involution. The aim to study these identities is to induce commutativity in MA-semirings.

Keywords: semirings; MA-semirings; derivations; involution
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1. Introduction and preliminaries

The notion of derivations is one of the well known aspects of algebra and its generalized structures. Derivations and its generalizations on some algebraic structure have become an interesting area of research as it enforces significant properties.

Javed et al. [1] introduced MA-Semirings that is an additive inverse semiring satisfying A_2 condition of Bandlet and Petrich [2]. MA-semirings is a generalized structure of rings and distributive lattices but in spite of semirings we can deal with lie theory in MA-semirings. For ready reference and see more we refer [1, 3–5]. Involution is one of the important and fundamental concepts studied in functional analysis and algebra. Rickart introduced B*-algebra in [6] and Segal gave the notion of C*-algebra [7], these are now well known concepts which are defined with involution. Later on many algebraists used this idea in groups, rings and semirings (see [8–16]). Several research papers have been produced for MA-semirings with involution, for further references see [3, 4, 17–22]. To discuss the results of rings with involution in MA-semirings with involution would be of great interest for readers and researchers.

Throughout this paper, S denotes a additive inverse semiring with absorbing zero '0' and commutative addition. The additive inverse of $s \in S$ is a unique element $s' \in S$ such that $s + s' + s = s$ and $s' + s + s' = s'$. A MA-Semiring is additive inverse semiring satisfying $s + s'$ contains in the centre of S, $\forall s \in S$. This is generalization of ring but every MA semiring may not be a ring and for examples (see [1, 5, 17]).

The involution is an additive mapping $* : S \longrightarrow S$ satisfyies $\forall s, t \in S$, $(s^*)^* = s$ and $(st)^* = t^*s^*$. An element $s \in S$ is hermitian (skew hermitian) if $s^* = s$ ($s^* = s'$) . The set of hermitian elements (skew hermitian) of S are denoted by H (K). Involution is of second kind if $Z(S) \nsubseteq H$. An additive mapping $\delta : S \longrightarrow S$ is a derivation if $\delta(st) = \delta(s)t + s\delta(t)$. The commutator is defined as $[s, t] = st + t's$. By Jordan product we mean $s \circ t = st + ts$ for all $s, t \in S$. A mapping $f : S \longrightarrow S$ is centralizing (commuting) if $[[f(s), s], r] = 0$ ($[f(s), s] = 0$), $\forall s, r \in S$. Now we include some identities of commutators and Jordan products which are frequently used in the sequel. For all $s, t, z \in S$, we have $[s, st] = s[s, t]$, $[st, z] = s[t, z] + [s, z]t$, $[s, tz] = [s, t]z + t[s, z]$, $[s, t] + [t, s] = t(s + s') = s(t + t')$, $(st)' = s't = st'$, $[s, t]' = [s, t'] = [s', t]$, $s \circ (t + z) = s \circ t + s \circ z$. For more detail one can see [1, 5].

In the following we recall a few results for MA-semirings with involution which are very useful for proving the main results.

Lemma 1. *[18] Let S be a semiprime MA-semiring with second kind involution $*$. Then $K \cap Z(S) \neq \{0\}$ and therefore $H \cap Z(S) \neq \{0\}$.*

Remark 1. *If S is an MA-semiring with second kind involution $*$, then:*
1). for any $k \in K$, $k^2 \in H$.
2). for any $h \in H \cap Z(S)$ and $h_o \in H$, $hh_o \in H$.

Lemma 2. *[18] Let S be a prime and 2-torsion free MA-semiring with second kind involution $*$. If $[u, u^*] = 0$ for all $u \in S$, then S is commutative.*

Lemma 3. *[18] Let S be a prime and 2-torsion free MA-semiring with second kind involution $*$ and non-zero derivatio δ satisfying $\delta(u \circ u^*) = 0$ for all $u \in S$, then S is commutative.*

Lemma 4. *[18] Let S be a prime and 2-torsion free MA-semiring with second kind involution $*$ and a non zero derivation δ satisfying $\delta[u, u^*] = 0$ for all $u \in S$, then S is commutative.*

Lemma 5. *[18] Let S be a semiprime and 2-torsion free MA-semiring with $a, b \in S$. If $aub + bua = 0, \forall u \in S$, then $ab = 0 = ab$. Further if S is prime, then $a = 0$ or $b = 0$.*

Proof. We have for all $u \in S$

$$aub + bua = 0 \tag{1.1}$$

which can be further written as

$$aub = bu'a \tag{1.2}$$

In (1.1) replacing u by ubv, we get $aubvb + bubva = 0$ and using (1.2) we obtain $bua'vb + bubva = 0$ and again using (1.2), we get $2bubva = bubva + bubva = 0$. Using 2-torsion freeness and then using (1.2) again, we have

$$buavb = 0 \tag{1.3}$$

Right multiplying (1.3) by ua, we obtain $buavbua = 0$ that is $buaSbua = \{0\}$ by the semiprimeness, we get

$$bua = 0 \tag{1.4}$$

Multiplying (1.4) by a from the left and b from the right, we get $abSab = \{0\}$ and therefore by the semiprimeness ab=0. Similarly we can show that $ba = 0$. From (1.4) it is quite clear that in case of primeness, we get either $a = 0$ or $b = 0$. □

Shakir Ali and others [23] proved some results on derivations satisfying certain identities on prime rings with second kind involution. The principal aim of this research article is to establish these results for prime MA-semirings with second kind involution.

2. Main results

Following result is a generalized version of Theorem 3.1 of [23].

Theorem 1. *Let S be a prime and 2-torsion free MA-semiring with second kind involution $*$ and let δ_1 and δ_2 be derivations of S such that at least one of them is nonzero. If*

$$[\delta_1(u), \delta_1(u^*)] + \delta_2(u \circ u^*) = 0, \ \forall u \in S, \tag{2.1}$$

then S is commutative.

Proof. If $\delta_1 = 0$ and $\delta_2 \neq 0$, then by Lemma 3 S is commutative. Secondly if $\delta_1 \neq 0$ and $\delta_2 = 0$, then (2.1) becomes

$$[\delta_1(u), \delta_1(u^*)] = 0, \ \forall u \in S \tag{2.2}$$

Linearizing (2.2) and using (2.2) again, we obtain

$$[\delta_1(u), \delta_1(v^*)] + [\delta_1(v), \delta_1(u^*)] = 0 \tag{2.3}$$

In (2.3) replacing v by v^*, we obtain

$$[\delta_1(u), \delta_1(v)] + [\delta_1(v^*), \delta_1(u^*)] = 0 \tag{2.4}$$

In (2.4) replacing v by $vh, h \in H \cap Z(S)$ and using (2.4) again, we obtain

$$[\delta_1(u), v\delta_1(h)] + [v^*\delta_1(h), \delta_1(u^*)] = 0 \tag{2.5}$$

In (2.5) replacing v by vk, $\forall k \in K \cap Z(S)$, we get $([\delta_1(u), v\delta_1(h)] + [v^*\delta_1(h), \delta_1(u^*)]')Sk = \{0\}$ and therefore by Lemma 1 we have $[\delta_1(u), v\delta_1(h)] + [v^*\delta_1(h), \delta_1(u^*)]' = 0$, which further implies

$$[\delta_1(u), v\delta_1(h)] = [v^*\delta_1(h), \delta_1(u^*)] \tag{2.6}$$

Using (2.6) into (2.5) and the 2-torsion freeness of S, we obtain $[\delta_1(u), v\delta_1(h)] = 0$ which further implies

$$[\delta_1(u), v]\delta_1(h) + v[\delta_1(u), \delta_1(h)] = 0 \tag{2.7}$$

In (2.7) replacing v by vw, we get

$$[\delta_1(u), v]w\delta_1(h) + v([\delta_1(u), w]\delta_1(h) + w[\delta_1(u), \delta_1(h)]) = 0.$$

Using (2.7) we obtain $[\delta_1(u), v]S\delta_1(h) = \{0\}$ which further implies, by the primeness, that either $[\delta_1(u), v] = 0$ or $\delta_1(h) = 0$. If $[\delta_1(u), v] = 0$, then by Theorem 2.2 of [18], S is commutative. Secondly if $\delta_1(h) = 0,\ \forall h \in H \cap Z(S)$, then $\delta_1(k) = 0,\ \forall k \in K \cap Z(S)$. In (2.4) replacing v by $yk,\ k \in K \cap Z(S)$ and using $\delta(k) = 0$, we obtain $([\delta_1(u), \delta_1(v)] + [\delta_1(v^*), \delta_1(u^*)]^{'})k = 0$ and therefore $([\delta_1(u), \delta_1(v)] + [\delta_1(v^*), \delta_1(u^*)]^{'})Sk = \{0\}$. In view of Lemma 1, by the primeness, we obtain $[\delta_1(u), \delta_1(v)] + [\delta_1(v^*), \delta_1(u^*)]^{'} = 0$ and hence

$$[\delta_1(u), \delta_1(v)] = [\delta_1(v^*), \delta_1(u^*)] \tag{2.8}$$

Using (2.8) into (2.4) and then 2-torsion freeness of S implies that

$$[\delta_1(u), \delta_1(v)] = 0 \tag{2.9}$$

In (2.9) replacing v by vw and using MA-semiring commutator identities, we obtain

$$\delta_1(v)[\delta_1(u), w] + [\delta_1(u), \delta_1(v)]w + v[\delta_1(u), \delta_1(w)] + [\delta_1(u), v]\delta_1(w) = 0$$

and using (2.9) again, we get

$$\delta_1(v)[\delta_1(u), w] + [\delta_1(u), v]\delta_1(w) = 0 \tag{2.10}$$

In (2.10) replacing v by $\delta(v)$ and using (2.9) again, we get

$$\delta_1^2(v)[\delta_1(u), w] = 0 \tag{2.11}$$

In (2.11) replacing w by rw and using (2.11) again, we get $\delta_1^2(v)S[\delta_1(u), w] = \{0\}$. Therefor by the primeness, we have $\delta_1^2(v) = 0$ or $[\delta_1(u), w] = 0$. If $[\delta_1(u), w] = 0$, then by Theorem 2.2 of [18], S is commutative. On the other hand, suppose that

$$\delta_1^2(v) = 0 \tag{2.12}$$

In (2.12) replacing v by vw and using (2.12) again and the 2-torsion freeness, we have $\delta_1(v)\delta_1(w) = 0$, which further gives $\delta_1(v)S\delta_1(w) = \{0\}$. By the primeness, we get $\delta_1 = 0$, a contradiction.
Now we consider that δ_1 and δ_2 both are nonzero. In (2.1) replacing u by u^*, we get

$$[\delta_1(u^*), \delta_1(u)] + \delta_2(u^* \circ u) = 0$$

which further gives

$$[\delta_1(u, \delta_1(u^*))]^{'} + \delta_2(u \circ u^*) = 0$$

and hence

$$[\delta_1(u), \delta_1(u^*)] = \delta_2(u \circ u^*) \tag{2.13}$$

Using (2.13) into (2.1) and then by the 2-torsion freeness, we have $\delta_2(u \circ u^*) = 0$. By Lemma 3, S is commutative. □

Following result is a generalized version of Theorem 3.2 of [23] which can be obtained through similar fashion of Theorem 1.

Theorem 2. *Let S be a prime and 2-torsion free MA-semiring with second kind involution $*$ and let δ_1 and δ_2 be derivations of S such that at least one of them is nonzero. If*

$$[\delta_1(u), \delta_1(u^*)] + \delta_2(u^{'} \circ u^*) = 0, \ \forall u \in S,$$

then S is commutative.

Following theorem is a generalized version of Theorem 3.3 of [23].

Theorem 3. *Let S be a prime and 2-torsion free MA-semiring with second kind involution $*$ and let δ_1 and δ_2 be derivations of S such that at least one of them is nonzero. If*

$$(\delta_1(u) \circ \delta_1(u^*)) + \delta_2[u, u^*] = 0, \ \forall u \in S, \tag{2.14}$$

then S is commutative.

Proof. In (2.14) replacing u by u^*, we obtain

$$(\delta_1(u^*) \circ \delta_1(u)) + \delta_2[u^*, u] = 0$$

and therefore

$$(\delta_1(u) \circ \delta_1(u^*)) + \delta_2[u, u^*]^{'} = 0$$

which further implies

$$(\delta_1(u) \circ \delta_1(u^*)) = \delta_2[u, u^*] \tag{2.15}$$

Using (2.15) into (2.14) and then by the 2-torsion freeness, we obtain $\delta_2[u, u^*] = 0$. By Lemma 4, S is commutative. □

Following result is a generalized version of Theorem 3.4 of [23] which can be obtained through similar fashion of Theorem 3.

Theorem 4. *Let S be a prime and 2-torsion free MA-semiring with second kind involution $*$ and let δ_1 and δ_2 be derivations of S such that at least one of them is nonzero. If*

$$(\delta_1(u) \circ \delta_1(u^*)) + \delta_2[u, u^*]^{'} = 0, \ \forall u \in S,$$

then S is commutative.

Following result is a generalized version of Theorem 3.5 of [23].

Theorem 5. *Let S be a prime and 2-torsion free MA-semiring with second kind involution $*$. If δ is a nonzero derivation of S satisfying*

$$\delta[u, u^*] + [\delta(u), \delta(u^*)] = 0, \ \forall u \in S, \tag{2.16}$$

then S is commutative.

Proof. In (2.16) replacing u by $u+v$ and using (2.16) again, we obtain

$$\delta[u,v^*]+\delta[v,u^*]+[\delta(u),\delta(v^*)]+[\delta(v),\delta(u^*)]=0 \tag{2.17}$$

In (2.17) replacing v by v^*, we obtain

$$\delta[u,v]+\delta[v^*,u^*]+[\delta(u),\delta(v)]+[\delta(v^*),\delta(u^*)]=0$$

and therefore

$$\delta[u,v]+[\delta(u),\delta(v)]+\delta[v^*,u^*]+[\delta(v^*),\delta(u^*)]=0 \tag{2.18}$$

In (2.18) replacing v by $vh,\ h\in H\cap Z(S)$, we get

$$\delta[u,v]h+[u,v]\delta(h)+[\delta(u),\delta(v)]h+[\delta(u),v\delta(h)]+\delta[v^*,u^*]h$$

$$+[v^*,u^*]\delta(h)+[\delta(v^*),\delta(u^*)]h+[v^*\delta(h),\delta(u^*)]=0$$

and therefore

$$(\delta[u,v]+[\delta(u),\delta(v)]+\delta[v^*,u^*]+[\delta(v^*),\delta(u^*)])h$$

$$+[u,v]\delta(h)+[\delta(u),v\delta(h)]+[v^*,u^*]\delta(h)+[v^*\delta(h),\delta(u^*)]=0$$

Using (2.18), we obtain

$$[u,v]\delta(h)+[\delta(u),v\delta(h)]+[v^*,u^*]\delta(h)+[v^*\delta(h),\delta(u^*)]=0 \tag{2.19}$$

In (2.19) replacing v by $vk,\ k\in K\cap Z(S)$, we get

$$([u,v]\delta(h)+[\delta(u),v\delta(h)]+([v^*,u^*]\delta(h)+[v^*\delta(h),\delta(u^*)])^{'})k=0$$

which further implies

$$([u,v]\delta(h)+[\delta(u),v\delta(h)]+([v^*,u^*]\delta(h)+[v^*\delta(h),\delta(u^*)])^{'})Sk=\{0\}$$

In view of Lemma 1, by primeness, we have

$$[u,v]\delta(h)+[\delta(u),v\delta(h)]+([v^*,u^*]\delta(h)+[v^*\delta(h),\delta(u^*)])^{'}=0$$

and hence

$$[u,v]\delta(h)+[\delta(u),v\delta(h)]=[v^*,u^*]\delta(h)+[v^*\delta(h),\delta(u^*)] \tag{2.20}$$

Using (2.20) into (2.19) and then by 2-torsion freeness, we obtain

$$[u,v]\delta(h)+[\delta(u),v\delta(h)]=0 \tag{2.21}$$

In (2.21) replacing v by vw, we get

$$v[u,w]\delta(h)+[u,v]w\delta(h)+v[\delta(u),w\delta(h)]+[\delta(u),v]w\delta(h)=0$$

and therefore after rearranging the terms, we obtain

$$v([u,w]\delta(h)+[\delta(u),w\delta(h)])+[u,v]w\delta(h)+[\delta(u),v]w\delta(h)=0$$

Using (2.21), we find $([u, v] + [\delta(u), v])S\delta(h) = \{0\}$ which further, by the primeness, implies that either $[u, v] + [\delta(u), v] = 0$ or $\delta(h) = 0$. If

$$[u, v] + [\delta(u), v] = 0 \tag{2.22}$$

In (2.22) taking $v = u$, we get

$$[u, u] + [\delta(u), u] = 0 \tag{2.23}$$

As $[u, u] = [u, u]'$, therefore $[u, u]' + [\delta(u), u] = 0$ and therefore

$$[u, u] = [\delta(u), u] \tag{2.24}$$

Using (2.24) into (2.23) and then by the 2-torsion freeness, we get $[\delta(u), u] = 0, \forall u \in S$. By Theorem 2.2 of [24], we conclude that S is commutative. On the other hand, if $\delta(h) = 0, \forall h \in H \cap Z(S)$, then $\delta(k) = 0, k \in K \cap Z(S)$.
In (2.17) replacing u by uk, $k \in K \cap Z(S)$ and using the fact $\delta(k) = 0$, we get

$$\delta[u, v^*]k + \delta[v, u^*]'k + [\delta(u), \delta(v^*)]k + [\delta(v), \delta(u^*)]'k = 0$$

and therefore in view of Lemma 1, using primeness, we obtain

$$\delta[u, v^*] + \delta[v, u^*]' + [\delta(u), \delta(v^*)] + [\delta(v), \delta(u^*)]' = 0$$

and hence

$$\delta[u, v^*] + [\delta(u), \delta(v^*)] = \delta[v, u^*] + [\delta(v), \delta(u^*)]$$

Replacing the last relation into (2.17) and then using 2-torsion freeness, we obtain

$$\delta[u, v^*] + [\delta(u), \delta(v^*)] = 0$$

and replacing v by v^*, we get

$$\delta[u, v] + [\delta(u), \delta(v)] = 0 \tag{2.25}$$

In (2.25) replacing v by vu, we obtain

$$\delta[u, v]u + [u, v]\delta(u) + [\delta(u), \delta(v)]u + \delta(v)[\delta(u), u] + [\delta(u), v]\delta(u) = 0$$

Rearranging the terms, we can write

$$(\delta[u, v]u + [\delta(u), \delta(v)]u) + [u, v]\delta(u) + \delta(v)[\delta(u), u] + [\delta(u), v]\delta(u) = 0$$

Using (2.25), we obtain

$$[u, v]\delta(u) + \delta(v)[\delta(u), u] + [\delta(u), v]\delta(u) = 0 \tag{2.26}$$

In (2.26) replacing v by uv and using (2.26), we obtain

$$\delta(u)v[\delta(u), u] + [\delta(u), u]v\delta(u) = 0 \tag{2.27}$$

In view of Lemma 5, we obtain from (2.27) either $\delta(u) = 0$ or $[\delta(u), u] = 0$. If $\delta(u) = 0$, then $\delta = 0$, a contradiction. Secondly, if $[\delta(u), u] = 0$, then by Theorem 2.2 of [18], S is commutative. □

Following result is a generalized version of Theorem 3.6 of [23].

Theorem 6. *Let S be a prime and 2-torsion free MA-semiring with second kind involution $*$. If δ is a nonzero derivation of S satisfying*

$$\delta(u \circ u^*) + \delta(u) \circ \delta(u^*) = 0, \ \forall u \in S, \tag{2.28}$$

then S is commutative.

Proof. In (2.28) replacing u by $u + v$ and using (2.28) again, we get

$$\delta(u \circ v^*) + \delta(v \circ u^*) + \delta(u) \circ \delta(v^*) + \delta(v) \circ \delta(u^*) = 0 \tag{2.29}$$

In (2.29) replacing v by $vh, \ h \in H \cap Z(S)$, we obtain
$\delta(u \circ v^*)h + (u \circ v^*)\delta(h) + \delta(v \circ u^*)h + (v \circ u^*)\delta(h)$

$$+(\delta(u) \circ \delta(v^*)h) + \delta(u) \circ ((v^*)\delta(h) + (\delta(v) \circ \delta(u^*))h + (v\delta(h)) \circ \delta(u^*) = 0$$

Using (2.29) again, we obtain

$$(u \circ v^*)\delta(h) + (v \circ u^*)\delta(h) + \delta(u) \circ (v^*\delta(h)) + (v\delta(h)) \circ \delta(u^*) = 0 \tag{2.30}$$

In (2.30) replacing y by $yk, \ k \in K \cap Z(S)$, we obtain

$$(u \circ v^*)^{'}\delta(h) + (v \circ u^*)\delta(h) + (\delta(u) \circ (v^*\delta(h)))^{'} + (v\delta(h)) \circ \delta(u^*)k = 0$$

and therefore

$$((u \circ v^*)^{'}\delta(h) + (v \circ u^*)\delta(h) + (\delta(u) \circ (v^*\delta(h)))^{'} + (v\delta(h)) \circ \delta(u^*))Sk = \{0\}$$

In view of Lemma 1 using primeness of S, we have

$$(u \circ v^*)^{'}\delta(h) + (v \circ u^*)\delta(h) + (\delta(u) \circ (v^*\delta(h)))^{'} + (v\delta(h)) \circ \delta(u^*) = 0$$

which further implies

$$(v \circ u^*)\delta(h) + (v\delta(h)) \circ \delta(u^*) = (u \circ v^*)\delta(h) + (\delta(u) \circ (v^*\delta(h))) \tag{2.31}$$

Using (2.31) into (2.29) and then by the 2-torsion freeness of S, we obtain

$$(v \circ u^*)\delta(h) + (v\delta(h)) \circ \delta(u^*) = 0$$

and replacing u by u^* it further gives $(v \circ u)\delta(h) + (v\delta(h)) \circ \delta(u) = 0$. Therefore

$$uv\delta(h) + vu\delta(h) + v\delta(h)\delta(u) + \delta(u)v\delta(h) = 0 \tag{2.32}$$

Replacing v by uv in (2.32), we get

$$u(uv\delta(h) + vu\delta(h) + v\delta(h)\delta(u)) + \delta(u)uv\delta(h) = 0 \tag{2.33}$$

From (2.32), using $uv\delta(h)+vu\delta(h)+v\delta(h)\delta(u) = \delta(u)v^{'}\delta(h)$ into (2.34), we get $u\delta(u)v^{'}\delta(h)+\delta(u)uv\delta(h) = 0$ and therefore $[\delta(u), u]v\delta(h) = 0$. By the primeness of S, we either $[\delta(u), u] = 0$ or $\delta(h) = 0$. If $[\delta(u), u] = 0$, then by Theorem 2.2 of [24], S is commutative. On the other hand, if $\delta(h) = 0$, then $\delta(k) = 0,\ \forall k \in K \cap Z(S)$.
In (2.29) replacing y by yk and using the fact that $\delta(k) = 0$, we obtain

$$(\delta(u \circ v^*)^{'} + \delta(v \circ u^*) + \delta(u) \circ \delta(v^*)^{'} + \delta(v) \circ \delta(u^*))Sk = \{0\}.$$

In view of Lemma 1, using primeness, we obtain

$$\delta(u \circ v^*)^{'} + \delta(v \circ u^*) + \delta(u) \circ \delta(v^*)^{'} + \delta(v) \circ \delta(u^*) = 0$$

and hence

$$\delta(v \circ u^*) + \delta(v) \circ \delta(u^*) = \delta(u \circ v^*) + \delta(u) \circ \delta(v^*) \tag{2.34}$$

Using (2.30) into (2.29) and then using 2-torsion freeness, we get

$$\delta(v \circ u^*) + \delta(v) \circ \delta(u^*) = 0$$

and by replacing u by u^* it further gives

$$\delta(v \circ u) + \delta(v) \circ \delta(u) = 0 \tag{2.35}$$

In (2.35) taking $y = h$ in particular and using the fact that $\delta(h) = 0$, we obtain $\delta(u)h = 0$ and therefore $\delta(u)Sh = \{0\}$. As $H \cap Z(S) \neq \{0\}$, by the primeness of S, $\delta = 0$, a contradiction. This completes the proof. □

3. Conclusions

This article presents derivations satisfying certain identities on a special class of semirings known as MA-semirings and involution is main idea of the identities discussed. The results proved in this article revolve around the notion of commutativity which is one of the powerful ideas of algebraic structures. This leads the way to study further conditions on semirings which enable to induce commutativity or other important characteristics by using different types of mappings. Further the conditions discussed here are furnishing open problems for researchers; how to control them through Lie and other certain ideals of semirings.

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Conflict of interest

The authors declare that they have no conflict of interest.

Authors contributions

All authors in revised version are agree and approve their authorships, and have equal contributions in preparing this paper.

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