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*Research article*

# Interpolative Chatterjea and cyclic Chatterjea contraction on quasi-partial $b$-metric space

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**Abstract:** The fixed point results for Chatterjea type contraction in the setting of Complete metric space exists in literature. Taking this approach forward Karapinar gave the concept of cyclic Chatterjea contraction mappings. Fan also worked on these cyclic mappings in a new setting of quasi-partial b-metric space. Motivated by the work of these researchers , we have introduced the notion of $qp_b$-cyclic Chatterjea contractive mappings and established fixed point results on them. The aim of this paper is to use an interpolative approach in the framework of quasi-partial b-metric space and to prove existence and uniqueness of fixed point theorem for $qp_b$-interpolative Chatterjea contraction mappings. The results are affirmed with applications based on them.

**Keywords:** quasi-partial $b$-metric space; fixed point; Chatterjea contraction; cyclic mapping; $qp_b$-cyclic Chatterjea contraction mapping; interpolation
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## 1. Introduction

In 1922, Banach [1] introduced one of the most fundamental and significant result called Banach contraction principle of non-linear analysis. It is a prominent result for solving existence problems in several branches of mathematical analysis. Picard theorem, non- linear volterra integral equations, Fredholm integral equations, etc. are the examples where Banach contraction principle is mostly used besides supporting the convergence of schemes in computational mathematics. Due to application potential, the notion of Banach contraction principle was investigated by several authors [2–5].

In 1968, Kannan [6] introduced a significant variant of Banach contraction principle which remove the continuity condition in [1]. i.e.,

**Theorem 1** ( [6]). *Let $(M, d)$ be a complete metric spaces and a self map $T : M \rightarrow M$ be a Kannan contraction mapping,*

$$d(T\mu, T\vartheta) \leq \rho[d(\mu, T\mu) + d(\vartheta, T\vartheta)]$$

*for all $\mu, \vartheta \in M$, where $\rho \in [0, \frac{1}{s})$. Then $T$ admits a unique fixed point in $M$.*

In correspond to the evolution of spaces, in 1972, Chatterjea [7] defined following contraction mapping on complete metric space.

**Theorem 2** ( [7]). *Let $(M, d)$ be a complete metric space. A self-mapping $T : M \rightarrow M$ be a Chatterjea type contraction*

$$d(T\mu, T\vartheta) \leq \rho[d(\mu, T\vartheta) + d(\vartheta, T\mu)]$$

*for all $\mu, \vartheta \in M$, where $\rho \in (0, \frac{1}{2})$. Then $T$ has a unique fixed point.*

The concept of cyclic contraction mapping was defined by Kirk et al. [8]. In 2011, Karapinar et al. [9] introduced Kannan type cyclic contraction which is as follows:

Let (M, d) be a metric space. A cyclic mapping $T : A \cup B \rightarrow A \cup B$ is said to be a Kannan type cyclic contraction if there exists $\lambda \in [0, 1/2)$ such that $d(T\mu, T\vartheta) \leq \lambda[d(\mu, T\mu) + d(\vartheta, T\vartheta)]$ for any $\mu \in$ A and $\vartheta \in B$.

Later on, in 2016, Fan [10] proved theorems on fixed point for some special cyclic mappings satisfying Banach contraction condition, Kannan contraction condition, and $\beta$-quasi contraction condition within the environment of a quasi-partial $b$-metric space.

Very recently in the year 2018, Karapinar [11] revisited the Kannan type contraction by adopting interpolative approach and dropped uniqueness of fixed point.

**Theorem 3** ( [11]). *In the framework of a complete metric space $(M, d)$, a mapping $T : M \rightarrow M$ forms an interpolative Kannan type contraction i.e. if there are constants $\rho \in [0, 1)$ and $\alpha \in (0, 1)$ such that*

$$d(T\mu, T\vartheta) \leq \rho[d(\mu, T\mu)]^{\alpha} \cdot [d(\vartheta, T\vartheta)]^{1-\alpha}$$

*for all $\mu, \vartheta \in M \setminus Fix(T)$, where $Fix(T) = \{z \in M, Tz = z\}$. Then it possesses a fixed point in $M$.*

In continuation, interesting work was done by many authors [12–28] which enriched this field.

Throughout this paper, $R^+$ denote the set of all non-negative real numbers.

In this paper, our aim is to investigate the validity of existence and uniqueness of fixed point via $qp_b$-cyclic Chatterjea contraction and interpolative Chatterjea contractions for quasi-partial $b$-metric space introduced by Gupta and Gautam [29].

**Definition 1** ( [30]). Let M$\neq \phi$. A partial metric is a function $p : M \times M \rightarrow R^+$ satisfying

(PM1) $p(\mu, \vartheta) = p(\vartheta, \mu)$,

(PM2) If $0 \leq p(\mu, \mu) = p(\mu, \vartheta) = p(\vartheta, \vartheta)$, then $\mu = \vartheta$,

(PM3) $p(\mu, \mu) \leq p(\mu, \vartheta)$,

(PM4) $p(\mu, \vartheta) - p(\delta, \delta) \leq p(\mu, \delta) + p(\delta, \vartheta)$

for all $\mu, \vartheta, \delta \in M$. The pair (M, $p$ ) is called partial metric space.

**Definition 2** ( [31]). A quasi-partial metric on a nonempty set M is a function $q : M \times M \to R^+$ such that

(QPM1) If $0 \leq q(\mu,\mu) = q(\mu,\vartheta) = q(\vartheta,\vartheta)$, then $\mu = \vartheta$,

(QPM2) $q(\mu,\mu) \leq q(\mu,\vartheta)$,

(QPM3)$q(\mu,\mu) \leq q(\mu,\vartheta)$,

(QPM4)$q(\mu,\vartheta) - q(\delta,\delta) \leq q(\mu,\delta) + q(\delta,\vartheta)$

for all $\mu, \vartheta, \delta \in M$. A quasi-partial metric space is a pair (M, $q$ ) such that M is an nonempty set and $q$ is a quasi-partial metric on M.

**Example 1.** $M = [0, \infty)$, $q : M \times M \to [0, \infty)$. Define $q(\mu,\vartheta) = \max\{\mu,\vartheta\} + |\mu - \vartheta|$ .
Here $q(\mu,\mu) = q(\mu,\vartheta) = q(\vartheta,\vartheta) \Rightarrow \mu = \vartheta$ as $\mu = \max\{\mu,\vartheta\} + |\mu - \vartheta| = \vartheta$ .
Again $q(\mu,\mu) \leq q(\mu,\vartheta)$ as $\max\{\mu,\mu\} + |\mu - \mu| \leq \max\{\mu,\vartheta\} + |\mu - \vartheta|$ and similarly $q(\mu,\mu) \leq q(\vartheta,\mu)$ .
Also $q(\mu,\vartheta) + q(\delta,\delta) \leq qp_b(\mu,\delta) + qp_b(\delta,\vartheta)$.
Let $\mu, \vartheta, \delta \in X$ . If $\mu \leq \vartheta \leq \delta$ , then

$$\begin{aligned} max\{\mu,\vartheta\} + |\mu - \vartheta| &\leq \vartheta + |\mu - \delta| + |\delta - \vartheta| \\ &\leq max\{\mu,\delta\} + |\mu - \delta| + max\{\delta,\vartheta\} + |\delta - \vartheta| - \delta. \end{aligned}$$

So ($QPM4$) holds. Thus ($M$,q) is a quasi-partial metric space.

**Definition 3** ( [32]). A quasi-partial $b$-metric on a nonempty set $M$ is a function $qp_b : M \times M \to R^+$ such that for some real number $s \geq 1$ and for all $\mu, \vartheta, \delta \in M$

(QP$b_1$)$qp_b(\mu,\mu) = qp_b(\mu,\vartheta) = qp_b(\vartheta,\vartheta) \Rightarrow \mu = \vartheta$,

(QP$b_2$)$qp_b(\mu,\mu) \leq qp_b(\mu,\vartheta)$,

(QP$b_3$)$qp_b(\mu,\mu) \leq qp_b(\vartheta,\mu)$,

(QP$b_4$)$qp_b(\mu,\vartheta) \leq s[qp_b(\mu,\delta) + qp_b(\delta,\vartheta)] - qp_b(\delta,\delta)$.

A quasi-partial $b$-metric space is a pair $(M, qp_b)$ such that $M$ is an nonempty set and $qp_b$ is a quasi-partial $b$-metric on $M$. The number $s$ is called the coefficient of $(M, qp_b)$.

Let $qp_b$ be a quasi-partial $b$-metric on the set $M$. Then

$$dqp_b(\mu,\vartheta) = qp_b(\mu,\vartheta) + qp_b(\vartheta,\mu) - qp_b(\mu,\mu) - qp_b(\vartheta,\vartheta)$$

is a $b$-metric on $M$.

**Example 2.** Let M = $\mathbb{R}$. Define the metric $qp_b(\mu,\vartheta) = |\mu - \vartheta| + |\mu| + |\mu - \vartheta|^2$
for any $(\mu,\vartheta) \in M \times M$ with $s \geq 2$.
It can be shown that $(M, qp_b)$ is a quasi-partial b-metric space.
In fact, if $qp_b(\mu,\mu) = qp_b(\vartheta,\vartheta) = qp_b(\mu,\vartheta)$

$\Rightarrow \mu = \vartheta$ which shows ($QPb_1$) is true.
Also $qp_b\ (\mu, \mu) \leq qp_b(\mu, \vartheta)$ which proves ($QPb_2$).
Now, $qp_b\ (\mu, \mu) = |\mu| \leq |\mu - \vartheta| + |\vartheta| + |\mu - \vartheta|^2$
Since,

$$\begin{aligned} |\mu| - |\vartheta| &\leq |(|\mu| - |\vartheta|)| \\ &\leq |\mu - \vartheta| \\ &\leq |\mu - \vartheta| + |\mu - \vartheta|^2 \end{aligned}$$

which proves ($QPb_3$). Now we will prove ($QPb_4$) with s = 2, that is
$qp_b\ (\mu, \vartheta) \leq 2[qp_b(\mu, \delta) + qp_b(\delta, \vartheta)] - qp_b(\delta, \delta)$
In addition, since
$|\mu - y|^2 \leq (|\mu - \delta| + |\delta - \vartheta|)^2 \leq 2(|\mu - \delta|^2 + |\delta - \vartheta|^2)$
We have $qp_b\ (\mu, \vartheta) + qp_b\ (\delta, \delta)$

$$\begin{aligned} &= |\mu - \vartheta| + |\mu| + |\mu - \vartheta|^2 + |\delta| \\ &\leq 2[|\mu - \delta| + |\delta - \vartheta| + |\mu| + |\delta| + |\mu - \delta|^2 + |\delta - \vartheta|^2 \end{aligned}$$

Rearranging proves ($QPb_4$).
Hence $(X, qp_b)$ is a Quasi-Partial b-metric space with $s = 2$.

**Definition 4** ( [33]). Let $(M, qp_b)$ be a quasi-partial $b$-metric. Then

(i) A sequence $\{\mu_n\} \subset M$ converges to $\mu \in M$ if and only if

$$qp_b(\mu, \mu) = \lim_{n\to\infty} qp_b(\mu, \mu_n) = \lim_{n\to\infty} qp_b(\mu_n, \mu).$$

(ii) A sequence $\{\mu_n\} \subset M$ is called a Cauchy sequence if and only if

$$\lim_{n,m\to\infty} qp_b(\mu_n, \mu_m) \ \text{and} \ \lim_{m,n\to\infty} qp_b(\mu_m, \mu_n) \ \text{exist(and are finite).}$$

(iii) The quasi partial $b$-metric space $(M, qp_b)$ is said to be complete if every Cauchy sequence $\{\mu_n\} \subset M$ converges with respect to $\tau qp_b$ to a point $\mu \in M$ such that

$$qp_b(\mu, \mu) = \lim_{n,m\to\infty} qp_b(\mu_n, \mu_m) = \lim_{m,n\to\infty} qp_b(\mu_m, \mu_n).$$

(iv) A mapping $f : M \to M$ is said to be continuous at $\mu_0 \in M$ if, for every $\epsilon > 0$, there exists $\delta > 0$ such that $f(B(\mu_0, \delta)) \subset B(f(\mu_0), \epsilon)$.

**Definition 5** ( [33]). Let $(M, qp_b)$ be a quasi-partial $b$-metric space and $T : M \to M$ be a given mapping. Then $T$ is said to be sequentially continuous at $z \in M$ if for each sequence $\{\mu_n\}$ in $M$ converging to $z$, we have $T\mu_n \to Tz$, that is, $\lim_{n\to\infty} qp_b(T\mu_n, Tz) = qp_b(Tz, Tz)$.

**Lemma 1** ( [34]). *Let $(M, qp_b)$ be a quasi-partial b-metric space and $(M, dqp_b)$ be the corresponding b-metric space. Then $(M, dqp_b)$ is complete if $(M, qp_b)$ is complete..*

**Lemma 2** ( [10]). *Let $(M, qp_b)$ be a quasi-partial b-metric space and $\{\mu_n\}_{n=0}^{\infty}$ be a sequence in M. If $\mu_n \xrightarrow{qp_b} \mu$, $\mu_n \xrightarrow{qp_b} \vartheta$ and $qp_b(\mu, \mu) = qp_b(\vartheta, \vartheta) = 0$ then $\mu = \vartheta$.*

## 2. $qp_b$-cyclic-Chatterjea mapping in quasi-partial b-metric spaces

In this section, we will introduce the notion of $qp_b$-cyclic-Chatterjea mapping in a quasi-partial $b$-metric space and state a condition on the contraction constant under which a self-map on a complete quasi-partial $b$-metric space obtains a fixed point.

**Definition 6.** Let $A$ and $B$ be nonempty subsets of a quasi-partial $b$-metric space $(M, qp_b)$ with coefficient $s \geq 1$. A cyclic mapping $T : A \cup B \to A \cup B$ is said to be a $qp_b$-cyclic-Chatterjea mapping if there exists $\alpha \in \mathbb{R}$, $0 \leq \rho < \frac{1}{s^2(s+1)}$ such that

$$qp_b(T\mu, T\vartheta) \leq \rho[qp_b(\mu, T\vartheta) + qp_b(\vartheta, T\mu)] \tag{2.1}$$

holds both for $\mu \in A$, $\vartheta \in B$ and for $\mu \in B$, $\vartheta \in A$.

**Remark 1.** The inequalities stated below follow from the condition

$$0 \leq \rho < \frac{1}{s^2(s+1)}$$

(i) $\rho < \frac{1}{s(s+1)}$
(ii) $\rho < \frac{1}{s}$
(iii) $\frac{s^2\rho}{1-s\rho} < 1$
(iv) $\frac{s\rho}{1-s\rho} < 1$
(v) $\rho < \frac{1}{2}$

**Theorem 4.** *Let $A$ and $B$ be two nonempty closed subsets of a complete quasi-partial b-metric space $(M, qp_b)$ and $T : A \cup B \to A \cup B$ be a cyclic mapping which is a $qp_b$-cyclic-Chatterjea mapping. Then $A \cap B$ is nonempty and $T$ has a unique fixed point in $A \cap B$.*

*Proof.* Let $\mu \in A$, considering condition (2.1) and then using $QPb_2$ we have,

$$\begin{aligned} qp_b(T\mu, T^2\mu) &\leq \rho[qp_b(\mu, T^2\mu) + qp_b(T\mu, T\mu)] \\ &\leq \rho[qp_b(\mu, T^2\mu) + qp_b(T\mu, T^2\mu)] \end{aligned} \tag{2.2}$$

Thus,

$$qp_b(T\mu, T^2\mu) \leq \frac{\rho}{1-\rho} qp_b(\mu, T^2\mu) \tag{2.3}$$

Again using (2.1), we get

$$\begin{aligned} qp_b(T^2\mu, T\mu) &\leq \rho[qp_b(T\mu, T\mu) + qp_b(\mu, T^2\mu)] \\ &\leq \rho[qp_b(T^2\mu, T\mu) + qp_b(\mu, T^2\mu)] \quad (\text{by } QPb_3) \end{aligned}$$

or

$$qp_b(T^2\mu, T\mu) \leq \frac{\rho}{1-\rho} qp_b(\mu, T^2\mu) \tag{2.4}$$

Let $\beta = qp_b(\mu, T^2\mu)$, we have from (2.3) and (2.4)

$$qp_b(T\mu, T^2\mu) \leq \frac{\rho\beta}{1-\rho} \leq \frac{\rho\beta}{1-s\rho} \text{ and } qp_b(T^2\mu, T\mu) \leq \frac{\alpha\beta}{1-\rho} \leq \frac{\rho\beta}{1-s\rho} \tag{2.5}$$

Again using (2.1) and $QPb_4$ we get,

$$\begin{aligned} qp_b(T^2\mu, T^3\mu) &\leq \rho[qp_b(T\mu, T^3\mu) + qp_b(T^2\mu, T^2\mu)] \\ &\leq \rho[s\left[qp_b(T\mu, T^2\mu) + qp_b(T^2\mu, T^3\mu)\right] - qp_b(T^2\mu, T^2\mu) + qp_b(T^2\mu, T^2\mu)] \\ &= s\rho[qp_b(T\mu, T^2\mu) + qp_b(T^2\mu, T^3\mu)] \end{aligned}$$

On rearranging,

$$qp_b(T^2\mu, T^3\mu) \leq \frac{s\rho}{1-s\rho} qp_b(T\mu, T^2\mu).$$

Using (2.5) in above inequality, we get

$$qp_b(T^2\mu, T^3\mu) \leq \frac{s\rho^2\beta}{(1-s\rho)^2} \tag{2.6}$$

Applying (2.1) again, using $QPb_4$ and (2.5)–(2.6), we have

$$\begin{aligned} qp_b(T^3\mu, T^2\mu) &\leq \rho[qp_b(T^2\mu, T^2\mu)] + qp_b(T\mu, T^3\mu)] \\ &\leq \rho[qp_b(T^2\mu, T^2\mu) + s\left[qp_b(T\mu, T^2\mu) + qp_b(T^2\mu, T^3\mu)\right] - qp_b(T^2\mu, T^2\mu)] \\ &= s\rho[qp_b(T\mu, T^2\mu) + qp_b(T^2\mu, T^3\mu)] \\ &\leq s\rho\left[\frac{\rho\beta}{1-s\rho} + \frac{s\rho^2\beta}{(1-s\rho)^2}\right] \\ &= \frac{s\rho^2\beta}{(1-s\rho)^2} \end{aligned} \tag{2.7}$$

Hence, on generalizing (2.7), we get

$$qp_b(T^n\mu, T^{n+1}\mu) \leq \frac{s^{n-1}\rho^n\beta}{(1-s\rho)^n} \quad \text{and} \quad qp_b(T^{n+1}\mu, T^n\mu) \leq \frac{s^{n-1}\rho^n\beta}{(1-s\rho)^n} \tag{2.8}$$

We claim that $\{T^n\mu\}_{n=1}^{\infty}$ is a Cauchy sequence in $(M, qp_b)$. For this, let $m, n \in \mathbb{N}$ such that $m < n$.

Using $QPb_4$ repeatedly and (2.8), we get

$$\begin{aligned} qp_b(T^m\mu, T^n\mu) &\leq s\left[qp_b(T^m\mu, T^{m+1}\mu) + qp_b(T^{m+1}\mu, T^n\mu)\right] - qp_b(T^{m+1}\mu, T^{m+1}\mu) \\ &\leq s\left[qp_b(T^m\mu, T^{m+1}\mu) + qp_b(T^{m+1}\mu, T^n\mu)\right] \\ &\leq sqp_b(T^m\mu, T^{m+1}\mu) + s^2qp_b(T^{m+1}\mu, T^{m+2}\mu) + s^2qp_b(T^{m+2}\mu, T^n\mu) \\ &\leq sqp_b(T^m\mu, T^{m+1}\mu) + s^2qp_b(T^{m+1}\mu, T^{m+2}\mu) + s^3qp_b(T^{m+2}\mu, T^{m+3}\mu) + \cdots + s^{n-m}qp_b(T^{n-1}\mu, T^n\mu) \\ &\leq \frac{s.s^{m-1}.\rho^m.\beta}{(1-s\rho)^m} + \frac{s^2.s^m.\rho^{m+1}.\beta}{(1-s\rho)^{m+1}} + \frac{s^3.s^{m+1}.\rho^{m+2}.\beta}{(1-s\rho)^{m+2}} + \cdots + \frac{s^{n-m}.s^{n-2}.\rho^{n-1}.\beta}{(1-s\rho)^{n-1}} \end{aligned}$$

$$= \frac{s^m\rho^m\beta}{(1-s\rho)^m} + \frac{s^{m+2}\rho^{m+1}\beta}{(1-s\rho)^{m+1}} + \frac{s^{m+4}\rho^{m+2}\beta}{(1-s\rho)^{m+2}} + \cdots + \frac{s^{2n-m-2}\rho^{n-1}\beta}{(1-s\rho)^{n-1}}$$
$$= \frac{s^m\rho^m\beta}{(1-s\rho)^m}\left[1 + \frac{s^2\rho}{(1-s\rho)} + \frac{s^4\rho^2}{(1-s\rho)^2} + \cdots + \frac{s^{2n-2m-2}\rho^{n-m-1}}{(1-s\rho)^{n-m-1}}\right].$$

By Remark 1, $\frac{s^2\rho}{1-s\rho} < 1$, therefore,

$$qp_b(T^m\mu, T^n\mu) \leq \frac{(\frac{s\rho}{1-s\rho})^m.\beta\left\{1-(\frac{s^2\rho}{1-s\rho})^{\text{n}-\text{m}}\right\}}{\left\{1-(\frac{s^2\rho}{1-s\rho})\right\}}$$
$$\leq \left(\frac{s\rho}{1-s\rho}\right)^m.\beta\left\{\frac{1}{1-\frac{s^2\rho}{1-s\rho}}\right\}$$
$$= \left(\frac{s\rho}{1-s\rho}\right)^m.\frac{\beta(1-s\rho)}{(1-s\rho-s^2\rho)}$$

Letting $m, n \to \infty$ and since by Remark 1, $\frac{s\rho}{1-s\rho} < 1$, we must have

$$\lim_{m,n\to\infty} qp_b(T^m\mu, T^n\mu) \leq 0$$

which implies

$$\lim_{m,n\to\infty} qp_b(T^m\mu, T^n\mu) = 0 \tag{2.9}$$

Similarly,

$$\lim_{m,n\to\infty} qp_b(T^n\mu, T^m\mu) = 0 \tag{2.10}$$

From the above two limits we have established that the sequence $\{T^n\mu\}_{n=1}^{\infty}$ is a cauchy sequence in $(M, qp_b)$.

By the completeness property, there exists $w \in X$, such that $\{T^n\mu\}_{n=1}^{\infty}$ converges to $w$ and

$$\begin{aligned} qp_b(w, w) &= \lim_{n\to\infty} qp_b(T^n\mu, w) \\ &= \lim_{n\to\infty} qp_b(w, T^n\mu) \\ &= \lim_{n,m\to\infty} qp_b(T^n\mu, T^m\mu) \\ &= \lim_{n,m\to\infty} qp_b(T^m\mu, T^n\mu) = 0 \end{aligned} \tag{2.11}$$

Observe that $\left\{T^{2n}\mu\right\}_{n=0}^{\infty}$ is a sequence in $A$ and $\left\{T^{2n-1}\mu\right\}_{n=1}^{\infty}$ is a sequence in $B$ in a way that both sequences converge to $w$. Also note that $A$ and $B$ are closed, so we have $w \in A \cap B$. It is also interesting to note that $Tw \in A \cap B$ since $T$ is cyclic.

On the other hand, we prove that sequence $\{T^n\mu\}_{n=1}^{\infty}$ also converges to $Tw$.

For,

$$qp_b(T^n\mu, Tw) \leq \rho\left[qp_b(T^{n-1}\mu, Tw) + qp_b(w, T^n\mu)\right]$$

Since $\rho < \frac{1}{2}$,

$$qp_b(T^n\mu, Tw) < \frac{1}{2}\left[qp_b(T^{n-1}\mu, Tw) + qp_b(w, T^n\mu)\right]$$

Letting $n \to \infty$ in the above inequality and using (2.11) we get

$$\lim_{n\to\infty} qp_b(T^n\mu, Tw) \leq \frac{1}{2}\lim_{n\to\infty} qp_b(T^{n-1}\mu, Tw)$$

which holds if and only if

$$\lim_{n\to\infty} qp_b(T^n\mu, Tw) = 0. \tag{2.12}$$

Similarly

$$\lim_{n\to\infty} qp_b(Tw, T^n\mu) = 0. \tag{2.13}$$

In addition, by contractive condition (2.1) and applying $(QPb_4)$ we obtain

$$\begin{aligned} qp_b(Tw, Tw) &\leq \rho[qp_b(w, Tw) + qp_b(w, Tw)] \\ &= 2\rho qp_b(w, Tw) \\ &\leq 2\rho[sqp_b(w, T^n\mu) + qp_b(T^n\mu, Tw) - qp_b(T^n\mu, T^n\mu)] \\ &\leq 2\rho s[qp_b(w, T^n\mu) + qp_b(T^n\mu, Tw)]. \end{aligned}$$

Letting $n \to \infty$ in the above inequality and using (2.11)–(2.12) we get

$$qp_b(Tw, Tw) = 0 \tag{2.14}$$

Eqs (2.12)–(2.14) together imply $\{T^n\mu\}_{n=1}^{\infty}$ also converges to $Tw$.

Since all the conditions of Lemma 2 hold, we must have $Tw = w$ which implies $w \in A \cap B$ is a fixed point of $T$.

To prove that $w$ is unique fixed point, let us assume that there exists another fixed point $w^*$ of $T$ in $A \cap B$, that is $Tw^* = w^*$, then from the contractive condition (2.1), we have

$$\begin{aligned} qp_b(w^*, w) &= qp_b(Tw^*, Tw) \\ &\leq \rho[qp_b(w^*, Tw) + qp_b(w, Tw^*)] \\ &= \rho[qp_b(w^*, w) + qp_b(w, w^*)] \end{aligned}$$

or,

$$qp_b(w^*, w) \leq \frac{\rho}{1-\rho} qp_b(w, w^*) \tag{2.15}$$

Similarly,

$$qp_b(w, w^*) \leq \frac{\rho}{1-\rho} qp_b(w^*, w) \tag{2.16}$$

Using (2.15) and (2.16) we can say

$$qp_b(w^*, w) \le \left(\frac{\rho}{1-\rho}\right)^2 .qp_b(w^*, w)$$

or

$$qp_b(w^*, w)\left[1 - \left(\frac{\rho}{1-\rho}\right)^2\right] \le 0.$$

But $\left[1 - \left(\frac{\rho}{1-\rho}\right)^2\right] > 0$ since $\rho < \frac{1}{2}$, so we must have

$$qp_b(w^*, w) = 0. \tag{2.17}$$

Note that by (2.1),

$$\begin{aligned} qp_b(w, w) &= qp_b(Tw, Tw) \\ &\le 2\rho qp_b(w, Tw) \\ &= 2\rho qp_b(w, w) \end{aligned}$$

or,

$$qp_b(w, w)\,[1 - 2\rho] \le 0.$$

Again since $\rho < \frac{1}{2}$, so we have

$$qp_b(w, w) = 0. \tag{2.18}$$

Similarly we obtain

$$qp_b(w^*, w^*) = 0. \tag{2.19}$$

The conditions (2.17)–(2.19) together with $QPb_1$ imply $w = w^*$.

Analogously, when $\mu \in B$, similar arguments may be given to prove the result. □

We now justify our result by illustrating it with an example below.

**Example 3.** Let $M = [0, 1]$ and $A = [0, 1]$ and $B = [0, 1/2)$.

Let us define $T : A \cup B \to A \cup B$ as $T\mu = \mu/4$. Define the quasi partial $b$-metric as

$$qp_b(\mu, \vartheta) = |\mu - \vartheta| + \mu \quad \text{for and } (\mu, \vartheta) \in M \times M.$$

We will verify that the mapping $T$ is $qp_b$-cyclic Chatterjea contraction mapping.

If, $\mu \in A$, then $T\mu \in (0, 1/4] = T(A) \subseteq B$ and if, $\mu \in B$, then $T\mu \in [0, 1/8) = T(B) \subseteq A$.

Hence the mapping $T$ is a cyclic map on $M$. Here $(M, qp_b)$ is a quasi-partial $b$-metric space with $s = 1$.

The $qp_b$-cyclic Chatterjea contraction condition with $\rho = \frac{1}{3}$ and $\frac{1}{3} < \frac{1}{s^2(s+1)}$ becomes

$$qp_b(\mu/4, \vartheta/4) \le \frac{1}{3}\left[qp_b\left(\mu, \frac{\vartheta}{4}\right) + qp_b\left(\vartheta, \frac{\mu}{4}\right)\right]$$

$$1/4\,|\mu-\vartheta|+\mu/4 \le 1/3[1/4[|4\mu-\vartheta|+4\mu+|4\vartheta-\mu|+4\vartheta]]$$

i.e.

$$|\mu-\vartheta|+\mu \le 1/3[|4\mu-\vartheta|+|4\vartheta-\mu|+4\mu+4\vartheta]$$

Let $Z = 1/3[|4\mu-\vartheta|+|4\vartheta-\mu|+4\mu+4\vartheta]$.

Then,

$$|\mu-\vartheta|+\mu \le Z. \tag{2.20}$$

To prove this fact let us consider these cases:

**Case 1:** When $\mu=\vartheta$; then $|\mu-\vartheta|+\mu=\mu$ and $z=14\mu/3$ which shows, $|\mu-\vartheta|+\mu \le Z$.

**Case 2:** When $\mu<\vartheta$ then, $|\mu-\vartheta|+\mu=\vartheta-\mu+\mu=\vartheta$ and

$$\begin{aligned} Z &= 1/3[|4\mu-\vartheta|+4\vartheta-\mu+4\mu+4\vartheta] \\ &= 1/3[|4\mu-\vartheta|+8\vartheta+3\mu]. \end{aligned} \tag{2.21}$$

Now two cases arise:

(a) If $4\mu<\vartheta$ then (2.20) reduces to $Z=1/3[\vartheta-4\mu+8\vartheta+3\mu]=1/3[9\vartheta-\mu]$ and which further shows, $|\mu-\vartheta|+\mu \le Z$ since, $\vartheta \le 1/3(9\vartheta-\mu)$

(b) If $4\mu>\vartheta$ then (2.21) reduces to $Z=1/3[4\mu-\vartheta+8\vartheta+3\mu]=1/3[7\vartheta+7\mu]$ which shows $|\mu-\vartheta|+\mu \le Z$ since, $\vartheta \le 1/3(7\mu+7\vartheta)$.

**Case 3:** When $\mu>\vartheta$, then, $|\mu-\vartheta|+\mu=\mu-\vartheta+\mu=2\mu-\vartheta$ and

$$Z = 1/3[|4y-\mu|+4\mu-\vartheta+4\mu+4\vartheta] = 1/3[[|4\vartheta-\mu|+8\mu+3\vartheta] \tag{2.22}$$

Now two cases arise:

(a) If $4\vartheta<\mu$ then (2.22) reduces to $Z=1/3[\mu-4\vartheta+3\vartheta+8\mu]=1/3[9\mu-\vartheta]$ which shows, $|\mu-\vartheta|+\mu \le Z$ since $2\mu-\vartheta \le 1/3[9\mu-\vartheta]$.

(b) If $4\vartheta>\mu$ then (2.22) reduces to $Z=1/3[4\vartheta-\mu+3\vartheta+8\mu]=1/3[7\vartheta+7\mu]$ which shows $|\mu-\vartheta|+\mu \le Z$ since, $2\mu-\vartheta \le 1/3(7\vartheta+7\mu)$.

Hence contradictive condition is true in all the three cases with $\alpha=\frac{1}{s^2(s+1)}$.

Here,

$$\begin{aligned} d_{qp_b}(\mu,\vartheta) &= qp_b(\mu,\vartheta)+qp_b(\vartheta,\mu)-qp_b(\mu,\mu)-qp_b(\vartheta,\vartheta) \\ &= |\mu-\vartheta|+\mu+|\vartheta-\mu|+\mu-\mu-\vartheta \\ &= 2\,|\mu-\vartheta| \end{aligned}$$

which is a complete metric. Hence $(M,qp_b)$ is a complete quasi-partial $b$-metric space. Therefore, all conditions of Theorem 4 are satisfied and so $T$ has a fixed point(which is $w=0\in A\cap B$).

**Example 4.** Following Example 1, Let $M = [0, 1]$ and $A = [0, 1/2]$ and $B = [1/2, 1]$.

Define $T : A \cup B \to A \cup B$ as $T\mu = \mu/4$. Here, $A \cap B = \{\frac{1}{2}\} \neq \phi$ and $T\mu = [0, \frac{1}{4}] \nsubseteq B$. Hence mapping T is not cyclic on M. Consider the quasi partial $b$-metric

$$qp_b(\mu, \vartheta) = |\mu - \vartheta| + \mu \quad \text{for and } (\mu, \vartheta) \in M \times M$$

is complete and T is $qp_b$- Chatterjea contraction mapping. Clearly T has a fixed point $0 \notin A \cap B$. Therefore, Theorem 4 is not applicable in non-cyclic case.

## 3. $qp_b$-interpolative-Chatterjea mapping in quasi-partial $b$-metric spaces

Our next result ensures the existence of fixed point for interpolative Chatterjea type contraction but dropped uniqueness property of fixed point in the setting of a quasi-partial $b$-metric space. We start our results by the generalization of the definition of Chatterjea type contraction via interpolation notion, as follows.

**Definition 7.** Let $(M, qp_b)$ be a complete quasi-partial $b$-metric space. We say that the self-mapping $T : M \to M$ is an interpolative Chatterjea type contraction if there exists $\rho \in \left[0, \frac{1}{s}\right)$, $\alpha \in (0, 1)$ such that

$$qp_b(T\mu, T\vartheta) \leq \rho[qp_b(\mu, T\vartheta)]^{\alpha} \left[\frac{1}{s^2} qp_b(\vartheta, T\mu)\right]^{1-\alpha} \tag{3.1}$$

for all $\mu, \vartheta \in M \setminus Fix(T)$.

**Theorem 5.** *Let $(M, qp_b)$ be a complete quasi-partial b-metric space and T be an interpolative Chatterjea type contraction. Then, T has a fixed point in M.*

*Proof.* Let $\mu_0 \in (M, qp_b)$. We shall set a constructive sequence $\{\mu_n\}$ by $\varepsilon_{n+1} = T^n(\mu_0)$ for all positive integer $n$. Without loss of generality, we assume that $\mu_n = \mu_{n+1}$ for each nonnegative integer $n$. Indeed, if there exist a nonnegative integer $n_0$ such that $\mu_{n_0} = \mu_{n_0+1} = T\mu_{n_0}$, then, $\mu_{n_0}$ forms a fixed point.

Thus, we have $qp_b(\mu_n, T\mu_n) = qp_b(\mu_n, \mu_{n+1}) > 0$, for each nonnegative integer $n$.

Let $\mu = \mu_n$ and $\vartheta = \mu_{n-1}$ in (3.1), we derive that

$$\begin{aligned} qp_b(\mu_{n+1}, \mu_n) &= qp_b(T\mu_n, T\mu_{n-1}) \\ &\leq \rho[qp_b(\mu_n, T\mu_{n-1})]^{\alpha} \cdot \left[\frac{1}{s^2} qp_b(\mu_{n-1}, T\mu_n)\right]^{1-\alpha} \\ &\leq \rho[qp_b(\mu_n, \mu_n)]^{\alpha} \cdot \left[\frac{1}{s^2} qp_b(\mu_{n-1}, \mu_{n+1})\right]^{1-\alpha} \\ &\leq \rho[qp_b(\mu_{n+1}, \mu_n)]^{\alpha} \cdot \left[\frac{1}{s^2}[s\{qp_b(\mu_{n-1}, \mu_n) + qp_b(\mu_n, \mu_{n+1})\} - qp_b(\mu_n, \mu_n)]\right]^{1-\alpha} \\ &\leq \rho[qp_b(\mu_{n+1}, \mu_n)]^{\alpha} \cdot \left[\frac{1}{s^2} s\{qp_b(\mu_{n-1}, \mu_n) + qp_b(\mu_n, \mu_{n+1})\}\right]^{1-\alpha} \\ &\leq \rho[qp_b(\mu_{n+1}, \mu_n)]^{\alpha} \cdot \left[\frac{1}{s}\{qp_b(\mu_{n-1}, \mu_n) + qp_b(\mu_n, \mu_{n+1})\}\right]^{1-\alpha} \end{aligned} \tag{3.2}$$

Suppose that $qp_b(\mu_{n-1}, \mu_n) < qp_b(\mu_n, \mu_{n+1})$ for some $n \geq 1$. Thus,

$$\frac{1}{s}\{qp_b(\mu_{n-1}, \mu_n) + qp_b(\mu_n, \mu_{n+1})\} \leq qp_b(\mu_n, \mu_{n+1})$$

Consequently, the inequality(3.2) yields that $qp_b(\mu_{n+1}, \mu_n) \leq qp_b(\mu_{n-1}, \mu_n)$ which is a contradiction. Thus, we have $qp_b(\mu_n, \mu_{n+1}) < qp_b(\mu_{n-1}, \mu_n)$ for all $n \geq 1$. Hence, $\{d(\mu_{n-1}, \mu_n)\}$ is a non-increasing sequence with positive terms. Set $L = \lim_{n\to\infty} qp_b(\mu_{n-1}, \mu_n)$. We have

$$\frac{1}{s}\{qp_b(\mu_{n-1}, \mu_n) + qp_b(\mu_n, \mu_{n+1})\} \leq qp_b(\mu_{n-1}, \mu_n)$$

By (3.2),

$$\begin{aligned} qp_b(\mu_{n+1}, \mu_n) &\leq \rho[qp_b(\mu_{n+1}, \mu_n)]^\alpha \cdot [\{qp_b(\mu_{n-1}, \mu_n)]^{1-\alpha} \\ [qp_b(\mu_{n+1}, \mu_n)]^{1-\alpha} &\leq \rho[qp_b(\mu_{n-1}, \varepsilon_n)]^{1-\alpha} \\ qp_b(\mu_{n+1}, \mu_n) &\leq \rho^{\frac{1}{1-\alpha}} qp_b(\mu_{n-1}, \mu_n) \\ qp_b(\mu_{n+1}, \mu_n) &\leq \rho qp_b(\mu_{n-1}, \mu_n) \leq \lambda^n qp_b(\mu_0, \mu_1) \end{aligned} \tag{3.3}$$

By taking $n \to \infty$ in the inequality (3.3), we get $L = 0$.

Now we will show that sequence $\{\mu_n\}$ is Cauchy.

Let $n, k \in N$

$$\begin{aligned} qp_b(\mu_n, \mu_{n+k}) &\leq s qp_b(\mu_n, \mu_{n+1}) + s^2 qp_b(\mu_{n+1}, \mu_{n+2}) + \cdots + s^k qp_b(\mu_{n+k-1}, \mu_{n+k}) \\ &\leq [s\rho^n + s^2\rho^{n+1} + \cdots + s^k\rho^{n+k-1}] qp_b(\mu_0, \mu_1) \\ &\leq s^k \sum_{i=n}^{n+k-1} \rho^i qp_b(\mu_0, \mu_1) \\ &\leq s^k \sum_{i=n}^{\infty} \rho^i qp_b(\mu_0, \mu_1) \ldots \end{aligned} \tag{3.4}$$

From (3.4),

$$qp_b(\mu_{n+m}, \mu_{n+m+k}) \leq s^k \sum_{i=m}^{\infty} \rho^i qp_b(\mu_n, \mu_{n+1})$$

$$\lim_{m\to\infty, n\to\infty} qp_b(\mu_{n+m}, \mu_{n+m+k}) \leq s^k \lim_{m\to\infty} \sum_{i=m}^{\infty} \lim_{n\to\infty} \rho^i qp_b(\mu_n, \mu_{n+1}) = 0$$

Therefore,

$$\lim_{n\to\infty} qp_b(\mu_n, \mu_{n+k}) = \lim_{m\to\infty, n\to\infty} qp_b(\mu_{n+m}, \mu_{n+m+k}) = 0 \tag{3.5}$$

Since $M$ is complete, so there exists $z \in M$ such that $\lim_{n\to\infty} \mu_n = z$.
Suppose that $\mu_n \neq T\mu_n$ for each $n \geq 0$,

$$\begin{aligned} qp_b(\mu_{n+1}, Tz) = qp_b(T\mu_n, Tz) &\leq \rho[qp_b(\mu_n, Tz)]^\alpha \cdot \left[\frac{1}{s^2} qp_b(z, T\mu_n)\right]^{1-\alpha} \\ &\leq \rho[qp_b(\mu_n, Tz)]^\alpha \cdot [qp_b(z, \mu_{n+1})]^{1-\alpha} \end{aligned}$$

Letting $n \to \infty$ in the inequality, we find that $qp_b(z, Tz) = 0$, which is a contradiction. Thus, $Tz = z$. □

**Example 5.** Let $M = \{1, 2, 3, 4\}$. Define complete quasi-partial $b$-metric as $qp_b(\mu, \vartheta) = \max\{\mu, \vartheta\} + |\mu - \vartheta|$. The evaluation of values of $qp_b$, $\mu$ and $\vartheta$ are given in Table 1.

**Table 1.** Values of $qp_b$, $\mu$ and $\vartheta$.

| $qp_b(\mu,\vartheta)$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| **1** | 1 | 3 | 5 | 7 |
| **2** | 3 | 2 | 4 | 6 |
| **3** | 5 | 4 | 3 | 5 |
| **4** | 7 | 6 | 5 | 4 |

We define self mappings $T$ on $M$ as $T : \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 1 & 2 \end{pmatrix}$ as shown in Figure 1.

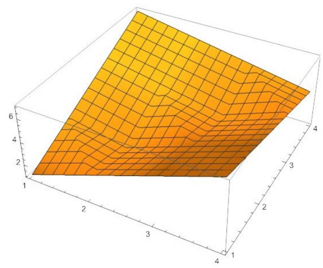

**Figure 1.** 1 and 3 are the fixed point of T.

Choose $\alpha = \frac{1}{2}, \rho = \frac{9}{10}$

**Case 1:** Let $(\mu, \vartheta) = (3, 1)$. Without loss of generality, we have

$$qp_b(T\mu, T\vartheta) \leq \rho[qp_b(\mu, T\vartheta)]^{\alpha} \left[\frac{1}{s^2} qp_b(\vartheta, T\mu)\right]^{1-\alpha}$$

$$qp_b(T3, T1) = 1 \leq \rho[qp_b(3, T1)]^{1/2} \left[\frac{1}{s^2} qp_b(1, T3)\right]^{1/2}$$

**Case 2:** Let $(\mu, \vartheta) = (3, 3)$

$$qp_b(T3, T3) = 1 \leq \rho[qp_b(3, T3)]^{1/2} \left[\frac{1}{s^2} qp_b(3, T3)\right]^{1/2}$$

**Case 3:** Let $(\mu, \vartheta) = (3, 4)$

$$qp_b(T3, T4) = 3 \leq \rho[qp_b(3, T4)]^{1/2} \left[\frac{1}{s^2} qp_b(4, T3)\right]^{1/2}$$

Hence we conclude that 1 and 3 are the fixed point of $T$ in the setting of interpolative Chatterjea type contraction. Thus $T$ can have more than one fixed point.

## 4. Open problem

Let $(M, qp_b)$ be a complete quasi-partial b-metric space. Consider a family of self-maps $T_n : X \longrightarrow X, n \geq 1$ and $s \geq 1$ such that

$$qp_b(T_i\mu, T_j\vartheta) \leq \rho_{i,j}[qp_b(\mu, T_i6\vartheta)]^{\alpha_i}[\frac{1}{s}qp_b(\vartheta, T_j\mu)]^{1-\alpha_i}$$

What are the conditions on $P_{i,j}$ and $\alpha_i$ for $T_n$ to have a fixed point?

## 5. Conclusions

The main contribution of this paper is to introduce two new approaches to obtain fixed point in contractive maps, one is $qp_b$–cyclic mapping and other is an interpolative approach on predefined Chatterjea contraction to ensure the existence of fixed points. In Several real world problems, sensitivity analysis of experimental signals and synthesis of scientific data is needed for the approximation of natural curves and surfaces. To model such problems, interpolation is required as an iterated function system. Common fixed points and coupled fixed points on similar type of interpolative contraction can be obtained in future. Also unique fixed point for these maps can be worked in further studies in development of nonlinear analysis.

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### Conflict of interest

The authors declare that they have no competing interests.

### Author’s contributions

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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