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Research article

Edge event-triggered control and state-constraint impulsive consensus for nonlinear multi-agent systems

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Abstract: In the paper, some consensus problems of multi-agent system controlled by edge event-triggered strategy and state-constraint impulsive are taken into account. For the state-constraint impulsive protocol, two types of control protocols which contain input saturation and double actuator saturation are put forward. With regard to the edge event-triggered control, we propose the rule of it and let the time of the edge event-triggered be impulsive time to avoid the Zeno-behavior. Then, compared to the control method of others, we can greatly reduce the cost in the process of exchanging information. Next, some sufficient conditions of the system are required to reach consensus. In the end, a few examples are exploited for testing and checking the theoretical analyses.

Keywords: edge event-triggered strategy; multi-agent system; impulsive protocol; input saturation; actuator saturation
Mathematics Subject Classification: 34, 37

1. Introduction

As the topic of multi-agent becomes hotter and hotter, for a long time in the past, there exist many researches put forward in the consensus problem in order to improve the theory. In [1–3], the model of integrator and first-order multi-agent systems which conclude linear and non-linear dynamics function had been studied. In [4–8], the consensus problem of second-order or high-order multi-agent systems were taken into consideration. In [9, 10, 12–14], the authors thought about the consensus problem of the multi-agent system which contains a leader-followers consensus or leaderless consensus in which the system was controlled by impulsive protocols. However, there is still an issue about how to reduce unnecessary information interaction on multi-agent system or how to make multi-agent system converge to a consensus with less cost. In this paper, edge event-triggered is an effective control means of saving energy.

With the same time, the theoretical analysis of impulsive system had also aroused the interest of many scholars [15–22]. In actual situation, there were many restrictions on actuators and inputs. In [18], the problem based on state-constraint impulsive protocol were taken into consideration. In [19], the issue of time-delay complex networks system based on the impulsive control were researched. On many occasions, impulsive control was regarded as a useful approach to handle some stability or synchronization problems. Then, it was evident that impulsive control is more beneficial in chaos from [23, 24] or complex networks [25–28] which compares with continuous control.

So far, there have been a great number of theorems and deductions about impulsive system. The sufficient conditions of the primary outcomes are obtained with high requirements. Nevertheless, there is no research on that state-constraint impulsive protocols are applied to nonlinear multi-agent systems via edge event-triggered control so far, then, we will discuss the problem in this paper.

The contributions of this article are twofold:

(1) In real life, it is a common phenomenon to have a limitation of the input or actuator. For the purpose of better closing to the actual situation, two kinds of impulsive control protocols which conclude input saturation and double actuator saturation are discussed in this paper, the sufficient conditions for system to reach consensus are obtained.

(2) Edge event-triggered strategy which is a novel control way of event-triggered can greatly reduce the energy in the process of exchanging information. In this paper, we combine impulse control with edge event trigger control and let the time triggered by the edge event be the impulse time to avoid the Zeno-behavior. When the state error of the agent is small at a certain impulse time, the information interaction can be eliminated. Then, compared to the single impulsive control [11], the number of information exchange has been reduced and the energy consumption of the whole system is correspondingly reduced.

The framework of the paper is as follows. Section II describes some preliminaries which conclude notations, graph theory, state constraints and edge event-triggered strategy are introduced. In section III, the models of the nonlinear dynamics and two types of impulsive control rules are formulated. In section IV, some theorems and their proofs are offered. For the purpose of validating the feasibility of the proposed methods, some numerical simulation are offered in section V. In section VI, conclusion of this paper are offered.

2. Preliminaries

2.1. Notations

R is defined as the set of real number and let N be a set of positive real number. The matrix inequality $A > B$ stands for that every element of A is bigger than B. Suppose that every eigenvalue of matrix A is real. $\lambda_{min}(C)$ and $\lambda_{max}(C)$ are the smallest eigenvalue and largest eigenvalue of matrix C. Then, $diag[a_1, a_2, \ldots, a_N]$ represents a diagonal matrix with elements a_i on the diagonal. I is an identity matrix and I_N denotes a N- dimensional identity matrix. $max(x_i)$ means the maximum of x_i when $x_i \in R$ and $i = 1, 2, \ldots, N$. $co\{u^j : j = 1, 2, \ldots, N\}$ represents a convex hull. I_N denotes an N-dimensional identity matrix.

2.2. *Graph theory*

In the paper, the symbol G=(v,ε) represents a graph in which $v = \{v_1, v_2, \ldots, v_N\}$ is a set of nodes and $\varepsilon \subseteq v \times v$ is a set of edge. Let $A = [a_{ij}]$ be a weighted adjacency matrix with nonnegative elements. For an undirect graph G, the element of adjacency matrix A which is a symmetric matrix is 1 if there exists an edge (v_i, v_j) between node v_i and node v_j, otherwise, the element is 0. The out-degree of node i is defined as $deg(i) = \sum_{j=1}^{N} a_{ij}$ and let matrix $\widetilde{D}$ be the degree matrix which is a diagonal matrix with the out-degree of each node along the principal diagonal. Then, the Laplacian matrix $L = [l_{ij}]$ and the expression is:

$$l_{ij} = \begin{cases} deg_{out}(i), & i = j \\ -a_{ij}, & j \in N_i \\ 0, & \text{otherwise.} \end{cases}$$

where N_i is a set which is made up of all the neighbors of node i.

Suppose there are ϱ edges in graph G and label them be $e_1, e_2, \ldots, e_\varrho$, then, it's obvious that each edge $e_g = (v_i, v_j)$ where $g \in [1, \varrho]$. Denote $D = [d_{ij}]$ be a incidence matrix and the elements of D is that

$$d_{ij} = \begin{cases} 1, & \text{if } v_i \text{ is the head node of the } j_{th} \text{ oriented edge} \\ -1, & \text{if } v_i \text{ is the tail node of the } j_{th} \text{ oriented edge} \\ 0, & \text{otherwise} \end{cases}$$

So, there exists a relationship between the Laplacian matrix L and the incidence matrix D is that $L = DWD^T$. Then, $W = [w_{ii}]$ is a diagonal matrix and define w_{ii} be weight value of i_{th} edge.

From now on, we assume that the topologies of multi-agent systems are all undirected graphs which are all connected.

2.3. *State-constraint strategy*

In practical industrial applications, the parameters will be limited by various physical conditions. So it is a common phenomenon to limit the actuator or input. For example, the cost of electronic devices during power transmission makes the input limited. In addition, almost all actual physical systems are subject to state constrains which conclude actuator saturation constraints and input saturation constraints. Then, define the saturation function be

$$sat(y) = \begin{cases} 1, & y > 1 \\ y, & -1 \leqslant y \leqslant 1 \\ -1, & y < -1 \end{cases} \tag{2.1}$$

where $y \in R$.

2.4. *Edge event-triggered strategy*

For a multi-agent system, we should first formulate some edge event-triggered rules. Then, at each time of sampling, whether the state of each agent and its neighbors are updated depend on the edge event is triggered or not. Namely, if agent j is a neighbor of agent i and the edge event is triggered by their communication link, the relative state of the two agents are all sampled and their controllers are also updated respectively. Else, the edge event is not triggered, their relative state don't renew and is the same as the one at the most recent sampling moment. For convenience, let $t_g^{(t)}$ be the event-triggered moment at the g_{th} time and $t_{g^{ij}(t)}$ be the time that is the last sampling time before t.

$$g^{ij}(t) = max\{g|t_g \in \{t_g \leqslant t\}\}$$

In this paper, we assume that the moment of event-trigger is the impulse constant.

3. Problem description

The following nonlinear system is considered in the paper and the dynamics of each agent in the system is expressed as follows:

$$\dot{x}_i(t) = f(t, x_i(t)) + b_i(t), \quad i = 1, 2, \ldots, N. \tag{3.1}$$

in which $x_i(t) \in R$ is the desired state of the ith agent. $f(t, x_i)$ is a nonlinear functions. The nominal control input of ith agent is represented by $b_i(t)$.

In the paper, the following two types of control protocols are formulated:

Control protocols

1: Input Saturation

$$b_i(t) = \sum_{k=1}^{\infty} sat(\sum_{j\in N_i} r_i a_{ij}\left(x_j\left(t_{g^{ij}(t_k)}\right) - x_i\left(t_{g^{ij}(t_k)}\right)\right))\delta(t - t_k) \tag{3.2}$$

2: Double Actuator Saturation

$$b_i(t) = \sum_{k=1}^{\infty}\sum_{j\in N_i} a_{ij}\left(sat[x_j\left(t_{g^{ij}(t_k)}\right)] - sat[x_i\left(t_{g^{ij}(t_k)}\right)]\right)\delta(t - t_k) \tag{3.3}$$

where r_i is the strength of impulsive. The constant $\{t_k\}$ satisfies the inequality $0 < t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$. We let $\lim_{k\to\infty} t_k = +\infty$ and denote $\delta(t - t_k)$ be a Dirac function. Then we assume that $\triangle\ x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$, $x_i(t_k) = x_i(t_k^-)$ and $x_i(t_k^-) = \lim_{t\to t_k^-} x_i(t)$, $x_i(t_k^+) = \lim_{t\to t_k^+} x_i(t)$.

Suppose that there exist m edges of graph G and label the m edges be $e_1, e_2, \ldots, e_m$. Then, for any edge $e_g = (\nu_i, \nu_j)$ in which $1 \leqslant g \leqslant m$, let $z_g = x_i(t) - x_j(t)$, $\tilde{z}_g(t_k^-) = x_i(t_{ij}(t_k^-)) - x_j(t_{ij}(t_k^-))$ and define $x(t) = [x_1(t), x_2(t), \ldots, x_N(t)]^T$, $z(t) = [z_1(t), z_2(t), \ldots, z_m(t)]^T$, $\tilde{z}(t_k^-) = [\tilde{z}_1(t_k^-), \tilde{z}_2(t_k^-), \ldots, \tilde{z}_m(t_k^-)]^T$. So, it's clear that $z(t) = D^T x(t)$.

Here, we bring forward the rule of edge event-triggered. For the system, the edge event e_g will be turned on at the time t_k unless the following inequality is invalid.

$$\begin{cases} \mu_g\|\tilde{z}_g(t_k)\| \leqslant \|z_g(t_k)\| \leqslant \sigma_g\|\tilde{z}_g(t_k)\| \\ z_g(t_k)\tilde{z}_g(t_k) \geqslant 0 \end{cases} \tag{3.4}$$

where μ_g and σ_g are all system parameters to be designed.

Then, the next definition and lemmas are shown for supporting derivation better:

Definition 1. The system achieves consensus with the control protocols while

$$\lim_{t\to\infty} |x_j(t) - x_i(t)| = 0$$

where $i, j = 1, 2, \cdots, N$.

Lemma 1. [18] Let $w^1, w^2, \ldots, w^U \in R^{n1}$, $m^1, m^2, \ldots, m^V \in R^{n2}$, $w = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$, $m = (m_1, m_2, \cdots, m_n)^{\mathrm{T}}$ and $U, V, n1, n2$ are positive integers. If $w \in co\{w^u : u = 1, 2, \ldots, U\}$ and $m \in co\{m^v : v = 1, 2, \ldots, V\}$, then

$$\begin{bmatrix} w \\ m \end{bmatrix} \in co\left\{\begin{bmatrix} w^u \\ m^v \end{bmatrix} : u = 1, 2, \ldots, U; v = 1, 2, \ldots, V\right\}. \tag{3.5}$$

Lemma 2. [18] Let w, $m \in R^n$, $w = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$, $m = (m_1, m_2, \cdots, m_n)^{\mathrm{T}}$, $n \in N^+$. E is a set of $n \times n$ diagonal matrices and its diagonal elements are either 1 or 0. Assume that E_i stands for every element of E. $i = 1, 2, \ldots 2^n$. Then, $E = \{E_i : i \in \{1, 2, \cdots, 2^n\}\}$. Denote $E_i^- = I - E_i$. It is obvious that E_i^- is also one element of E when $E_i \in E$. When $|m_i| \leqslant 1$, $sat(w) \in co\{E_i w + E_i^- m : i \in \{1, 2, \cdots, 2^n\}\}$.

For example, if $n = 2$, then

$$E = \left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$

If $x \in R^n$ and $T, H \in R^{n\times n}$ are two matrices, when $\|Hx\|_\infty \leqslant 1$, we can get $sat(Tx) \in co\{E_i Tx + E_i^- Hx : i \in \{1, 2, \cdots, 2^n\}\}$. There exists $0 \leqslant \varrho_i \leqslant 1$ and satisfies $\sum_{i=1}^{2^n} \varrho_i = 1$. Then $sat(Tx) = \sum_{i=1}^{2^n} \varrho_i (E_i T + E_i^- H)x$.

Lemma 3. Let $x = [x_1, x_2, \ldots, x_n]^T$, $SAT(x) = [sat(x_1), sat(x_2), \ldots, sat(x_n)]^T$ and $H = diag[h_1, h_2, \ldots, h_n]$. Let $V = diag[v_1, v_2, \ldots, v_n]$, $W = diag[w_1, w_2, \ldots, w_n]$ and $0 \leqslant v_i < 1$. Denote $W = I - V$, it is clearly that $0 < w_i \leqslant 1$. Then, for $SAT(x)$, there exists a matrix H such that $SAT(x) = Vx + WHx$ when $\|Hx\|_\infty \leqslant 1$.

Proof. Let $V = diag[v_1, v_2, \ldots, v_n]$, $W = diag[w_1, w_2, \ldots, w_n]$, $H = diag[h_1, h_2, \ldots, h_n]$. We assume $0 \leqslant v_i < 1$and $v_i + w_i = 1$, then $0 < w_i \leqslant 1$. So there exists a constant h_i such that $sat(x_i) = v_i x_i + w_i h_i x_i$. For example, we choose $v_i = a$, $w_i = 1 - a$, then $sat(x_i) = a x_i + (1 - a) h_i x_i$. So we let $h_i = 1$ if $-1 \leqslant x_i \leqslant 1$ and let $h_i = \frac{sign(x_i) - a x_i}{(1-a) x_i}$ when $|x_i| > 1$. Then, we can easily obtain that $SAT(x) = Vx + WHx$ when $\|Hx\|_\infty \leqslant 1$. The proof is completed.

Lemma 4. [3] If there exist two matrices P and B belong to $R^{n\times n}$, which are all positive definite and symmetric, then for any $x \in R^n$, the following inequation holds

$$\lambda_{min}(P^{-1}B) x^T P x \leqslant x^T B x \leqslant \lambda_{max}(P^{-1}B) x^T P x. \tag{3.6}$$

Remark 1. For Lemma 4, If matrix P is a identity matrix I_n, then the following inequation holds

$$\lambda_{min}(B) x^T x \leqslant x^T B x \leqslant \lambda_{max}(B) x^T x. \tag{3.7}$$

Assumption 1. Nonlinear function $f(\cdot)$ satisfies a condition that there exists a positive constant τ such that:

$$|f(t, a) - f(t, b)| \leqslant \tau |a - b|. \tag{3.8}$$

4. Main results

4.1. Input saturation

Consider a consensus problem of system (3.1) based on protocol 1 (shown as (3.2)). Then the system can be considered as follows:

$$\begin{cases} \dot{x}_i(t) = f(t, x_i(t)), \quad t \neq t_k \\ \triangle\ x_i(t_k) = sat[\sum_{j\in N_i} r_i a_{ij}\left(x_j(t_{g^{ij}(t_k^-)}) - x_i(t_{g^{ij}(t_k^-)})\right)], t = t_k \end{cases} \tag{4.1}$$

According to Lemma 2, we can get:

$$\triangle\ x(t_k) = (\sum_{i=1}^{2^n} \varrho_i(E_iR + E_i^- H))(-Lx(t_{g^{ij}(t_k^-)})) \tag{4.2}$$

denote $R = diag[r_1, r_2, \ldots, r_N]$. Assume $S = \sum_{i=1}^{2^n} \varrho_i(E_iR + E_i^- H)$ and if the matrix H is selected for a diagonal matrix, S is also a diagonal matrix.

Then, obviously

$$\triangle\ x(t_k) = -SLx(t_{g^{ij}(t_k^-)}) \tag{4.3}$$

Furthermore, the system can be described as follow

$$\begin{cases} \dot{x}(t) = F(t, x(t)), \quad t \neq t_k \\ x(t_k^+) = x(t_k^-) - SLx(t_{g^{ij}(t_k^-)}), \quad t = t_k \end{cases} \tag{4.4}$$

define $F(t, x(t)) = (f(t, x_1(t)), f(t, x_2(t)), \ldots, f(t, x_N(t)))^T$.

Theorem 1. Suppose that there exists a matrix H such that $||HLx||_\infty \leqslant 1$ under the impulsive control protocol 1, if Assumption 1 holds and there exist two constants $\hat{\xi}$, $\hat{\gamma}$ that make the following inequities hold:
(i) There exists a constant $\hat{\varpi}$ to make the expression $0 < t_{k+1} - t_k \leqslant \hat{\varpi}, k \in N^+$ holds.
(ii) $0 < 1 - \hat{\gamma} \leqslant \hat{\xi}$;
(iii) $\ln(\hat{\xi}) + 2\tau\hat{\varpi} \leqslant 0$;
The system (3.1) will be aligned under protocol 1.

Proof. Consider the following Lyapunov function:

$$V(x(t)) = x^T(t)Lx(t) = \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{N} a_{ij}(x_j(t) - x_i(t))^2. \tag{4.5}$$

When $t \neq t_k$, we can easily obtain that the derivative of V is:

$$\begin{aligned}\dot{V}(t) &= \sum_{j=1}^{N}\sum_{i=1}^{N} a_{ij}[(x_j(t)-x_i(t))(\dot{x}_j(t)-\dot{x}_i(t))]\\ &= \sum_{j=1}^{N}\sum_{i=1}^{N} a_{ij}[x_j(t)-x_i(t)][f(t,x_j(t))-f(t,x_i(t))]\\ &\leqslant \sum_{j=1}^{N}\sum_{i=1}^{N} a_{ij}|x_j(t)-x_i(t)||f(t,x_j(t))-f(t,x_i(t))|\\ &\leqslant \tau\sum_{j=1}^{N}\sum_{i=1}^{N} a_{ij}|x_j(t)-x_i(t)||x_j(t)-x_i(t)|\\ &\leqslant 2\tau V(t).\end{aligned} \tag{4.6}$$

When $t = t_k$, we can get that

$$\begin{aligned}V(x(t_k^+)) &= x^T(t_k^+)Lx(t_k^+)\\ &= [x(t_k^-) - SLx(t_{g^{ij}(t_k^-)})]^T L[x(t_k^-) - SLx(t_{g^{ij}(t_k^-)})]\\ &= [x^T(t_k^-) - x^T(t_{g^{ij}(t_k^-)})LS]L[x(t_k^-) - SLx(t_{g^{ij}(t_k^-)})]\\ &= x^T(t_k^-)Lx(t_k^-) - x^T(t_k^-)LSLx(t_{g^{ij}(t_k^-)}) - x(t_k^-)LSLx^T(t_{g^{ij}(t_k^-)}) + x^T(t_{g^{ij}(t_k^-)})LSLSLx(t_{g^{ij}(t_k^-)})\\ &= x^T(t_k^-)Lx(t_k^-) - 2x^T(t_k^-)LSLx(t_{g^{ij}(t_k^-)}) + x^T(t_{g^{ij}(t_k^-)})LSLSLx(t_{g^{ij}(t_k^-)})\\ &= V(t_k^-) - V_1(t_k^-)\end{aligned} \tag{4.7}$$

where

$$V_1(t_k^-) = 2x^T(t_k^-)LSLx(t_{g^{ij}(t_k^-)}) - x^T(t_{g^{ij}(t_k^-)})LSLSLx(t_{g^{ij}(t_k^-)})$$

For the convenience of the next, the case that the edge events labeled by 1 to g are triggered at $t = t_k$ is assumed. Also, we can get $L = DWD^T$, $z(t) = D^T x(t)$ and denote $z = z(t)$, $\tilde{z} = \tilde{z}(t_k^-)$. Then, according to Lemma 4 and Remark 1, it's easy to obtain that

$$\begin{aligned}V_1(t_k^-) &= 2x^T(t_k^-)LSLx(t_{g^{ij}(t_k^-)}) - x^T(t_{g^{ij}(t_k^-)})LSLSLx(t_{g^{ij}(t_k^-)})\\ &= 2z^TWD^TSDW\tilde{z} - \tilde{z}^TWD^TSLSDW\tilde{z}\\ &\geqslant 2\hat{\alpha}z^TW\tilde{z} - \hat{\beta}\tilde{z}^TW\tilde{z}\\ &= 2\hat{\alpha}(w_1z_1\tilde{z}_1 + w_2z_2\tilde{z}_2 + \ldots + w_mz_m\tilde{z}_m) - \hat{\beta}(w_1\tilde{z}_1^2 + w_2\tilde{z}_2^2 + \ldots + w_m\tilde{z}_m^2)\\ &\geqslant (2\hat{\alpha}-\hat{\beta})(w_1z_1^2 + w_2z_2^2 + \ldots + w_gz_g^2) + (2\hat{\alpha}\mu_{g+1}-\hat{\beta})w_{g+1}\tilde{z}_{g+1}^2 + \ldots + (2\hat{\alpha}\mu_m-\hat{\beta})w_m\tilde{z}_m^2\\ &\geqslant (2\hat{\alpha}-\hat{\beta})(w_1z_1^2 + w_2z_2^2 + \ldots + w_gz_g^2) + (2\hat{\alpha}\mu_{g+1}-\hat{\beta})w_{g+1}\frac{1}{\sigma_{g+1}^2}z_{g+1}^2 + \ldots + (2\hat{\alpha}\mu_m-\hat{\beta})w_m\frac{1}{\sigma_m^2}z_m^2\\ &= z^TQWz\\ &\geqslant \hat{\gamma}V(t_k^-)\end{aligned} \tag{4.8}$$

denote

$$\hat{\alpha} = \lambda_{min}(D^T S DW)$$

$$\hat{\beta} = \lambda_{max}(D^T S LS DW)$$

$$\hat{\gamma} = \lambda_{min}(Q)$$

and Q is a diagonal matrix in which g elements are $2\hat{\alpha} - \hat{\beta}$ and $m - g$ elements are $\frac{2\hat{\alpha}\mu_p - \hat{\beta}}{\sigma_p^2}$, where $p = m - g, m - g + 1, \ldots, m$.

Based on (4.7) and (4.8), it's clearly that

$$\begin{aligned} V(x(t_k^+)) &= V(t_k^-) - V_1(t_k^-) \\ &\leqslant (1 - \hat{\gamma})V(t_k^-) \\ &\leqslant \hat{\xi} V(x(t_k^-)) \end{aligned} \tag{4.9}$$

From (4.6) and (4.9), one obtains

$$\begin{cases} \dot{V}(x(t)) \leqslant 2\tau V(x(t)), \ t \neq t_k \\ V(x(t_k^+)) \leqslant \hat{\xi} V(x(t_k^-)), \ t = t_k \end{cases} \tag{4.10}$$

When $t \in [t_0, t_1)$, in view of inequalities (4.10), we can get

$$\begin{cases} V(x(t)) \leqslant e^{2\tau(t-t_0)} V(x(t_0)) \\ V(x(t_1^+)) \leqslant \hat{\xi} e^{2\tau(t_1-t_0)} V(x(t_0)) \end{cases} \tag{4.11}$$

According to mathematical deduction, when $t \in [t_k, t_{k+1})$, it implies that

$$\begin{cases} V(x(t)) \leqslant e^{2\tau(t-t_k)} V(x(t_k^+)), \\ V(x(t_k^+)) \leqslant \hat{\xi} V(x(t_k^-)), \end{cases} \tag{4.12}$$

From (4.11) and (4.12), one gets

$$\begin{aligned} V(x(t)) &\leqslant e^{2\tau(t-t_k)} V(x(t_k^+)) \\ &\leqslant \hat{\xi} e^{2\tau(t-t_k)} V(x(t_k^-)) \\ &\leqslant \hat{\xi} e^{2\tau(t-t_k)} e^{2\tau(t_k-t_{k-1})} V(x(t_{k-1})) \\ &\leqslant \hat{\xi} e^{2\tau(t-t_{k-1})} V(x(t_{k-1})) \\ &\leqslant \ldots \\ &\leqslant \hat{\xi}^k e^{2\tau(t-t_0)} V(x(t_0)) \end{aligned} \tag{4.13}$$

Notice that $\ln(\hat{\xi}) + 2\tau\hat{\varpi} \leqslant 0$ from (*iii*) in Theorem 1, one obtains

$$0 < \hat{\xi} e^{2\tau(t-t_{k-1})} \leqslant 1. \tag{4.14}$$

It is obvious that the system (4.1) which controlled by protocol 1 can reach consensus. The proof is completed.

4.2. Double actuator saturation

Discuss a consensus problem of the system (3.1) under control protocol 2 whose expression is (3.3). For the sake of convenience, suppose this situation that the edge events labeled by 1 to g are triggered at the moment $t = t_k$. Then multi-agent system can be rewritten as follows:

$$\begin{cases} \dot{x}_i(t) = f(t, x_i(t)), \quad t \neq t_k \\ \triangle\ x_i(t_k) = \sum_{j\in N_i} a_{ij}\left(sat(t_{g^{ij}(t_k^-)}) - sat(x_i(t_{g^{ij}(t_k^-)}))\right), t = t_k \end{cases} \tag{4.15}$$

According to Lemma 3, we can get the impulsive instant expression of agent is

$$\begin{aligned} \triangle\ x(t_k) &= -LSAT(x(t_{g^{ij}(t_k^-)})) \\ &= -L((V + W\bar{H})x(t_{g^{ij}(t_k^-)})) \end{aligned} \tag{4.16}$$

where $SAT(x(t)) = (sat(x_1(t)), sat(x_2(t)), \ldots, sat(x_N(t)))^T$ and $\bar{H} = diag(h_1, h_2, \ldots, h_N)^T$. We let $O = V + W\bar{H}$, it's clearly that O is a diagonal matrix. Then

$$\triangle\ x(t_k) = -LOx(t_{g^{ij}(t_k^-)}) \tag{4.17}$$

Then the system can be reformulated as:

$$\begin{cases} \dot{x}(t) = F(t, x(t)), \quad t \neq t_k \\ x(t_k^+) = x(t_k^-) - LOx(t_{g^{ij}(t_k^-)}), \quad t = t_k \end{cases} \tag{4.18}$$

Theorem 2. Assume that there exists a matrix $\bar{H}$ such that $\|\bar{H}x\|_\infty \leqslant 1$ under the impulsive control protocol 2, if Assumption 1 holds and there exist two constants $\bar{\xi}$, $\bar{\gamma}$ that make the following inequities hold
(i) There exists a constant $\bar{\varpi}$ to make the inequation $0 < t_{k+1} - t_k \leqslant \bar{\varpi}, k \in N^+$ holds.
(ii) $0 < 1 - \bar{\gamma} \leqslant \bar{\xi}$;
(iii) $\ln(\bar{\xi}) + 2\tau\bar{\varpi} \leqslant 0$;
Then, system (3.1) can achieve consensus based on the action of protocol 2.

Proof. Taking the following Lyapunov function into account:

$$V(x(t)) = x^T(t)Lx(t) = \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{N} a_{ij}(x_j(t) - x_i(t))^2. \tag{4.19}$$

When $t \neq t_k$, the proof is the same as Theorem 1, so we can easily obtain:

$$\dot{V}(t) \leqslant 2\tau V(t). \tag{4.20}$$

When $t = t_k$, one can gather

$$\begin{aligned} V(x(t_k^+)) &= x^T(t_k^+)Lx(t_k^+) \\ &= [x(t_k^-) - LOx(t_{g^{ij}(t_k^-)})]^T L[x(t_k^-) - LOx(t_{g^{ij}(t_k^-)})] \\ &= [x^T(t_k^-) - x^T(t_{g^{ij}(t_k^-)})OL]L[x(t_k^-) - LOx(t_{g^{ij}(t_k^-)})] \\ &= x^T(t_k^-)Lx(t_k^-) - 2x^T(t_k^-)LLOx(t_{g^{ij}(t_k^-)}) + x^T(t_{g^{ij}(t_k^-)})OLLLOx(t_{g^{ij}(t_k^-)}) \\ &= V(t_k^-) - V_2(t_k^-) \end{aligned} \tag{4.21}$$

where

$$V_2(t_k^-) = 2x^T(t_k^-)LLOx(t_{g^{ij}(t_k^-)}) - x^T(t_{g^{ij}(t_k^-)})OLLLOx(t_{g^{ij}(t_k^-)})$$

Then, according to the above, it's easy to obtain that

$$\begin{aligned}
V_2(t_k^-) &= 2x^T(t_k^-)LLOx(t_{g^{ij}(t_k^-)}) - x^T(t_{eij}^{(t_k^-)})OLLLOx(t_{g^{ij}(t_k^-)}) \\
&\geqslant 2\zeta x^T(t_k^-)LLx(t_{g^{ij}(t_k^-)}) - \delta^2 x^T(t_{g^{ij}(t_k^-)})LLLx(t_{g^{ij}(t_k^-)}) \\
&= 2\zeta z^T WD^T DW\tilde{z} - \delta^2 \tilde{z}^T WD^T LDW\tilde{z} \\
&\geqslant 2\zeta\bar{\alpha} z^T W\tilde{z} - \delta^2\bar{\beta}\tilde{z}^T W\tilde{z} \\
&= 2\zeta\bar{\alpha}(w_1 z_1\tilde{z}_1 + w_2 z_2\tilde{z}_2 + \ldots + w_m z_m\tilde{z}_m) - \delta^2\bar{\beta}(w_1\tilde{z}_1^2 + w_2\tilde{z}_2^2 + \ldots + w_m\tilde{z}_m^2) \\
&\geqslant (2\zeta\bar{\alpha} - \delta^2\bar{\beta})(w_1 z_1^2 + w_2 z_2^2 + \ldots + w_g z_g^2) + (2\zeta\bar{\alpha}\mu_{g+1} - \delta^2\bar{\beta})w_{g+1}\tilde{z}_{g+1}^2 + \ldots \\
&\quad + (2\zeta\bar{\alpha}\mu_m - \delta^2\bar{\beta})w_m\tilde{z}_m^2 \\
&\geqslant (2\zeta\bar{\alpha} - \delta^2\bar{\beta})(w_1 z_1^2 + w_2 z_2^2 + \ldots + w_g z_g^2) + (2\zeta\bar{\alpha}\mu_{g+1} - \delta^2\bar{\beta})w_{g+1}\frac{1}{\sigma_{g+1}^2}z_{g+1}^2 + \ldots \\
&\quad + (2\zeta\bar{\alpha}\mu_m - \delta^2\bar{\beta})w_m\frac{1}{\sigma_m^2}z_m^2 \\
&= z^T\bar{Q}Wz \\
&\geqslant \bar{\gamma}V(t_k^-)
\end{aligned} \tag{4.22}$$

where

$$\zeta = \lambda_{min}(O)$$

$$\delta = \lambda_{max}(O)$$

$$\bar{\alpha} = \lambda_{min}(D^T DW)$$

$$\bar{\beta} = \lambda_{max}(D^T LDW)$$

$$\bar{\gamma} = \lambda_{min}(\bar{Q})$$

and $\bar{Q}$ is a diagonal matrix in which g elements are $2\zeta\bar{\alpha} - \delta^2\bar{\beta}$ and $m-g$ elements are $\frac{2\zeta\bar{\alpha}\mu_p - \delta^2\bar{\beta}}{\sigma_p^2}$ in which $p = m-g, m-g+1, \ldots, m$.

Focus on (4.21) and (4.22), it's clearly that

$$\begin{aligned}
V(x(t_k^+)) &= V(t_k^-) - V_1(t_k^-) \\
&\leqslant (1-\bar{\gamma})V(t_k^-) \\
&\leqslant \bar{\xi}V(x(t_k^-))
\end{aligned} \tag{4.23}$$

From (4.20) and (4.23), we can observe

$$\begin{cases} \dot{V}(x(t)) \leqslant 2\tau V(x(t)), \ t \neq t_k \\ V(x(t_k^+)) \leqslant \bar{\xi}V(x(t_k^-)), \ t = t_k \end{cases} \tag{4.24}$$

When $t \in [t_0, t_1)$, from the inequality of (4.24), we can get

$$\begin{cases} V(x(t)) \leqslant e^{2\tau(t-t_0)} V(x(t_0)) \\ V(x(t_1^+)) \leqslant \bar{\xi} e^{2\tau(t_1-t_0)} V(x(t_0)) \end{cases} \tag{4.25}$$

Thus, we can easily obtain that

$$\begin{cases} V(x(t)) \leqslant e^{2\tau(t-t_k)} V(x(t_k^+)), t \in [t_k, t_{k+1}) \\ V(x(t_k^+)) \leqslant \bar{\xi} V(x(t_k^-)), \ t = t_k \end{cases} \tag{4.26}$$

According to (4.26), by mathematical deduction, it holds that

$$\begin{aligned} V(x(t)) &\leqslant e^{2\tau(t-t_k)} V(x(t_k^+)) \\ &\leqslant \bar{\xi} e^{2\tau(t-t_k)} V(x(t_k^-)) \\ &\leqslant \bar{\xi} e^{2\tau(t-t_k)} e^{2\tau(t_k-t_{k-1})} V(x(t_{k-1})) \\ &\leqslant \bar{\xi} e^{2\tau(t-t_{k-1})} V(x(t_{k-1})) \\ &\leqslant \ldots \\ &\leqslant \bar{\xi}^k e^{2\tau(t-t_0)} V(x(t_0)) \end{aligned} \tag{4.27}$$

From (iii) in Theorem 2, we can receive $\ln(\bar{\xi}) + 2\tau\bar{\varpi} \leqslant 0$. Then,

$$0 < \bar{\xi} e^{2\tau(t-t_{k-1})} \leqslant 1. \tag{4.28}$$

It's evident that the system (4.15) which controlled by protocol 2 reach consensus. The proof is completed.

5. Numerical simulations

Example 1. We consider the multi-agent system (4.1) under the control of protocol 1.

$$\dot{x}_i(t) = f(t, x_i(t)) + b_i(t), i = 1, 2, 3, 4 \tag{5.1}$$

For the system (4.1), the undirect graph is chosen as the Figure 1.

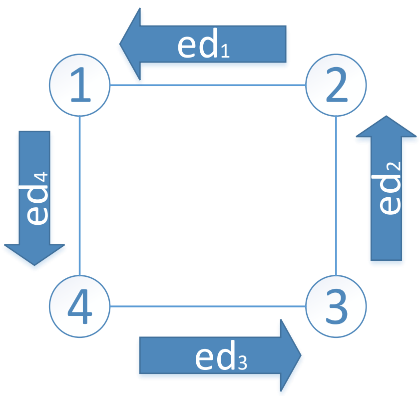

Figure 1. The network topology in Example 1.

From the graph of Figure 1, we can clearly acquire the incidence matrix

$$D = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

And we select that nonlinear functions are as follows: $f(t, x_i(t)) = cos^2(x_i(t)) - |sin(x_i(t))|$. And choose $x_1(0) = 3, x_2(0) = -0.4, x_3(0) = -2.7, x_4(0) = 1.3$ and $\tau = 1.2$.

With regard to Theorem 1, let's define the step size to be 0.001, $R = diag[0.5, 0.7, 0.6, 0.4]$, $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 0.8$, $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = 1.2$, $\varrho_1 = 0.2, \varrho_2 = 0.1, \varrho_3 = 0.3, \varrho_4 = 0.1, \varrho_5 = 0.3$, the rest of ϱ_i are all zero and $\hat{\xi} = 0.86$, $t_k - t_{k-1} = 0.2$, $k \geqslant 1$. According to calculation, the conditions of $(ii), (iii)$ in Theorem 1 are all hold. Then, Figure 2 displays the state value of ith agent under the control of impulsive and event-triggered, Figure 3 indicates the error between every two agents, Figure 4 reveals the event-triggered time of each edge.

We can obviously see the system is consensus under the control of protocol 1 from Figure 2 and Figure 3.

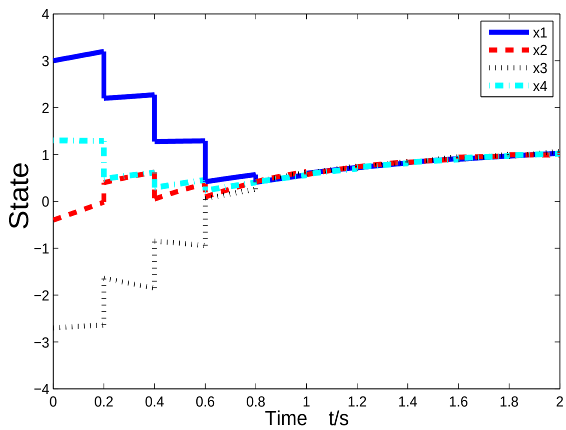

Figure 2. The state of every agent with impulsive in Example 1.

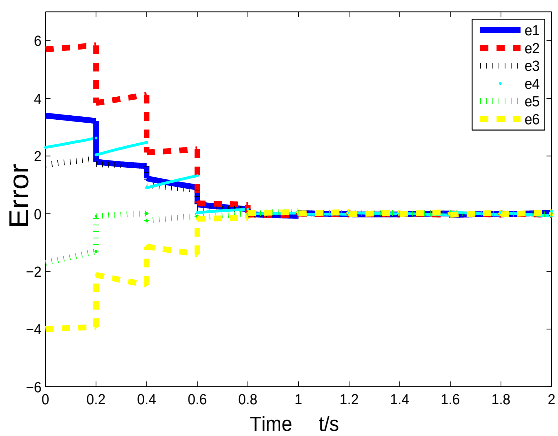

Figure 3. The error between every two agents in Example 1.

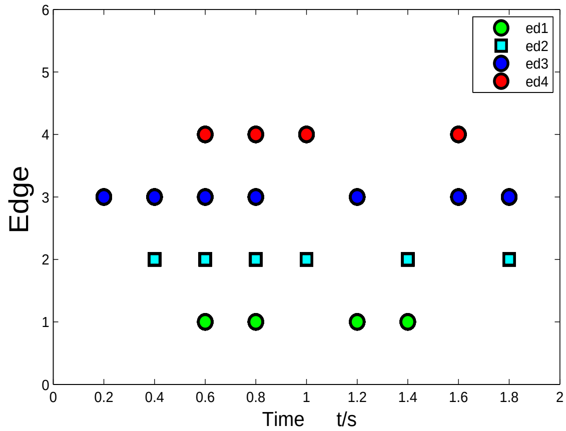

Figure 4. The event-triggered time of each edge in Example 1.

Example 2. We take the multi-agent system (4.15) into consideration which is controlled by protocol 2.

$$\dot{x}_i(t) = f(t, x_i(t)) + b_i(t), i = 1, 2, 3, 4 \tag{5.2}$$

As the system (4.15), we let the graph and the direction of every edge as follows in Figure 5:

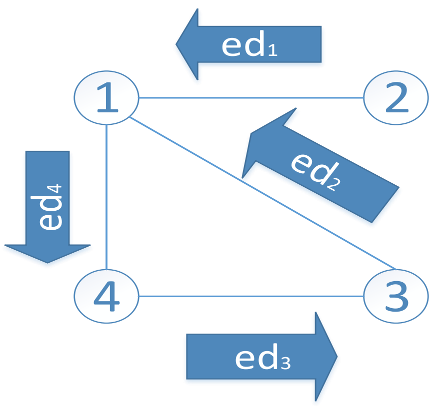

Figure 5. The network topology in Example 2.

So, the incidence matrix D is

$$D = \begin{bmatrix} 1 & 1 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

Then we assume that the nonlinear function of the dynamics of each agent are $f(t, x(t)) \cos^2(x(t)) - |0.4\cos(x(t)) - 1|$ and choose $x(0) = [3, -0.4, -2.28, 0.3]$, $\tau = 1.6$.

About Theorem 2, suppose step size be 0.001, $R = diag[0.4, 0.6, 0.6, 0.4]$, $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 0.72$, $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = 1.46$ $\bar{\xi} = 0.79$, $\varrho_1 = 0.3, \varrho_3 = 0.2, \varrho_5 = 0.2, \varrho_7 = 0.1, \varrho_9 = 0.2$, the rest of ϱ_i are all zero and $t_k - t_{k-1} = 0.2$, $k \geqslant 1$. By calculation, we can verify that the conditions of Theorem

2 are established. Then, the state of ith agent with the impulsive control is manifested in Figure 6 and the error between every two agents is evident in Figure 7. The event-triggered moment of each edge is displayed in Figure 8.

It is clear that the multi-agent system reach consensus under the control of protocol 2 from Figure 6 and Figure 7.

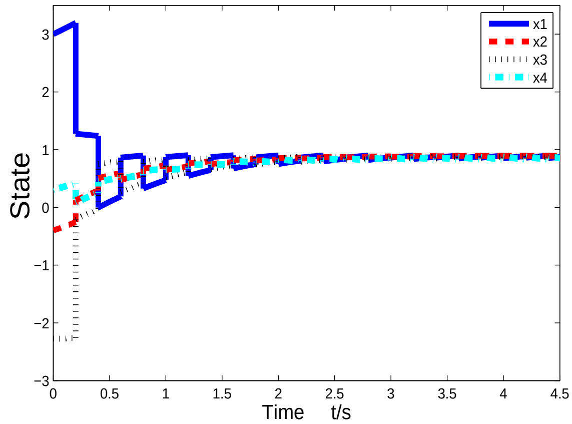

Figure 6. The state of each agent with impulsive in Example 2.

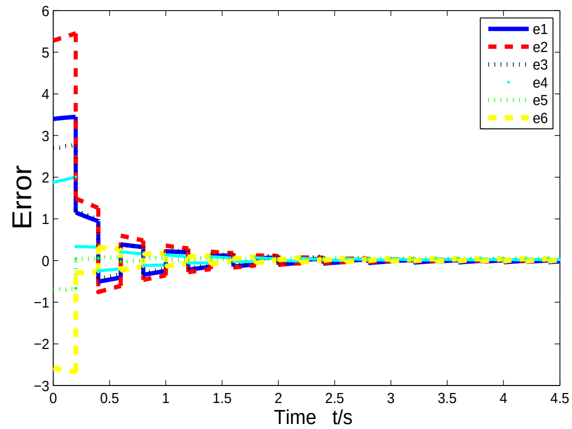

Figure 7. The error between every two agents in Example 2.

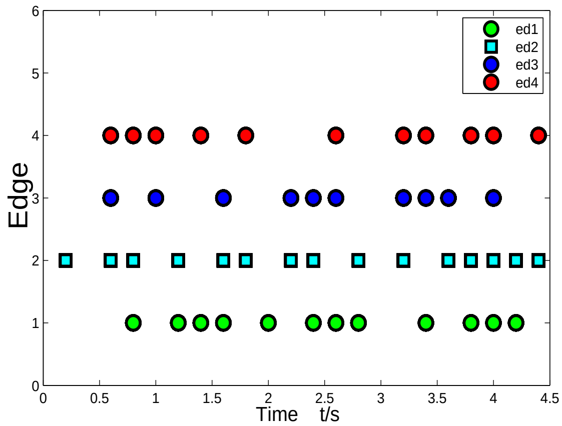

Figure 8. The event-triggered time of each edge in Example 2.

6. Conclusions

We discuss the nonlinear multi-agent consensus issue under the control state-constrain impulsive and edge event-triggered tactics in the above. Impulsive protocol based on the relative information between the agents and their neighbors has been adopted to deal with the problem of consensus. Edge event-triggered strategy can reduce cost by reducing the number of information exchange. According to theoretical analysis, we can gather sufficient conditions to ensure the consensus of the system in this paper. Numerical simulations have verified that the consensus problem are solved by the control protocols. The asynchronous event-based problem of multi-agent systems will be taken into consideration.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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