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Research article

An application of improved Bernoulli sub-equation function method to the nonlinear conformable time-fractional SRLW equation

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Abstract: The nonlinear conformable time-fractional Symmetric Regularized Long Wave (SRLW) equation plays an important role in physics. This equation is an interesting model to describe ion-acoustic and space change waves with weak nonlinearity. In this paper, we solve the SRLW equation via the Improved Bernoulli Sub-Equation Function Method (IBSEFM). New exact solutions are constructed and by the aid of software mathematics programme, 2D and 3D graphs of the solutions according to the parameters are plotted. The results show that IBSEFM is a powerful mathematical tool to solve nonlinear conformable time-fractional equations arising in mathematical physics.

Keywords: conformable time-fractional SRLW equation; IBSEFM
Mathematics Subject Classification: 35C08, 34K20, 32W50

1. Introduction

The investigation of exact travelling wave solutions of nonlinear partial differential equations (NLPDEs) is important to understand the nonlinear physical process that appears in many areas of scientific fields such as nonlinear optics [1, 2], plasma physics [3, 4], biophysics [5] and discrete electrical transmission lines [6–8].

To find the exact solutions of these equations, many powerful methods have been applied in the literature by mathematicians and physicists. Some of these effective methods are extended tanh method [9,10], first integral method [11,12], sine-cosine method [13], dynamical system method [14], modified simple equation method [15, 16], tanh method [17], generalized tanh function method [18], improved F expansion method with Riccati equation [19, 20], modified $\exp(\Omega(\xi))$-expansion function method [21,22], Kudryashov method [46] and improved Bernoulli sub-equation function method [23,24].

In recent years, the fractional differential equations have become a useful tool for describing nonlinear phenomena of science and engineering models. Many of techniques which applied to the nonlinear partial differential equations have been adapted for fractional nonlinear partial differential equations to find exact solutions. For example the functional variable method [25], the first integral method [26], the exp-function method [27], Kudryashov method [46], modified extended tanh method [47], fractional Riccati expansion method [28], modified extended tanh method [29, 30] and many others.

In [31] a new simple well behaved definition of the fractional derivative called conformable fractional derivative is introduced. The conformable fractional derivative is theoretically easier than fractional derivative to handle. Also the conformable fractional derivative obeys some conventional properties that can't be satisfied by the existing fractional derivatives, for instance; the chain rule [32]. The conformable fractional derivative has the weakness that the fractional derivative of any differentiable function at the point zero is equal to zero. So that in [32, 33], it is proposed a suitable fractional derivative that allows us to escape the lack of the conformable fractional derivative. During the last few years, many of techniques applied to find exact solutions for conformable fractional nonlinear partial differential equations [34–47].

In this paper, we obtain the exact solutions of conformable time fractional Symmetric Regularized Long Wave (SRLW) equation by using IBSEFM. We consider

$$D_{tt}^{2\alpha}u + pu_{xx} + quD_t^{\alpha}u_x + qu_xD_t^{\alpha}u + rD_{tt}^{2\alpha}u_{xx} = 0, \quad t > 0, \tag{1.1}$$

where $u = u(x,t), p, q$ and r real parameters, $D_{tt}^{2\alpha}$ is the fractional differential operator of order 2α in conformable sense. This equation is an interesting model to describe ion-acoustic and space charge waves with weak non linearity [47]. Before beginning to the solution procedure, we should give some significant properties of conformable fractional derivative. We explain the solution procedure in the third section and in the fourth section we give the implementation of the proposed procedure to conformable time fractional SRLW equation.

2. Conformable fractional derivative

In this section, we give some basic definitions and theorems of the conformable fractional derivative.

The conformable derivative of order α with respect to the independent variable t is defined as [31]

$$D_t^{a}(y(t)) = \lim_{\tau\to 0}\frac{y(t+\tau t^{1-\alpha}) - y(t)}{\tau}, \ t > 0, \ \alpha \in (0,1],$$

for a function $y = y(t) : [0,\infty) \to \mathbb{R}$.

Theorem 2.1. *Assume that the order of the derivative $\alpha \in (0,1]$ and suppose that $u = u(t)$ and $v = v(t)$ are $\alpha-$ differentiable for all positive t. Then,*

1. $D_t^a(c_1u + c_2v) = c_1D_t^a(u) + c_2D_t^a(v)$, *for* $\forall\ c_1, c_2 \in \mathbb{R}$.

2. $D_t^a(t^k) = kt^{k-a}, \forall\ k \in \mathbb{R}$.

3. $D_t^a(\lambda) = 0$, *for all constant function* $u(t) = \lambda$.

4. $D_t^a(uv) = uD_t^a(v) + vD_t^a(u)$.

5. $D_t^a\left(\frac{u}{v}\right) = \frac{vD_t^a(u) - uD_t^a(v)}{v^2}$.

6. $D_t^a(u)(t) = t^{1-\alpha}\frac{du}{dt}$.

Conformable fractional differential operator satisfies some critical fundamental properties like the chain rule, Taylor series expansion and Laplace transform.

Theorem 2.2. *Let $u = u(t)$ be an $\alpha-$ conformable differentiable function and assume that v is differentiable and defined in the range of u. Then,*

$$D_t^{\alpha}(u \circ v)(t) = t^{1-\alpha} v'(t) u'(v(t)).$$

The proofs of these theorems are given in [32] and in [35] respectively.

3. Basic properties of the improved Bernoulli sub-equation function method (IBSEFM)

In this section let us give the fundamental properties of the IBSEFM. We present the four main steps of the IBSEFM below the following:

Step 1: Let us consider the following conformable time-fractional PDE of the form

$$P(u, D_t^{\alpha} u, u_x, D_{tt}^{2\alpha} u, u_{xx}, ...) = 0. \tag{3.1}$$

The goal is converting (3.1) with a suitable fractional transformation into the nonlinear ordinary differential equation. The wave transformation as;

$$u(x,t) = U(\eta), \quad \eta = \left(x - \frac{v t^{\alpha}}{\alpha} \right), \tag{3.2}$$

where v is a constant to be determined later. Using chain rule and substituting (3.2) in (3.1), we obtain the following nonlinear ordinary differential equation;

$$N(U, U', U'', ...) = 0. \tag{3.3}$$

Step 2: We hypothesize that the solution of (3.3) may be presented below;

$$U(\eta) = \frac{\sum\limits_{i=0}^{n} a_i F^i(\eta)}{\sum\limits_{j=0}^{m} b_j F^j(\eta)} = \frac{a_0 + a_1 F(\eta) + a_2 F^2(\eta) + ...a_n F^n(\eta)}{b_0 + b_1 F(\eta) + b_2 F^2(\eta) + ...b_m F^m(\eta)}, \tag{3.4}$$

where $a_0, a_1, ..., a_n$ and $b_0, b_1, ..., b_m$ are coefficients which will be determined later. $m \neq 0, n \neq 0$ are arbitrary constants that chosen according to the balance principle and considering the form of Bernoulli differential equation below the following;

$$F'(\eta) = \sigma F(\eta) + d F^M(\eta), \ \sigma \neq 0, d \neq 0, \ M \in \mathbb{R} - \{0, 1, 2\}, \tag{3.5}$$

where $F(\eta)$ is polynomial. Substituting (3.4) and (3.5) in (3.3), it yields us an equation of polynomial $\Omega(F)$ of F as following:

$$\Omega(F(\eta)) = \rho_s F(\eta)^s + ... + \rho_1 F(\eta) + \rho_0 = 0.$$

According to the balance principle that is the highest order derivative term and nonlinear term in (3.3), we can determine the relationship between n, m and M.

Step 3: The coefficients of $\Omega(F(\eta))$ all be zero will give us an algebric system of equations;

$$\rho_i = 0, \ \ i = 0, ..., s.$$

Solving this system, we can specify the values of $a_0, a_1, ..., a_n$ and $b_0, b_1, ..., b_m$.

Step 4: When we solve differential equation (3.5), we obtain the following two situations according to σ and d;

$$F(\eta) = \left[\frac{-d}{\sigma} + \frac{E}{e^{\sigma(M-1)\eta}}\right]^{\frac{1}{1-M}}, \ \ \sigma \neq d, \tag{3.6}$$

$$F(\eta) = \left[\frac{(E-1) + (E+1)\tanh(\sigma(1-M))\frac{\eta}{2}}{1 - \tanh\left(\sigma(1-M)\frac{\eta}{2}\right)}\right], \sigma = d, \ E \in \mathbb{R}.$$

Using a complete discrimination system for polynomial of $F(\eta)$, we obtain the analytical solutions of (3.3) with the help of software programme and classify the exact solutions of (3.3). For a better interpretations of obtained results, we can plot two and three dimensional figures of analytical solutions by considering suitable values of parameters.

4. Application of the IBSEFM method

In this section, the application of the IBSEFM to the conformable time-fractional SRLW equation is presented. Consider the following wave transform

$$u(x,t) = U(\eta), \ \eta = a\left(x - v\frac{t^{\alpha}}{\alpha}\right), \tag{4.1}$$

where a is the wave height and v is the wave velocity.

Substituting (4.1) into (1.1) and integrating once respect to ξ, we get nonlinear ordinary differential equation;

$$\left(a^2v^2 + a^2p\right)U + rv^2a^4U'' - qva^2\frac{U^2}{2} = C, \tag{4.2}$$

where C is constant of integration and assume that $C = 0$ to reduce the complexity of the solutions. When we reconsider (4.2) for balance principle, considering between U^2 and U'', we obtain the following relationship:

$$2M + m = n + 2.$$

This relationship of m, n and M gives us different versions of the solutions to (4.2) and we can obtain some analytical solutions as follows:

If we take $M = 3, n = 5, m = 1$ for (3.4) and (3.5), then we can write the following equations;

$$U(\eta) = \frac{a_0 + a_1F(\eta) + a_2F^2(\eta) + a_3F^3(\eta) + a_4F^4(\eta) + a_5F^5(\eta)}{b_0 + b_1F(\eta)} = \frac{\Upsilon(\eta)}{\Psi(\eta)}, \tag{4.3}$$

$$U'(\eta) = \frac{\Upsilon'(\eta)\Psi(\eta) - \Upsilon(\eta)\Psi'(\eta)}{\Psi^2(\eta)}, \tag{4.4}$$

and

$$U''(\eta) = \frac{\Upsilon'(\eta)\Psi(\eta) - \Upsilon(\eta)\Psi'(\eta)}{\Psi^2(\eta)} - \frac{[\Upsilon(\eta)\Psi'(\eta)]'\Psi^2(\eta) - 2\Upsilon(\eta)[\Psi'(\eta)]^2\Psi(\eta)}{\Psi^4(\eta)}, \tag{4.5}$$

where $F' = \sigma F + dF^3$, $a_2 \neq 0, b_1 \neq 0, \sigma \neq 0, d \neq 0$. When we use (4.3)-(4.5) in (4.2), we obtain a system of algebraic equations from coefficients of polynomial of F. By solving the algebraic system of equations with the help of mathematics software programme, it yields us the following coefficients:

Case 1:

$$\begin{aligned} a_0 &= 0,\ a_1 = 0,\ a_2 = \frac{\sigma a_4}{d},\ a_3 = \frac{\sigma a_5}{d}, a_4 = a_4, a_5 = a_5, \\ b_0 &= \frac{q\left(-1 + \sqrt{1 - 16a^2 pr\sigma^2}\right) a_4}{96a^2 d^2 pr},\ b_1 = -\frac{q\left(-1 + \sqrt{1 - 16a^2 pr\sigma^2}\right) a_5}{96a^2 d^2 pr}, \\ v &= -\frac{1 + \sqrt{1 - 16a^2 pr\sigma^2}}{8a^2 r\sigma^2}. \end{aligned}$$

Substituting these coefficients along with (3.6) in (4.3) we obtain the following solution of the conformable time fractional SRLW equation;

$$u_1(x,t) = -\frac{6d \exp\left\{2\sigma\left(ax + \frac{t^\alpha(1+\sqrt{1-16a^2 pr\sigma^2})}{8ar\alpha\sigma^2}\right)\right\} E\sigma\left(1 + \sqrt{1 - 16a^2 pr\sigma^2}\right)}{qd\left(\exp\left\{2\sigma\left(ax + \frac{t^\alpha(1+\sqrt{1-16a^2 pr\sigma^2})}{8ar\alpha\sigma^2}\right)\right\} - E\sigma\right)^2},$$

where $d, \sigma, a, r, E, q, d, \alpha$ are constants and not zero (Figure 1).

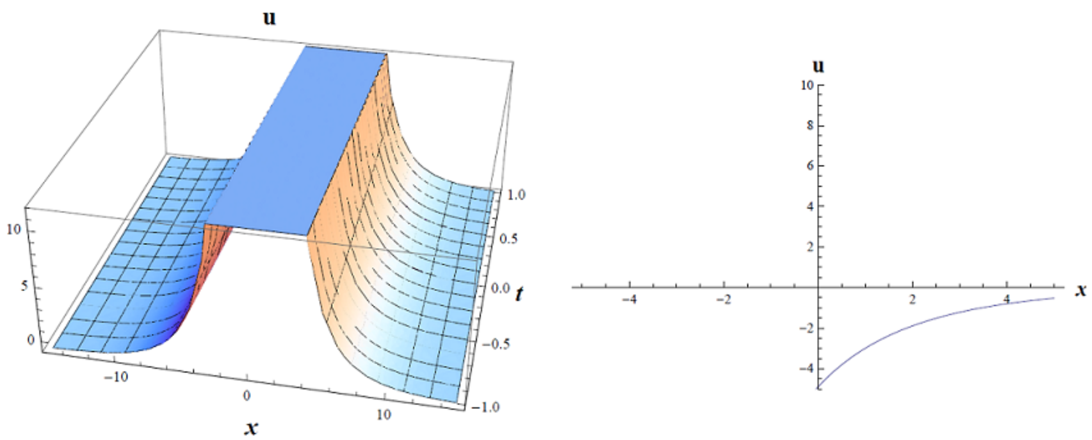

Figure 1. The 3D and 2D surfaces of $u_1(x,t)$. By considering the values $r = 0.1$; $d = 0.2$; $p = 0.5$; $a = 0.2$; $v = 0.3$; $\sigma = 1$; $\alpha = 1$; $q = 0.4$; $E = 0.3$; $-15 < x < 15$, $-1 < t < 1$ for 3D surface and $-5 < x < 5$; $t = 0.1$ for 2D surface.

Case 2:

$$\begin{aligned} a_0 &= \frac{(p+v)a_4}{24a^2 d^2 rv^2},\ a_1 = \frac{(p+v)a_5}{24a^2 d^2 rv^2},\ a_2 = \frac{\sqrt{p+v}a_4}{2ad\sqrt{rv}},\ a_3 = \frac{\sqrt{p+v}a_5}{2ad\sqrt{rv}}, \\ a_4 &= a_4,\ a_5 = a_5, b_0 = \frac{qa_4}{48a^2 d^2 rv},\ b_1 = \frac{qa_5}{48a^2 d^2 rv},\ \sigma = \frac{\sqrt{p+v}}{2a\sqrt{rv}}. \end{aligned}$$

Substituting these coefficients along with (3.6) in (4.3) we obtain following solution of the conformable time fractional SRLW equation;

$$u_2(x,t) = \frac{2\left(p + v\left(1 + \frac{12ad\exp\left\{\frac{\sqrt{p+v}(t^{\alpha}v - xa)}{\sqrt{r}va}\right\}\sqrt{r}(p+v)^{\frac{3}{2}}E}{\left(-2ad\sqrt{r}v + \exp\left\{\frac{\sqrt{p+v}(t^{\alpha}v - xa)}{\sqrt{r}va}\right\}\sqrt{p+v}E\right)^2}\right)\right)}{qv},$$

where $a, p, E, q, d, v, r, \alpha$ are constants and not zero (Figure 2).

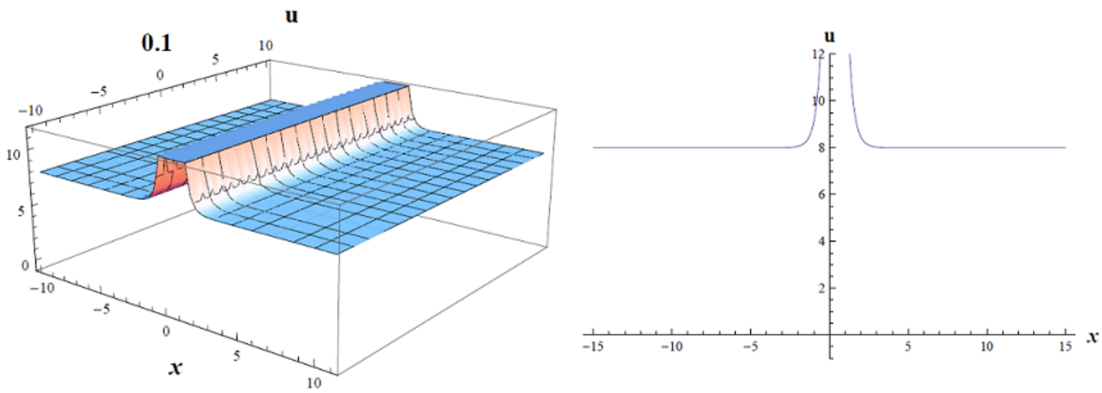

Figure 2. The 3D and 2D surfaces of $u_2(x,t)$. By considering the values $r = 0.5$; $d = 0.3$; $p = 0.2$; $a = 1$; $v = 0.5$; $\sigma = 1$; $\alpha = 1$; $q = 0.5$; $E = 0.5$; $-10 < x < 10$, $-10 < t < 10$ for 3D surface and $-15 < x < 15$; $t = 0.1$ for 2D surface.

Case 3:

$$a_0 = \frac{\sigma^2 a_4}{6d^2}, a_1 = \frac{\sigma^2 a_5}{6d^2}, a_2 = \frac{\sigma a_4}{d}, a_3 = \frac{\sigma a_5}{d},\ b_1 = \frac{a_5 b_0}{a_4}, r = \frac{a_4 q}{48a^2 d^2 v b_0}, p = -v + \frac{qv\sigma^2 a_4}{12d^2 b_0}.$$

Substituting these coefficients along with (3.6) in (4.3) we obtain following solution of the conformable time fractional SRLW equation;

$$u_3(x,t) = \frac{\left(\sigma^2 + \frac{6E^2\sigma^4}{\left(de^{2a\left(x - \frac{t^{\alpha}v}{\alpha}\right)\sigma} - E\sigma\right)^2} + \frac{6E\sigma^3}{\left(de^{2a\left(x - \frac{t^{\alpha}v}{\alpha}\right)\sigma} - E\sigma\right)^2}\right)a_4}{6d^2 b_0},$$

where $a, E, d, b_0, a_4, v, \sigma, \alpha$ are constants and not zero (Figures 3–7).

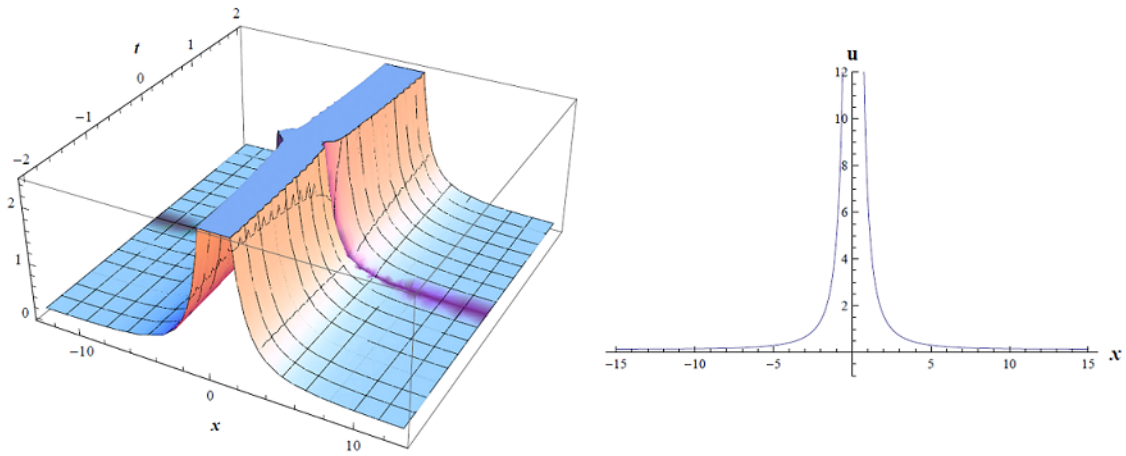

Figure 3. The 3D and 2D surfaces of $u_3(x,t)$ for $\alpha = 0.2$. By considering the values $\alpha = 0.2$; $E = 0.5$; $\sigma = 0.4$; $d = 0.3$; $a = 0.5$; $b_0 = 1$; $a_4 = 0.5$; $v = 0.3$; $-13 < x < 13$, $-2 < t < 2$ for 3D surface and $-15 < x < 15$; $t = 0.2$ for 2D surface.

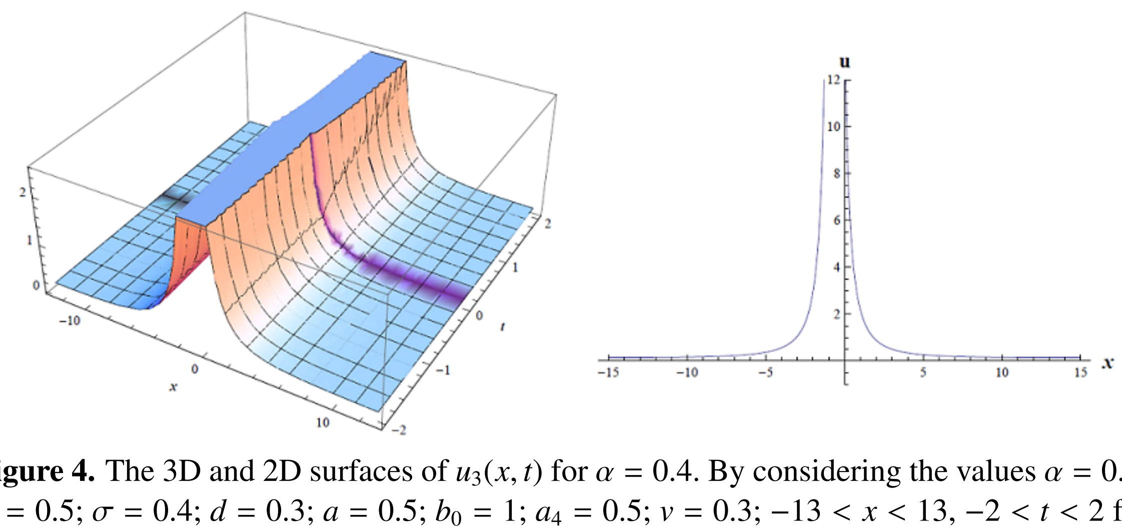

Figure 4. The 3D and 2D surfaces of $u_3(x,t)$ for $\alpha = 0.4$. By considering the values $\alpha = 0.4$; $E = 0.5$; $\sigma = 0.4$; $d = 0.3$; $a = 0.5$; $b_0 = 1$; $a_4 = 0.5$; $v = 0.3$; $-13 < x < 13$, $-2 < t < 2$ for 3D surface and $-15 < x < 15$; $t = 0.2$ for 2D surface.

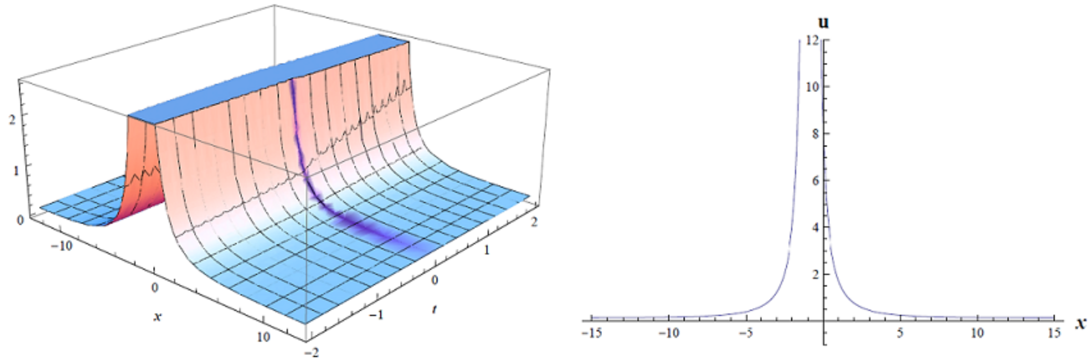

Figure 5. The 3D and 2D surfaces of $u_3(x, t)$ for $\alpha = 0.6$. By considering the values $\alpha = 0.6$; $E = 0.5$; $\sigma = 0.4$; $d = 0.3$; $a = 0.5$; $b_0 = 1$; $a_4 = 0.5$; $v = 0.3$; $-13 < x < 13$, $-2 < t < 2$ for 3D surface and $-15 < x < 15$; $t = 0.2$ for 2D surface.

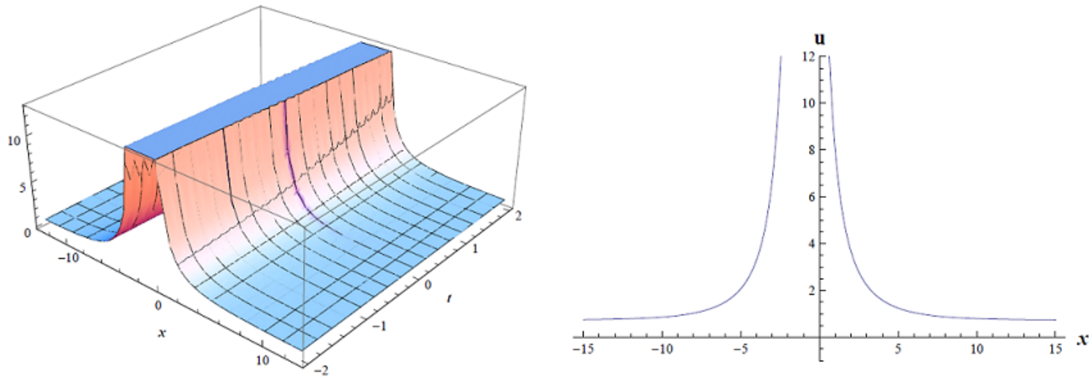

Figure 6. The 3D and 2D surfaces of $u_3(x, t)$ for $\alpha = 0.8$. By considering the values $\alpha = 0.8$; $E = 0.5$; $\sigma = 0.4$; $d = 0.3$; $a = 0.5$; $b_0 = 1$; $a_4 = 0.5$; $v = 0.3$; $-13 < x < 13$, $-2 < t < 2$ for 3D surface and $-15 < x < 15$; $t = 0.2$ for 2D surface.

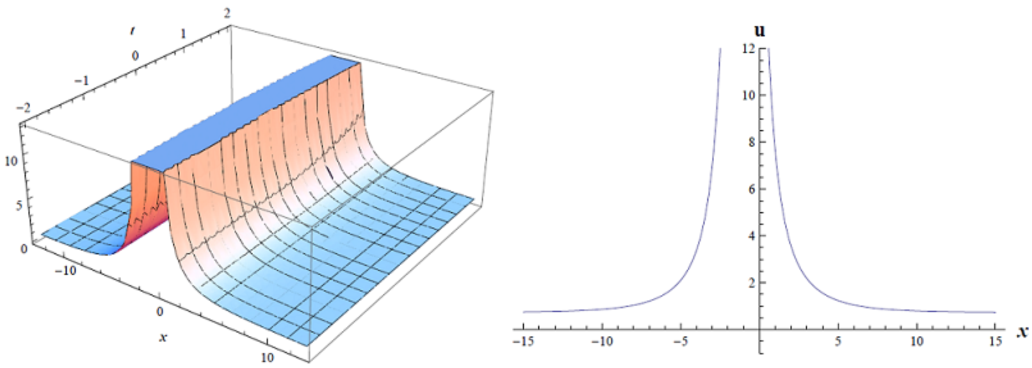

Figure 7. The 3D and 2D surfaces of $u_3(x, t)$ for $\alpha = 1$. By considering the values $\alpha = 1$; $E = 0.5$; $\sigma = 0.4$; $d = 0.3$; $a = 0.5$; $b_0 = 1$; $a_4 = 0.5$; $v = 0.3$; $-13 < x < 13$, $-2 < t < 2$ for 3D surface and $-15 < x < 15$; $t = 0.2$ for 2D surface.

5. Conclusion and Discussion

In this article, we have successfully applied the IBSEFM to the nonlinear conformable time-fractional SRLW equation to investigate some new exact solutions. It has been observed that all analytical solutions examined in this paper verify the nonlinear ordinary differential equation (4.2) which is obtained from nonlinear comformable time-fractional SRLW equation under the terms of wave transformation. All necessary computational calculations and graphs have been acquired by using software mathematics programme. According to the figures, one can see that the formats of travelling wave solutions in two and three dimensional surfaces are similar to the physical meaning of results. If we take more values of coefficients, we can obtain more travelling wave solutions for this model. This method is very reliable, efficient and submits new travelling wave solutions. Therefore, the IBSEFM can be applied to the other nonlinear fractional differential models.

Conflict of interest

All authors declare no conflicts of interest in this paper.

References

1. F. Maucher, D. Buccoliero, S. Skupin, et al. *Tracking azimuthons in nonlocal nonlinear media*, Opt. Quant. Electron., **41** (2009), 337–348.
2. M. Alidou, A. Kenfack-Jiotsa, T. C. Kofane, *Modulational instability and spatiotemporal transition to chaos*, Chaos, Solitons Fractals, **27** (2006), 914–925.
3. R. Nath, P. Pedri, L. Santos, *Stability of Dark Solitons in Three Dimensional Dipolar Bose-Einstein Condensates*, Phys. Rev. Lett., **101** (2008), 210402.
4. M. G. Prahović, R. D. Hazeltine, P. J. Morrison, *Exact solutions for a system of nonlinear plasma fluid equations*, Physics of Fluids B: Plasma Physics, **4** (1992), 831–840.
5. P. B. Ndjoko, J. M. Bilbault, S. Binzcak, et al. *Compact-envelope bright solitary wave in a DNA double strand*, Phys. Rev. E, **85** (2012).
6. S. B. Yamgoue, F. B. Pelap, *Comment on Compact envelope dark solitary wave in a discrete nonlinear electrical transmission line*, Phys. Lett. A., **380** (2016), 2017–2020.
7. G. R. Deffo, S. B. Yamgoue, F. B. Pelap, *Modulational instability and peak solitary wave in a discrete nonlinear electrical transmission line described by the modified extended nonlinear Schrodinger equation*, The Eurupean Phys. J., **91** (2018).
8. E. Kengne, R. Vaillancourt, *Transmission of solitary pulse in dissipative nonlinear transmission lines*, Communications in Nonlinear Science and Numerical Simulation, **14** (2009), 3804–3810.
9. E. Fan, Y. C. Hon, *Applications of extended tanh method to special types of nonlinear equations,* Appl. Math. Comput., **141** (2003), 351–358.
10. A. M. Wazwaz, *The extended tanh method for abundant solitary wave solutions of nonlinear wave equations,* Appl. Math. Comput., **187** (2007), 1131–1142.
11. Z. Feng, *On explicit exact solutions to the compound Burgers-Kdv equation*, Phys. Lett., **293** (2002), 57–66.

12. K. Hosseini, P. Gholamin, *Feng's first integral method for analytic treatment of two higher dimensional nonlinear partial differantial equations,* Differantial Equations and Dynamical Systems, **23** (2015), 317–325.
13. A. M. Wazwaz, *Sine-cosine method for handling nonlinear wave equations,* Math. Comput. Model., **40** (2004), 499–508.
14. Y. Fu, J. Li, *Exact stationary -wave solutions in the standart model of the Kerr-nonlinear optical fiber with the Bragggrating*, J. Appl. Anal. Comput., **7** (2017), 1177–1184.
15. N. Tanghizadeh, M. Mirzazadeh, A. S. Paghaleh, et al. *Exact solutions of nonlinear evolution equations by using the modified simple equation method*, Ain Shams Eng. J., **3** (2012), 321–325.
16. M. Mirzazadeh, *Modified simple equation method and its applications to nonlinear partial differantial equations*, Inf. Sci. Lett., **3** (2014), 1–9.
17. A. J. M. Jawad, *Soliton solutions for nonlinear systems (2+1) dimensional equations,* IOSR Journal of Mathematics, **1** (2012), 27–34.
18. I. E. Inan, D. Kaya, *Exact solutions of some nonlinear partial differential equations*, Physica A, **381** (2007), 104–115.
19. M. A. Akbar, N. H. M. Ali, *The improved F-expansion method with Riccati equation and its applications in mathematical physics,* Cogent Mathematics, **4** (2017), 1–19.
20. Y. M. Zhao, *F-expansion method and its application for finding new exact solutions to the Kudryashov– Sinelshchikov equation*, J. Appl. Math., **2013** (2013).
21. F. Ozpinar, H. M. Baskonus, H. Bulut, *On the complex and hyperbolic structures for the (2+1)-dimensional boussinesq water equation,* Entropy, **17** (2015), 8267–8277.
22. H. M. Baskonus, M. Askin, *Travelling Wave Simulations to the Modified Zakharov-Kuzentsov Model Arising In Plasma Physics*, 6th International Youth Science Forum, Computer Science and Engineering, (2016) Lviv, Ukraine, 24-26 November.
23. H. M. Baskonus, H. Bulut, *Exponential prototype structure for (2+1) dimensional Boiti Leon Pempinelli systems in mathematical physics*, Waves Random Complex Media, **26** (2016), 189–196.
24. F. Dusunceli, *Solutions for the Drinfeld-Sokolov Equation Using an IBSEFM Method, MSU Journal of Science,* J. Amer. Math. Soc., **6** (2018), 505–510.
25. W. Liu and K. Chen, *The functional variable method for finding exact solutions of some nonlinear timefractional differential equations*, Pramana, **81** (2013), 377–384.
26. B. Lu, *The first integral method for some time fractional diferential equations*, J. Math. Anal. Appl., **395** (2012), 684–693.
27. Z. Bin, *Exp-function method for solving fractional partial dierential equations*, The Sci. World J., **2013** (2013), 1–8.
28. A. Emad, B. Abdel-Salam, A. Y. Eltayeb, *Solution of nonlinear space-time fractional diffeerential equations using the fractional Riccati expansion method*, Math. Probl. Eng., **2013** (2013), 1–6.
29. H. Bulut, G. Yel, H. M. Baskonus, *An Application of Improved Bernoulli Sub-Equation Function Method to The Nonlinear Time-Fractional Burgers Equation*, Turk. J. Math. Comput. Sci., **5** (2016), 1–7.
30. D. G. Prakasha, P. Veeresha, H. M. Baskonus, *Analysis of the dynamics of hepatitis E virus using the Atangana-Baleanu fractional derivative,* The European Physical Journal Plus, **134** (2019), 241.
31. R. Khalil, M. Al Horani, A. Yousef, et al. *A new definition of fractional derivative*, J. Comput. Appl. Math., **264** (2014), 65–70.

32. T. Abdeljawad, *On conformable fractional calulus,* J. Comput. Appl. Math., **279** (2015), 57–66.
33. A. Atangana, D. Baleanu, A. Alsaedi, *New properties of conformable derivative*, Open Math., **13** (2015), 1–10.
34. A. Atangana, *A novel model for the lassa hemorrhagic fever, deathly disease for pregnant women*, Neural. Comput. Appl., **26** (2015), 1895–1903.
35. A. Korkmaz, K. Hosseini, *Exact solutions of a nonlinear conformable time fractional parabolic equation with exponential nonlinearity using reliable methods*, Opt. Quant. Electron., **49** (2017), 278.
36. A. Korkmaz, *Exact solutions of space-time fractional EW and modified EW equations*, Chaos, Solitons Fractals, **96** (2017), 132–138.
37. A. Korkmaz, *Exact solutions to (3 + 1) conformable time fractional Jimbo-Miwa, Zakharov-Kuznetsov and modified Zakharov-Kuznetsov equations*, Commun. Theor. Phys., **67** (2017), 479–482.
38. K. Hosseini, R. Ansari, *New exact solutions of nonlinear conformable timefractional Boussinesq equations using the modified Kudryashov method*, Waves Random Complex Media, **27** (2017), 628–636.
39. D. Kumar, J. Singh, D. Baleanu, *Analysis of regularized long-wave equation associated with a new fractional operator with Mittag-Leffler type kernel*, Physica A, **492** (2018), 155–167.
40. D. Kumar, J. Singh, D. Baleanu, *A new analysis for fractional model of regularized long wave equation arising in ion acoustic plasma waves,* Math. Methods Appl. Sci., **40** (2017), 5642–5653.
41. T. A. Sulaiman, M. Yavuz, H. Bulut, et al. *Investigation of the fractional coupled viscous Burgers' equation involving Mittag-Leffler kernel*, Physica A: Statistical Mechanics and its Applications, **527** (2019), 121–126.
42. H. Rezazadeh, D. Kumar, T. A. Sulaiman, et al. *New complex hyperbolic and trigonometric solutions for the generalized conformable fractional Gardner equation*, Mod. Phys. Lett. B, **33** (2019), 1950196.
43. H. Rezazadeh, A. Korkmaz, M. Eslami, et al. *Traveling wave solution of conformable fractional generalized reaction Duffing model by generalized projective Riccati equation method,* Opt. Quant. Electron., **50** (2018), 150.
44. G. Yel, H. M. Baskonus, *Solitons in conformable time-fractional Wu–Zhang system arising in coastal design,* Pramana, **93** (2019), 57.
45. C. E. Seyler, D. L. Fenstermacher, *A symmetric regularized-long-wave equation,* Phys. Fluids, **27** (1984), 4–7.
46. R. I. Nuruddeen, A. M. Nass, *Exact solitary wave solution for the fractional and classical GEW-Burgers equations: an application of Kudryashov method*, Journal of Taibah University for Science, **12** (2018), 309–314.
47. K. K. Ali, R. I. Nuruddeen, K. R. Raslan, *New structures for the space-time fractional simplified MCH and SRLW equations*, Chaos Soliton. Fractal., **106** (2018), 304–309.

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