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Research article

Optical solitons for Triki-Biswas equation by two analytic approaches

Aliyu Isa Aliyu[1], **Ali S. Alshomrani**[2], **Mustafa Inc**[3,*] **and Dumitru Baleanu**[4,5]

[1] Department of Mathematics, Sun Yat-sen University, Guangzhou 510275, P. R. China
[2] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[3] Department of Mathematics, Firat University, 23119 Elazig, Turkey
[4] Department of Mathematics, Cankaya University, Ankara, Turkey
[5] Institute of Space Sciences, Magurele, Romania

* **Correspondence:** Email: minc@firat.edu.tr; Tel: +905468036564.

Abstract: The present study is devoted to using two analytic approaches to study the Triki-Biswas equation (TBE). The TBE model plays a vital role in propagation of short pulses of width around regions of sub-10 fs in optical. The analytic approaches used are the sine-Gordon expansion (SGEM) and the Riccatti Bernoulli sub-ODE (RBSO) methods. Chirped kink-type, bright envelope and singular solitons are formally derived.

Keywords: TBE; Sine-Gordon expansion method; Riccatti Bernoulli sub-ODE method; optical soliton; chirped kink-type; bright solitons
Mathematics Subject Classification: 35C08, 35C15, 32W50

1. Introduction

A lot of nonlinear wave propagations in physics are described by a Schrödinger equations. The Schrödinger equations appear in different fields of physical, biological and engineering sciences. Several important concepts, e.g. processing, control acoustics, electro-magnetic, electro-chemistry are very much described by Schrödinger equations. To understanding these nonlinear terminologies, mathematicians and physicists have made a giant effort to find out more about the behavior of the models, especially their solutions. Therefore, several powerful integration approaches have utilized to study many equations [1–27]. In this paper, we will consider the following TBE [3–6]:

$$i\psi_t + \alpha\psi_{xx} + i\beta(|\psi|^{2n}\psi)_x = 0, \tag{1.1}$$

where ψ is the variable representing the wave profile, ψ_{xx} represents the group velocity dispersion (GVD) and alpha is the coefficient of GVD, $(|\psi|^{2n}\psi)_x$ is the term representing the non-Kerr dispersion (NKD) for $n > 2$ and β is the coefficient of NKD. In the event when $n = 1$, (1.1) transforms to the well known Kaup-Newell equation [7]. (1.1) has been solved by several authors using different methods and the results have been reported in [3–6].

In the current study, we aim to apply the SGEM [14] and the RBSO [15] to investigate the chirped envelope solitons of (1.1) which includes the bright, kink-type and singular solitons.

2. Mathematical analysis sand derivation of chirped parameters

Our aim is to acquire a solution in the following form

$$\psi(x,t) = R(\xi)e^{i(\phi(\xi)-\omega t)}, \quad \xi = x - vt \tag{2.1}$$

where ω is the frequency and v represent the velocity. The chirp is represented by

$$\delta g(x,t) = -\frac{\partial}{\partial x}[\phi(\xi) - \omega t] = \phi'(\xi). \tag{2.2}$$

Putting (2.1) into (1.1) and separating the result into real and imaginary components, we obtain the following equations

$$gR + v\phi' R + \alpha R'' - \alpha R(\phi')^2 - \beta\phi' R^{2n+1} = 0, \tag{2.3}$$

and

$$\alpha R\phi'' + 2\alpha R'\phi' - vR' + \beta(2n+1)R^{2n}R' = 0. \tag{2.4}$$

To solve the above two equations, we apply the following ansatz solution

$$R' = \eta_1 R^{2n} + \eta_2, \tag{2.5}$$

where η_1 is a constant and η_2 is the nonlinear chirp parameter. Thus, we obtain

$$\delta g(x,t) = -(\eta_1 R^{2n} + \eta_2). \tag{2.6}$$

On inserting (2.5) into (2.4), we obtain the chirped parameters given by

$$\eta_1 = -\frac{\beta(2n+1)}{2\alpha(n+1)}, \quad \eta_2 = \frac{v}{2\alpha}. \tag{2.7}$$

Putting (2.5) into (2.3), we acquire

$$R'' + b_1 R^{4n+1} + b_2 R^{2n+1} + b_3 R = 0, \tag{2.8}$$

where $b_1 = \frac{\beta^2(2n+1)}{4\alpha^2(n+1)^2}, \quad b_2 = \frac{v\beta}{2\alpha^2}, \quad b_3 = \frac{4g\alpha+v^2}{4\alpha^2}$.

Equation (2.8) is an elliptic equation describing the dynamics of field amplitude in the concept of nonlinear media. Thus, (2.8) can be written in another form as

$$(R')^2 + \frac{b_1}{2n+1}R^{4n+1} + \frac{b_2}{n+1}R^{2n+1} + b_3 R^2. \tag{2.9}$$

Applying the following transformation

$$R(\xi) = U(\xi)^{\frac{1}{2}} \tag{2.10}$$

(2.9) reduces to

$$U'' + \delta U + \sigma U^{n+1} + \gamma U^{2n+1} = 0, \tag{2.11}$$

where
$\delta = 4b_3$, $\sigma = \frac{2b_2(n+2)}{n+1}$, $\gamma = \frac{4b_1(n+1)}{2n+1}$. To derive the solutions of (2.11), we utilize the following change of variable

$$U(\xi) = u(\xi)^{\frac{1}{n}} \tag{2.12}$$

to reduce (2.11) to

$$n^2\delta u^2 + n^2\sigma u^3 + n^2\gamma u^4 + (1-n)u'^2 + n\mathrm{uu}'' = 0. \tag{2.13}$$

3. Description of methods and derivation of optical solitons

3.1. Optical solitons by SGEM

Consider the following partial differential equation (PDE) represented by

$$P(\psi, \psi_t, \psi_x, \psi_{tt}, \psi_{xx}, \psi_{xt}, ...) = 0. \tag{3.1}$$

Applying the transformation

$$\psi(x,t) = u(\xi), \quad \xi = x - vt. \tag{3.2}$$

(3.1) can be reduced to the following ordinary differential equation (ODE) ODE

$$P(u, u', u'', ...) = 0. \tag{3.3}$$

Consider the following ODE derived from the integration of a Sine-Gordon equation ([8,9])

$$y'(\xi) = \sin(y(\xi)). \tag{3.4}$$

(3.4) possesses the following solutions

$$\sin(y(\xi)) = \mathrm{sech}(\xi) \quad \text{or} \quad \cos(y(\xi)) = \tanh(\xi), \tag{3.5}$$

and

$$\sin(y(\xi)) = i\mathrm{csch}(\xi) \quad \text{or} \quad \cos(y(\xi)) = \coth(\xi). \tag{3.6}$$

We use the following ansatz to retrieve the solutions of (3.3):

$$u(\xi) = \sum_{j=1}^{n} \cos^{j-1}(y) \times \Big[B_j \sin(y) + A_j \cos(y)\Big] + A_0. \tag{3.7}$$

n in (3.7) is derived using the balancing formulae. Inserting n into (3.7) and performing all necessary algebraic computations, the solution of (3.2) can be derived and subsequently, the solutions of (3.1).

Now we Apply the SGEM to study (2.13), Balancing u^4 and uu'' from (2.13), we obtain $n = 1$. Inserting $n = 1$ into (2.13), we acquire

$$u = B_1 \sin(y) + A_1 \cos(y) + A_0. \tag{3.8}$$

Inserting (3.8) into (2.13) using (3.4), we acquire the following system of equations upon collecting terms of similar coefficients:

constants:

$$n^2 A_0^2 \left(\delta + \sigma A_0 + \gamma A_0^2\right) = 0, \tag{3.9}$$

$\mathbf{cos}^2(y)$ **:**

$$n^2\, A_1^2 \left(\delta + 3\sigma A_0 + 6\gamma A_0^2 + \gamma A_1^2\right) = 0, \tag{3.10}$$

$\mathbf{cos}(y)$ **:**

$$n^2\, A_1 \left(3\sigma A_0^2 + 4\gamma A_0^3 + \sigma A_1^2 + 2A_0 \left(\delta + 2\gamma A_1^2\right)\right) = 0, \tag{3.11}$$

$\mathbf{sin}(y)\mathbf{cos}(y)$ **:**

$$2n^2\, \left(\delta + 3\sigma A_0 + 6\gamma A_0^2\right) A_1 B_1 = 0, \tag{3.12}$$

$\mathbf{cos}^3(y)\mathbf{sin}(y)$ **:**

$$n\, A_1 \left(1 + 4n\gamma A_1^2\right) B_1 = 0, \tag{3.13}$$

$\mathbf{sin}^2(y)$**:**

$$A_1 B_1 \left(-2 - n + 4n^2 \gamma B_1^2\right) = 0, \tag{3.14}$$

$\mathbf{cos}(y)\mathbf{sin}^3(y)$ **:**

$$-n\, A_1 \left(n\sigma \left(A_1^2 - 3B_1^2\right) + 2A_0 \left(1 + 2n\gamma A_1^2 - 6n\gamma B_1^2\right)\right) = 0, \tag{3.15}$$

$\mathbf{cos}(y)\mathbf{sin}^2(y)$ **:**

$$n\, B_1 \left(n\sigma \left(3A_1^2 - B_1^2\right) + 2A_0 \left(1 + 6n\gamma A_1^2 - 2n\gamma B_1^2\right)\right) = 0, \tag{3.16}$$

$\mathbf{cos}^2(y)\mathbf{sin}(y)$ **:**

$$\left(-(-1 + n)A_1^2 + nB_1^2 \left(-1 + n\delta + 3n\sigma A_0 + 6n\gamma A_0^2 + n\gamma B_1^2\right)\right) = 0, \tag{3.17}$$

sin(y)**:**

$$n\,B_1\left(3n\sigma A_0^2+4n\gamma A_0^3+n\sigma B_1^2+A_0\left(-1+2n\delta+4n\gamma B_1^2\right)\right)=0, \tag{3.18}$$

sin$^2(y)$**cos**$^2(y)$**:**

$$\left(-n^2\gamma A_1^4+B_1^2\left(1+n-n^2\gamma B_1^2\right)+A_1^2\left(-1-n+6n^2\gamma B_1^2\right)\right)=0. \tag{3.19}$$

Solution of equations (23)- (3.19) gives the following family

Family 1:

$$\begin{aligned}A_1=0, A_0=\frac{\delta(-19+8\delta)(-1+8\delta)\sigma}{270\gamma}, B_1=\sqrt{\frac{\delta(-1+8\delta)}{15\gamma}},\\ \delta=\frac{5}{4},\gamma=\frac{3\sigma^2}{16},n=\frac{1}{3}(11-4\delta),\end{aligned} \tag{3.20}$$

Family 2:

$$\sigma=\frac{6}{A_1},\delta=-4,\gamma=-\frac{2}{A_1^2},n=1,B_1=0,A_0=A_1. \tag{3.21}$$

3.1.1. Bright optical soliton

Using (3.20), we retrieve the bright soliton represented by

$$\psi(x,t)=\left(-\frac{2}{\sigma}+\frac{2\text{sech}[x-vt]}{\sigma}\right)^{\frac{1}{2n}}\times e^{(\phi(x-vt)-\omega t)}. \tag{3.22}$$

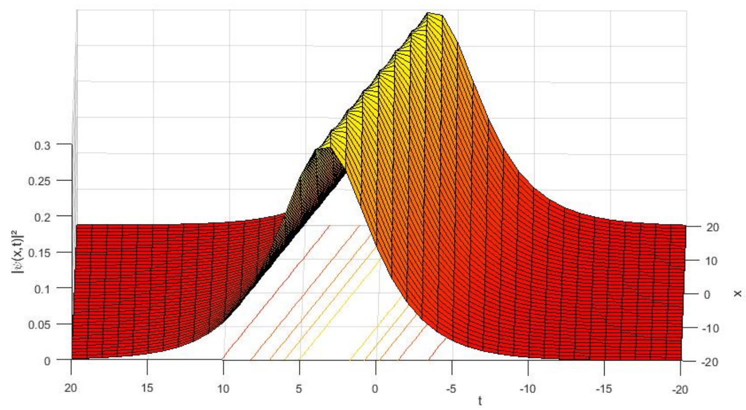

Figure 1. Surface profile of bright soliton (3.22) by setting $\sigma=0.1, n=3$.

3.1.2. Dark optical soliton

Using (3.20), we retrieve the dark optical soliton represented by

$$\psi(x,t) = \left(A_1 - A_1 \tanh[x - vt]\right)^{\frac{1}{2n}} \times e^{(\phi(x-vt)-\omega t)}. \tag{3.23}$$

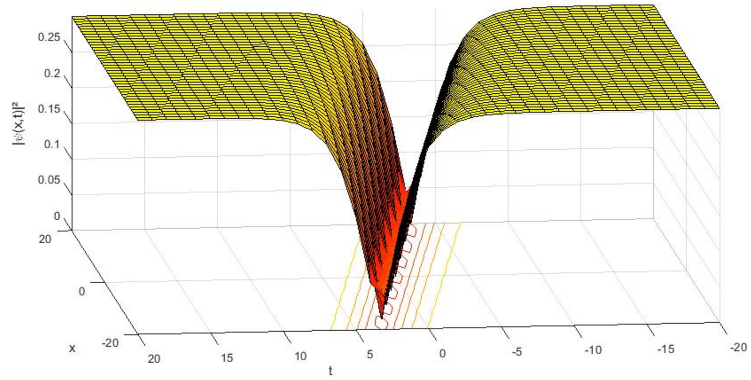

Figure 2. Surface profile of kink-type soliton (3.23) by setting $\sigma = 0.1, n = 3$.

3.1.3. Singular optical solitons

Using (3.20) gives the following singular soliton

$$\psi(x,t) = \left(-\frac{2}{\sigma} - \frac{2\text{csch}[x - vt]}{\sigma}\right)^{\frac{1}{2n}} e^{(\phi(x-vt)-\omega t)}. \tag{3.24}$$

while (3.21) gives the dark-singular soliton represented by

$$\psi(x,t) = \left(A_1 - A_1 \coth[x - vt]\right)^{\frac{1}{2n}} \times e^{(\phi(x-vt)-\omega t)}. \tag{3.25}$$

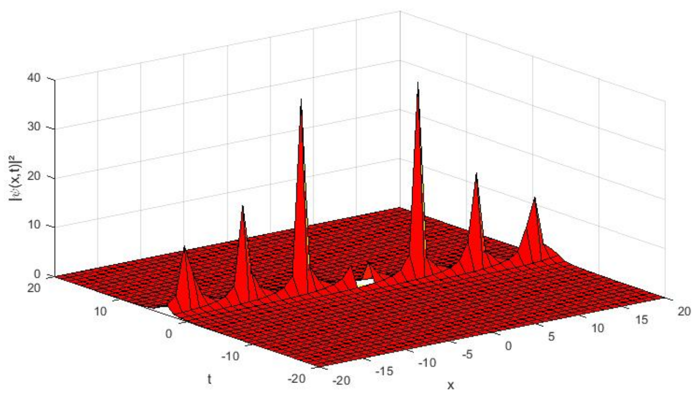

Figure 3. Surface profile of singular soliton (3.24) by setting $\sigma = 0.8, n = 3$.

3.2. Optical solitons by RBSOM

Suppose that (3.3) is the solution of the following Riccati-Bernoulli equation (RBE) [15]:

$$u' = bu + au^{2-m} + cu^{m}, \tag{3.26}$$

with a, b, c, and m being any constants. Differentiating (3.26) once, we get

$$u'' = u^{-1-2m}\left(au^2 + cu^{2m} + bu^{1+m}\right)\left(-a(-2+m)u^2 + cmu^{2m} + bu^{1+m}\right). \tag{3.27}$$

Putting (3.26) and (3.27) into (2.13), we obtain

$$\begin{aligned} n^2\delta u^2 + n^2\sigma u^3 + n^2\gamma u^4 + (1-n)\left(bu + au^{2-m} + cu^{m}\right)^2 + \\ nu^{-2m}\left(au^2 + cu^{2m} + bu^{1+m}\right)\left(-a(-2+m)u^2 + \right. \\ \left. cmu^{2m} + bu^{1+m}\right) = 0. \end{aligned} \tag{3.28}$$

Substituting $m = 0$ in (3.28) gives

$$\begin{aligned} c^2 - c^2n + 2bcu - bcnu + b^2u^2 + 2acu^2 + n^2\delta u^2 + \\ 2abu^3 + abnu^3 + n^2\sigma u^3 + a^2u^4 + a^2nu^4 + n^2\gamma u^4 = 0. \end{aligned} \tag{3.29}$$

Collecting all the coefficients of $u^i (i = 0, 1..., 4)$ and performing all the required algebraic calculations, we obtain the following independent set of parametric equations:

u^0 **coeff:**

$$-c^2(-1+n) = 0, \tag{3.30}$$

u^1 **coeff:**

$$-bc(-2+n) = 0, \tag{3.31}$$

u^2 **coeff:**

$$\left(b^2 + 2ac + n^2\delta\right) = 0, \tag{3.32}$$

u^3 **coeff:**

$$\left(ab(2+n) + n^2\sigma\right) = 0, \tag{3.33}$$

u^4 **coeff:**

$$\left(a^2(1+n) + n^2\gamma\right) = 0. \tag{3.34}$$

Solution of equations (44)-(48) gives the values of the constants represented by

$$n = 1, c = 0, \delta = -b^2, \gamma = -\frac{2\sigma^2}{9b^2}, a = -\frac{\sigma}{3b}. \tag{3.35}$$

From the solutions of the RBE (3.26) given in [15], we obtain the following kink-type and singular soliton solutions of (1.1) represented by

$$\psi(x,t)=\left(\frac{3b^2}{2\sigma}+\frac{3b^2\tanh\left[\frac{1}{2}b(C-tv+x)\right]}{2\sigma}\right)^{\frac{1}{2n}}\times e^{(\phi(x-vt)-\omega t)}, \tag{3.36}$$

$$\psi(x,t)=\left(\frac{3b^2}{2\sigma}+\frac{3b^2\coth\left[\frac{1}{2}b(C-tv+x)\right]}{2\sigma}\right)^{\frac{1}{2n}}\times e^{(\phi(x-vt)-\omega t)}. \tag{3.37}$$

4. Conclusions

The current work treated the systematic examination of the TBE. The SGEM and RBSO were utilized for fabricating the chirped bright, kink-type and singular solitons. Illustration of the physical behavior of solutions are displayed by assigning several values to the arbitrary constants, which might be significant for clarification. The reported solutions may have many applications in the fields of physics and various other branches of physical sciences. In Figures 1–3, we showed the properties of the acquired solutions of Eq. (1.1). The SGEM and RBSO and the can be applied to study other NPDEs in mathematical physics. Although, all the solutions derived by RBSO method were recovered by the SGEM, this made the SGEM a more powerful method than the RBSO.

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Conflict of interest

The authors declare that they have no conflict of interest.

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