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*Research article*

# Complexiton solutions and periodic-soliton solutions for the (2+1)-dimensional BLMP equation

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**Abstract:** The (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation is studied, which describes the incompressible fluid. By virtue of an ansätz functions and the bilinear form, many entirely new complexiton solutions and periodic-soliton solutions are derived. With the aid of symbolic computation, their dynamical behaviors are demonstrated in some three-dimensional plots by choosing different values of the parameters.

**Keywords:** bilinear form; complexiton solutions; incompressible fluid; dynamical behaviors; periodic solutions
**Mathematics Subject Classification:** 35C08, 68M07, 33F10

## 1. Introduction

The nonlinear evolution equations (NLEEs) can be used to describe many physical models [1–5]. The investigation on exact solutions and numerical solutions of NLEEs has become one of the most important areas in the study of nonlinear physical phenomena [6–9]. Via symbolic computation [10–17], many effective methods are presented [18–25].

In this paper, a (2+1)-dimensional Boiti-Leon-Manna-Pempinelli (BLMP) equation is investigated as [26–35]

$$u_{yt} + u_{xxxy} - 3\, u_x\, u_{xy} - 3\, u_{xx}\, u_y = 0, \tag{1.1}$$

where $u = u(x, y, t)$. Some exact solutions including kinky periodic solitary-wave solutions, periodic soliton solutions and kink solutions were discussed [26]. Bilinear form was presented via using the binary Bell polynomials [27]. The variable separable solutions and some novel localized excitations were obtained [28]. New solutions were derived via wronskian formalism and the Hirota method [29,30]. The periodic-soliton solutions are investigated [31] and so on [32–35]. But so far, complexiton solutions and double periodic-soliton solutions for Eq. (1) have not been obtained.

The organization of this paper is as follows. Section 2 obtains many new complexiton solutions and double periodic-soliton solutions based the bilinear form and an ansätz function, their dynamical behaviors are demonstrated in some three-dimensional plots by selecting different values of the parameters. Section 3 gives the conclusions.

## 2. Complexiton solutions and double periodic-soliton solutions

Under the transformation

$$u(x, y, t) = -2\,[\ln\Psi(x, y, t)]_x, \tag{2.1}$$

the Eq. (1) is transformed into the bilinear form

$$(D_yD_t + D_yD_x^3)\Psi\cdot\Psi = 0. \tag{2.2}$$

Eq. (3) is equivalent to

$$-\Psi_t\Psi_y - \Psi_{xxx}\Psi_y + 3\Psi_{xy}\Psi_{xx} - 3\Psi_x\Psi_{xxy} + \Psi\left(\Psi_{yt} + \Psi_{xxxy}\right) = 0. \tag{2.3}$$

Supposing Eq. (4) has the following form of solution:

$$\begin{aligned}\Psi &= e^{\Psi_1}[\Theta_1\cos(\Psi_2) + \Theta_2\sin(\Psi_2)] + k_1e^{2\Psi_1}\\ &+ e^{\Psi_3}[\Theta_3\cos(\Psi_4) + \Theta_4\sin(\Psi_4)] + k_2e^{\Psi_4},\end{aligned} \tag{2.4}$$

where $\Psi_i = \iota_i\,x + \rho_i\,y + \varsigma_i\,t, i = 1, 2, 3, 4$ and $\iota_i$, $\rho_i$ and $\varsigma_i$ are unknown constants. Substituting Eq. (5) into Eq. (4) and equating corresponding coefficients of $e^{\Psi_1}$, $e^{\Psi_3}$, $e^{\Psi_4}$, $\cos\Psi_2$, $\sin\Psi_2$, $\cos\Psi_4$, and $\sin\Psi_4$ to zero, a set of algebraic equations for $\iota_i$, $\rho_i$ and $\varsigma_i$ can be presented as follows

**Case (1)**

$$\begin{aligned}k_1 &= \rho_4 = \rho_2 = 0, \rho_1 = \rho_3, \varsigma_4 = \iota_4\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right),\\ \varsigma_2 &= \iota_2\left(-3\iota_1^2 + 12\iota_4\iota_1 + \iota_2^2 - 12\iota_4^2\right), \varsigma_3 = -\iota_3^3 + 15\iota_4^2\iota_3 - 20\iota_4^3,\\ \varsigma_1 &= -\iota_1^3 + 6\iota_4\iota_1^2 + 3\iota_2^2\iota_1 - 12\iota_4^2\iota_1 - 14\iota_4^3\\ &+ 24\iota_3\iota_4^2 - 6\iota_2^2\iota_4 - 6\iota_3^2\iota_4,\end{aligned} \tag{2.5}$$

$$\begin{aligned}\Psi &= e^{2\left(x\iota_4 + t\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right)\iota_4\right)}k_2 + e^{x\iota_1 + y\rho_3 + t\varsigma_1}[\cos[x\iota_2 + t(-3\iota_1^2\\ &+ 12\iota_4\iota_1 + \iota_2^2 - 12\iota_4^2)\iota_2]\Theta_1 + \sin[x\iota_2 + t(-3\iota_1^2 + 12\iota_4\iota_1 + \iota_2^2\end{aligned}$$

$$
\begin{aligned}
- &\quad 12\iota_4^2)\iota_2]\Theta_2] + e^{x\iota_3+t\left(-\iota_3^3+15\iota_4^2\iota_3-20\iota_4^3\right)+y\rho_3}[\cos[x\iota_4 + t(-3\iota_3^2 \\
+ &\quad 12\iota_4\iota_3 - 11\iota_4^2)\iota_4]\Theta_3 + \sin[x\iota_4 + t\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right)\iota_4]\Theta_4]. \qquad (2.6)
\end{aligned}
$$

$$
\begin{aligned}
u_1 &= -[2[2e^{2[x\iota_4+t\left(-3\iota_3^2+12\iota_4\iota_3-11\iota_4^2\right)\iota_4]}k_2\iota_4 + e^{x\iota_1+y\rho_3+t\varsigma_1}\iota_1[\cos[x\iota_2 \\
&+ t\left(-3\iota_1^2 + 12\iota_4\iota_1 + \iota_2^2 - 12\iota_4^2\right)\iota_2]\Theta_1 + \sin[x\iota_2 + t(-3\iota_1^2 + 12\iota_4\iota_1 \\
&+ \iota_2^2 - 12\iota_4^2)\iota_2]\Theta_2] + e^{x\iota_1+y\rho_3+t\varsigma_1}[\cos[x\iota_2 + t(-3\iota_1^2 + 12\iota_4\iota_1 + \iota_2^2 \\
&- 12\iota_4^2)\iota_2]\iota_2\Theta_2 - \sin[x\iota_2 + t\left(-3\iota_1^2 + 12\iota_4\iota_1 + \iota_2^2 - 12\iota_4^2\right)\iota_2]\iota_2\Theta_1] \\
&+ e^{x\iota_3+t\left(-\iota_3^3+15\iota_4^2\iota_3-20\iota_4^3\right)+y\rho_3}\iota_3[\cos[x\iota_4 + t(-3\iota_3^2 + 12\iota_4\iota_3 \\
&- 11\iota_4^2)\iota_4]\Theta_3 + \sin\left(x\iota_4 + t\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right)\iota_4\right)\Theta_4] \\
&+ e^{x\iota_3+t\left(-\iota_3^3+15\iota_4^2\iota_3-20\iota_4^3\right)+y\rho_3}[\cos[x\iota_4 + t\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right)\iota_4] \\
&* \iota_4\Theta_4 - \sin[x\iota_4 + t\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right)\iota_4]\iota_4\Theta_3]]] \\
&/ [e^{2[x\iota_4+t\left(-3\iota_3^2+12\iota_4\iota_3-11\iota_4^2\right)\iota_4]}k_2 + e^{x\iota_1+y\rho_3+t\varsigma_1}[\cos[x\iota_2 + t(-3\iota_1^2 \\
&+ 12\iota_4\iota_1 + \iota_2^2 - 12\iota_4^2)\iota_2]\Theta_1 + \sin[x\iota_2 + t(-3\iota_1^2 + 12\iota_4\iota_1 + \iota_2^2 \\
&- 12\iota_4^2)\iota_2]\Theta_2] + e^{x\iota_3+t\left(-\iota_3^3+15\iota_4^2\iota_3-20\iota_4^3\right)+y\rho_3}[\cos[x\iota_4 + t(-3\iota_3^2 + 12\iota_4 \\
&* \iota_3 - 11\iota_4^2)\iota_4]\Theta_3 + \sin[x\iota_4 + t\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right)\iota_4]\Theta_4]]. \qquad (2.7)
\end{aligned}
$$

The dynamical behavior to Eq. (8) is demonstrated in Figure 1.

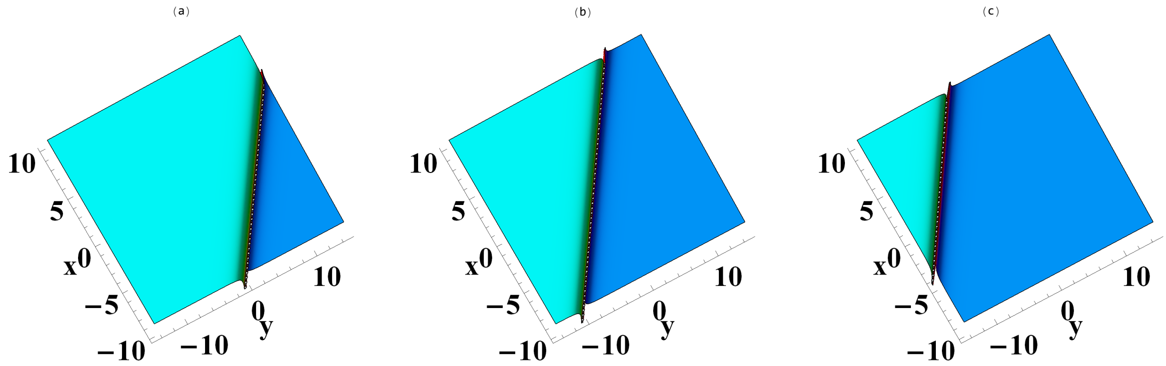


**Figure 1.** $\iota_1 = k_2 = \iota_3 = \Theta_2 = -1$, $\iota_2 = \iota_4 = 0$, $\Theta_1 = \rho_3 = \Theta_3 = \Theta_4 = 1$, (a) $t = -10$, (b) $t = 0$ and (c) $t = 10$.

**Case (2)**

$$
\begin{aligned}
k_1 &= \rho_4 = \iota_2 = 0, \varsigma_2 = 4\iota_2^3, \iota_1 = 2\iota_4, \varsigma_4 = \iota_4\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right), \\
\varsigma_1 &= -22\iota_4^3 + 24\iota_3\iota_4^2 - 6\iota_3^2\iota_4, \varsigma_3 = -\iota_3^3 + 15\iota_4^2\iota_3 - 20\iota_4^3, \qquad (2.8)
\end{aligned}
$$

$$
\begin{aligned}
\Psi &= e^{2[x\iota_4+t\left(-3\iota_3^2+12\iota_4\iota_3-11\iota_4^2\right)\iota_4]}k_2 + e^{2x\iota_4+t\left(-22\iota_4^3+24\iota_3\iota_4^2-6\iota_3^2\iota_4\right)+y\rho_1}[\cos \\
&\quad (y\rho_2)\Theta_1 + \sin(y\rho_2)\Theta_2] + e^{x\iota_3+t\left(-\iota_3^3+15\iota_4^2\iota_3-20\iota_4^3\right)+y\rho_3}[\cos[x\iota_4 + t(-3\iota_3^2 \\
&+ 12\iota_4\iota_3 - 11\iota_4^2)\iota_4]\Theta_3 + \sin[x\iota_4 + t\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right)\iota_4]\Theta_4]. \qquad (2.9)
\end{aligned}
$$

$$
\begin{aligned}
u_2 &= -[2[2e^{2[x\iota_4+t\left(-3\iota_3^2+12\iota_4\iota_3-11\iota_4^2\right)\iota_4]}k_2\iota_4 + 2[\cos(y\rho_2)\Theta_1 + \sin[y\rho_2]\Theta_2]\iota_4 \\
& e^{2x\iota_4+t\left(-22\iota_4^3+24\iota_3\iota_4^2-6\iota_3^2\iota_4\right)+y\rho_1} + e^{x\iota_3+t\left(-\iota_3^3+15\iota_4^2\iota_3-20\iota_4^3\right)+y\rho_3}\iota_3 \\
& [\cos[x\iota_4 + t\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right)\iota_4]\Theta_3 + \sin[x\iota_4 + t(-3\iota_3^2 \\
&+ 12\iota_4\iota_3 - 11\iota_4^2)\iota_4]\Theta_4] + e^{x\iota_3+t\left(-\iota_3^3+15\iota_4^2\iota_3-20\iota_4^3\right)+y\rho_3}[\cos[x\iota_4 \\
&+ t\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right)\iota_4]\iota_4\Theta_4 - \sin[x\iota_4 + t(-3\iota_3^2 + 12\iota_4\iota_3 \\
&- 11\iota_4^2)\iota_4]\iota_4\Theta_3]]]/[e^{2[x\iota_4+t\left(-3\iota_3^2+12\iota_4\iota_3-11\iota_4^2\right)\iota_4]}k_2 + [\cos(y\rho_2)\Theta_1 \\
&+ \sin(y\rho_2)\Theta_2]e^{2x\iota_4+t\left(-22\iota_4^3+24\iota_3\iota_4^2-6\iota_3^2\iota_4\right)+y\rho_1} + [\cos[x\iota_4 + t(-3\iota_3^2 \\
&+ 12\iota_4\iota_3 - 11\iota_4^2)\iota_4]\Theta_3 + \sin[x\iota_4 + t\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right)\iota_4]\Theta_4]] \\
&* e^{x\iota_3+t\left(-\iota_3^3+15\iota_4^2\iota_3-20\iota_4^3\right)+y\rho_3}. \qquad (2.10)
\end{aligned}
$$

The dynamical behavior to Eq. (11) is demonstrated in Figure 2.

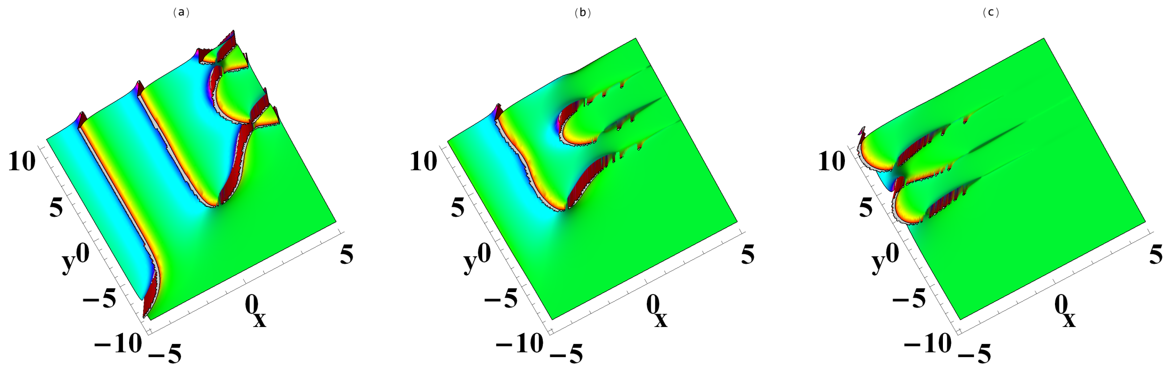


**Figure 2.** $\Theta_3 = \Theta_2 = -1$, $k_2 = 2$, $\Theta_1 = \iota_3 = \iota_4 = \rho_1 = \rho_2 = \rho_3 = \Theta_4 = 1$, (a) $t = -2$, (b) $t = 0$ and (c) $t = 2$.

**Case (3)**

$$
\begin{aligned}
k_1 &= \rho_2 = \iota_4 = 0, \varsigma_2 = \iota_2\left(-3\iota_1^2 + 6\iota_3\iota_1 + \iota_2^2 - 3\iota_3^2\right), \rho_1 = 2\rho_4, \varsigma_4 = 4\iota_4^3, \\
\varsigma_1 &= [-\Theta_2\iota_1^3 + 3(\iota_2\Theta_1 + \iota_3\Theta_2)\iota_1^2 + 3\left(\left(\iota_2^2 - \iota_3^2\right)\Theta_2 - 2\iota_2\iota_3\Theta_1\right)\iota_1 - \iota_2^3\Theta_1 \\
&+ 3\iota_2\iota_3^2\Theta_1 - 3\iota_2^2\iota_3\Theta_2 + \Theta_1\varsigma_2]/\Theta_2, \varsigma_3 = -\iota_3^3, \qquad (2.11)
\end{aligned}
$$

$$
\begin{aligned}
\Psi &= e^{2y\rho_4}k_2 + e^{x\iota_1+2y\rho_4+t\varsigma_1}[\cos[x\iota_2 + t\left(-3\iota_1^2 + 6\iota_3\iota_1 + \iota_2^2 - 3\iota_3^2\right)\iota_2]\Theta_1 \\
&+ \sin[x\iota_2 + t\left(-3\iota_1^2 + 6\iota_3\iota_1 + \iota_2^2 - 3\iota_3^2\right)\iota_2]\Theta_2] \\
&+ e^{-t\iota_3^3+x\iota_3+y\rho_3}[\cos(y\rho_4)\Theta_3 + \sin(y\rho_4)\Theta_4]. \qquad (2.12)
\end{aligned}
$$

$$
\begin{aligned}
u_3 &= -[2[e^{x\iota_1+2y\rho_4+t\varsigma_1}\iota_1[\cos[x\iota_2 + t\left(-3\iota_1^2 + 6\iota_3\iota_1 + \iota_2^2 - 3\iota_3^2\right)\iota_2]\Theta_1 \\
&+ \sin[x\iota_2 + t\left(-3\iota_1^2 + 6\iota_3\iota_1 + \iota_2^2 - 3\iota_3^2\right)\iota_2]\Theta_2] + e^{x\iota_1+2y\rho_4+t\varsigma_1}[\cos[x\iota_2 \\
&+ t\left(-3\iota_1^2 + 6\iota_3\iota_1 + \iota_2^2 - 3\iota_3^2\right)\iota_2]\iota_2\Theta_2 - \sin[x\iota_2 + t(-3\iota_1^2 + 6\iota_3\iota_1
\end{aligned}
$$

$$
\begin{aligned}
&+\quad \iota_2^2 - 3\iota_3^2)\iota_2]\iota_2\Theta_1] + e^{-t\iota_3^3+x\iota_3+y\rho_3}\iota_3[\cos(y\rho_4)\Theta_3 + \sin(y\rho_4)\Theta_4]]] \\
&/\quad [e^{2y\rho_4}k_2 + e^{x\iota_1+2y\rho_4+t\varsigma_1}[\cos[x\iota_2 + t\left(-3\iota_1^2 + 6\iota_3\iota_1 + \iota_2^2 - 3\iota_3^2\right)\iota_2]\Theta_1 \\
&+\quad \sin[x\iota_2 + t\left(-3\iota_1^2 + 6\iota_3\iota_1 + \iota_2^2 - 3\iota_3^2\right)\iota_2]\Theta_2] \\
&+\quad e^{-t\iota_3^3+x\iota_3+y\rho_3}\left(\cos(y\rho_4)\Theta_3 + \sin(y\rho_4)\Theta_4\right)].
\end{aligned}
\tag{2.13}
$$

The dynamical behavior to Eq. (14) is demonstrated in Figure 3.

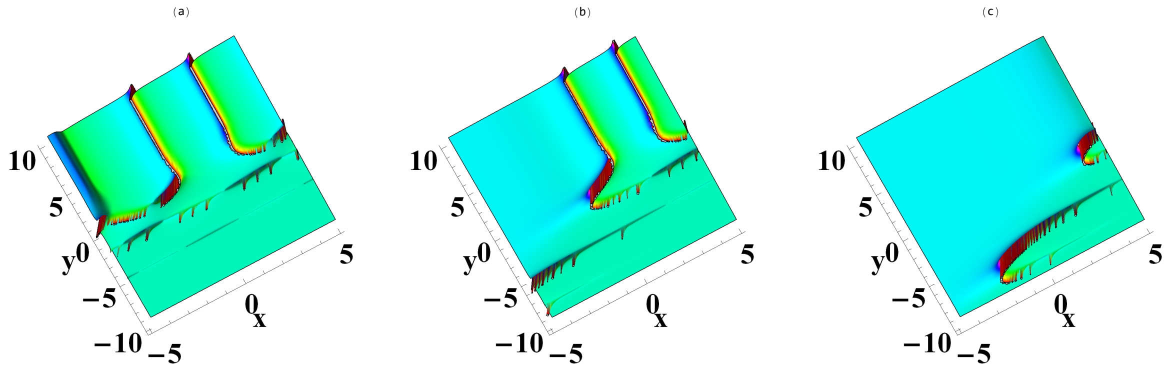


**Figure 3.** $\Theta_3 = \Theta_2 = -1$, $k_2 = 2$, $\Theta_1 = \iota_3 = \iota_2 = \iota_1 = \rho_4 = \rho_3 = \Theta_4 = 1$, (a) $t = -5$, (b) $t = 0$ and (c) $t = 5$.

**Case (4)**

$$
\begin{aligned}
k_1 &= \rho_2 = \varsigma_3 = \iota_3 = \iota_4 = \varsigma_4 = 0, \\
\varsigma_1 &= 3\iota_1\iota_2^2 - \iota_1^3, \varsigma_2 = \iota_2\left(\iota_2^2 - 3\iota_1^2\right),
\end{aligned}
\tag{2.14}
$$

$$
\begin{aligned}
\Psi &= e^{2y\rho_4}k_2 + e^{x\iota_1+t\left(3\iota_1\iota_2^2-\iota_1^3\right)+y\rho_1}[\cos[x\iota_2 + t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2]\Theta_1 + \sin[x\iota_2 \\
&+ t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2]\Theta_2] + e^{y\rho_3}[\cos(y\rho_4)\Theta_3 + \sin(y\rho_4)\Theta_4].
\end{aligned}
\tag{2.15}
$$

$$
\begin{aligned}
u_4 &= -[2[e^{x\iota_1+t\left(3\iota_1\iota_2^2-\iota_1^3\right)+y\rho_1}\iota_1[\cos\left(x\iota_2 + t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2\right)\Theta_1 + \sin[x\iota_2 \\
&+ t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2]\Theta_2] + e^{x\iota_1+t\left(3\iota_1\iota_2^2-\iota_1^3\right)+y\rho_1}[\cos[x\iota_2 + t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2] \\
&* \iota_2\Theta_2 - \sin[x\iota_2 + t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2]\iota_2\Theta_1]]]/[[\cos[x\iota_2 + t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2]\Theta_1 \\
&+ \sin\left(x\iota_2 + t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2\right)\Theta_2]e^{2y\rho_4}k_2 + e^{x\iota_1+t\left(3\iota_1\iota_2^2-\iota_1^3\right)+y\rho_1} \\
&+ e^{y\rho_3}[\cos(y\rho_4)\Theta_3 + \sin(y\rho_4)\Theta_4]].
\end{aligned}
\tag{2.16}
$$

The dynamical behavior to Eq. (17) is demonstrated in Figure 4.

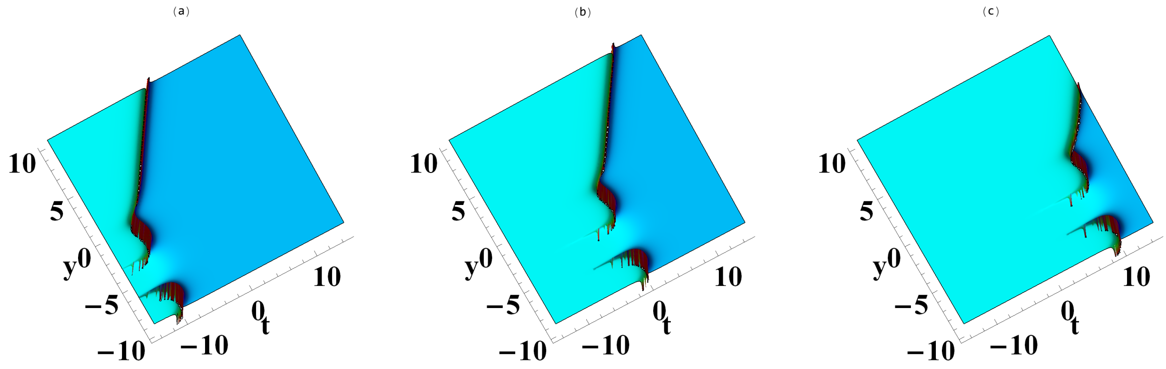


**Figure 4.** $\iota_1 = \Theta_2 = -1$, $k_2 = -2$, $\Theta_1 = \rho_1 = \rho_3 = \rho_4 = \Theta_3 = \Theta_4 = 1$, $\iota_2 = 0$, (a) $x = -10$, (b) $x = 0$ and (c) $x = 10$.

**Case (5)**

$$k_1 = \varsigma_2 = \iota_2 = \iota_4 = \varsigma_4 = 0, \iota_3 = \iota_1, \varsigma_3 = -\iota_1^3, \varsigma_1 = -\iota_1^3, \tag{2.17}$$

$$\begin{aligned} \Psi &= e^{2y\rho_4}k_2 + e^{-t\iota_1^3+x\iota_1+y\rho_1}[\cos(y\rho_2)\,\Theta_1 + \sin(y\rho_2)\,\Theta_2] \\ &+ e^{-t\iota_1^3+x\iota_1+y\rho_3}[\cos(y\rho_4)\,\Theta_3 + \sin(y\rho_4)\,\Theta_4]. \end{aligned} \tag{2.18}$$

$$\begin{aligned} u_5 &= -[2[e^{-t\iota_1^3+x\iota_1+y\rho_1}\iota_1[\cos(y\rho_2)\,\Theta_1 + \sin(y\rho_2)\,\Theta_2] + e^{-t\iota_1^3+x\iota_1+y\rho_3} \\ &* \iota_1[\cos(y\rho_4)\,\Theta_3 + \sin(y\rho_4)\,\Theta_4]]]/[e^{2y\rho_4}k_2 + e^{-t\iota_1^3+x\iota_1+y\rho_1}[\cos(y\rho_2)\,\Theta_1 \\ &+ \sin(y\rho_2)\,\Theta_2] + e^{-t\iota_1^3+x\iota_1+y\rho_3}[\cos(y\rho_4)\,\Theta_3 + \sin(y\rho_4)\,\Theta_4]]. \end{aligned} \tag{2.19}$$

The dynamical behavior to Eq. (20) is demonstrated in Figure 5.

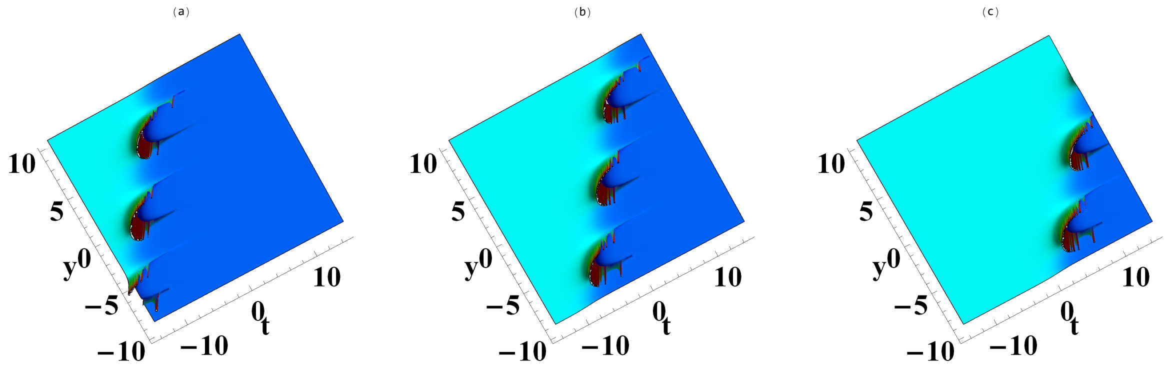


**Figure 5.** $\iota_1 = \Theta_2 = -1$, $k_2 = -2$, $\Theta_1 = \rho_1 = \rho_2 = \rho_3 = \rho_4 = \Theta_3 = \Theta_4 = 1$, (a) $x = -10$, (b) $x = 0$ and (c) $x = 10$.

**Case (6)**

$$k_2 = \varsigma_2 = \rho_4 = \iota_2 = 0, \varsigma_3 = -\iota_3^3 + 3\iota_1\iota_3^2 - 3\iota_1^2\iota_3 + 3\iota_4^2\iota_3 - 3\iota_1\iota_4^2,$$

$$\varsigma_4 = \iota_4^3 - 3(\iota_1-\iota_3)^2\iota_4, \varsigma_1 = -\iota_1^3, \rho_3 = 2\rho_1, \tag{2.20}$$

$$\begin{aligned}\Psi &= e^{2\left(-t\iota_1^3+x\iota_1+y\rho_1\right)}k_1 + e^{-t\iota_1^3+x\iota_1+y\rho_1}[\cos(y\rho_2)\Theta_1+\sin(y\rho_2)\Theta_2]\\ &+ e^{x\iota_3+t\left(-\iota_3^3+3\iota_1\iota_3^2-3\iota_1^2\iota_3+3\iota_4^2\iota_3-3\iota_1\iota_4^2\right)+2y\rho_1}[\cos[x\iota_4+t\varsigma_4]\Theta_3\\ &+ \sin[x\iota_4+t\left(\iota_4^3-3(\iota_1-\iota_3)^2\iota_4\right)]\Theta_4].\end{aligned} \tag{2.21}$$

$$\begin{aligned}u_6 &= -[2[2e^{2\left(-t\iota_1^3+x\iota_1+y\rho_1\right)}k_1\iota_1 + e^{-t\iota_1^3+x\iota_1+y\rho_1}[\cos(y\rho_2)\Theta_1+\sin(y\rho_2)\\ &* \Theta_2]\iota_1 + e^{x\iota_3+2y\rho_1+t\varsigma_3}\iota_3(\sin(x\iota_4+t\varsigma_4)+\cos(x\iota_4+t\varsigma_4)\Theta_3)\\ &+ e^{x\iota_3+2y\rho_1+t\varsigma_3}[\cos(x\iota_4+t\varsigma_4)\iota_4-\sin(x\iota_4+t\varsigma_4)\iota_4\Theta_3]]]\\ &/ [e^{2\left(-t\iota_1^3+x\iota_1+y\rho_1\right)}k_1+e^{-t\iota_1^3+x\iota_1+y\rho_1}[\cos(y\rho_2)\Theta_1+\sin(y\rho_2)\Theta_2]\\ &+ e^{x\iota_3+2y\rho_1+t\varsigma_3}[\sin(x\iota_4+t\varsigma_4)+\cos(x\iota_4+t\varsigma_4)\Theta_3]].\end{aligned} \tag{2.22}$$

The dynamical behavior to Eq. (23) is demonstrated in Figure 6.

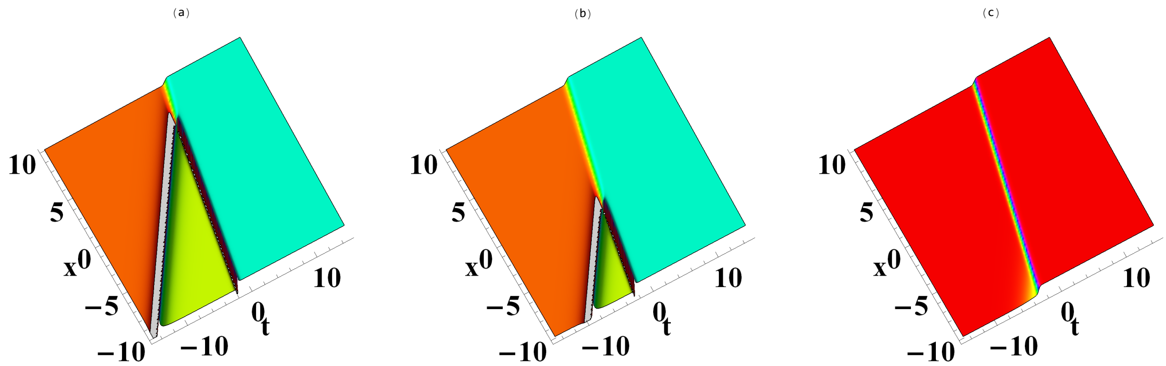


**Figure 6.** $\iota_3 = \Theta_2 = \Theta_1 = -1$, $k_1 = 2$, $\iota_1 = \rho_1 = \rho_2 = \Theta_3 = \Theta_4 = 1$, (a) $y = -5$, (b) $y = 0$ and (c) $y = 5$.

**Case (7)**

$$k_2 = \varsigma_2 = \varsigma_1 = \rho_4 = \iota_2 = \iota_1 = 0, \varsigma_3 = 3\iota_3\iota_4^2 - \iota_3^3, \varsigma_4 = \iota_4^3 - 3\iota_3^2\iota_4, \tag{2.23}$$

$$\begin{aligned}\Psi &= e^{2y\rho_1}k_1 + e^{y\rho_1}[\cos(y\rho_2)\Theta_1+\sin(y\rho_2)\Theta_2] + e^{x\iota_3+t\left(3\iota_3\iota_4^2-\iota_3^3\right)+y\rho_3}\\ &* [\cos[x\iota_4+t\left(\iota_4^3-3\iota_3^2\iota_4\right)]\Theta_3+\sin[x\iota_4+t\left(\iota_4^3-3\iota_3^2\iota_4\right)]\Theta_4].\end{aligned} \tag{2.24}$$

$$\begin{aligned}u_7 &= -[2[e^{x\iota_3+t\left(3\iota_3\iota_4^2-\iota_3^3\right)+y\rho_3}\iota_3[\cos[x\iota_4+t\left(\iota_4^3-3\iota_3^2\iota_4\right)]\Theta_3+\sin[x\iota_4\\ &+ t\left(\iota_4^3-3\iota_3^2\iota_4\right)]\Theta_4] + e^{x\iota_3+t\left(3\iota_3\iota_4^2-\iota_3^3\right)+y\rho_3}[\cos[x\iota_4+t\left(\iota_4^3-3\iota_3^2\iota_4\right)]\\ &* \iota_4\Theta_4 - \sin[x\iota_4+t\left(\iota_4^3-3\iota_3^2\iota_4\right)]\iota_4\Theta_3]]]/[e^{2y\rho_1}k_1+e^{y\rho_1}[\cos(y\rho_2)\Theta_1\\ &+ \sin(y\rho_2)\Theta_2] + e^{x\iota_3+t\left(3\iota_3\iota_4^2-\iota_3^3\right)+y\rho_3}[\cos[x\iota_4+t\left(\iota_4^3-3\iota_3^2\iota_4\right)]\Theta_3\\ &+ \sin[x\iota_4+t\left(\iota_4^3-3\iota_3^2\iota_4\right)]\Theta_4]].\end{aligned} \tag{2.25}$$

The dynamical behavior to Eq. (26) is demonstrated in Figure 7.

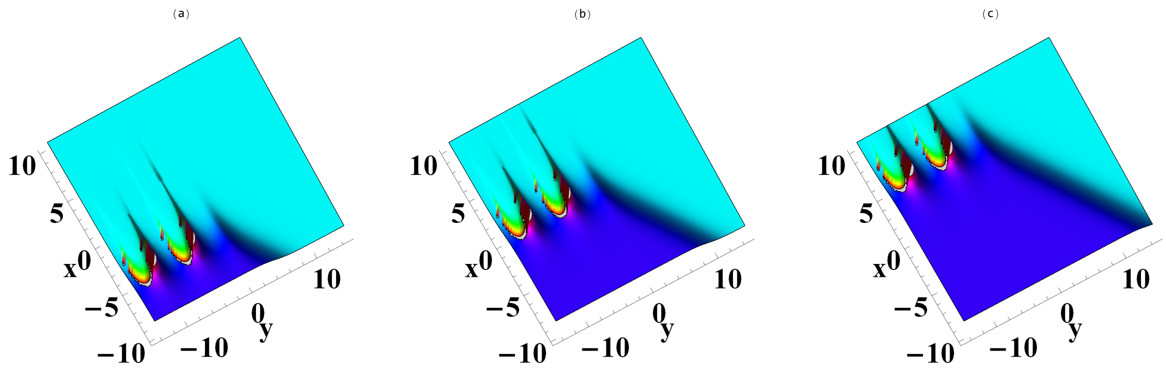


**Figure 7.** $\iota_3 = \Theta_2 = \Theta_1 = -1$, $k_1 = 2$, $\iota_4 = 0$, $\rho_1 = \rho_2 = \rho_3 = \Theta_3 = \Theta_4 = 1$, (a) $t = -5$, (b) $t = 0$ and (c) $t = 5$.

**Case (8)**

$$\begin{aligned} k_2 &= \rho_4 = \rho_2 = 0, \rho_1 = \rho_3, \varsigma_1 = 3\iota_1\iota_2^2 - \iota_1^3, \varsigma_2 = \iota_2^3 - 3\iota_1^2\iota_2, \\ \varsigma_3 &= 6\iota_1^3 - 12\iota_3\iota_1^2 + 6\iota_2^2\iota_1 + 6\iota_3^2\iota_1 - 6\iota_4^2\iota_1 - \iota_3^3 + 3\iota_3\iota_4^2, \\ \varsigma_4 &= \iota_4^3 - 3\left(\iota_3 - 2\iota_1\right)^2\iota_4, \end{aligned} \tag{2.26}$$

$$\begin{aligned} \Psi &= e^{2[x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_3]}k_1 + e^{x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_3}[\cos[x\iota_2 + t(\iota_2^3 \\ &- 3\iota_1^2\iota_2)]\Theta_1 + \sin[x\iota_2 + t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\Theta_2] + e^{x\iota_3 + y\rho_3 + t\varsigma_3} \\ &* [\sin\left(x\iota_4 + t\varsigma_4\right) + \cos\left(x\iota_4 + t\varsigma_4\right)\Theta_3]. \end{aligned} \tag{2.27}$$

$$\begin{aligned} u_8 &= -[2[2e^{2[x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_3]}k_1\iota_1 + e^{x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_3}[\cos[x\iota_2 \\ &+ t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\Theta_1 + \sin[x\iota_2 + t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\Theta_2]\iota_1 \\ &+ e^{x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_3}[\cos[x\iota_2 + t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\iota_2\Theta_2 - \sin[x\iota_2 \\ &+ t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\iota_2\Theta_1] + e^{x\iota_3 + y\rho_3 + t\varsigma_3}\iota_3[\sin\left(x\iota_4 + t\varsigma_4\right) + \cos(x\iota_4 \\ &+ t\varsigma_4)\Theta_3] + e^{x\iota_3 + y\rho_3 + t\varsigma_3}[\cos\left(x\iota_4 + t\varsigma_4\right)\iota_4 - \sin\left(x\iota_4 + t\varsigma_4\right)\iota_4\Theta_3]]] \\ &/ \; [e^{2\left(x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_3\right)}k_1 + e^{x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_3}[\cos[x\iota_2 \\ &+ t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\Theta_1 + \sin\left(x\iota_2 + t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)\right)\Theta_2] \\ &+ e^{x\iota_3 + y\rho_3 + t\varsigma_3}\left(\sin\left(x\iota_4 + t\varsigma_4\right) + \cos\left(x\iota_4 + t\varsigma_4\right)\Theta_3\right)]. \end{aligned} \tag{2.28}$$

The dynamical behavior to Eq. (29) is demonstrated in Figure 8.

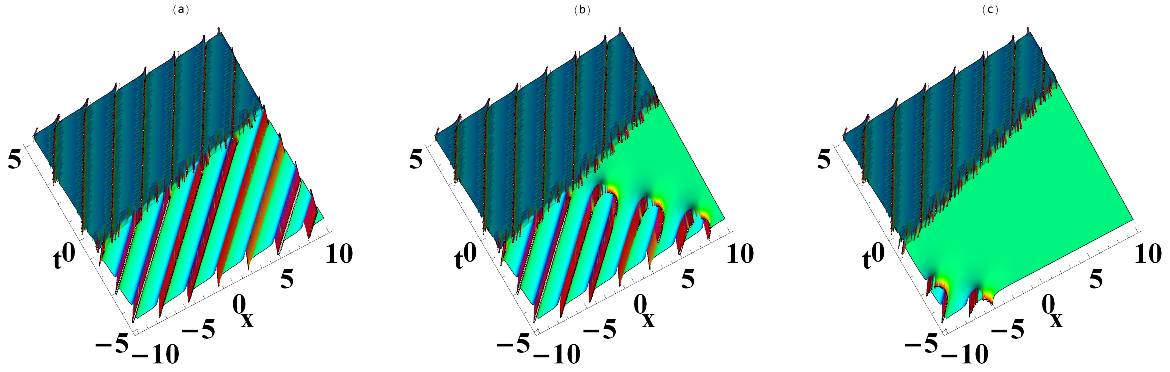


**Figure 8.** $\iota_1 = \rho_3 = \Theta_3 = \Theta_2 = 1, \Theta_4 = 1, \iota_3 = \iota_4 = \iota_2 = k_1 = \Theta_1 = -1$,
(a) $y = -15$, (b) $y = 0$ and (c) $y = 15$.

**Case (9)**

$$k_2 = \varsigma_4 = \varsigma_2 = \iota_2 = \iota_4 = 0, \iota_1 = \iota_3, \varsigma_1 = -\iota_3^3, \varsigma_3 = -\iota_3^3, \tag{2.29}$$

$$\begin{aligned}\Psi &= e^{2\left(-t\iota_3^3+x\iota_3+y\rho_1\right)}k_1 + e^{-t\iota_3^3+x\iota_3+y\rho_1}[\cos(y\rho_2)\Theta_1 + \sin(y\rho_2)\Theta_2] \\ &+ e^{-t\iota_3^3+x\iota_3+y\rho_3}[\cos(y\rho_4)\Theta_3 + \sin(y\rho_4)\Theta_4].\end{aligned} \tag{2.30}$$

$$\begin{aligned}u_9 &= -[2[2e^{2\left(-t\iota_3^3+x\iota_3+y\rho_1\right)}k_1\iota_3 + e^{-t\iota_3^3+x\iota_3+y\rho_1}[\cos(y\rho_2)\Theta_1 + \sin(y\rho_2)\Theta_2] \\ &* \iota_3 + e^{-t\iota_3^3+x\iota_3+y\rho_3}[\cos(y\rho_4)\Theta_3 + \sin(y\rho_4)\Theta_4]\iota_3]]/[e^{2\left(-t\iota_3^3+x\iota_3+y\rho_1\right)}k_1 \\ &+ e^{-t\iota_3^3+x\iota_3+y\rho_1}[\cos(y\rho_2)\Theta_1 + \sin(y\rho_2)\Theta_2] \\ &+ e^{-t\iota_3^3+x\iota_3+y\rho_3}[\cos(y\rho_4)\Theta_3 + \sin(y\rho_4)\Theta_4]].\end{aligned} \tag{2.31}$$

The dynamical behavior to Eq. (32) is demonstrated in Figure 9.

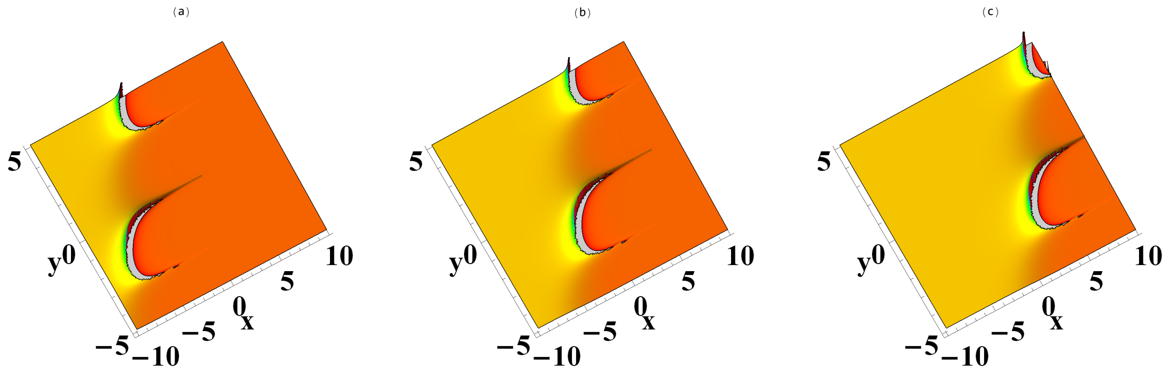


**Figure 9.** $\rho_1 = \rho_4 = \rho_2 = k_1 = 1, \rho_3 = \Theta_3 = \Theta_2 = \Theta_4 = 1, \iota_3 = \Theta_1 = -1$,
(a) $t = -5$, (b) $t = 0$ and (c) $t = 5$.

**Case (10)**

$$k_2 = \varsigma_4 = \rho_2 = \iota_4 = 0, \iota_3 = 2\iota_1, \varsigma_1 = 3\iota_1\iota_2^2 - \iota_1^3,$$

$$\varsigma_2 = \iota_2^3 - 3\iota_1^2\iota_2, \varsigma_3 = 6\iota_1\left(4\iota_1^2 + \iota_2^2\right) - 26\iota_1^3, \tag{2.32}$$

$$\begin{aligned}\Psi &= e^{2[x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_1]}k_1 + e^{x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_1}[\cos[x\iota_2 + t(\iota_2^3 \\ &- 3\iota_1^2\iota_2)]\Theta_1 + \sin[x\iota_2 + t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\Theta_2] \\ &+ e^{2x\iota_1 + t[6\iota_1\left(4\iota_1^2 + \iota_2^2\right) - 26\iota_1^3] + y\rho_3}[\cos(y\rho_4)\Theta_3 + \sin(y\rho_4)\Theta_4].\end{aligned} \tag{2.33}$$

$$\begin{aligned}u_{10} &= -[2[2e^{2[x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_1]}k_1\iota_1 + e^{x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_1}[\cos[x\iota_2 \\ &+ t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\Theta_1 + \sin[x\iota_2 + t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\Theta_2]\iota_1 + 2e^{2x\iota_1 + y\rho_3 + t\varsigma_3} \\ &* [\cos(y\rho_4)\Theta_3 + \sin(y\rho_4)\Theta_4]\iota_1 + e^{x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_1}[\cos[x\iota_2 \\ &+ t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\iota_2\Theta_2 - \sin[x\iota_2 + t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\iota_2\Theta_1]]] \\ &/ [e^{2[x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_1]}k_1 + e^{x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_1}[\cos[x\iota_2 \\ &+ t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\Theta_1 + \sin[x\iota_2 + t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\Theta_2] \\ &+ e^{2x\iota_1 + y\rho_3 + t\varsigma_3}[\cos(y\rho_4)\Theta_3 + \sin(y\rho_4)\Theta_4]].\end{aligned} \tag{2.34}$$

The dynamical behavior to Eq. (35) is demonstrated in Figure 10.

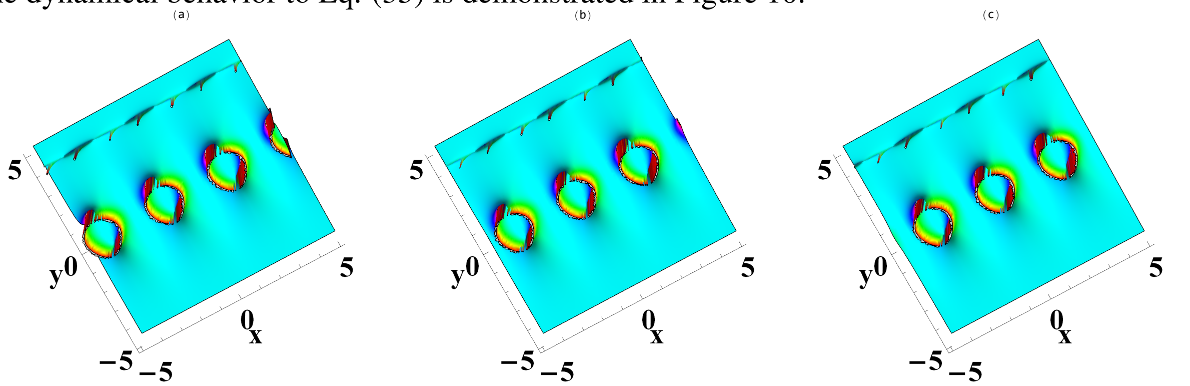


**Figure 10.** $\rho_1 = \Theta_4 = k_1 = 1, \iota_1 = 0, \rho_4 = \Theta_3 = \Theta_2 = \Theta_1 = 1, \iota_2 = -2$, (a) $t = -25$, (b) $t = 0$ and (c) $t = 25$.

**Case (11)**

$$\begin{aligned}\rho_1 &= \rho_4 = \varsigma_2 = \iota_2 = \rho_3 = 0, \iota_1 = \frac{3\iota_3^2 - 5\iota_4^2}{6(\iota_3 - \iota_4)}, \varsigma_1 = -\frac{\left(3\iota_3^2 - 5\iota_4^2\right)^3}{216(\iota_3 - \iota_4)^3}, \\ \varsigma_3 &= \frac{-3\iota_3^5 + 6\iota_4\iota_3^4 + 6\iota_4^2\iota_3^3 - 24\iota_4^3\iota_3^2 + 41\iota_4^4\iota_3 - 30\iota_4^5}{12(\iota_3 - \iota_4)^2}, \\ \varsigma_4 &= -\frac{\iota_4\left(9\iota_3^4 - 36\iota_4\iota_3^3 + 54\iota_4^2\iota_3^2 - 36\iota_4^3\iota_3 + 13\iota_4^4\right)}{12(\iota_3 - \iota_4)^2},\end{aligned} \tag{2.35}$$

$$\Psi = e^{2[\frac{x\left(3\iota_3^2 - 5\iota_4^2\right)}{6(\iota_3 - \iota_4)} - \frac{t\left(3\iota_3^2 - 5\iota_4^2\right)^3}{216(\iota_3 - \iota_4)^3}]}k_1 + e^{2(x\iota_4 + t\varsigma_4)}k_2 + e^{\frac{x\left(3\iota_3^2 - 5\iota_4^2\right)}{6(\iota_3 - \iota_4)} - \frac{t\left(3\iota_3^2 - 5\iota_4^2\right)^3}{216(\iota_3 - \iota_4)^3}}$$

$$
\begin{aligned}
&* \quad [\cos(y\rho_2)\Theta_1 + \sin(y\rho_2)\Theta_2] + e^{x\iota_3+t\varsigma_3}[\cos(x\iota_4+t\varsigma_4)\Theta_3 \\
&+ \quad \sin(x\iota_4+t\varsigma_4)\Theta_4].
\end{aligned} \tag{2.36}
$$

$$
\begin{aligned}
u_{11} &= -[2[2e^{2(x\iota_4+t\varsigma_4)}k_2\iota_4 + \frac{e^{2[\frac{x(3\iota_3^2-5\iota_4^2)}{6(\iota_3-\iota_4)}-\frac{t(3\iota_3^2-5\iota_4^2)^3}{216(\iota_3-\iota_4)^3}]}k_1\left(3\iota_3^2-5\iota_4^2\right)}{3(\iota_3-\iota_4)} \\
&+ \frac{e^{\frac{x(3\iota_3^2-5\iota_4^2)}{6(\iota_3-\iota_4)}-\frac{t(3\iota_3^2-5\iota_4^2)^3}{216(\iota_3-\iota_4)^3}}\left(3\iota_3^2-5\iota_4^2\right)(\cos(y\rho_2)\Theta_1+\sin(y\rho_2)\Theta_2)}{6(\iota_3-\iota_4)} \\
&+ e^{x\iota_3+t\varsigma_3}\iota_3(\cos(x\iota_4+t\varsigma_4)\Theta_3+\sin(x\iota_4+t\varsigma_4)\Theta_4) \\
&+ e^{x\iota_3+t\varsigma_3}[\cos(x\iota_4+t\varsigma_4)\iota_4\Theta_4-\sin(x\iota_4+t\varsigma_4)\iota_4\Theta_3]]] \\
&/ \quad [e^{2\left(\frac{x(3\iota_3^2-5\iota_4^2)}{6(\iota_3-\iota_4)}-\frac{t(3\iota_3^2-5\iota_4^2)^3}{216(\iota_3-\iota_4)^3}\right)}k_1 + e^{2(x\iota_4+t\varsigma_4)}k_2 \\
&+ e^{\frac{x(3\iota_3^2-5\iota_4^2)}{6(\iota_3-\iota_4)}-\frac{t(3\iota_3^2-5\iota_4^2)^3}{216(\iota_3-\iota_4)^3}}(\cos(y\rho_2)\Theta_1+\sin(y\rho_2)\Theta_2) \\
&+ e^{x\iota_3+t\varsigma_3}[\cos(x\iota_4+t\varsigma_4)\Theta_3+\sin(x\iota_4+t\varsigma_4)\Theta_4]].
\end{aligned} \tag{2.37}
$$

The dynamical behavior to Eq. (38) is demonstrated in Figure 11.

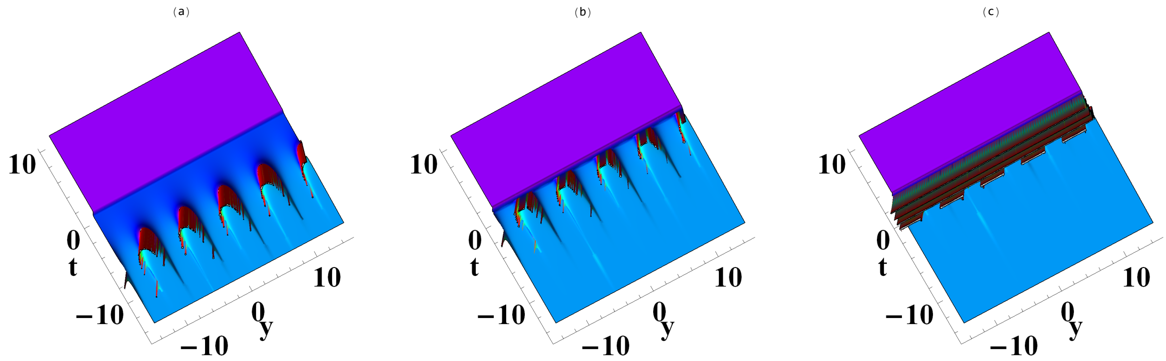


**Figure 11.** $\iota_3 = k_2 = k_1 = \Theta_1 = 1, \iota_4 = -2, \rho_2 = \Theta_3 = \Theta_4 = 1,$
(a) $x = -5$, (b) $x = 0$ and (c) $x = 5$.

**Case (12)**

$$
\rho_1 = \rho_4, \iota_2 = \iota_4 = \varsigma_4 = \varsigma_2 = 0, \iota_1 = \iota_3, \varsigma_3 = -\iota_3^3, \varsigma_1 = -\iota_1^3, \tag{2.38}
$$

$$
\begin{aligned}
\Psi &= e^{2\left(-t\iota_3^3+x\iota_3+y\rho_4\right)}k_1 + e^{2y\rho_4}k_2 + e^{-t\iota_3^3+x\iota_3+y\rho_4}[\cos(y\rho_2)\Theta_1 \\
&+ \sin(y\rho_2)\Theta_2] + e^{-t\iota_3^3+x\iota_3+y\rho_3}[\cos(y\rho_4)\Theta_3+\sin(y\rho_4)\Theta_4].
\end{aligned} \tag{2.39}
$$

$$
\begin{aligned}
u_{12} &= -[2[2e^{2\left(-t\iota_3^3+x\iota_3+y\rho_4\right)}k_1\iota_3 + e^{-t\iota_3^3+x\iota_3+y\rho_4}[\cos(y\rho_2)\Theta_1 \\
&+ \sin(y\rho_2)\Theta_2]\iota_3 + e^{-t\iota_3^3+x\iota_3+y\rho_3}[\cos(y\rho_4)\Theta_3+\sin(y\rho_4)\Theta_4]\iota_3]]
\end{aligned}
$$

$$
\begin{aligned}
/ \quad & [e^{2\left(-t\iota_3^3+x\iota_3+y\rho_4\right)}k_1 + e^{2y\rho_4}k_2 + e^{-t\iota_3^3+x\iota_3+y\rho_4}[\cos(y\rho_2)\,\Theta_1 \\
+ \quad & \sin(y\rho_2)\,\Theta_2] + e^{-t\iota_3^3+x\iota_3+y\rho_3}[\cos(y\rho_4)\,\Theta_3 + \sin(y\rho_4)\,\Theta_4]]. \qquad (2.40)
\end{aligned}
$$

The dynamical behavior to Eq. (41) is demonstrated in Figure 12.

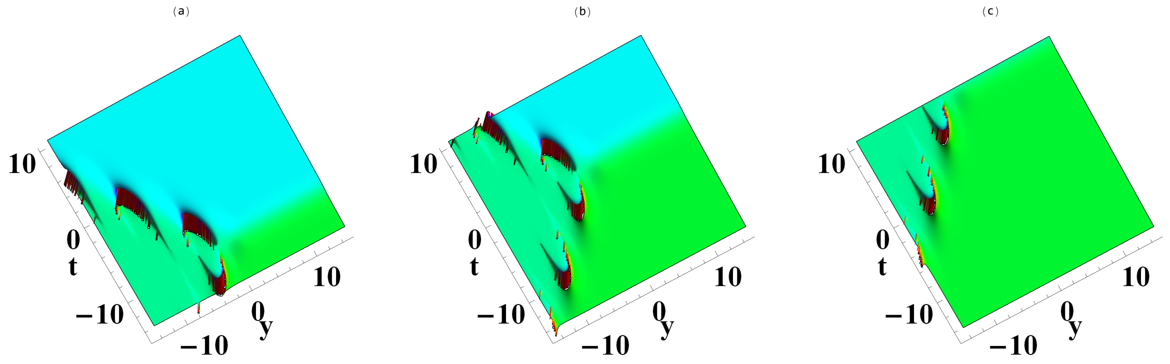


**Figure 12.** $\iota_3 = k_2 = k_1 = \Theta_1 = 1, \iota_4 = -2, \rho_2 = \rho_3 = \rho_4 = \Theta_3 = \Theta_4 = 1, \Theta_2 = -1,$
(a) $x = -10$, (b) $x = 0$ and (c) $x = 10$.

**Case (13)**

$$\rho_1 = \rho_4, \rho_2 = \iota_3 = \iota_1 = \varsigma_4 = \iota_4 = \varsigma_1 = 0, \varsigma_3 = -\iota_3^3, \varsigma_2 = \iota_2^3, \qquad (2.41)$$

$$
\begin{aligned}
\Psi \quad = \quad & e^{2y\rho_4}k_1 + e^{2y\rho_4}k_2 + e^{y\rho_4}[\cos\left(t\iota_2^3 + x\iota_2\right)\Theta_1 + \sin\left(t\iota_2^3 + x\iota_2\right)\Theta_2] \\
+ \quad & e^{y\rho_3}[\cos(y\rho_4)\,\Theta_3 + \sin(y\rho_4)\,\Theta_4]. \qquad (2.42)
\end{aligned}
$$

$$
\begin{aligned}
u_{13} \quad = \quad & -[2e^{y\rho_4}[\cos\left(t\iota_2^3 + x\iota_2\right)\iota_2\Theta_2 - \sin\left(t\iota_2^3 + x\iota_2\right)\iota_2\Theta_1]]/[e^{2y\rho_4}k_1 \\
+ \quad & e^{2y\rho_4}k_2 + e^{y\rho_4}[\cos\left(t\iota_2^3 + x\iota_2\right)\Theta_1 + \sin\left(t\iota_2^3 + x\iota_2\right)\Theta_2] \\
+ \quad & e^{y\rho_3}[\cos(y\rho_4)\,\Theta_3 + \sin(y\rho_4)\,\Theta_4]]. \qquad (2.43)
\end{aligned}
$$

The dynamical behavior to Eq. (44) is demonstrated in Figure 13.

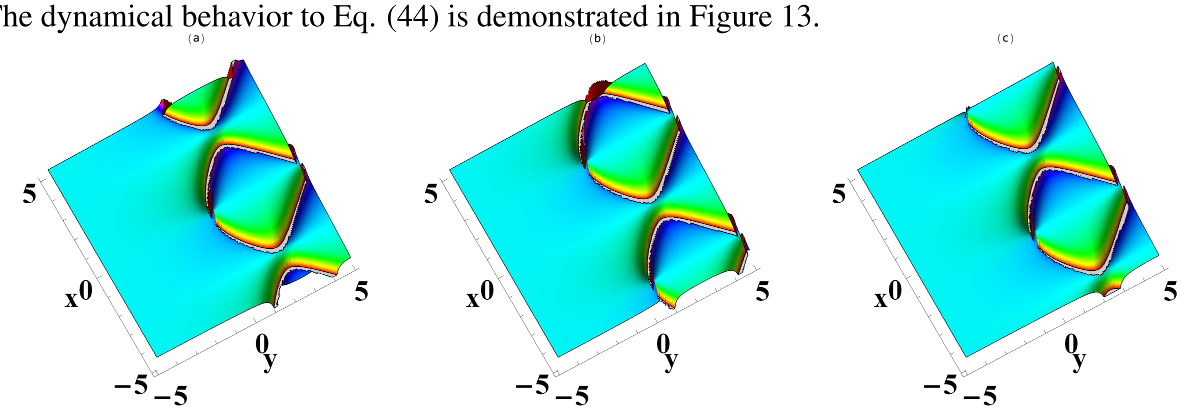


**Figure 13.** $\iota_2 = k_2 = k_1 = \Theta_1 = 1, \Theta_3 = \Theta_4 = 1, \rho_3 = \rho_4 = \Theta_2 = -1,$
(a) $t = -10$, (b) $t = 0$ and (c) $t = 10$.

**Case (14)**

$$\begin{aligned}\rho_1 &= \rho_4, \rho_2 = \Theta_3 = \Theta_4 = \iota_4 = \varsigma_4 = 0,\\ \varsigma_1 &= 3\iota_1\iota_2^2 - \iota_1^3, \varsigma_2 = \iota_2\left(\iota_2^2 - 3\iota_1^2\right),\end{aligned} \tag{2.44}$$

$$\begin{aligned}\Psi &= e^{2[x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_4]}k_1 + e^{2y\rho_4}k_2 + e^{x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_4}[\cos[x\iota_2\\ &+ t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2]\Theta_1 + \sin[x\iota_2 + t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2]\Theta_2].\end{aligned} \tag{2.45}$$

$$\begin{aligned}u_{14} &= -[2[2e^{2[x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_4]}k_1\iota_1 + e^{x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_4}[\cos[x\iota_2\\ &+ t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2]\Theta_1 + \sin[x\iota_2 + t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2]\Theta_2]\iota_1\\ &+ e^{x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_4}[\cos[x\iota_2 + t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2]\iota_2\Theta_2 - \sin[x\iota_2\\ &+ t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2]\iota_2\Theta_1]]]/[e^{2[x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_4]}k_1 + e^{2y\rho_4}k_2\\ &+ e^{x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_4}[\cos\left(x\iota_2 + t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2\right)\Theta_1\\ &+ \sin\left(x\iota_2 + t\left(\iota_2^2 - 3\iota_1^2\right)\iota_2\right)\Theta_2]].\end{aligned} \tag{2.46}$$

The dynamical behavior to Eq. (47) is demonstrated in Figure 14.

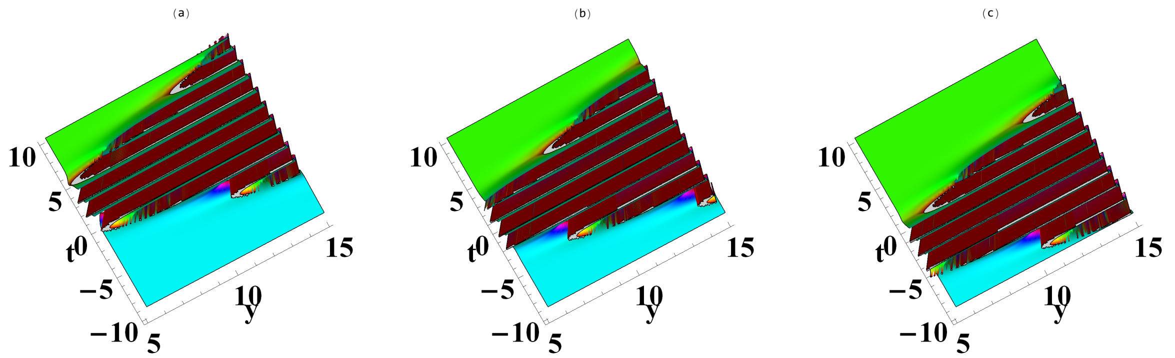


**Figure 14.** $\iota_2 = k_2 = k_1 = \iota_1 = 1$, $\Theta_1 = 1$, $\rho_4 = \Theta_2 = -1$, (a) $x = -5$, (b) $x = 0$ and (c) $x = 5$.

**Case (15)**

$$\begin{aligned}k_2 &= \iota_4 = \varsigma_4 = \rho_2 = 0, \varsigma_2 = \iota_2^3 - 3\iota_1^2\iota_2, \varsigma_1 = 3\iota_1\iota_2^2 - \iota_1^3, \iota_3 = \iota_1 + i\iota_2,\\ \Theta_2 &= i\Theta_1, \Theta_4 = \frac{(\rho_1 - \rho_3)\Theta_3}{\rho_4}, \varsigma_3 = 6\iota_1^3 - 12\iota_3\iota_1^2 + 6\left(\iota_2^2 + \iota_3^2\right)\iota_1 - \iota_3^3,\end{aligned} \tag{2.47}$$

$$\begin{aligned}\Psi &= e^{2[x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_1]}k_1 + e^{x\iota_1 + t\left(3\iota_1\iota_2^2 - \iota_1^3\right) + y\rho_1}[\cos[x\iota_2 + t(\iota_2^3\\ &- 3\iota_1^2\iota_2)]\Theta_1 + i\sin\left(x\iota_2 + t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)\right)\Theta_1] + e^{x(\iota_1 + i\iota_2) + y\rho_3 + t\varsigma_3}\\ &* [\cos(y\rho_4)\Theta_3 + \frac{\sin(y\rho_4)(\rho_1 - \rho_3)\Theta_3}{\rho_4}].\end{aligned} \tag{2.48}$$

$$\begin{aligned}
u_{15} &= -[2[2e^{2[x\iota_1+t\left(3\iota_1\iota_2^2-\iota_1^3\right)+y\rho_1]}k_1\iota_1 + e^{x\iota_1+t\left(3\iota_1\iota_2^2-\iota_1^3\right)+y\rho_1}[\cos[x\iota_2\\
&+ t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\Theta_1 + i\sin\left(x\iota_2 + t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)\right)\Theta_1]\iota_1\\
&+ e^{x\iota_1+t\left(3\iota_1\iota_2^2-\iota_1^3\right)+y\rho_1}[i\cos[x\iota_2 + t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\iota_2\Theta_1 - \sin[x\iota_2\\
&+ t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\iota_2\Theta_1] + e^{x(\iota_1+i\iota_2)+y\rho_3+t\varsigma_3}\left(\iota_1 + i\iota_2\right)[\cos\left(y\rho_4\right)\Theta_3\\
&+ \frac{\sin\left(y\rho_4\right)\left(\rho_1-\rho_3\right)\Theta_3}{\rho_4}]]]/[e^{2\left(x\iota_1+t\left(3\iota_1\iota_2^2-\iota_1^3\right)+y\rho_1\right)}k_1 + [\cos[x\iota_2\\
&+ t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\Theta_1 + i\sin[x\iota_2 + t\left(\iota_2^3 - 3\iota_1^2\iota_2\right)]\Theta_1]\\
&* e^{x\iota_1+t\left(3\iota_1\iota_2^2-\iota_1^3\right)+y\rho_1} + e^{x(\iota_1+i\iota_2)+y\rho_3+t\varsigma_3}\\
&* [\cos\left(y\rho_4\right)\Theta_3 + \frac{\sin\left(y\rho_4\right)\left(\rho_1-\rho_3\right)\Theta_3}{\rho_4}]]. \qquad (2.49)
\end{aligned}$$

**Case (16)**

$$\begin{aligned}
k_1 &= \rho_4 = 0, \Theta_1 = i\Theta_2, \varsigma_4 = \iota_4\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right), \rho_1 = i\rho_2 + \rho_3,\\
\varsigma_3 &= [-2\Theta_3\iota_3^3 - 6\iota_4\left(\Theta_4 - 2\Theta_3\right)\iota_3^2 - 6\iota_4^2\left(3\Theta_3 - 4\Theta_4\right)\iota_3 + 2\iota_4^3\left(2\Theta_3 - 11\Theta_4\right)\\
&+ \left(2\Theta_3 - \Theta_4\right)\left(\left(\iota_1 - i\iota_2 - 2\iota_4\right)^3 + \varsigma_1 - i\varsigma_2\right)]/(2\Theta_3),\\
\varsigma_1 &= -\iota_1^3 + 6\iota_4\iota_1^2 + 3\iota_2^2\iota_1 - 12\iota_4^2\iota_1 - 14\iota_4^3 + 24\iota_3\iota_4^2 - 6\iota_2^2\iota_4 - 6\iota_3^2\iota_4\\
&+ i\left(-\iota_2^3 + 3\iota_1^2\iota_2 + 12\iota_4^2\iota_2 - 12\iota_1\iota_4\iota_2 + \varsigma_2\right), \qquad (2.50)
\end{aligned}$$

$$\begin{aligned}
\Psi &= e^{2\left(x\iota_4+t\left(-3\iota_3^2+12\iota_4\iota_3-11\iota_4^2\right)\iota_4\right)}k_2 + e^{x\iota_1+y(i\rho_2+\rho_3)+t\varsigma_1}[i\cos(x\iota_2 + y\rho_2\\
&+ t\varsigma_2)\Theta_2 + \sin\left(x\iota_2 + y\rho_2 + t\varsigma_2\right)\Theta_2] + e^{x\iota_3+y\rho_3+t\varsigma_3}[\cos[x\iota_4 + t(-3\iota_3^2 + 12\\
&* \iota_4\iota_3 - 11\iota_4^2)\iota_4]\Theta_3 + \sin\left(x\iota_4 + t\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right)\iota_4\right)\Theta_4]. \qquad (2.51)
\end{aligned}$$

$$\begin{aligned}
u_{16} &= -[2[2e^{2[x\iota_4+t\left(-3\iota_3^2+12\iota_4\iota_3-11\iota_4^2\right)\iota_4]}k_2\iota_4 + e^{x\iota_1+y(i\rho_2+\rho_3)+t\varsigma_1}\iota_1[i\cos\\
&\left(x\iota_2 + y\rho_2 + t\varsigma_2\right)\Theta_2 + \sin\left(x\iota_2 + y\rho_2 + t\varsigma_2\right)\Theta_2] + e^{x\iota_1+y(i\rho_2+\rho_3)+t\varsigma_1}\\
&* [\cos\left(x\iota_2 + y\rho_2 + t\varsigma_2\right)\iota_2\Theta_2 - i\sin\left(x\iota_2 + y\rho_2 + t\varsigma_2\right)\iota_2\Theta_2]\\
&+ e^{x\iota_3+y\rho_3+t\varsigma_3}\iota_3[\cos[x\iota_4 + t\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right)\iota_4]\Theta_3 + \sin[x\iota_4\\
&+ t\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right)\iota_4])\Theta_4] + e^{x\iota_3+y\rho_3+t\varsigma_3}[\cos[x\iota_4 + t(-3\iota_3^2\\
&+ 12\iota_4\iota_3 - 11\iota_4^2)\iota_4]\iota_4\Theta_4 - \sin[x\iota_4 + t(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2)\iota_4]\iota_4\\
&* \Theta_3]]]/[e^{2[x\iota_4+t\left(-3\iota_3^2+12\iota_4\iota_3-11\iota_4^2\right)\iota_4]}k_2 + e^{x\iota_1+y(i\rho_2+\rho_3)+t\varsigma_1}[i\cos(x\iota_2\\
&+ y\rho_2 + t\varsigma_2)\Theta_2 + \sin\left(x\iota_2 + y\rho_2 + t\varsigma_2\right)\Theta_2] + e^{x\iota_3+y\rho_3+t\varsigma_3}[\cos[x\iota_4\\
&+ t\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right)\iota_4]\Theta_3\\
&+ \sin[x\iota_4 + t\left(-3\iota_3^2 + 12\iota_4\iota_3 - 11\iota_4^2\right)\iota_4]\Theta_4]]. \qquad (2.52)
\end{aligned}$$

**Case (17)**

$$\rho_1 = \rho_4, \iota_2 = \varsigma_2 = 0, \Theta_4 = i\Theta_3, \varsigma_1 = -\left(\iota_3 + i\iota_4\right)^3, \iota_1 = \iota_3 + i\iota_4,$$

$$\varsigma_4 = -4\iota_4^3 + 6\iota_1\iota_4^2 - 3\iota_1^2\iota_4, \varsigma_3 = -\iota_3^3 - (3+6i)\iota_4^2\iota_3 + (6+2i)\iota_4^3, \tag{2.53}$$

$$\begin{aligned}\Psi &= e^{2\left(-t(\iota_3+i\iota_4)^3+x(\iota_3+i\iota_4)+y\rho_4\right)}k_1 + e^{2(x\iota_4+y\rho_4+t\varsigma_4)}k_2 + [\cos(y\rho_2)\Theta_1 \\ &+ \sin(y\rho_2)\Theta_2]e^{-t(\iota_3+i\iota_4)^3+x(\iota_3+i\iota_4)+y\rho_4} + [\cos(x\iota_4+y\rho_4+t\varsigma_4)\Theta_3 \\ &+ i\sin(x\iota_4+y\rho_4+t\varsigma_4)\Theta_3]e^{x\iota_3+t\left(-\iota_3^3-(3+6i)\iota_4^2\iota_3+(6+2i)\iota_4^3\right)+y\rho_3}.\end{aligned} \tag{2.54}$$

$$\begin{aligned}u_{17} &= -[2[2e^{2[-t(\iota_3+i\iota_4)^3+x(\iota_3+i\iota_4)+y\rho_4]}k_1(\iota_3+i\iota_4) + [\cos(y\rho_2)\Theta_1 \\ &+ \sin(y\rho_2)\Theta_2](\iota_3+i\iota_4)e^{-t(\iota_3+i\iota_4)^3+x(\iota_3+i\iota_4)+y\rho_4} + 2e^{2(x\iota_4+y\rho_4+t\varsigma_4)}k_2 \\ &* \iota_4 + e^{x\iota_3+t\left(-\iota_3^3-(3+6i)\iota_4^2\iota_3+(6+2i)\iota_4^3\right)+y\rho_3}\iota_3[\cos(x\iota_4+y\rho_4+t\varsigma_4)\Theta_3 \\ &+ i\sin(x\iota_4+y\rho_4+t\varsigma_4)\Theta_3] + e^{x\iota_3+t\left(-\iota_3^3-(3+6i)\iota_4^2\iota_3+(6+2i)\iota_4^3\right)+y\rho_3} \\ &* [i\cos(x\iota_4+y\rho_4+t\varsigma_4)\iota_4\Theta_3 - \sin(x\iota_4+y\rho_4+t\varsigma_4)\iota_4\Theta_3]]] \\ &/ [e^{2[-t(\iota_3+i\iota_4)^3+x(\iota_3+i\iota_4)+y\rho_4]}k_1 + e^{2(x\iota_4+y\rho_4+t\varsigma_4)}k_2 + [\cos(y\rho_2)\Theta_1 \\ &+ \sin(y\rho_2)\Theta_2]e^{-t(\iota_3+i\iota_4)^3+x(\iota_3+i\iota_4)+y\rho_4} + [\cos(x\iota_4+y\rho_4+t\varsigma_4)\Theta_3 \\ &+ i\sin(x\iota_4+y\rho_4+t\varsigma_4)\Theta_3]e^{x\iota_3+t\left(-\iota_3^3-(3+6i)\iota_4^2\iota_3+(6+2i)\iota_4^3\right)+y\rho_3}].\end{aligned} \tag{2.55}$$

**Case (18)**

$$\begin{aligned}\rho_1 &= \rho_4, \rho_2 = \iota_1 = \iota_4 = \varsigma_4 = \varsigma_1 = 0, \varsigma_2 = \iota_2^3, \\ \varsigma_3 &= -\iota_3^3, \Theta_1 = -\frac{\iota_3\Theta_2}{\iota_2}, \iota_3 = i\iota_2,\end{aligned} \tag{2.56}$$

$$\begin{aligned}\Psi &= e^{2y\rho_4}k_1 + e^{2y\rho_4}k_2 + e^{y\rho_4}[\sin\left(t\iota_2^3+x\iota_2\right)\Theta_2 - i\cos\left(t\iota_2^3+x\iota_2\right)\Theta_2] \\ &+ e^{it\iota_2^3+ix\iota_2+y\rho_3}[\cos(y\rho_4)\Theta_3 + \sin(y\rho_4)\Theta_4].\end{aligned} \tag{2.57}$$

$$\begin{aligned}u_{18} &= -[2[e^{y\rho_4}[\cos\left(t\iota_2^3+x\iota_2\right)\iota_2\Theta_2 + i\sin\left(t\iota_2^3+x\iota_2\right)\iota_2\Theta_2] \\ &+ ie^{it\iota_2^3+ix\iota_2+y\rho_3}\iota_2[\cos(y\rho_4)\Theta_3 + \sin(y\rho_4)\Theta_4]]]/[e^{2y\rho_4}k_1 + e^{2y\rho_4}k_2 \\ &+ e^{y\rho_4}\left(\sin\left(t\iota_2^3+x\iota_2\right)\Theta_2 - i\cos\left(t\iota_2^3+x\iota_2\right)\Theta_2\right) \\ &+ e^{it\iota_2^3+ix\iota_2+y\rho_3}[\cos(y\rho_4)\Theta_3 + \sin(y\rho_4)\Theta_4]].\end{aligned} \tag{2.58}$$

**Case (19)**

$$\begin{aligned}\rho_1 &= \rho_4 = \rho_2 = 0, \iota_2 = i\iota_1, \iota_4 = \iota_3 - \iota_1, \Theta_3 = -\frac{3\iota_1\Theta_4}{5\iota_1 - 2\iota_3}, \Theta_1 = i\Theta_2, \\ \varsigma_1 &= (3\iota_1 - 2\iota_3)(3\iota_1^2 - 6\iota_3\iota_1 + 2\iota_3^2), \varsigma_3 = 20\iota_1^3 - 45\iota_3\iota_1^2 + 30\iota_3^2\iota_1 \\ &- 6\iota_3^3, \varsigma_4 = (\iota_1 - \iota_3)\left(11\iota_1^2 - 10\iota_3\iota_1 + 2\iota_3^2\right), \\ \varsigma_2 &= i(3\iota_1 - 2\iota_3)\left(3\iota_1^2 - 6\iota_3\iota_1 + 2\iota_3^2\right),\end{aligned} \tag{2.59}$$

$$\begin{aligned}\Psi &= e^{2(x\iota_1+t\varsigma_1)}k_1 + e^{2[x(\iota_3-\iota_1)+t\varsigma_4]}k_2 + e^{x\iota_1+t\varsigma_1}[\cosh(x\iota_1 - it\varsigma_2)\Theta_1 \\ &+ i\sinh(x\iota_1 - it\varsigma_2)\Theta_2] + e^{x\iota_3+y\rho_3+t\varsigma_3}[\cos(x(\iota_3-\iota_1)+t\varsigma_4)\Theta_3 \\ &+ \sin(x(\iota_3-\iota_1)+t\varsigma_4)\Theta_4]. \end{aligned} \tag{2.60}$$

$$\begin{aligned}u_{19} &= -[2[2e^{2(x\iota_1+t\varsigma_1)}k_1\iota_1 + e^{x\iota_1+t\varsigma_1}[\cosh(x\iota_1 - it\varsigma_2)\Theta_1 + i\sinh(x\iota_1 \\ &- it\varsigma_2)\Theta_2]\iota_1 + 2e^{2(x(\iota_3-\iota_1)+t\varsigma_4)}k_2(\iota_3-\iota_1) + e^{x\iota_1+t\varsigma_1}[\sinh(x\iota_1 - it\varsigma_2) \\ &* \iota_1\Theta_1 + i\cosh(x\iota_1 - it\varsigma_2)\iota_1\Theta_2] + e^{x\iota_3+y\rho_3+t\varsigma_3}\iota_3[\cos[x(\iota_3-\iota_1) \\ &+ t\varsigma_4]\Theta_3 + \sin[x(\iota_3-\iota_1)+t\varsigma_4]\Theta_4] + e^{x\iota_3+y\rho_3+t\varsigma_3}[\cos[x(\iota_3-\iota_1) \\ &+ t\varsigma_4](\iota_3-\iota_1)\Theta_4 - \sin[x(\iota_3-\iota_1)+t\varsigma_4](\iota_3-\iota_1)\Theta_3]]]/[e^{2(x\iota_1+t\varsigma_1)}k_1 \\ &+ e^{2(x(\iota_3-\iota_1)+t\varsigma_4)}k_2 + e^{x\iota_1+t\varsigma_1}[\cosh(x\iota_1 - it\varsigma_2)\Theta_1 + i\sinh(x\iota_1 - it\varsigma_2) \\ &* \Theta_2] + e^{x\iota_3+y\rho_3+t\varsigma_3}[\cos(x(\iota_3-\iota_1)+t\varsigma_4)\Theta_3 \\ &+ \sin(x(\iota_3-\iota_1)+t\varsigma_4)\Theta_4]]. \end{aligned} \tag{2.61}$$

**Case (20)**

$$\begin{aligned}\rho_1 &= \rho_4 = \rho_2 = 0, \iota_2 = i\iota_1, \iota_4 = \iota_1, \Theta_3 = -\frac{3(\iota_1-\iota_3)\Theta_4}{5\iota_1 - 3\iota_3}, \Theta_1 = i\Theta_2, \\ \varsigma_1 &= \frac{-11\Theta_4\iota_1^3 + 12\iota_3\Theta_4\iota_1^2 - 3\iota_3^2\Theta_4\iota_1}{\Theta_4}, \varsigma_2 = -i\iota_1\left(11\iota_1^2 - 12\iota_3\iota_1 + 3\iota_3^2\right), \\ \varsigma_4 &= \iota_1\left(-11\iota_1^2 + 12\iota_3\iota_1 - 3\iota_3^2\right), \varsigma_3 = -20\iota_1^3 + 15\iota_3\iota_1^2 - \iota_3^3, \end{aligned} \tag{2.62}$$

$$\begin{aligned}\Psi &= e^{2(x\iota_1+t\varsigma_1)}k_1 + e^{2(x\iota_1+t\varsigma_4)}k_2 + e^{x\iota_1+t\varsigma_1}[\cosh(x\iota_1 - it\varsigma_2)\Theta_1 \\ &+ i\sinh(x\iota_1 - it\varsigma_2)\Theta_2] + e^{x\iota_3+y\rho_3+t\varsigma_3}[\cos(x\iota_1+t\varsigma_4)\Theta_3 \\ &+ \sin(x\iota_1+t\varsigma_4)\Theta_4]. \end{aligned} \tag{2.63}$$

$$\begin{aligned}u_{20} &= -[2[2e^{2(x\iota_1+t\varsigma_1)}k_1\iota_1 + 2e^{2(x\iota_1+t\varsigma_4)}k_2\iota_1 + e^{x\iota_1+t\varsigma_1}[\cosh(x\iota_1 - it\varsigma_2)\Theta_1 \\ &+ i\sinh(x\iota_1 - it\varsigma_2)\Theta_2]\iota_1 + e^{x\iota_1+t\varsigma_1}[\sinh(x\iota_1 - it\varsigma_2)\iota_1\Theta_1 + i\cosh(x\iota_1 \\ &- it\varsigma_2)\iota_1\Theta_2] + e^{x\iota_3+y\rho_3+t\varsigma_3}\iota_3[\cos(x\iota_1+t\varsigma_4)\Theta_3 + \sin(x\iota_1+t\varsigma_4)\Theta_4] \\ &+ e^{x\iota_3+y\rho_3+t\varsigma_3}[\cos(x\iota_1+t\varsigma_4)\iota_1\Theta_4 - \sin(x\iota_1+t\varsigma_4)\iota_1\Theta_3]]]/[e^{2(x\iota_1+t\varsigma_1)} \\ &* k_1 + e^{2(x\iota_1+t\varsigma_4)}k_2 + e^{x\iota_1+t\varsigma_1}[\cosh(x\iota_1 - it\varsigma_2)\Theta_1 + i\sinh(x\iota_1 - it\varsigma_2) \\ &* \Theta_2] + e^{x\iota_3+y\rho_3+t\varsigma_3}[\cos(x\iota_1+t\varsigma_4)\Theta_3 + \sin(x\iota_1+t\varsigma_4)\Theta_4]]. \end{aligned} \tag{2.64}$$

## 3. Conclusion

In this paper, the (2+1)-dimensional BLMP equation is discussed, which describes the incompressible fluid. Based on the bilinear form and an ansätz functions, many entirely new complexiton solutions and double periodic-soliton solutions are obtained. The dynamical behaviors are demonstrated in some three-dimensional plots by setting different values of the parameters. The ansätz function is very effective in solving the periodic solutions and complexiton solutions of the higher order nonlinear evolution equations. In Figs. 1-14, it is obvious that the waves are repeated at intervals of time or distance. All the solutions have been verified to be correct by symbolic computation software Mathematica.

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**Conflict of interest**

The authors declare that there is no conflict of interests regarding the publication of this article.

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**Symbolic program**

$$
\begin{aligned}
&In[1] := \Psi[x,y,t] = Exp[\iota_1\, x + \rho_1\, y + \varsigma_1\, t][\Theta_1 \cos[\iota_2\, x + \rho_2\, y + \varsigma_2\, t] \\
&+\Theta_2 \sin[\iota_2\, x + \rho_2\, y + \varsigma_2\, t]] + k_1 Exp[2(\iota_1\, x + \rho_1\, y + \varsigma_1\, t)] \\
&+Exp[\iota_3\, x + \rho_3\, y + \varsigma_3\, t][\Theta_3 \cos[\iota_4\, x + \rho_4\, y + \varsigma_4\, t] \\
&+\Theta_4 \sin[\iota_4\, x + \rho_4\, y + \varsigma_4\, t]] + k_2 Exp[\iota_4\, x + \rho_4\, y + \varsigma_4\, t] \\
&In[2] := M = Expand[-\Psi_t\Psi_y - \Psi_{xxx}\Psi_y + 3\Psi_{xy}\Psi_{xx} \\
&-3\Psi_x\Psi_{xxy} + \Psi\left(\Psi_{yt} + \Psi_{xxxy}\right)] \\
&In[3] := M1 = FullSimplify[Coefficient[M, Exp[2(\iota_1\, x + \rho_1\, y + \varsigma_1\, t) \\
&+2(\iota_4\, x + \rho_4\, y + \varsigma_4\, t)]]] \\
&In[4] := M2 = FullSimplify[Coefficient[M, Exp[3(\iota_1\, x + \rho_1\, y + \varsigma_1\, t)]]] \\
&In[5] := M3 = FullSimplify[Coefficient[M, Sin[\iota_2\, x + \rho_2\, y + \varsigma_2\, t]]] \\
&In[6] := M4 = FullSimplify[Coefficient[M, Cos[\iota_2\, x + \rho_2\, y + \varsigma_2\, t]]] \\
&In[7] := M5 = FullSimplify[Coefficient[M, Sin[\iota_4\, x + \rho_4\, y + \varsigma_4\, t]]] \\
&In[8] := M6 = FullSimplify[Coefficient[M, Cos[\iota_4\, x + \rho_4\, y + \varsigma_4\, t]]]
\end{aligned}
$$

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