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Research article

The price adjustment equation with different types of conformable derivatives in market equilibrium

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Abstract: In the current study, price adjustment equation which takes an important place in market equilibrium is presented in consideration of truncated M-derivative including Mittag-Leffler function, beta-derivative and conformable derivative defined in the form of limit for α-differentiable functions. These popular limit-based derivative and integral definitions enable α to vary between (0, 1], whereupon we can observe the intrinsic behavior of the competitive market at different times. The reason of popularity of the underlying definitions is that the natural appearances of their applications. Due to their similarity to classical derivative, it can be taken a good deal of advantages of them in terms of applicability to the diverse governing models. Hence we derive some novel solutions of the market equilibrium models which are big parts of our lives and in order to solve the linear ordinary differential equations in the sense of M-derivative and beta derivative, the solution methods are given. Moreover, we carry out simulation analysis in order to confirm the usefulness of results obtained.

Keywords: stability; time paths; M-derivative; beta-derivative; market equilibrium
Mathematics Subject Classification: 97M30, 97M10, 97M99

1. Introduction

Mathematical economic models representing economic processes by some variables enable economists to constitute expressive and favorable claims about controversial circumstances. The application of mathematical approach in economics dates back a long time. Further, mathematical tools bring tons of advantage to the economic models simplifying the reality that allow us better understanding and prediction about economic behavior. Market equilibrium and economic growth both take an important place for real world problems and also competition in the market play a crucial role for buyers and sellers of product. Buyers and sellers are two economic agents of economy concerning about commodities whose prices may increase or decrease rapidly. Each product has own price and buyers and sellers have not any control over price of a product. The attention can be taken to the fact that these agents are in so small relation with size of the economy, which shows us their effects as a consumer or a producer on market prices are neglectable. It is supposed that buyers achieve satisfaction over the economic products that they can buy with their income, and sellers derive a high profit from their production in a competitive manner. Details about economic models can be observed in [1,2].

In real life, it is difficult to determine whether buyers choose cheap products or expensive products because snob effect can cause buyers to choose expensive items. Hence there is a substantial complexity about buyer's choice. Nevertheless, important results can be obtained without taking individual satisfaction into consideration. In addition, competitive market is also concerned with competitive equilibrium which is a state of the whole quantity demanded by buyers of items and the whole quantity supplied by sellers of items are equivalent to each other. The quantity demanded by buyers is taken as a function called demand function and similarly, the quantity supplied by sellers is also taken as a function called supply function. These functions of product price are given as:

$$q_d = d_0 - d_1 p \quad and \quad q_s = -s_0 + s_1 p, \tag{1.1}$$

respectively, where p is the product price, q_d is the quantity demanded, q_s is the quantity supplied and d_0, d_1, s_0 and s_1 are positive constants. One can see easily that the equilibrium price can be obtained by taking $q_d = q_s$ and it is as follows

$$p^* = \frac{(d_0 + s_0)}{(d_1 + s_1)}. \tag{1.2}$$

Economists generally suppose that markets are in equilibrium which means supply of a product is exactly equal to demand of product. Hence there is not surplus or shortage in the market in this situation and price tends to remain stable. For instance, let us consider basic price adjustment equation:

$$\frac{dp}{dt} = \lambda(q_d - q_s), \tag{1.3}$$

where $\lambda > 0$ is the speed of adjustment constant. This linear model shows that when demand exceeds supply, the price rises and when supply exceeds demand, the price falls. Plugging the Eq. (1.1) in Eq. (1.3), one gets

$$p' + \lambda(d_1 + s_1)p = \lambda(d_0 + s_0), \tag{1.4}$$

and if we solve this linear differential equation, then we obtain

$$p(t) = p^* + [p(0) - p^*]e^{-\lambda(d_1+s_1)t}. \tag{1.5}$$

On the other hand, it can be considered a model that allows the expectations of agents. In this case, demand and supply functions are written as

$$q_d(t) = d_0 - d_1 p(t) + d_2 p'(t) \quad and \quad q_s(t) = -s_0 + s_1 p(t) - s_2 p'(t), \tag{1.6}$$

respectively, where $p(t)$ is the product price, $q_d(t)$ is the quantity demanded, $q_s(t)$ is the quantity supplied and d_0, d_1, d_2, s_0, s_1 and s_2 are positive constants. For $q_d(t) = q_s(t)$, following equation is obtained

$$p'(t) - \frac{(d_1 + s_1)}{(d_2 + s_2)} p(t) = -\frac{(d_0 + s_0)}{(d_2 + s_2)}, \tag{1.7}$$

and the solution of this linear equation as below:

$$p(t) = p^* + [p(0) - p^*]e^{\frac{(d_1+s_1)}{(d_2+s_2)}t}. \tag{1.8}$$

Note that, as product prices increase, demanders want to buy more before prices increase further. Similarly, when product prices increase, suppliers are in tendency to offer less to get benefit from higher prices in future. In this model, $p(t)$ satisfy $q_d(t) = q_s(t)$ restriction for all $t \geq 0$ and situation of prices can be observed by this way. Moreover, when $p'(t) = 0$ for all $t \geq 0$, it can be said that market is in dynamic equilibrium which means equilibrium in a changing economy. Market equilibrium model mentioned above is seen in [3].

In recent years, some natural and beneficial derivative and integral definitions allowing to investigate the analysis of α-differentiable functions have been presented. One of the most popular of these proposed by Khalil et al [4] is conformable derivative which brings the crucial properties into conformity with the common properties of calculus. This efficient derivative resemble classical derivative in many aspects such as derivative of the product and quotient of two functions, chain rule as well as integration by parts formulas, Taylor power series and Laplace transform of some functions are presented by Abdeljawad [5–7]. Further, a physical application about projectile motion in a resisting medium by preserving the dimensionality of physical quantities given by Alharbi et al [8] makes conformable derivative more interesting in terms of applicability to physical problems, and also other interesting studies can be seen in [9–21]. It is well-known that differential equations have a great importance in modelling problems because even a simple differential equation enables to explain complicated natural phenomena. In addition, it can be seen easily that real world problems become more complicated day by day and so it is possible to need novel mathematical tools. Accordingly, Atangana introduced any other local derivative called beta-derivative to solve Hunter-Saxton partial differential equation [22, 23]. This derivative is the modified version of conformable derivative definition by inserting $(t + \frac{1}{\Gamma(\alpha)})^{1-\alpha}$, $0 < \alpha < 1$ into the limit. On the other hand, Katugampola [24] has introduced Katugampola derivative including exponential function inside the limit as an alternative definition to the conformable derivative. This derivative is defined as below:

$$D^\alpha f(t) = \lim_{\varepsilon \to 0} \frac{f(te^{\varepsilon t^{-\alpha}}) - f(t)}{\varepsilon}. \tag{1.9}$$

More recently, Sousa et al [25] have defined a novel truncated M-derivative containing Mittag-Leffler function instead of exponential function in Katugampola derivative. Due to the additional parameter γ inside Mittag-Leffler function, it can be considered as a generalized form of local derivatives for α-differentiable functions. Note that all of these limit-based local derivatives without memory should not be called fractional derivatives owing to lack of semigroup property and because we do not get the original function, when α is in tendency to be zero. Also, all these definitions have almost identical properties.

The manuscript has been prepared as follows: In Section 2, some vital theorems and definitions about our study are submitted. In Section 3, we analyze the solution method of linear ordinary differential equations based on truncated M-derivative and beta derivative and solve the price adjustment equation by means of truncated M-derivative, beta-derivative and conformable derivative in case of expectations of agents ignored and expectations of agents taken into account. In Section 4, some figures presented for much more understandable of the economic models studied in this paper.

2. Preliminaries

In this portion, we provide some necessary definitions, theorems and properties about some local derivatives which will serve the purpose of the main results of the manuscript.

2.1. On M-derivative and integral

Definition 2.2. [25] The truncated Mittag-Leffler function with one parameter is defined by

$$ {}_iE_\gamma(z) = \sum_{k=0}^{i} \frac{z^k}{\Gamma(\gamma k + 1)} \tag{2.1} $$

with $\gamma > 0$ and $z \in \mathbb{C}$.

Definition 2.3. [25] The truncated M-derivative for $0 < \alpha < 1$ is given by

$$ {}_i\mathcal{D}_M^{\alpha,\gamma} f(t) = \lim_{\varepsilon \to 0} \frac{f(t\,{}_iE_\gamma(\varepsilon t^{-\alpha})) - f(t)}{\varepsilon}, \tag{2.2} $$

$f : [0, \infty) \to \mathbb{R}$, $\forall t > 0$ and ${}_iE_\gamma(.)$, $\gamma > 0$ is the truncated Mittag-Leffler function defined in Eq. (2.1).

Theorem 2.1. *[25] If $0 < \alpha < 1$, $\gamma > 0$, $a, b \in \mathbb{R}$ and f, g are α-differentiable functions at a $t > 0$ point, then*

1. ${}_i\mathcal{D}_M^{\alpha,\gamma}(af + bg)(t) = a\,{}_i\mathcal{D}_M^{\alpha,\gamma} f(t) + b\,{}_i\mathcal{D}_M^{\alpha,\gamma} g(t)$.
2. ${}_i\mathcal{D}_M^{\alpha,\gamma}(f.g)(t) = f(t)\,{}_i\mathcal{D}_M^{\alpha,\gamma} g(t) + g(t)\,{}_i\mathcal{D}_M^{\alpha,\gamma} f(t)$.
3. ${}_i\mathcal{D}_M^{\alpha,\gamma}(\frac{f}{g})(t) = \frac{g(t)\,{}_i\mathcal{D}_M^{\alpha,\gamma} f(t) - f(t)\,{}_i\mathcal{D}_M^{\alpha,\gamma} g(t)}{[g(t)]^2}$.
4. ${}_i\mathcal{D}_M^{\alpha,\gamma}(c) = 0$, c is a constant.
5. Let f is differentiable function, then ${}_i\mathcal{D}_M^{\alpha,\gamma} f(t) = \frac{t^{1-\alpha}}{\Gamma(\gamma+1)} \frac{df(t)}{dt}$.
6. (Chain rule) ${}_i\mathcal{D}_M^{\alpha,\gamma}(f \circ g)(t) = f'(g(t))\,{}_i\mathcal{D}_M^{\alpha,\gamma} g(t)$, f is a differentiable function.

Definition 2.4. [25] Let $a \geq 0$, $t \geq a$, f is defined in $(a, t]$ and $0 < \alpha < 1$. Then, the M-integral is defined by

$$ {}_M\mathcal{I}_a^{\alpha,\gamma} f(t) = \Gamma(\gamma + 1) \int_a^t \frac{f(x)}{x^{1-\alpha}} dx, \tag{2.3} $$

where $\gamma > 0$.

2.5. On beta-derivative and integral

Definition 2.6. [22] Let $f : [0, \infty) \to \mathbb{R}$ and $\beta \in (0, 1]$, then Atangana's beta-derivative is defined as below:

$$ {}_0^A\mathcal{D}_t^\beta f(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon(t + \frac{1}{\Gamma(\beta)})^{1-\beta}) - f(t)}{\varepsilon}. \tag{2.4} $$

Theorem 2.2. *[23] Let f is differentiable and β-differentiable function on the (a, b), then*

$$ {}_0^A\mathcal{D}_t^\beta f(t) = \left(t + \frac{1}{\Gamma(\beta)}\right)^{1-\beta} \frac{df(t)}{dt}. \tag{2.5} $$

Theorem 2.3. *[22] Let f and g are two β-differentiable functions with $\beta \in (0, 1]$, then*

1. ${}_0^A\mathcal{D}_t^\beta(af + bg)(t) = a\,{}_0^A\mathcal{D}_t^\beta f(t) + b\,{}_0^A\mathcal{D}_t^\beta g(t)$.
2. ${}_0^A\mathcal{D}_t^\beta(f.g)(t) = f(t)\,{}_0^A\mathcal{D}_t^\beta g(t) + g(t)\,{}_0^A\mathcal{D}_t^\beta f(t)$.
3. ${}_0^A\mathcal{D}_t^\beta(\frac{f}{g})(t) = \frac{g(t)\,{}_0^A\mathcal{D}_t^\beta f(t) - f(t)\,{}_0^A\mathcal{D}_t^\beta g(t)}{[g(t)]^2}$.
4. ${}_0^A\mathcal{D}_t^\beta(c) = 0$, *$c$ is a constant.*

Definition 2.7. [22] Let $f : [0, \infty) \to \mathbb{R}$, then Atangana beta-integral of f function is defined by

$$ {}_0^A\mathcal{I}_t^\beta f(t) = \int_0^t \left(x + \frac{1}{\Gamma(\beta)}\right)^{\beta-1} f(x)dx. \tag{2.6}$$

2.8. On conformable derivative and integral

Definition 2.9. [4] Assuming that $f : [0, \infty) \to \mathbb{R}$, then conformable derivative is given by

$$\mathcal{D}_\alpha f(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}, \tag{2.7}$$

where $t > 0$ and $0 < \alpha < 1$.

Theorem 2.4. *[4] Let f, g are α-differentiable at a point $t > 0$ and $0 < \alpha < 1$. Then we get*

1. $\mathcal{D}^\alpha(af + bg)(t) = a\mathcal{D}^\alpha f(t) + b\mathcal{D}^\alpha g(t)$, *for all $a, b \in \mathbb{R}$.*
2. $\mathcal{D}^\alpha(t^r) = rt^{r-\alpha}$, *for all $r \in \mathbb{R}$.*
3. $\mathcal{D}(c) = 0$, *c is a constant.*
4. $\mathcal{D}^\alpha(f.g)(t) = f(t)\mathcal{D}^\alpha g(t) + g(t)\mathcal{D}^\alpha f(t)$.
5. $\mathcal{D}^\alpha(\frac{f}{g})(t) = \frac{g(t)\mathcal{D}^\alpha f(t) - f(t)\mathcal{D}^\alpha g(t)}{[g(t)]^2}$.
6. *Let f is a differentiable function, then $\mathcal{D}^\alpha f(t) = t^{1-\alpha}\frac{df}{dt}(t)$.*

Theorem 2.5. *[4] Let $0 < \alpha < 1$ and $a, r \in \mathbb{R}$. Then following results are obtained.*

1. $\mathcal{D}^\alpha(e^{at}) = at^{1-\alpha}e^{at}$.
2. $\mathcal{D}^\alpha(\sin at) = at^{1-\alpha}\cos at$.
3. $\mathcal{D}^\alpha(\cos at) = -at^{1-\alpha}\sin at$.
4. $\mathcal{D}^\alpha(\frac{t^\alpha}{\alpha}) = 1$.

Theorem 2.6. *[4] Let $0 < \alpha < 1$ and $t > 0$. Then*

1. $\mathcal{D}^\alpha(\sin\frac{t^\alpha}{\alpha}) = \cos\frac{t^\alpha}{\alpha}$.
2. $\mathcal{D}^\alpha(\cos\frac{t^\alpha}{\alpha}) = -\sin\frac{t^\alpha}{\alpha}$.
3. $\mathcal{D}^\alpha(e^{\frac{t^\alpha}{\alpha}}) = e^{\frac{t^\alpha}{\alpha}}$.

Definition 2.10. [4] Let $0 < \alpha < 1$, $f : [a, \infty) \to \mathbb{R}$, $a > 0$ then conformable integral is defined by

$$\mathcal{I}_a^t f(t) = \int_a^t \frac{f(x)}{x^{1-\alpha}}dx, \tag{2.8}$$

where the Riemann improper integral exists.

3. Main results

In this portion, our aim is to submit main finding and results for economic models which are investigated in detailed by means of truncated M-derivative, beta-derivative and conformable derivative. The crucial claim is to get much better results via underlying derivatives and integrals for α-differentiable functions.

3.1. Analysis of linear differential equation via M-derivative

In order to find solutions of problems encountered in every branch of science, there is a need to establish a mathematical model with the characteristic properties of the problem. In this process, differential equations have a great significance in real world problems because especially linear differential equations are the proper tool to define reality. In accordance with this purpose, we have been inspired to submit first-order linear ordinary differential equations in terms of M-derivative including important Mittag-Leffler function.

The general first-order linear differential equation based on M-derivative can be expressed by

$$ {}_i\mathcal{D}_M^{\alpha,\gamma} y(x) + P(x)y(x) = Q(x), \tag{3.1}$$

where $P(x)$ and $Q(x)$ are α-differentiable functions and y(x) is an unknown function. If we use the property (5) given in Theorem 2.1 and by applying to the Eq. (3.1), we can express Eq. (3.1) as below:

$$\frac{x^{1-\alpha}}{\Gamma(\gamma+1)} y'(x) + P(x)y(x) = Q(x), \tag{3.2}$$

$$y'(x) + \frac{\Gamma(\gamma+1)}{x^{1-\alpha}} P(x)y(x) = \frac{\Gamma(\gamma+1)}{x^{1-\alpha}} Q(x), \tag{3.3}$$

and the general solution of Eq. (3.3) is obtained as the following

$$y(x) = e^{-\int \frac{\Gamma(\gamma+1)}{x^{1-\alpha}} P(x)dx} \int \frac{\Gamma(\gamma+1)}{x^{1-\alpha}} Q(x) e^{\int \frac{\Gamma(\gamma+1)}{x^{1-\alpha}} P(x)dx} dx + c e^{-\int \frac{\Gamma(\gamma+1)}{x^{1-\alpha}} P(x)dx}. \tag{3.4}$$

If we use definition of M-integral given in Eq. (2.3) and Eq. (3.4), then we have the following general solution of Eq. (3.1),

$$y(x) = e^{-{}_M\mathcal{I}_a^{\alpha,\gamma}(P(x))} [{}_M\mathcal{I}_a^{\alpha,\gamma}(Q(x) e^{-{}_M\mathcal{I}_a^{\alpha,\gamma}(P(x))})] + c e^{-{}_M\mathcal{I}_a^{\alpha,\gamma}(P(x))}. \tag{3.5}$$

Consequently, we get Eq. (3.5) as a general solution of Eq. (3.1).

3.2. Price adjustment equation via M-derivative

- *Market equilibrium model without taking into account the expectations of agents*:

Eq. (1.4) can be written by means of M-derivative as below:

$$ {}_i\mathcal{D}_M^{\alpha,\gamma} p(t) + \lambda(d_1 + s_1)p(t) = \lambda(d_0 + s_0), \quad \alpha \in (0,1). \tag{3.6}$$

If we apply the method given in subsection 3.1, then

$$p(t) = \frac{(d_0 + s_0)}{(d_1 + s_1)} + \left[p(0) - \frac{(d_0 + s_0)}{(d_1 + s_1)}\right] e^{-\lambda(d_1+s_1)\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}. \tag{3.7}$$

If we put p^* given in Eq. (1.2) instead of $\frac{(d_0+s_0)}{(d_1+s_1)}$, then we get

$$p(t) = p^* + [p(0) - p^*] e^{-\lambda(d_1+s_1)\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}. \tag{3.8}$$

- *Market equilibrium model taking into account the expectations of agents*:

Eq. (1.7) with Mderivative can be given by

$${}_i\mathcal{D}_M^{\alpha,\gamma} p(t) - \frac{(d_1 + s_1)}{(d_2 + s_2)} p(t) = -\frac{(d_0 + s_0)}{(d_2 + s_2)}, \quad \alpha \in (0, 1). \tag{3.9}$$

If we solve Eq. (3.9) with the help of method given in subsection 3.1, then we obtain the following solution

$$p(t) = p^* + [p(0) - p^*] e^{\frac{(d_1+s_1)}{(d_2+s_2)}\Gamma(\gamma+1)\frac{t^\alpha}{\alpha}}, \tag{3.10}$$

where $p^* = \frac{d_0+s_0}{d_1+s_1}$.

3.3. Analysis of linear differential equation via beta-derivative

The general first-order linear differential equation with beta-derivative can be expressed by

$${}_0^A\mathcal{D}_x^\beta y(x) + P(x)y(x) = Q(x), \tag{3.11}$$

where $P(x)$ and $Q(x)$ are β-differentiable functions and y(x) is an unknown function. If we use the following property of beta-derivative

$${}_0^A\mathcal{D}_t^\beta f(t) = \left(t + \frac{1}{\Gamma(\beta)}\right)^{1-\beta} \frac{df(t)}{dt}, \tag{3.12}$$

and applying to the Eq. (3.11), we get

$$\left(x + \frac{1}{\Gamma(\beta)}\right)^{1-\beta} y'(x) + P(x)y(x) = Q(x), \tag{3.13}$$

$$y'(x) + \left(x + \frac{1}{\Gamma(\beta)}\right)^{\beta-1} P(x)y(x) = \left(x + \frac{1}{\Gamma(\beta)}\right)^{\beta-1} Q(x), \tag{3.14}$$

and we get the general solution of Eq. (3.14) as below:

$$y(x) = e^{-\int (x+\frac{1}{\Gamma(\beta)})^{\beta-1} P(x)dx} \int \left(x + \frac{1}{\Gamma(\beta)}\right)^{\beta-1} Q(x) e^{(x+\frac{1}{\Gamma(\beta)})^{\beta-1} P(x)dx} dx + c e^{-\int (x+\frac{1}{\Gamma(\beta)})^{\beta-1} P(x)dx}, \tag{3.15}$$

$$y(x) = e^{{}_0^A\mathcal{I}_x^\beta(P(x))} [{}_0^A\mathcal{I}_x^\beta(Q(x) e^{{}_0^A\mathcal{I}_x^\beta(P(x))})] + c e^{{}_0^A\mathcal{I}_x^\beta(P(x))}. \tag{3.16}$$

3.4. *Price adjustment equation via beta-derivative*

- *Market equilibrium model without taking into account the expectations of agents*:

Eq. (1.4) with beta-derivative can be written by

$$ {}_0^A\mathcal{D}_t^\beta p(t) + \lambda(d_1 + s_1)p(t) = \lambda(d_0 + s_0). \tag{3.17} $$

Applying the method given in subsection 3.1, we get

$$ p(t) = \frac{(d_0 + s_0)}{(d_1 + s_1)} + c_1 e^{-\lambda(d_1+s_1)\frac{(t+\frac{1}{\Gamma(\beta)})^\beta}{\beta}}. \tag{3.18} $$

If we put p^* instead of $\frac{(d_0+s_0)}{(d_1+s_1)}$, then

$$ p(t) = p^* + c_1 e^{-\lambda(d_1+s_1)\frac{(t+\frac{1}{\Gamma(\beta)})^\beta}{\beta}}, \tag{3.19} $$

where $c_1 = [p(0) - p^*]e^{\lambda(d_1+s_1)(\frac{1}{\Gamma(\beta)})^\beta}$ is the integration constant.

- *Market equilibrium model taking into account the expectations of agents*:

Eq. (1.7) can be written by the means of beta-derivative as the following

$$ {}_0^A\mathcal{D}_t^\beta p(t) - \frac{(d_1 + s_1)}{(d_2 + s_2)}p(t) = -\frac{(d_0 + s_0)}{(d_2 + s_2)}, \quad 0 < \beta < 1. \tag{3.20} $$

If we solve Eq. (3.20) with the help of method presented in subsection 3.1, then we obtain

$$ p(t) = p^* + c_2 e^{\frac{(d_1+s_1)}{(d_2+s_2)}\frac{(t+\frac{1}{\Gamma(\beta)})^\beta}{\beta}}, \tag{3.21} $$

where $p^* = \frac{d_0+s_0}{d_1+s_1}$ and $c_2 = [p(0) - p^*]e^{-\frac{(d_1+s_1)}{(d_2+s_2)}\frac{(\frac{1}{\Gamma(\beta)})^\beta}{\beta}}$.

3.5. *Price adjustment equation via conformable derivative*

- *Market equilibrium model without taking into account the expectations of agents*:

Eq. (1.4) can be presented by way of conformable derivative as below:

$$ \mathcal{D}^\alpha p(t) + \lambda(d_1 + s_1)p(t) = \lambda(d_0 + s_0), \quad 0 < \alpha < 1. \tag{3.22} $$

Applying the same method mentioned above to get general solution of price adjustment equation, we get

$$ p(t) = p^* + [p(0) - p^*]e^{-\lambda(d_1+s_1)\frac{t^\alpha}{\alpha}}. \tag{3.23} $$

- *Market equilibrium model taking into account the expectations of agents*:

Eq. (1.7) can be given by the following form in the terms of conformable derivative

$$ \mathcal{D}^\alpha p(t) - \frac{(d_1 + s_1)}{(d_2 + s_2)}p(t) = -\frac{(d_0 + s_0)}{(d_2 + s_2)}, \quad 0 < \alpha < 1. \tag{3.24} $$

If we solve Eq. (3.24), we obtain the following general solution

$$ p(t) = p^* + [p(0) - p^*]e^{\frac{(d_1+s_1)}{(d_2+s_2)}\frac{t^\alpha}{\alpha}}. \tag{3.25} $$

4. Discussion

This portion contains the vital performances of market equilibrium models addressed by means of truncated M-derivative, beta-derivative and conformable derivative depending on basic limit definition of integer-order derivative. Firstly, we want to draw attention to these local derivatives containing different terms in the limit and each term provides separate advantages to the governing models. These derivatives allowed α to vary between $(0, 1]$ are an alternative to the classical limit-based derivative. They are a modified version of each other and also they have several characteristics in common, which enable a large amount of convenience in calculations. Even if we don't call them as a fractional derivative, they involve non-integer order and it is clear that they have taken an important place in modelling many physical and biological problems in recent years. For this reason, we feel motivated to provide market equilibrium model by means of these well-behaved derivatives. To this aim, we start by plotting the demand and supply curves which have a strong correlation in Figure 1. What we want to show in this figure is to consider both demand and supply for a product at different prices. It is well known that the vertical axis is the price axis while the horizontal axis is the quantity of the product. Besides, we have to set a period to understand the price change process rather better. Let us think about potential supply and demand curves for the market. If the price of the product is too high, what happens to consumer mass? In this case, it is clear that consumers' demand will decrease and so the quantity demanded will be low. Note that the difference between demand and quantity demanded. The former is the whole relationship and the latter means that the actual specific quantity. Contrary to this case where price is too high, if price is too low, the quantity demanded will become really high. Moreover, if we consider the supply curve, it can be easily said that at some low price, the suppliers in the market are not willing to produce the product. As the price increases, the suppliers desire to produce much more.

It is possible to consider different scenarios, for instance, the suppliers or producers of the product may consider to sell the product at low price for whatever reason or unlike, the price and quantity may increase so much. What happens if these scenarios come true? In the first situation, the quantity demanded will be too high owing to the low price and people are faced with a shortage because suppliers will not be able to produce enough product. As it continues this way, the prices are going to start increase. If we look at from the producers point of view, it can be easily seen that the prices and quantity will go up. So, shortage will start to decrease over time. Let us think the other situation where price and quantity are too high. Accordingly, in essence, people will face with overshoots or surplus due to much lower quantity demanded because consumers desire to buy an item that they can afford to buy. So, producers will tend to cut prices to attract some consumers and maybe in the next period, there is a less of overshoots. Eventually, it can be seen that there is an intersection point in Figure 1 where we demonstrate the equilibrium price and equilibrium quantity. At that point, the quantity supplied and the quantity demanded equals one another and intersecting supply and demand curves is exactly desirable situation in the market because there is neither shortage or surplus of item. However, increasing revenue does not mean that the profit will increase so the main purpose is to increase the profit, not revenue. In addition, not only one company but also great numbers of company and consumers have to participate in the market to achieve market equilibrium.

Furthermore, in Figures 2–5 we compare the M-derivative, beta-derivative and conformable derivative with classical derivative when $\alpha = 0.9, \beta = 0.9$ and $\gamma = 0.02, 0.95, 1.04, 1.08$, respectively.

Due to the changing values of γ, one can clearly observe that the behavior of M-derivative with appropriate values of α and γ. Figure 6 demonstrates comparison analysis between underlying derivatives when $\alpha = 0.98, \beta = 0.98$ and $\gamma = 1.04$. It can be properly seen that as α and β approaches one, M-derivative, beta-derivative and conformable derivative approach integer order. In this manner, we illustrate the features of market equilibrium models involving M-derivative, beta-derivative and conformable derivative with the suitable values of α, β and γ. In similar way, Figures 7-9 exhibit the comparison analysis as α and β change.

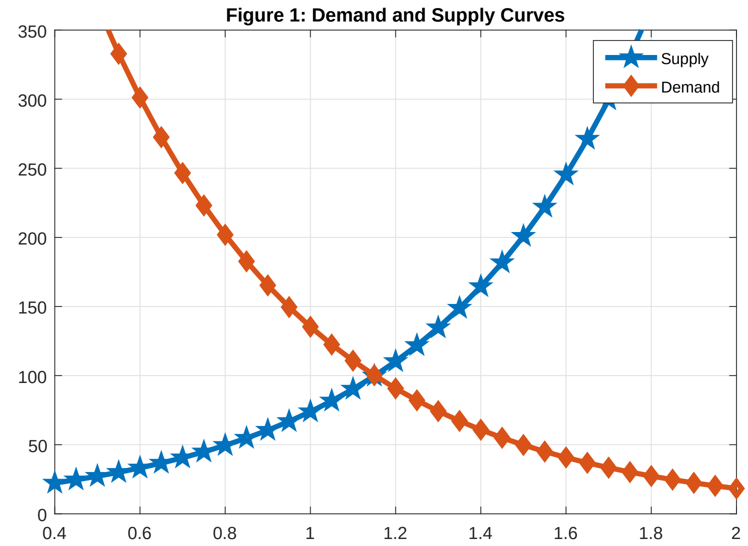

Figure 1. Demand and supply curves.

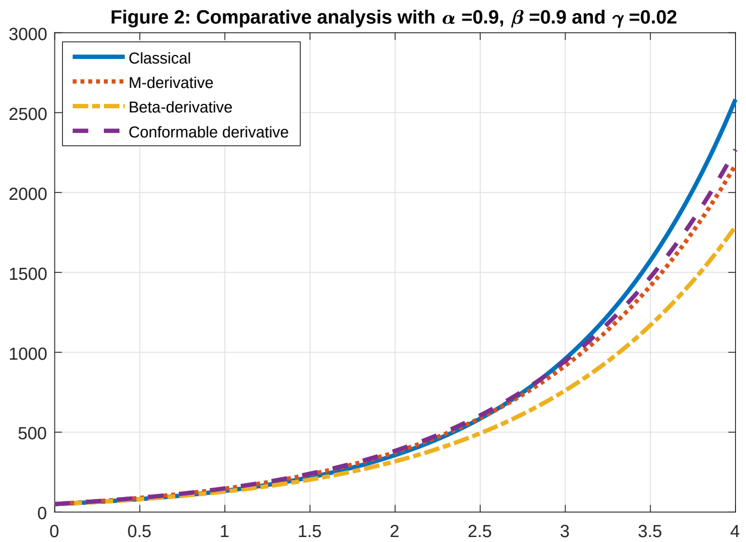

Figure 2. Comparative analysis with $\alpha = 0.9, \beta = 0.9$ and $\gamma = 0.02$.

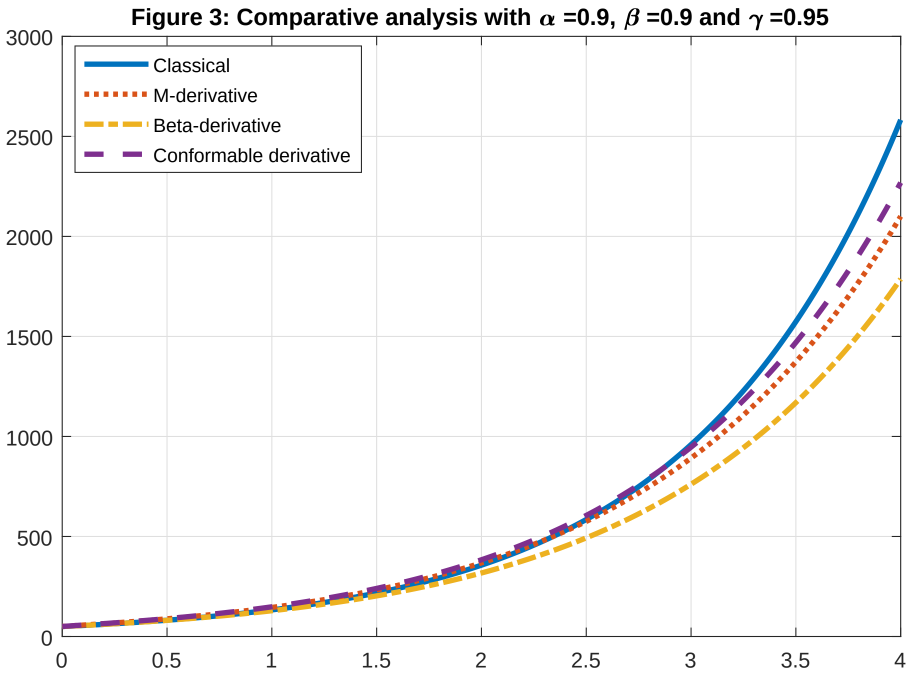

Figure 3. Comparative analysis with $\alpha = 0.9, \beta = 0.9$ and $\gamma = 0.95$.

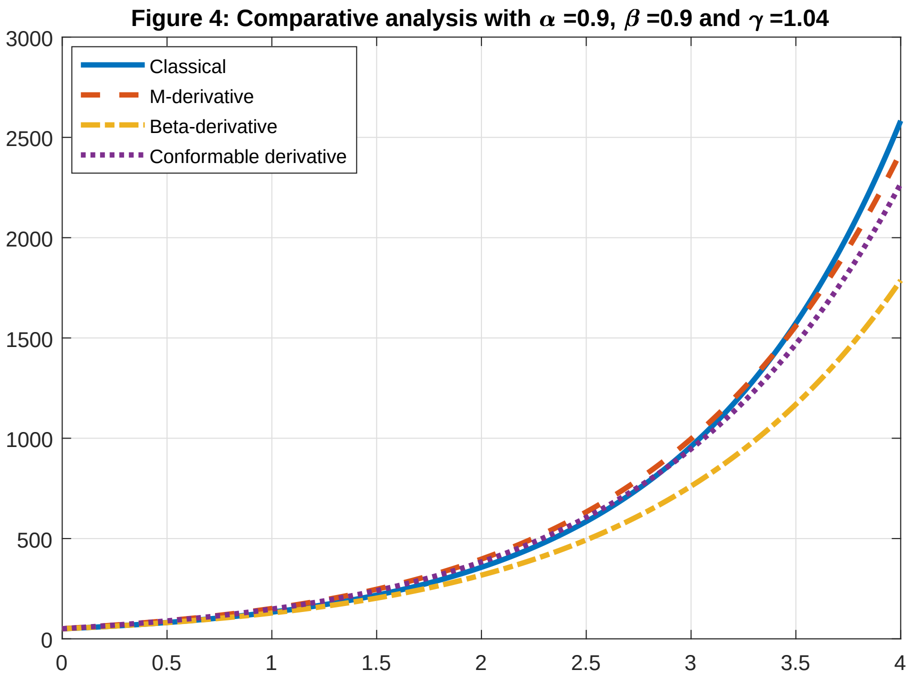

Figure 4. Comparative analysis with $\alpha = 0.9, \beta = 0.9$ and $\gamma = 1.04$.

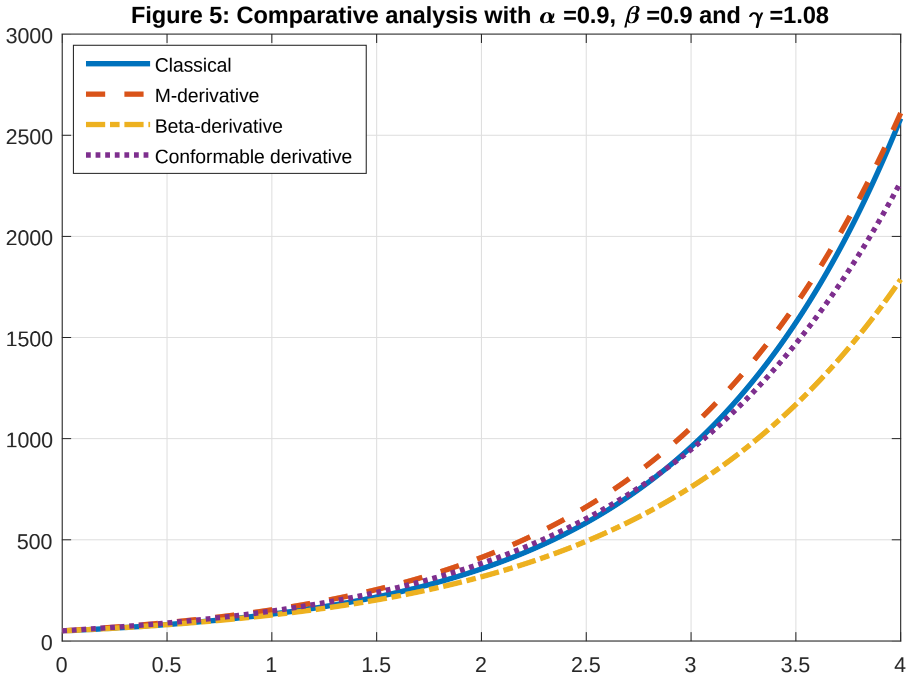

Figure 5. Comparative analysis with $\alpha = 0.9, \beta = 0.9$ and $\gamma = 1.08$.

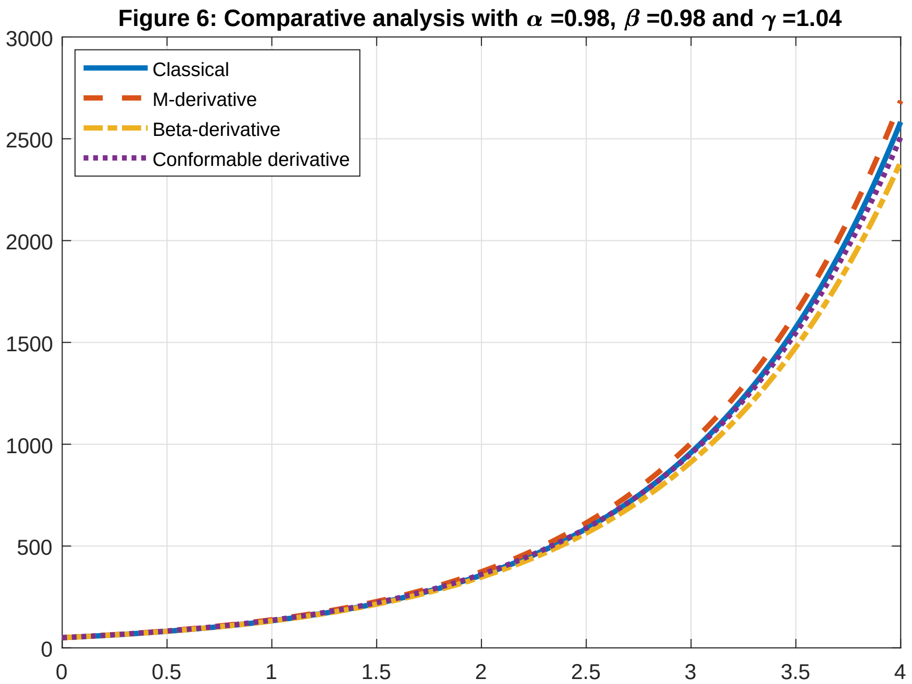

Figure 6. Comparative analysis with $\alpha = 0.98, \beta = 0.98$ and $\gamma = 1.04$.

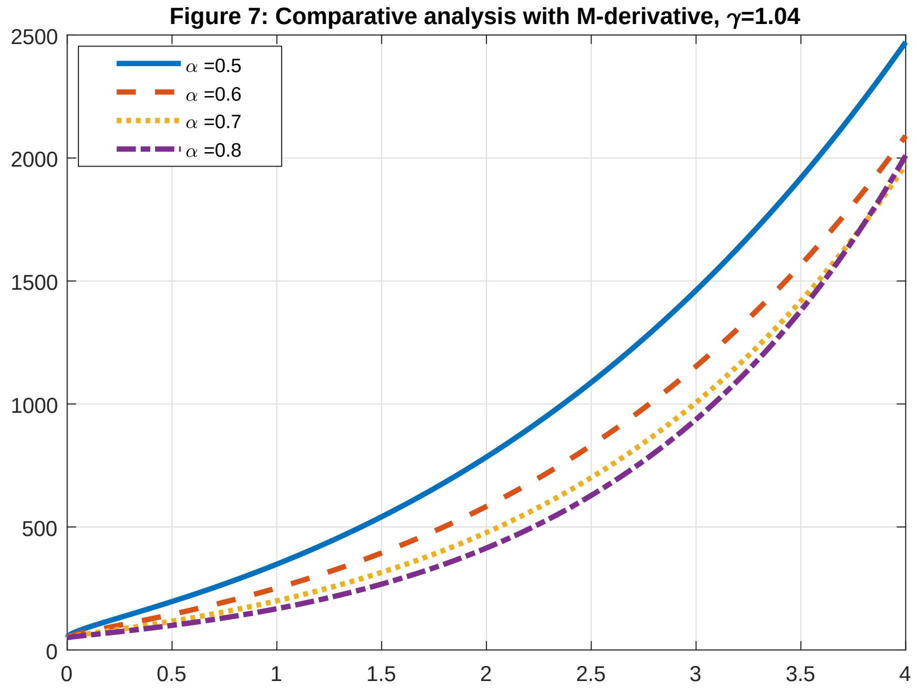

Figure 7. Comparative analysis with M-derivative, $\gamma = 1.04$.

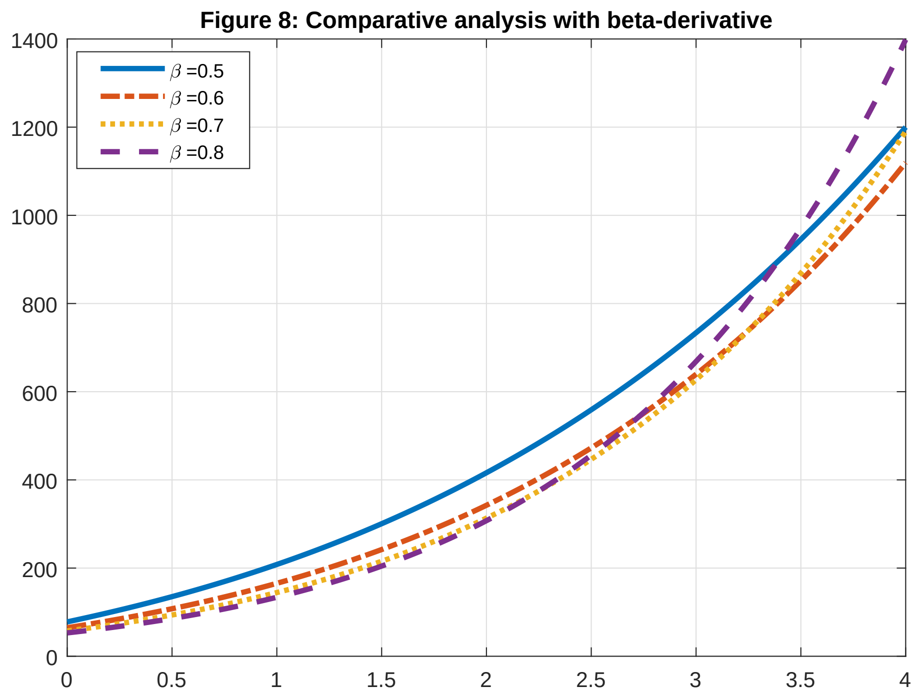

Figure 8. Comparative analysis with beta-derivative.

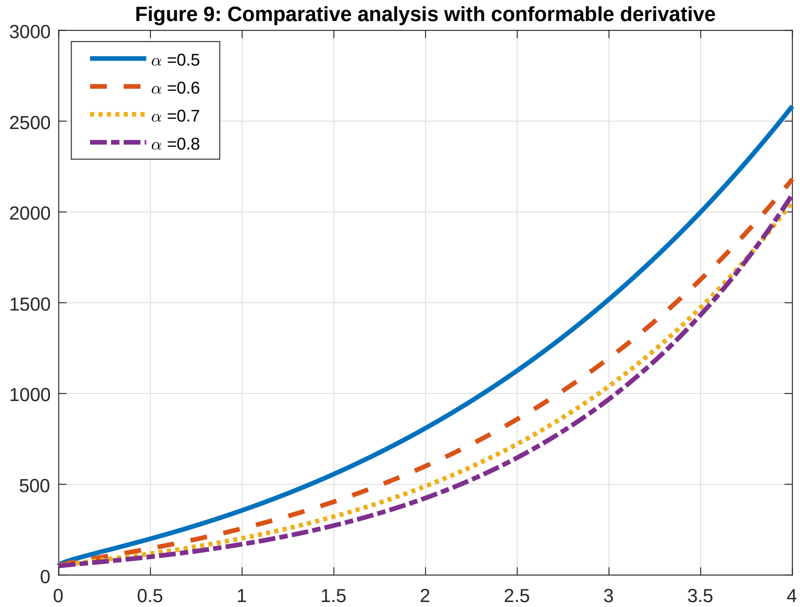

Figure 9. Comparative analysis with conformable derivative.

5. Conclusion

We have studied market equilibrium models via truncated M-derivative, beta-derivative and conformable derivative for α-differentiable functions when $\alpha \in (0, 1]$. One of the great advantages of these models over their counter classical versions is that they have arbitrary orders of derivative. To this aim, after providing the methods for linear ordinary differential equations by means of M-derivative and beta-derivative, we have introduced some crucial results with the help of these methods. Moreover, we have analyzed them in detailed by the graphical explanations and comparison analysis. As a result, the observations based on underlying derivatives for α-differentiable functions have matched those obtained in the classical case, but the models including arbitrary orders have made the models more convenient. So, we have investigated these governing models from the viewpoint of some kinds of local derivatives having the conformity with traditional derivative but as a future direction it can be stressed that these models will be also analyzed by virtue of the non-local fractional derivatives allowing to take advantage of the memory effect.

Conflict of interest

All authors declare no conflicts of interest in this paper.

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