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Research article

Asset pricing models in South Africa: A comparative of regression analysis and the Bayesian approach

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Abstract: This study investigated the risk-return relationship using the Capital Asset Pricing Model (CAPM), Carhart four-factor Model (C4FM) and Fama and French Multifactor Models (FFMMs): F3FM and F5FM. This study analyzed the JSE ALSI returns of the South African market, in relation to the risk factors constructed by the data of twenty-six emerging markets, over the sample period from October 2000 to October 2021. The methodology employed was a comparison of the conventional regression analysis and novel Bayesian approach. Regression analysis estimated by Ordinary Least Squares (OLS) is one of the foremost methods used in South Africa. However, such an approach does not take into account the properties of the price data, namely, the asymmetric, volatile and random nature. While the Newey-West adjustment is one way to assist in capturing autocorrelation and volatility, it fails to consider asymmetry. The Bayesian approach accounts for all the aforementioned properties, overcoming the fundamental flaws of regression analysis. The additional model diagnostics highlighted in this study improved the Bayesian approach' robustness. The risk factors of the FFMMs estimated by regression analysis, with and without the Newey-West adjustment, were insignificant, whereas the Bayesian test results were significant. This finding clearly highlighted that model choice impacts the significance of parameter estimation and the financial decisions of investors, firms and policymakers. Jensen's alpha revealed CAPM to be optimal but none of the asset pricing models fully captured the risk premia. Thus, returns should be investigated with different risk measures for future research purposes.

Keywords: risk-return relationship; asset pricing models; Capital Asset Pricing Model; Fama and French Multifactor Models; Fama and French three-factor Model; Carhart four-factor Model; Fama and French five-factor Model; regression analysis; Bayesian approach; asymmetry

JEL Codes: C22, C58, G12, G32

1. Introduction

The field of asset pricing models is broad and constantly evolving with new factors and ways to explain returns. A fundamental model, the Capital Asset Pricing Model (CAPM) by Markowitz (1952) and Sharpe (1964), attempted to explain returns by systematic risk - the risk exposure that arises from the market. However, the CAPM was critiqued for being a single-index model (Charteris et al., 2018; Cox and Britten, 2019; Molele and Mukuddem-Petersen, 2020). This gave rise to multifactor models, such as the Fama and French Multifactor Models (FFMMs), which were considered more empirically viable. The authors found that firm factors increased the explanatory power and significantly impacted expected returns, in addition to the original systematic risk (Fama and French, 1996, 2015).

In South African literature, there exists conflicting evidence on the optimal asset pricing model due to different variations and extended versions of the CAPM. For example, Peerbhai and Strydom (2018) took into account global and exchange rate factors, Charteris et al. (2018) the momentum factor, Molele and Mukuddem-Petersen (2020) and Mpoha and Bonga-Bonga (2020) exchange rate risk. More importantly, the foremost methods used to estimate the risk-return relationship were regression analysis (Charteris et al. 2018; Steyn and Theart, 2019; Molele and Mukuddem-Petersen, 2020) and the Fama and Macbeth (1973) regression (Peerbhai and Strydom, 2018; Cox and Britten, 2019).

Regression analysis and the Fama and Macbeth regression assume that price data is normally distributed. Despite the normality assumption, adjustments were made to improve the modelling approaches. For example, Charteris et al. (2018) applied the Newey-West adjustment, which captured autocorrelation and volatility, but failed to take into account the asymmetry. Moreover, Gow et al. (2010) and González-Sánchez (2021) found that the Newey-West and Fama-Macbeth corrections are suboptimal and lead to misspecified results. Therefore, the problem statement is that due to the normality assumption of regression analysis, the risk-return relationship is inaccurately captured, leading to inaccurate financial decisions and unreliable contributions made to the field of asset pricing models.

Consequently, this study focused on the methodology used to investigate the optimal asset pricing model. This is in line with Dutta (2019), who emphasized the power of methods used to capture the risk-return relationship. It follows that the more robust the methodology employed, the more practical the understanding, insight and applicability of the test results and factors involved. Thus, more optimal conclusions can be drawn and a solid contribution can be made to the field of asset pricing models. Hence, this study employed the novel Bayesian approach by Jensen and Maheu (2018), the only known study to the best of the authors knowledge, to apply this method to the risk-return relationship topic. The Bayesian approach, based on Bayes (1763) theorem, is defined as the probability estimation of an event given prior information. In this study, the event is the relationship between risk and return, and the prior refers to the inherent nature of financial data - the asymmetric, volatile and random properties of the risk-return variables.

South African studies by Cox and Britten (2019) and Charteris et al. (2018) used conventional model measures, such as the adjusted-R^2 to report on model diagnostics after model estimation. On the other hand, the Bayesian approach has additional statistical measures, that can be analyzed during model estimation, to improve the accuracy of the final test results (Mackenzie et al. 2018; Karabatsos, 2017). Given the amount of South African studies, that still employ the conventional regression analysis, this study aims to highlight the novel Bayesian approach over regression analysis by a comparative analysis of the test results which has economic implications to investors, businesses and policymakers.

While the developed United States (US) continues to explore novel methods, such as the Bayesian approach by Jensen and Maheu (2018) and relevant up-to-date software by Karabatsos (2017), South African studies continue to employ methods, based on model conventionality rather than robustness. Therefore, the aim of this study is to investigate the optimal asset pricing model by comparing the conventional regression analysis and the novel Bayesian approach. This study investigates the CAPM, Fama and French three-factor Model (F3FM), Carhart (1997) four-factor Model (C4FM) and Fama and French five-factor Model (F5FM).

This study consists of five parts. First, the background of asset pricing models is introduced, and the aim of this study is stated, which is to investigate the optimal model. Second, the literature review covers mainly South African studies and concludes with the identification of gaps found in the literature. Third, the methodology of this study outlines the asset pricing models investigated, the sample data, regression analysis, the Bayesian approach and respective model diagnostics. Fourth, the empirical results are discussed along with their implications. Finally, the key findings of this study are concluded.

2. Literature review

Molele and Mukuddem-Petersen (2020) focused on taking into account foreign exchange exposure in asset pricing models. The authors employed Jorion's CAPM, where returns are modelled as a function of the market portfolio and exchange rate factor. The study also employed C4FM, F3FM and F5FM over the sample period from January 2002 to November 2015. The market portfolio was the Johannesburg Stock Exchange All Share Index (JSE ALSI) and the exchange rate used was the Rand to the US Dollar, British Pound, Euro and trade-weighted exchange rate index. The estimation method employed was a Seemingly Unrelated Regression (SUR), which is equivalent to Ordinary Least Squares (OLS) in the context of the study. Molele and Mukuddem-Petersen (2020) concluded that foreign exchange exposure was significant for the extended CAPM. However, results produced by models, such as regression analysis by OLS, based on the normality assumption are unreliable, according to Jensen and Maheu (2018) and Karabatsos (2017). By not taking into account higher moment properties, which are the inherent nature of financial data, the risk-return relationship is being inaccurately captured.

Peerbhai and Strydom (2018) employed the Fama-Macbeth (1973) two-pass regression model to three forms of the CAPM. The study investigated the domestic CAPM (DCAPM) using the JSE ALSI as the market portfolio index and a 90-day Treasury Bill (T-Bill) rate. The international CAPM (ICAPM) used the MSCI All Country World Index (ACWI) as the world market index. The Multifactor CAPM (MCAPM) used exchange rates as proxies for the market index. Results revealed the order of optimality in predicting the risk-return relationship as MCAPM, ICAPM and DCAPM, particularly for large firms, highlighting global factors and exchange rates in the investigation of the CAPM. However, the Fama and Macbeth approach is based on the assumption that the price data of risk are constant. Peerbhai and Strydom (2018) stated that an advantage of the Fama-Macbeth approach is that it allows for the time variation of betas. The time-variation property is allowed because a rolling regression is incorporated into the Fama-Macbeth approach but it is still unreliable, as discussed by Mpoha and Bonga-Bonga (2020).

Mpoha and Bonga-Bonga (2020) investigated foreign exchange exposure using the Jorion model for the sample period from 01 January 2009 to 01 July 2019. The study employed an OLS rolling window regression with a step size of one and a window size of twenty-five. Such a method is used as the first step of the Fama and Macbeth two-pass regression method. The advantage of the rolling window regression is to account for stochasticity and parameter instability. The rolling window regression makes its estimations based on a step size and window size, of which the value selected is questionable. A small window can provide greater output than a large window, but this leads to questioning the trade-off between window size and accurate estimates, according to Mpoha and Bonga-Bonga (2020). The study concluded an overall strong presence of the exchange rate risk premium; positive for emerging South Africa and negative for the developed US.

Emerging markets are generally expected to have higher risk-return relationships due to higher levels of volatility, according to González-Sánchez (2021). The author found that the market risk premium is higher for emerging markets than for developed markets over different term structures. The study investigated the risk factors of the F5FM, from 2004 to 2019, in a comparative study between emerging and developed markets. The study obtained the data from the French website. The significant risk factors were the market risk premium, profitability, investment and momentum for both types of markets. Overall, the risk premium was the most significant, value for the developed markets and size for the emerging markets.

Cox and Britten (2019) conducted an investigation to explain returns on the basis of the size and value factors. The authors employed the CAPM, F3FM and F5FM to the data of the companies listed on the JSE, for the period 1994 to 2017, using the Fama-Macbeth (1973) tests as the main method of investigation. The study concluded that the model's robustness improved when risk factors were added to the original CAPM, in line with Charteris et al. (2018). It was noteworthy that the CAPM was extended to global factors and exchange rates in the previous studies by Peerbhai and Strydom (2018), Molele and Mukuddem-Petersen (2020) and Mpoha and Bonga-Bonga (2020). However, the extended CAPM is still considered incomplete, given the argument drawn from Charteris et al. (2018). The authors stated that one of the notable influences of the test results of asset pricing models is the different market dynamics stemming from the level of development, such as market integration. There are several other factors to consider, such as firm size, liquidity, investor behavior, political forces and the effects of foreign markets. Consequently, the CAPM should be extended to take into account the aforementioned factors.

Cox and Britten (2019) and Charteris et al. (2018) found that the FFMMs outperformed the CAPM. However, Foye (2018) stated that the F3FM is considered incomplete, while Bouzinne et al. (2019) explained that the choice behind the size and value risk factors is considered "vague." Another drawback of the F3FM is its inability to explain momentum (short-term return continuation). Fama and French (1996) stated that momentum is subject to the time period analyzed and changes over time. In addition, their model focuses on the forecast of long-term returns and not short-term. From an econometric viewpoint, Dutta (2019) emphasizes the power of methods to capture long-term anomalies, highlighting the prediction of long-term factors affecting returns. This lends to the significance of using a more robust model to efficiently and effectively capture the risk-return relationship.

Charteris et al. (2018) documented that there have been only two other studies, Reisinger and van Heerden (2014) and Boamah (2015), that aimed to understand the momentum phenomenon in the

South African market. Charteris et al. (2018) investigated momentum in the context of the CAPM, F3FM, C4FM, alternative three-factor model (AL3M) by Chen et al. (2011) and F5FM for the sample period from July 2000 to April 2013. Results were estimated using regression analysis by OLS with the Newey-West adjustment. Charteris et al. (2018) reported that the CAPM, F3FM and C4FM failed to explain momentum, while results were significant for the AL3M and F5FM. Such a finding is in line with Butt et al. (2021), who found that the momentum phenomenon is lower in emerging markets due to low volatility periods.

Steyn and Theart (2019) took a more traditional approach by focusing on an aggregate market level. The authors investigated the CAPM, for the sample period July 2004 to September 2018, using basic regression analysis. The study analyzed beta and an additional risk measure, standard deviation. It was found that shares with a high beta resulted in negative returns but were statistically insignificant, whereas the standard deviation was significant. This point highlighted that different risk measures affected the significance and relationship with returns. On that note, Jensen and Maheu (2018) investigated monthly excess returns and realized variance of the US market from January 1885 to December 2011. The authors also took into account volatility feedback, a stronger measure of volatility and source of asymmetry. In contrast to the conventional regression methods, a nonparametric Bayesian approach was employed to account for higher moment properties and uncertainty. The study found a positive and nonlinear risk-return relationship. Thus, a model that effectively takes asymmetry into account is more likely to estimate an accurate result of the risk-return relationship.

In conclusion, the first gap identified in literature is the conflicting evidence on which asset pricing model optimally estimates the risk-return relationship. This is due to different extensions of the CAPM and methods used. The main method of investigation employed in South African literature is regression analysis, which is based on the normality assumption. This is because South African studies continue to employ methods, based on model conventionality rather than robustness. There is a further lack of comparative testing to ensure whether the test results are actually being improved and whether the empirical nature of the financial data is being adequately captured. This presents the second gap to employ a more robust model; specifically, to conduct a comparative analysis of regression analysis (with and without the Newey-West adjustment) and the Bayesian approach. A comparative analysis will allow for a clear comparison of test results and make a reliable finding in the field of asset pricing models. The third gap is the lack of literature on the FFMMs in the emerging market, South Africa, as noted by Charteris et al. (2018) and Foye (2018). This could be due to data challenges that arise from the different accounting and legal environments used to construct the risk factors of the FFMMs. To overcome these challenges, this study employs data from the French website and extends its analysis to twenty-six emerging markets, following González-Sánchez (2021), with primary focus on South Africa.

3. Methodology

In this study, the asset pricing models (CAPM, F3FM, C4FM and the F5FM) are investigated. The data of twenty-six emerging markets are obtained, with primary focus on the South African market in the current analysis. The asset pricing models are estimated by conventional regression analysis, with and without the Newey-West adjustment, and the novel Bayesian approach. The test results and model diagnostics of both approaches are then compared to provide a meaningful conclusion.

Figure 1 shows the methodological steps to investigate the optimal asset pricing model.

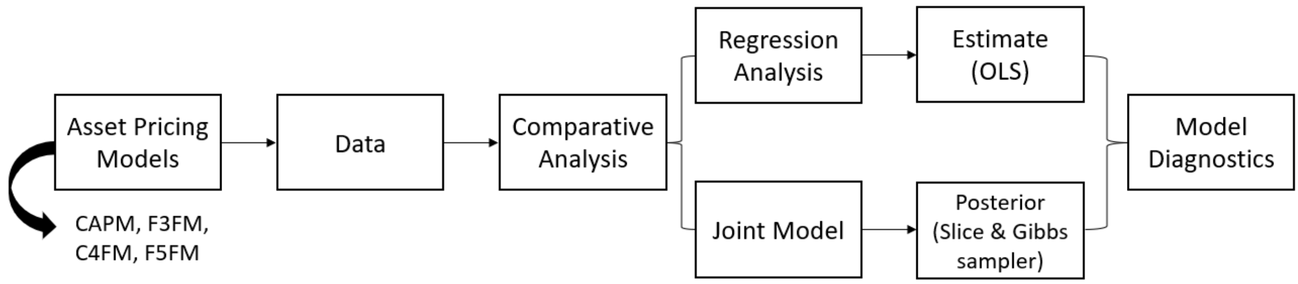

Figure 1. Methodological steps to find the optimal asset pricing model.

3.1. Asset pricing models

The Modern Portfolio Theory (MPT) by Markowitz (1952) is an essential tool used to construct portfolios, tailored to the needs and preferences of investors, by focusing on the average return (mean) and total risk (variance). Extended by Sharpe (1964), the CAPM modelled expected returns by systematic risk only and is shown as Equation 1:

$$E(R_i) = R_f + \beta_i\,[E(R_m) - R_f] \tag{1}$$

where $E(R_i)$ = expected return of security i, R_m = market returns, R_f = risk-free rate, β_i = systematic risk, also known as undiversifiable risk, market risk, or volatility.

According to literature, if the intercepts are zero, the risk factors have adequately captured returns (Charteris et al. 2018; Cox and Britten, 2019). The intercept is also referred to as Jensen's alpha and indicates the average stock return in excess of the forecasted expected return by the CAPM (Steyn and Theart, 2019).

Several studies argued that returns could not be sufficiently explained by a single variable, as cited by Charteris et al. (2018), Cox and Britten (2019) and Molele and Mukuddem-Petersen (2020). Consequently, the single-index model is extended to multifactor models which take into account different risk factors. In this study, the FFMMs consist of the F3FM, C4FM and F5FM. Equation 2 shows the F3FM by Fama and French (2015):

$$R_i - R_f = \alpha_0 + \beta_{i,MKT}MKT_t + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + e_{i,t} \tag{2}$$

where α_0 = intercept, MKT = excess market returns, SMB = size factor, HML = value factor and $e_{i,t}$ = error of security i for period t.

SMB (small minus big) is the size premium that captures the firm's risk exposure, where the stock of small firms is more sensitive because it is riskier than large firms. Hasnawati (2020) stated that large firms are often more stable due to several factors, such as a well-planned debt payment plan, greater access to funding, a committed customer and employee base. However, despite the high risks involved, small-cap firms can still outperform the large-cap firms. The risk-return trade-off holds, where the high-risk small firms earn higher returns than large firms (Fama and French, 2015; Hasnawati, 2020).

HML (high minus low) is the value premium and is also referred to as the book-to-market (BM) ratio variable, according to Munawaroh and Sunarish (2020). A firm's BM ratio is often quantified in relation to the value one. Stock above one is known as a value stock, whereas stock below one is called growth stock. Value stock means that the firm is trading at a low-cost relative to the firm's actual performance and can be treated as a mispricing in the market. Hence, a potential arbitrage opportunity for investors. Growth stocks indicate a firm trading at high cost and suggest an overpricing where investors would be willing to pay more for stock than what it is actually worth. Firms of growth stock usually have financially sound forecasts and tend to expand.

Due to the lack of literature on the momentum phenomenon in the South African market, as noted by Charteris et al. (2018), this study investigates the C4FM and explicitly takes into account the momentum factor. Equation 3 shows the C4FM:

$$R_i - R_f = \alpha_0 + \beta_{i,MKT}MKT_t + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \beta_{i,MOM}MOM_t + e_{i,t} \quad (3)$$

where MOM = the momentum factor.

Momentum refers to the rate of movement of a security's price - either an increase or decrease - over a period of time. The momentum phenomenon is when investors capitalize on such a movement. There are two fundamental concepts by Jegadeesh and Titman (1993), that describe the price movements, namely, winner and loser stocks. Winner stocks earn maximum returns, whereas loser stocks earn minimum returns over the previous two to twelve months, respectively.

According to Charteris et al. (2018), the F5FM has the ability to explain the momentum factor by reflecting a similar relationship pattern (winner and loser stocks). Equation 4 shows the F5FM by Fama and French (2015):

$$R_i - R_f = \alpha_0 + \beta_{i,MKT}MKT_t + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \beta_{i,RMW}RMW_t + \beta_{i,CMA}CMA_t + e_{i,t} \quad (4)$$

where RMW = profitability factor and CMA = investment factor.

RMW (robust minus weak) is the difference between diversified portfolio returns containing stocks with robust and weak profitability. According to Munawaroh and Sunarish (2020), profitability is determined by the amount of a firm's corporate profits. A high profit is translated to a high rate of return earned by an investor. Fama and French (2015) state a positive relationship between profitability and average returns.

CMA (conservative minus aggressive) is the difference between diversified portfolio returns containing stocks of conservative and aggressive firms. A high CMA value indicates abnormal returns and an improvement in the firm's growth.

In summary, the FF5M states a positive relationship between investment and returns and a negative relationship between profitability and returns. Following Charteris et al. (2018), in the context of momentum, for winner stocks, there is a positive correlation with profitability and investment. Similarly, for loser stocks, there is a negative correlation with profitability and investment.

3.2. Sample data

González-Sánchez (2021) investigated indices in relation to market risk factors of both emerging and developed markets. This study focuses on South Africa and therefore investigates the JSE ALSI following convention in South African literature. Since South Africa is an emerging market, the study investigates the ALSI in relation to the group of emerging markets consisting of twenty-six emerging countries, following González-Sánchez (2021). Additional countries and a comparison between the emerging and developed markets will be investigated in future research. In this study, an investigation on South Africa only is made to highlight the fundamental difference in parameter estimation that different models produce.

The monthly ALSI price data are obtained from IRESS for the sample period from September 2000 to October 2021. Following Charteris et al. (2018) and Cox and Britten (2019), monthly frequency is selected based on data availability. The start date of the post-apartheid regime in South Africa follows Charteris et al. (2018), who indicated that the market was adequately integrated into the global economy. Following standard literature, price data is converted to returns by $R_i = \ln\left(\frac{P_t}{P_{t-1}}\right)$, where R_i = ALSI market returns, P_t = price for the current month and P_{t-1} = price for the previous month.

The monthly risk factors, constructed by the data of twenty-six emerging markets, are obtained from the French website for the sample period from October 2000 to October 2021. The sample includes the emerging markets of the countries: Argentina, Brazil, Chile, China, Colombia, Czech Republic, Egypt, Greece, Hungary, India, Indonesia, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Qatar, Russia, Saudi Arabia, South Africa, South Korea, Taiwan, Thailand, Turkey and the United Arab Emirates. For a detailed explanation of the calculation of the ratios, see the French website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_5emerging.html.

Since the data is in simple returns (denoted by R_t), it is converted to log returns (r_t) by the formula $r_t = \log(R_t + 1)$, following Hudson and Gregoriou (2015). Since all the data on the French website is in USD, the data is converted to South African ZAR. The monthly exchange rate ZAR/USD, obtained from IRESS, is converted to log returns by $R_e = \ln\left(\frac{E_t}{E_{t-1}}\right)$ where R_e = market exchange rates and E_t = exchange rate for the current month and E_{t-1} = exchange rate for the previous month.

The US risk-free rate is replaced with the three-month T-bill, obtained from the South African Reserve Bank, in line with Charteris et al. (2018) and Cox and Britten (2019). The data is tested to ensure a valid time series by the Augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and Kwiatkowski, Phillips, Schmidt and Shin (KPSS) stationarity tests. The data is further tested for normality, heteroscedasticity and randomness.

3.3. Regression analysis

Following Steyn and Theart (2019), Charteris et al. (2018) and Molele and Mukuddem-Petersen (2020), the asset pricing models are estimated by regression analysis. Equation 5 shows a standard linear regression equation:

$$y = \sum \beta_i x_i + \beta_0 \tag{5}$$

y = returns, β_i = slope coefficient, x_i = risk factor(s) of security i, β_0 = constant.

Following Charteris et al. (2018) and Molele and Mukuddem-Petersen (2020), the estimation technique employed is OLS, with and without the Newey-West adjustment, to provide an unbiased estimation of test results. If the risk-return premia of the asset pricing models are optimally captured, the intercepts are expected to be zero (Cox and Britten, 2019). Although not within the scope of this study, see https://www.real-statistics.com/multiple-regression/autocorrelation/breusch-godfrey-and-newey-west-tool/ by Zaiontz (2022) for the theoretical explanation of the Newey-West adjustment.

3.4. Bayesian approach

Based on Bayes (1763) theorem, the Bayesian approach consists of a prior and posterior. The prior is the initial probability estimation, and, in this study, it refers to the fundamental properties of the financial data - the asymmetric, volatile and random nature. Once the prior information has been taken into account, this results in an updated probability estimation known as the posterior. Following literature by Jensen and Maheu (2018) and Karabatsos (2017), the Bayesian approach can be estimated in a parametric or nonparametric framework. A parametric model is defined as a finite model and is based on the normality assumption. Examples include regression analysis and the GARCH approach. A nonparametric model is defined as an infinite model and relaxes modelling assumptions, such as normality. Thus, a nonparametric model is more robust than a parametric model by definition.

A parametric Bayesian model is more robust than regression analysis and the GARCH approach because of clustering analysis. This means that, although the model assumes the normality assumption, it has the ability to consider asymmetric forms of the risk-return relationship, to a greater extent than traditional models. The nonparametric Bayesian approach has the ability to consider infinite asymmetric forms of the risk-return relationship.

In the context of this study, clustering analysis refers to the clustering mixture of parameters, as will be shown in the method procedure below. The cluster is a component of a mixture of, in this case, weights and parameters. According to Karabatsos (2017), a nonparametric Bayesian model is referred to as an infinite-mixture model. An infinite-mixture model describes a model that takes into account an infinite number of clusters. The nonparametric Bayesian model assumes an infinite number of clusters, whereas the parametric Bayesian model assumes a finite number of clusters. This study employs the parametric Bayesian approach because it is computationally possible through available Bayesian software by Karabatsos (2017). An analysis made by the nonparametric Bayesian approach can be made in the future, when it is computationally possible, due to advancements made by experts in programming and software.

Following Jensen and Maheu (2018), the first step is to model the joint distribution of returns in relation to the risk factors. Let $R_t = R_i - R_f$ where R_t = excess returns and $R_i - R_f$ is the difference between returns of security i and the risk-free rate. The joint model is given as:

$$P(R_t, x|I_{t-1}, \Omega, \Theta) = \sum_{j=1}^{\infty} w_j f(R_t, x|\theta_j, I_{t-1}) \tag{6}$$

where R_t = excess returns, x = risk factor(s), I_{t-1} = information set that contains the risk and return variables, $\Omega = \{w_j\}$ set of probability weights and $\Theta = \{\theta_j\}$ set of parameters, for j = mixture clusters.

For simplification purposes, Ω and Θ are dropped. By the law of total probability, the joint distribution is equivalent to the product of the marginal and conditional distribution. Hence, the model for the asset pricing models is as follows, respectively:

$$P(R_t, x\,|\theta_j, I_{t-1}) = f(R_t|\theta_j, I_{t-1}) * f(x|\theta_j, I_{t-1}) \tag{7}$$

where $\boldsymbol{\theta_j} = \{\beta_{i,\ MKT}\}$ and $\boldsymbol{I_{t-1}} = \{R_t, MKT_t\}$ for the CAPM,
$\boldsymbol{\theta_j} = \{\beta_{i,\ MKT}, \beta_{i,\ SMB}, \beta_{i,\ HML}\}$ and $\boldsymbol{I_{t-1}} = \{R_t, MKT_t, SMB_t, HML_t\}$ for the F3FM,
$\boldsymbol{\theta_j} = \{\beta_{i,\ MKT}, \beta_{i,\ SMB}, \beta_{i,\ HML}, \beta_{i,\ MOM}\}$ and $\boldsymbol{I_{t-1}} = \{R_t, MKT_t, SMB_t, HML_t, MOM_t\}$ for the C4FM,

$\boldsymbol{\theta_j} = \{\beta_{i,\ MKT}, \beta_{i,\ SMB}, \beta_{i,\ HML}, \beta_{i,RMW}, \beta_{i,\ CMA}\}$ and $\boldsymbol{I_{t-1}} = \{R_t, MKT_t, SMB_t, HML_t, RMW_t, CMA_t\}$ for the F5FM.

The function of the above variables is conditional on the prior information - the inherent properties of the financial data - $\boldsymbol{I_{t-1}}$ the information set which contains all the variables, and $\boldsymbol{\theta_j}$ which gives rise to asymmetric forms of the parameters by clustering analysis. Equation 8 shows the posterior model specification:

$$p(R_t, x, u_t | \theta_j, I_{t-1}) = \sum_{j=1}^{\infty} \mathbf{1}\,(u_t < w_j) * f(R_t, x | \theta_j, I_{t-1}) \quad (8)$$

Following Jensen and Maheu (2018), to estimate an updated probability estimation, the Gibbs sampling technique and slice sampler by Kalli et al. (2011) are applied. The slice sampler eliminates all weights of zero by the additional auxiliary variable u_t. The Gibbs sampler ensures a robust posterior by inferring from the initial probability estimation. In other words, the prior and posterior are subject to the same properties, such as asymmetry.

3.5. Model diagnostics

3.5.1. Regression analysis

Cox and Britten (2019) and Charteris et al. (2018) focus on the conventional model measures, such as R^2 and adjusted-R^2 to report on model diagnostics. The R^2 value is a statistical measure that explains the data variation. In the context of this study, R^2 is the percentage value that indicates the robustness of the risk factors in explaining stock returns. The value can be from zero to one, where the closer it is to one, the more robust the model is in explaining returns. For example, an R^2 of 0.823 means that the model fit explains 82.3% of the total variation about the mean.

An increase in the number of risk factors can lead to an increase in R^2 without improving the actual model's robustness. Therefore, the adjusted-R^2 value is relevant in the case of multifactor models, as it takes into account the model's degrees of freedom. An increase in the measure indicates that the additional risk factors improve the model fit, whereas for a decrease, the opposite is true.

The final three values are Sum of Squares Regression (SSR), Sum of Squares Error (SSE) and Sum of Squares Total (SST). SSR captures the variance of the return variable that the variance of the risk factors can explain. Hence, the SSR shows the optimality of the model fit of the risk-return relationship. SSE measures the difference between the observed and predicted risk-return relationship. SST indicates the amount of variation found in the return variable. It is quantified as the squared differences between returns and the associated mean.

3.5.2. Bayesian approach

The two similar statistical measures in regression analysis and the Bayesian approach are the R^2 and SSE values. As stated, SSE measures the difference between the observed and predicted risk-return relationship. Following Karabatsos (2017), this study analyzes the SSE with respect to the model's goodness-of-fit denoted as Gof(m) and the model posterior predictive denoted as D(m). The posterior predictive refers to the updated distribution of the risk-return parameters. The simulation of data from the posterior predictive distribution allows for further model diagnostics, such as normality tests

(Mackenzie et al. 2018). Following Karabatsos (2017), this study uses the Kolmogorov-Smirnov (KS) test and quantile-quantile (QQ) plots to investigate whether asymmetry has been adequately captured. For the KS test, if the *p*-value is less than 5%, the null hypothesis that the ALSI returns follow a normal distribution is rejected at the relevant level of significance. For the QQ plots, if there is a major deviation between the theoretical and empirical distribution, this indicates that asymmetry has been inadequately captured. If asymmetry is adequately captured, this would confirm accurate and reliable Bayesian test results.

According to Mackenzie et al. (2018), the two procedural ways to investigate the model diagnostics of Bayesian analysis are the posterior predictive and 'Bayesian *p*-value.' The Bayesian *p*-value is referred to as the Monte Carlo (MC) mixing value by Karabatsos (2017). If the MC mixing value is 0.5, this would mean that the fitted risk-return data is in line with the Bayesian model. The model is inadequate if the Bayesian *p*-value is zero or one; more specifically, the model is underfitted if close to zero and overfitted if close to one (Mackenzie et al. 2018). Penalty (predictive variance), denoted as P(m) by Karabatsos (2017), measures the error associated with making a forecast using a regression model. An underfitted model is indicated by a high predictive variance and SSE, whereas for an overfitted model, the values would be exceptionally high (Gelfand and Ghosh, 1998; Sahu, 2006). Finally, the error variance σ^2 reveals the unexplained variance from sources, such as uncertainty, stochasticity and measurement errors (Jensen and Maheu, 2018). A value of zero or approximately zero would mean that the σ^2 has been adequately captured (Karabatsos, 2017). Thus, the lowest D(m), Gof(m), P(m) and σ^2 indicate the optimal model.

4. Empirical results and discussion

4.1 Data exploration

The monthly ALSI price data obtained from IRESS for the sample period September 2000 to October 2021 are converted to market returns. The monthly data of the twenty-six emerging markets for the sample period October 2000 to October 2021 are obtained from the data library on the French website. The data in USD is converted to South African ZAR using the monthly USD/ZAR exchange rate obtained from IRESS. The US T-bill is replaced with a three-month T-bill following Charteris et al. (2018) and Cox and Britten (2019). A total of 253 observations are analyzed. Overall, the data is confirmed to be stationary by the ADF, PP and KPSS tests. The data is further confirmed to be asymmetric and random in nature but volatility is found to be absent in the data. The absence of volatility contrasts to the expectations of emerging markets which are generally considered high-risk and thus lead to potential superior returns. However, the finding of absent volatility is in line with the low-risk anomaly found by Steyn and Theart (2019). The asymmetric and random nature of the data substantiates the application of the Bayesian approach over regression analysis.

4.2 Asset pricing models test results

The ALSI excess returns are regressed on the risk factors using Excel. Table 1 shows the regression test results of the asset pricing models. The standard error values are shown in brackets.

Table 1. Regression test results of the CAPM, F3FM, C4FM and F5FM.

Model	Intercept	MKT	SMB	HML	MOM	RMW	CMA
CAPM	0.103 **	3.372 **					
	(0.025)	(0.627)					
F3FM	0.114 **	3.706 **	3.983	−4.037			
	(0.026)	(0.664)	(3.442)	(3.019)			
C4FM	0.128 **	4.070 **	4.346	−3.785	1.698		
	(0.030)	(0.785)	(3.469)	(3.034)	(1.947)		
F5FM	0.123 *	3.941 **	3.957	−5.494	-	−2.109	1.910
	(0.048)	(1.260)	(3.586)	(4.162)		(5.584)	(4.709)

NOTE: *, ** means the p-value is significant at a 5% and 1% level of significance, respectively

From Table 1, it can be seen that the majority of the intercepts are positive and statistically significant at the relevant levels of significance, 1 and 5%. Since the intercepts are greater than zero, the risk factors have inadequately captured returns. The positive and significant Jensen's alpha indicates that the stock returns earn more than the predicted CAPM over the sample period. The MKT coefficient is greater than one for all the asset pricing models, confirming that the risk-return relationship has been inadequately captured. Regarding the risk factors, none are found to be significant by the FFMMs, thus indicating no relationship with returns. This means that the additional risk factors do not improve the CAPM, in contrast to Charteris et al. (2018) and Cox and Britten (2019).

Following Charteris et al. (2018), the Newey-West adjustment is employed with the regression analysis to improve the test results. The ALSI excess returns are regressed on the risk factors using Excel, and an additional resource by Zaiontz (2022) to allow for the Newey-West adjustment. Table 2 shows the regression test results of the asset pricing models with the Newey-West adjustment.

Table 2. Regression test results of the CAPM, F3FM, C4FM and F5FM with the Newey-West adjustment.

Model	Intercept	MKT	SMB	HML	MOM	RMW	CMA
CAPM	0.103 **	3.372 **					
	(0.035)	(0.900)					
F3FM	0.114 **	3.706 **	3.983	−4.037			
	(0.036)	(0.911)	(4.276)	(4.264)			
C4FM	0.128 **	4.070 **	4.346	−3.785	1.698		
	(0.026)	(0.643)	(4.390)	(4.350)	(3.395)		
F5FM	0.123 **	3.941 **	3. 957	−5.494	-	−2.109	1.910
	(0.072)	(1.873)	(4.613)	(4.921)		(4.497)	(7.358)

From Table 2, it can be seen that the intercepts and risk factor values are identical to Table 1 above, suggesting that the Newey-West adjustment does not impact parameter estimation. This contrasts with Charteris et al. (2018), who used the adjustment to improve the asset pricing test results. In this study, the only difference is that most of the standard error values are higher with the Newey-West adjustment. A lower standard error is preferable, as it would indicate a smaller difference between the data and fitted values. Model diagnostics can provide more insight into this finding.

The unreliability of the Newey-West adjustment is in line with Gow et al. (2010) and González-Sánchez (2021). This motivates the application of the novel Bayesian approach. The ALSI excess returns are regressed on the risk factors using the Bayesian software by Karabatsos (2017). The implementation of the Bayesian approach is made in accordance with model diagnostics to ensure a more robust model, specifically with an approximate error variance of zero and optimal MC mixing values that were all approximately 0.5. Table 3 shows the Bayesian test results of the asset pricing models. The MC mixing values are shown in brackets.

Table 3. Bayesian test results of the CAPM, F3FM, C4FM and F5FM.

Model	Intercept	MKT	SMB	HML	MOM	RMW	CMA
CAPM	0.102 **	3.358 **					
	(0.502)	(0.503)					
F3FM	0.115 **	3.720 **	4.076 **	−4.068 **			
	(0.495)	(0.496)	(0.507)	(0.507)			
C4FM	0.127 **	4.053 **	4.297 **	−3.748 **	1.685 **		
	(0.507)	(0.510)	(0.494)	(0.5143)	(0.487)		
F5FM	0.123 **	3.941 **	3.913 **	−5.531 **	-	−2.124 *	1.845 *
	(0.492)	(0.486)	(0.513)	(0.506)		(0.506)	(0.493)

NOTE: *, ** means the posterior is significant in the 97.5% and 75% Bayesian intervals, respectively

Although not within the scope of this paper, the model specifications for the parametric Bayesian model are as follows: The prior variance of the slope parameters is 1000000000 and the prior inverse gamma distribution of the error variance is 1e-05/2. The posterior parameter estimates are determined by 20 000 MCMC sampling iterations, a burn-in period of 5000 and a thin number of 5. The model specification values of the Bayesian approach are selected in accordance with the model diagnostics to ensure an optimal model. More specifically, with minimum Gof(m), D(m), P(m) and σ^2 values. The MCMC sampling iterations and burn-in period values follow Jensen and Maheu (2018), while the thin number is based on the default value by Karabatsos (2016). The MCMC sampling iterations refer to the repetitive resampling process used to determine the posterior parameter estimates. The burn-in period refers to samples from the initial stages that are discarded due to no longer being able to accurately represent the required distribution. The thin number of 5 means that every fifth sampling iterate of the 20 000 MCMC sampling iterations is collected to determine the posterior estimates.

From Table 3, in major contrast to the test results of the conventional regression analysis, all the risk factors of the FFMMs are statistically significant in the Bayesian intervals, 75 and 97.5%. SMB is found to be significant and positive across the FFMMs, where smaller firms earn more than larger firms, which is in line with Fama and French (2015) expectations. In South Africa, this shows the significance of small, medium and micro enterprises (SMMEs), which provide at least 50% of employment opportunities and contributes to at least 34% of GDP, as reported by the National Treasury (2021). HML is significant and negative for the FFMMs, which indicates growth stocks, suggesting positive growth for the South African economy.

The strong presence of MOM is in line with González-Sánchez (2021), but contrasts with Charteris et al. (2018) and Butt et al. (2021). In the F5FM, RMW and CMA are significant in the 97.5% Bayesian interval, negative and positive, respectively. RMW indicates a low rate of return due to low profits earned by the firm's corporate profits. However, there are abnormal returns and an improvement in the firm's growth, as indicated by the positive CMA. Following Charteris et al. (2018), this study finds that investors who use the momentum strategy ought to consider a firm's investment policy in their decision-making process. Since the posterior estimates have MC mixing values of approximately 0.5, the findings of the Bayesian test results are optimal and reliable.

The regression test results with and without the Newey-West adjustment are identical, with the only difference being the standard error values. Consequently, the Newey-West adjustment provides no statistical advantage in improving forecast accuracy. On the other hand, the Bayesian test results found all the risk factors of the FFMMs to be significant. Such a contrast in test results clearly illustrates that the method used can affect parameter estimation and thus impact the financial decisions of firms and market participants.

It should be noted that the asset pricing models model diagnostics are an unrefined method to fully capture the relationship between risk and return. In other words, the model diagnostics measures, such as the adjusted- R^2, are not treated as a “formal” test, in line with Charteris et al. (2018). According to convention in literature, the low alpha indicates an optimal model. However, the nonzero alpha found by the regression and Bayesian approach test results, indicates that risk has been inadequately captured. Moreover, the JSE ALSI is considered a well-diversified value-weighted portfolio and the highly significant alphas indicate uncaptured risk by the risk factors. Therefore, different risk factors are needed to be considered. This study analyzes the model diagnostics following the South African studies by Cox and Britten (2019) and Charteris et al. (2018).

4.2. Model diagnostics

The model diagnostics are produced after the regression analysis and adjustments cannot be made to improve the model’s forecast accuracy. Table 4 shows the regression model diagnostics of the CAPM, F3FM, C4FM and F5FM with and without the Newey-West adjustment.

Table 4. Model diagnostics of regression analysis with and without Newey-West adjustment.

Model	R^2	Adjusted-R^2	SSR	SSE	SST
CAPM	0.103	0.100	0.059	0.509	0.568
F3FM	0.113	0.102	0.064	0.504	0.568
C4FM	0.115	0.101	0.066	0.502	0.568
F5FM	0.114	0.096	0.065	0.503	0.568

From Table 4, while the R^2 shows a marginal improvement for each model, none of the models indicate an optimal fit. CAPM is the least optimal, followed by F3FM and F5FM, whereas C4FM has the highest R^2 value but at just 11.5%. The latter means that the model explains only 11.5% of the variation of the risk-return relationship. For the adjusted-R^2, the order of most to least optimal are the F3FM, C4FM, CAPM and F5FM. This contrasts to Cox and Britten (2019), who found FF5M to be the most optimal and CAPM as the least optimal. It was expected by Charteris et al. (2018) and Cox and Britten (2019), that CAPM should be the least robust as the additional risk factors improve the model.

Table 5. Model diagnostics of the Bayesian approach.

Model	Gof(m)	D(m)	P(m)	R^2	σ^2
CAPM	0.510	1.027	0.518	0.103	0.002
F3FM	0.503	1.020	0.516	0.113	0.002
C4FM	0.503	1.019	0.517	0.115	0.002
F5FM	0.504	1.022	0.518	0.112	0.002

The variation found in the return variable is the same for all the models as indicated by the SST. Relative to the FFMMs, CAPM has the lowest SSR and the highest SSE. The low SSR means that the systematic risk exposure least explains the variance of the returns. The high SSE indicates that the quantification of the observed and predicted CAPM has the highest discrepancy. These results can be attributed to the low alpha value, where risk has not been fully captured by the CAPM. Table 5 shows the Bayesian model diagnostics of the CAPM, F3FM, C4FM and F5FM.

The implications of the model diagnostics of the Bayesian approach are similar to the regression analysis. CAPM is the least optimal, followed by F5FM and F3FM, with C4FM having the highest R^2. Since CAPM has the lowest alpha, the model diagnostics confirm that risk has not been fully captured by the model. The error variance σ^2 is approximately zero, indicating that the unexplained variance from sources, such as uncertainty, stochasticity and measurement errors has been adequately captured.

With regards to the posterior predictive D(m), the normality tests are employed to investigate if the intercepts and risk factors have adequately captured asymmetry. Figures 2 to 18 show the QQ plot and KS test for the asset pricing model's posterior intercepts and risk factors.

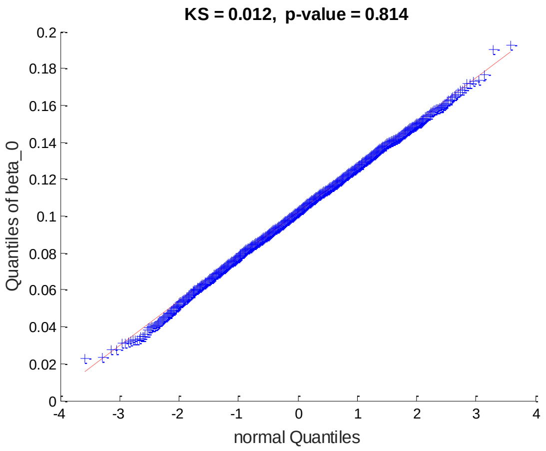

Figure 2. Normality tests for the posterior intercept of CAPM.

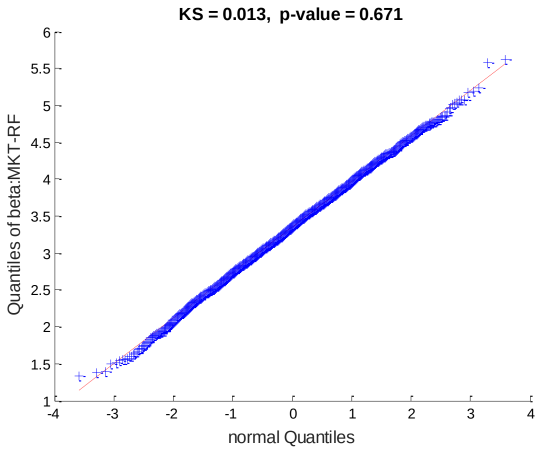

Figure 3. Normality tests for the posterior market risk premium of CAPM.

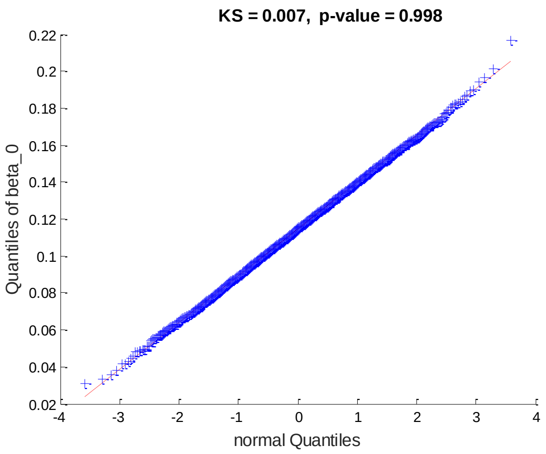

Figure 4. Normality tests for the posterior intercept of F3FM.

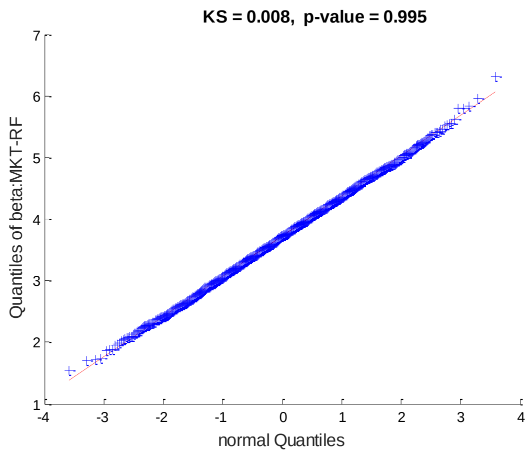

Figure 5. Normality tests for the posterior market risk premium of F3FM.

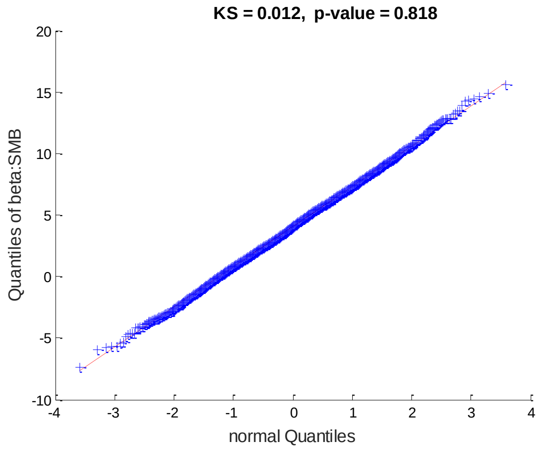

Figure 6. Normality tests for the posterior size risk premium of F3FM.

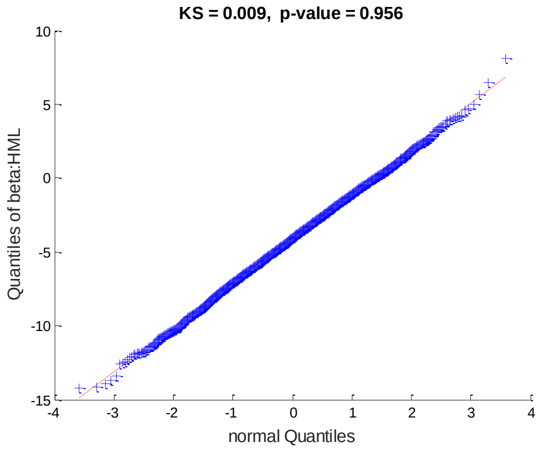

Figure 7. Normality tests for the posterior value risk premium of F3FM.

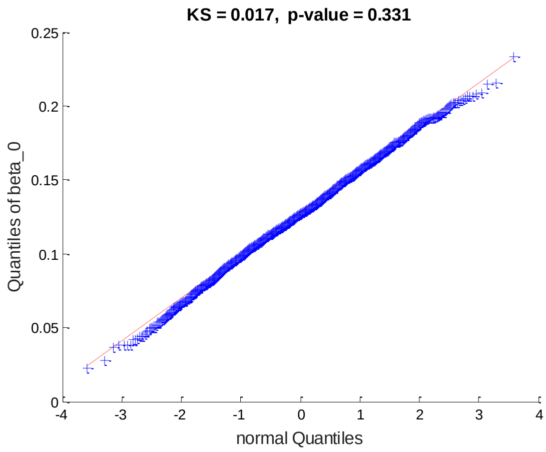

Figure 8. Normality tests for the posterior intercept of C4FM.

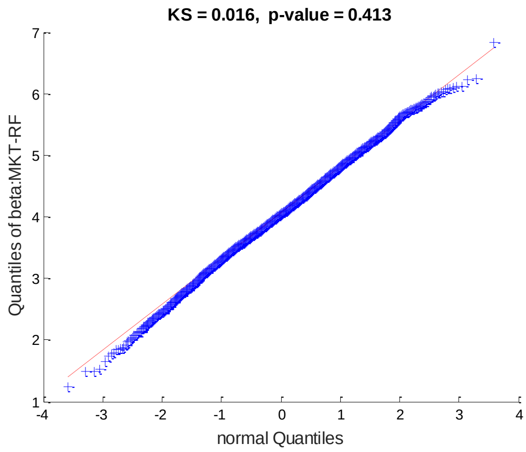

Figure 9. Normality tests for the posterior market risk premium of C4FM.

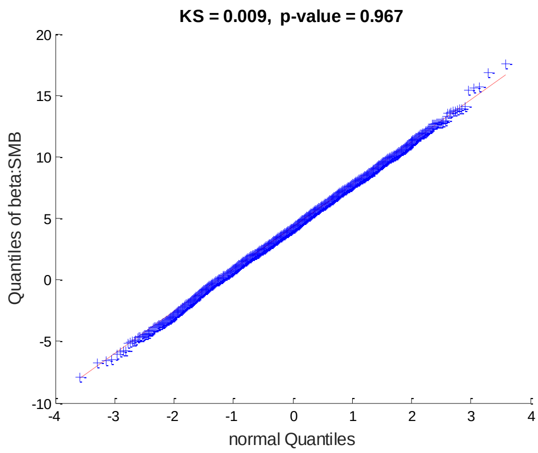

Figure 10. Normality tests for the posterior size risk premium of C4FM.

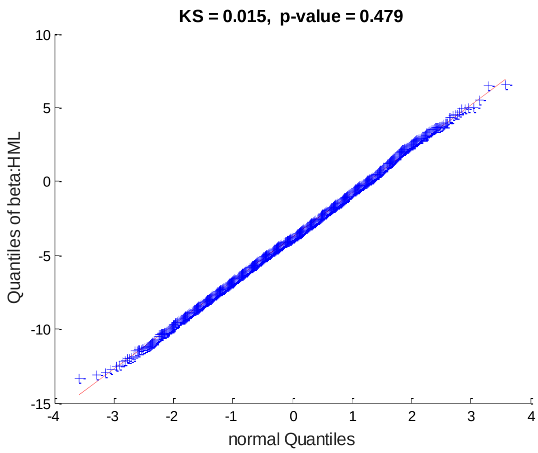

Figure 11. Normality tests for the posterior value risk premium of C4FM.

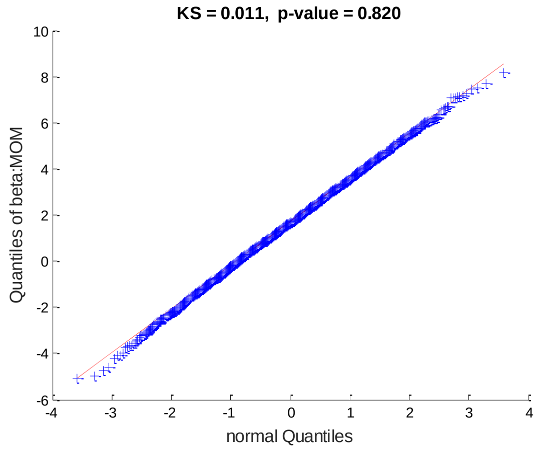

Figure 12. Normality tests for the posterior momentum risk premium of C4FM.

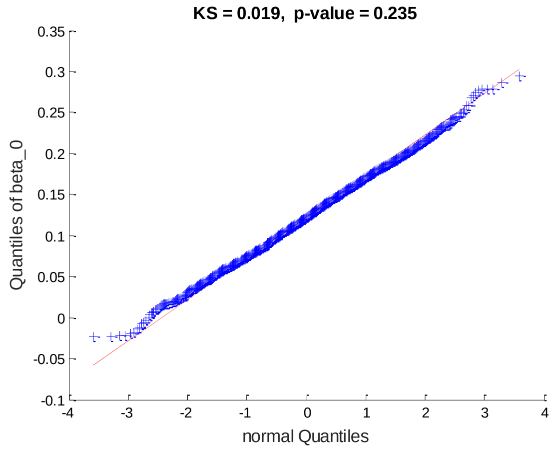

Figure 13. Normality tests for the posterior intercept of F5FM.

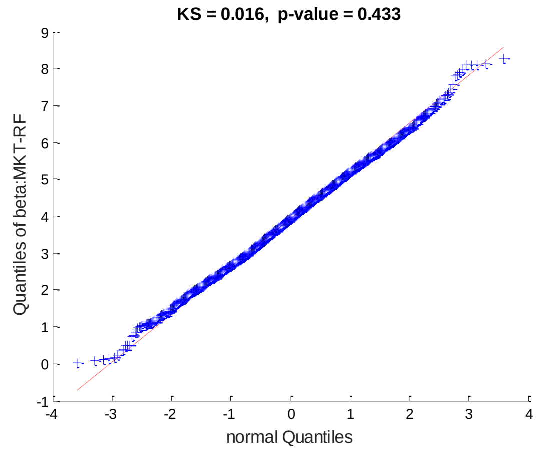

Figure 14. Normality tests for the posterior market risk premium of F5FM.

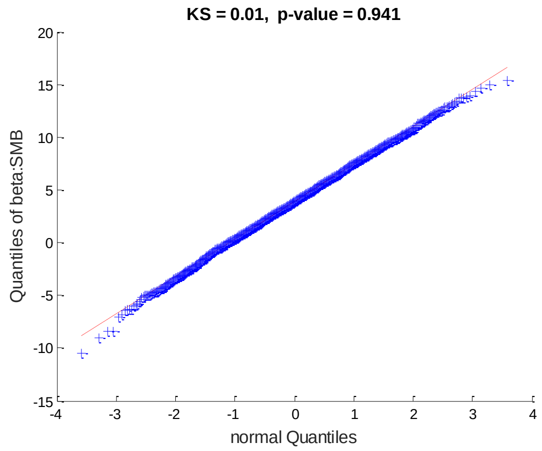

Figure 15. Normality tests for the posterior size risk premium of F5FM.

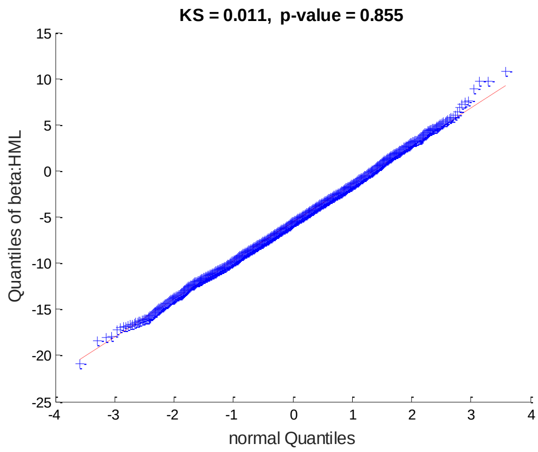

Figure 16. Normality tests for the posterior value risk premium of F5FM.

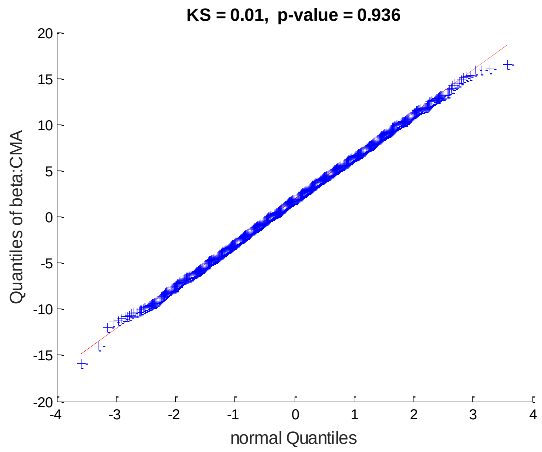

Figure 17. Normality tests for the posterior profitability risk premium of F5FM.

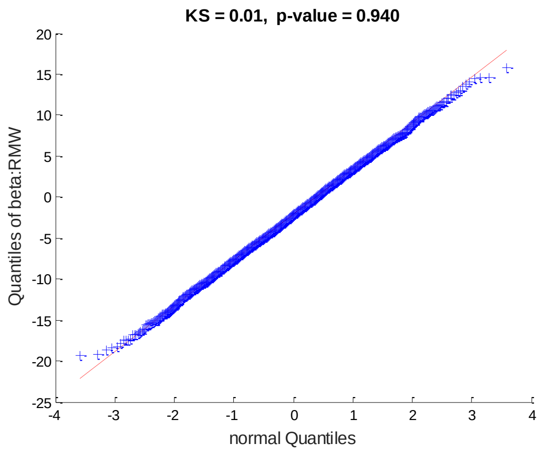

Figure 18. Normality tests for the posterior investment risk premium of F5FM.

From Figures 2 to 18, the QQ plot shows little to no major deviation between the theoretical normal quantiles and the empirical fitted posterior parameters. Since the *p*-values are greater than the 1 and 5% levels of significance, the null hypothesis of normality is not rejected. Thus, it can be concluded by the numerical and graphical analysis that the posterior parameters have adequately captured asymmetry.

5. Conclusions

This study aimed to address several gaps identified in literature on asset pricing models. First, this study contributed to the lack of literature on the topic in the emerging market, South Africa, as sufficed by Foye (2018) and Charteris et al. (2018). Second, this study highlighted the limitations of one of the foremost methods used in South Africa to investigate the risk-return relationship. That is regression analysis, with and without the Newey-West adjustment, following Steyn and Theart (2019), Charteris et al. (2018) and Molele and Mukuddem-Petersen (2020). Third, this study showed the significant impact of using a more robust methodology - the Bayesian approach by Jensen and Maheu (2018). The additional model diagnostics of the Bayesian approach, following Mackenzie et al. (2018) and Karabatsos (2017), assisted in estimating more accurate test results. This was the most significant contribution as the FFMMs risk factors of the Bayesian approach were all found to be significant, whereas regression analysis found the factors to be insignificant.

From the Bayesian test results, the significant size factor showed the contribution of SMMEs to the South African economy as these enterprises earn more than larger firms. The value factor revealed that larger corporations are still well established as the low BM ratio indicated growth stocks, hence, firms' tendency towards overpricing and expansion. Given this result, policy intervention, as reported by the National Treasury (2021), ensures that SMMEs are supported to continue developing, creating employment opportunities and contributing to the GDP. The regression test results showed no relationship between returns with the size and value risk premia, respectively, so no policy interventions would be motivated. Such a misinterpretation could lead to the detriment of the growth and development of the country's economy.

From the Bayesian test results, the weak presence of the profitability and investment factors indicated that the F5FM does not outperform the F3FM, in contrast to Fama and French (2015). Following Charteris et al. (2018), this study investigated the profitability and investment factors in the context of momentum. The weak presence of both factors could suggest an absence of momentum; however, the C4FM confirmed the presence of momentum, in contrast to Charteris et al. (2018) and Butt et al. (2021). Therefore, indicating that investors can use the momentum strategy in the South African market and earn a superior rate of return. In contrast, following the regression test results, such an investment strategy would be avoided, as the results found no relationship with returns. Overall, the lowest Jensen's alpha was found for the CAPM. Since the value was nonzero, this indicated that risk was not fully captured, as was confirmed by model diagnostics.

For future research purposes, this study makes three recommendations. Given the lack of literature on the momentum phenomenon in the South African market (Charteris et al. 2018), more studies should be undertaken on this risk factor. González-Sánchez (2021) investigated indices in relation to market risk factors of both emerging and developed markets. This study focused on South Africa, in relation to emerging markets, given the lack of literature noted by Charteris et al. (2018) and Foye (2018). Therefore, the second recommendation is a geographical extension to other emerging markets of interest, such as BRICS, the MSCI emerging index or any relevant group of emerging markets. The third recommendation is to employ a nonparametric Bayesian approach by Jensen and Maheu (2018). A nonparametric approach relaxes the normality assumption and can, thus, provide a more robust estimation. Since this study found that the asset pricing models inadequately explained returns, the final recommendation is to investigate different risk measures.

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Conflict of interest

The author declares no conflict of interest.

Data

The secondary price dataset used in this study can be obtained from the IRESS database.

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