AIMS Materials Science, 7(4): 381–398.
DOI: 10.3934/matersci.2020.4.381
Received: 11 April 2020
Accepted: 09 June 2020
Published: 23 June 2020

http://www.aimspress.com/journal/Materials

Research article

The effect of modified Ohm's and Fourier's laws in generalized magneto-thermo viscoelastic spherical region

Alaa. K. Khamis[1], Allal Bakali[1] , A.A.El-Bary[2,*] and Haitham. M. Atef[3]

[1] Mathematics Department-Faculty of Science-Northern Border University, Arar, Saudi Arabia.
[2] Basic and Applied Science Institute, Arab Academy for Science, Technology and Maritime Transport , P.O. Box 1029, Alexandria, Egypt
[3] Department of Mathematics, Faculty of science, Damanhur University, Damanhur, Egypt

* **Correspondence:** Email: aaelbary@aast.edu; Tel: +201007382198.

Abstract: This paper is dealing the modified Ohm's law, including the temperature gradient and charge thickness impacts, and the generalized Fourier's law, including the current density impact, the conditions of generalized thermo-viscoelasticity for a thermally, isotropic and electrically leading unbounded body with a spherical cavity is given. The detailing is applied to the generalized thermo elasticity dependent on Green–Naghdi (G-N II) and (G-N III) theory, where there is an underlying magnetic field corresponding to the plane limit, because of the utilization of the magnetic field, it results an incited magnetic and electric fields in the medium. The state space investigation is applied to acquire the temperature, displacement, stresses, induced electric field, instigated magnetic field and current density. Application is utilized to our concern to get the arrangement in the total structure. The considered variables are introduced graphically and discussions are made.

Keywords: generalized thermoelasticity; viscosity; magnetic field; spherical cavity; modified Ohm's and Fourier's laws

Abbreviations: λ_e, μ_e: Lame elastic constants; ρ: Density; C_E: Specific heat at constant strain; K: Thermal conductivity; α_t : Coefficient of linear thermal expansion; γ_e : $(3\lambda_e + 2\mu_e)\alpha_t$; γ_o : $(3\lambda_e\alpha_o + 2\mu_e\alpha_1)\alpha_t/\gamma_e$; α_o, α_1: Viscoelastic relaxation time; t: Time; q_i: Components of heat flux vector; σ_{ij}: Components of stress tensor; e_{ij}: Components of strain tensor; u_i: Components of displacement vector; T_o: Reference temperature;θ: Temperature increment; δ_{ij}: Kronicker delta; e: Cubical dilatation; R: Radius of the shell; μ_o: Magnetic permittivity; E: Electric displacement vector; J: Current density vector; H: Total magnetic intensity vector; h: Induced magnetic field vector; H_o: Initial uniform magnetic field; F_i: Components of Lorentz body force; π_o: Coefficient connecting the current density with the heat flow density; k_o: Coefficient connecting the temperature gradient and electric current density

1. Introduction

The traditional uncoupled hypothesis of thermo elasticity predicts two marvels not good with physical perceptions. In the first place, the condition of heat conduction of this hypothesis doesn't contain any elastic terms; second, the heat condition is of an parabolic kind, anticipating unending paces of spread for heat waves.

Biot [1] presented the hypothesis of coupled thermo elasticity to conquer the principal weakness. The overseeing conditions for this hypothesis are coupled, dispensing with the main oddity of the old style hypothesis. In any case, the two hypotheses share the second inadequacy since the heat equation for the coupled hypothesis is likewise parabolic.

Two generalizations to the coupled hypothesis were presented. The first is because of Lord and Shulman [2], who acquired a wave-type heat equation by proposing another law of heat equation to supplant the old style Fourier's law. Since the heat equation of this hypothesis is of the wave type, it consequently guarantees limited velocities of spread for heat and elastic waves. The staying administering equations for this hypothesis, to be specific, the equations of motion and constitutive relations, continue as before as those for the coupled and the uncoupled speculations. The second speculation to the coupled hypothesis of elasticity is what is known as the hypothesis of thermo elasticity with two relaxation times or the hypothesis of temperature-rate-dependent thermo elasticity. Müller [3], in a survey of the thermodynamics of thermo elasticity solids, proposed an entropy creation imbalance, with the assistance of which he thought about limitations on a class of constitutive equations. A generalization of this imbalance was proposed by Green and Laws [4]. Green and Lindsay got an express form of the constitutive equations in [5]. These equations were additionally gotten autonomously by Shuhubi [6] has acquired the fundamental solution for this hypothesis. This hypothesis contains two constants that go about as relaxation times and alter all the equation of the coupled hypothesis, not just the heat equation. The old style Fourier's law of heat equation isn't disregarded if the medium viable has a focal point of balance.

Later Green and Naghdi [7–9] proposed three hypotheses of generalized thermo elasticity. The primary model (G-N I) is actually equivalent to Biot's hypothesis [1]. The second and third models are named as G-N II and G-N III model. In G-N II and G-N III models, the thermal wave engenders with limited rates which concur with physical circumstances. A significant component of G-N II hypothesis is that this hypothesis doesn't suits dissipation of thermal energy though G-N III hypothesis obliges dissemination of dissipation of thermal energy.

With the quick advancement of polymer science and plastic industry, just as the wide utilization of materials under high temperature in present day innovation and use of science and topography in designing, the hypothetical examination and applications in viscoelastic material has become a significant errand for strong mechanics.

The hypothesis of thermo-viscoelasticity and the solutions of some boundary value problems of thermo-viscoelasticity were researched by Illyushin's and Pobedria [10]. Crafted by Biot [11,12], Morland and Lee [13] and Tanner [14] mate extraordinary walks in the most recent decade in discovering answers for limit esteem issues for linear viscoelasticity materials including temperature varieties for both semi static and dynamic issues. Drozdov [15] inferred a constitutive model in thermo-viscoelasticity which represents changes in elastic moduli and relaxation times. Tasteless [16] connected the arrangement of linear viscoelasticity issues to comparing linear elastic solutions. Lion [17] studied the large deformation behavior of reinforced rubber at different temperatures. Thermo-viscoelastic experimental characterization and numerical modelling of VHB polymer were induced by Liao et al. [18].

The hypothesis of electro-magneto-thermo-viscoelasticity has stimulated a lot of enthusiasm for some mechanical applications especially in atomic gadget. Where there exists an essential magnetic field. Different examinations have been completed by thinking about the connection between attractive, magnetic, thermal and strain fields. Examinations of such issue additionally impact different applications in biomedical building just as in various geometric investigations. Fish et al. [19] has studied modeling and simulation of nonlinear electro-thermo-mechanical continua with application to shape memory polymeric medical devices.

Numerous applications of state space approach created for various sort of issues in electro-magneto-thermo-viscoelasticity [20–34].

2. Basic governing equation

We shall consider a homogeneous isotropic thermo-viscoelastic medium occupying the region $R \ll r < \infty$ of a perfect electrically conductivity permeated by an initial constant magnetic field H_o, where R is the radius of the shell.

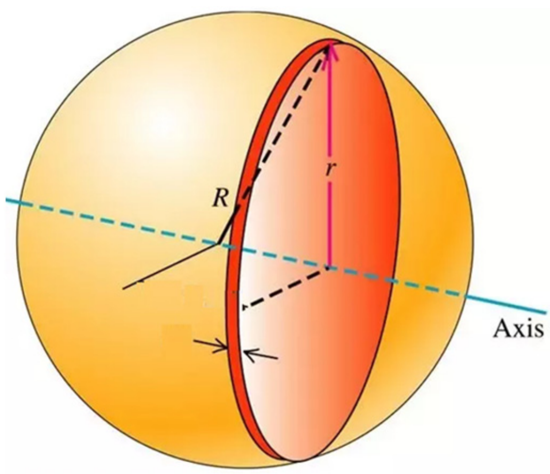

Figure 1. Show the spherical symmetry of the shell.

Due to the effect of this magnetic field there arises in the conducting medium an induced magnetic field h and induced electric field E. Also, there arises a force F (the Lorentz Force). Due to the effect of this force, points of the medium undergo a displacement u, which gives rise to a temperature.

The linearized equations of electromagnetism for slowly moving media are:

$$\text{curl}\, h = J + \varepsilon_o \frac{\partial E}{\partial t} \tag{1}$$

$$\text{curl}\, E = -\mu_o \frac{\partial E}{\partial t} \tag{2}$$

$$B = \mu_o H \tag{3}$$

$$\text{div}\, B = 0 \tag{4}$$

The above field equations are supplemented by constitutive equations which consist first of modified ohm's law:

$$E = -\mu_o \frac{\partial u}{\partial t} \times H_o + k_o grad\, \theta \tag{5}$$

The second constitutive equation is the one for the Lorenz force which is

$$F = J \times B \tag{6}$$

The third constitutive equation is the stress-displacement-temperature relation for viscoelastic medium of Kelvin–Voigt type:

$$\sigma_{ij} = 2\mu_e \left(1 + \alpha_1 \frac{\partial}{\partial t}\right) e_{ij} + \lambda_e \left(1 + \alpha_o \frac{\partial}{\partial t}\right) e\delta_{ij} - \gamma_e \left(1 + \gamma_o \frac{\partial}{\partial t}\right) \theta\delta_{ij} \tag{7}$$

The equation of motion is given by:

$$\rho \frac{\partial^2 u_i}{\partial^2 t} = \left(2\mu_e \left(1 + \alpha_1 \frac{\partial}{\partial t}\right) + \lambda_e \left(1 + \alpha_o \frac{\partial}{\partial t}\right)\right) u_{j,ij} - \gamma_e \left(1 + \gamma_o \frac{\partial}{\partial t}\right) \theta_{,i} + \mu_o (J \times H_o) \tag{8}$$

The generalized heat conduction equation is given by

$$K\theta_{,ii} + K^* \dot{\theta}_{,ii} = \rho C_E \ddot{\theta} + \gamma_e T_o \left(1 + \gamma_o \frac{\partial}{\partial t}\right) \ddot{e} + \pi_o div\, J \tag{9}$$

The strain displacement relation is given by

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{10}$$

Together with the previous equations, constitute a complete system of generalized-magneto-thermo-viscoelasticity equations for a medium with a perfect conductivity.

Let (r, ψ, ϕ) denote the radial coordinates, the co-latitude, and the longitude of a spherical coordinates system, respectively. Due to spherical symmetry, all the considered function will be functions of r and t only.

The components of the displacement vector will be taken the form:

$$u_r = u(r,t), u_\psi = u_\phi = 0 \tag{11}$$

The strain tensor components are thus given by

$$e_{rr} = \frac{\partial u}{\partial r}, e_{\psi\psi} = e_{\phi\phi} = \frac{u}{r}, e_{r\phi} = e_{\phi\psi} = 0 \tag{12}$$

$$e = \frac{\partial u}{\partial r} + \frac{2u}{r} = \frac{1}{r^2}\frac{\partial(r^2 u)}{\partial r} \tag{13}$$

From Eq 7 we obtain the components of the stress tensor as

$$\sigma_{rr} = 2\mu_e\left(1 + \alpha_1\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial r} + \lambda_e\left(1 + \alpha_o\frac{\partial}{\partial t}\right)e - \gamma_e\left(1 + \gamma_o\frac{\partial}{\partial t}\right)\theta \tag{14}$$

$$\sigma_{\phi\phi} = \sigma_{\psi\psi} = 2\mu_e\left(1 + \alpha_1\frac{\partial}{\partial t}\right)\frac{u}{r} + \lambda_e\left(1 + \alpha_o\frac{\partial}{\partial t}\right)e - \gamma_e\left(1 + \gamma_o\frac{\partial}{\partial t}\right)\theta \tag{15}$$

$$\sigma_{r\phi} = \sigma_{r\psi} = \sigma_{\psi\phi} = 0 \tag{16}$$

Assume now that the initial magnetic field acts in the ϕ -direction and has the components$(0, 0, H_o)$. The induced magnetic field h will have one component h in the ϕ-direction, while the induced electric field E will have one component E in the ψ-direction.

Then, Eqs 1, 2 and 5 yield

$$J = H_o\frac{\partial e}{\partial r} + \frac{k_o}{\mu_o}\frac{\partial \theta}{\partial r} \tag{17}$$

$$h = -H_o\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) - \frac{k_o}{\mu_o}\frac{\partial \theta}{\partial r} \tag{18}$$

$$E = \mu_o H_o\frac{\partial u}{\partial t} + k_o\frac{\partial \theta}{\partial r} \tag{19}$$

From Eqs 17 and 6, we get that the Lorentz force has only one component F_r in the r-direction:

$$F_r = \mu_o H_o^2\frac{\partial e}{\partial r} + k_o H_o\frac{\partial \theta}{\partial r} \tag{20}$$

Also, we arrived at

$$\rho\frac{\partial^2 u}{\partial t^2} = \left(2\mu_e\left(1 + \alpha_1\frac{\partial}{\partial t}\right) + 2\lambda_e\left(1 + \alpha_o\frac{\partial}{\partial t}\right) + \mu_o H_o^2\right)\frac{\partial e}{\partial r} - \gamma_e\left(1 + \gamma_o\frac{\partial}{\partial t}\right)\frac{\partial e}{\partial r} + k_o H_o\frac{\partial \theta}{\partial r} \tag{21}$$

Equation 21 is to be supplemented by the constitutive Eq 13 and the heat conduction equation

$$K\nabla^2\theta + K^*\nabla^2\dot{\theta} = \rho C_E\ddot{\theta} + \gamma_e T_o\left(1+\gamma_o\frac{\partial}{\partial t}\right)\ddot{e} + \pi_o div\, J \tag{22}$$

Where ∇^2 is Laplaces operator in spherical coordinates which is given by

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 sin\psi}\left(sin\psi\frac{\partial}{\partial\psi}\right) + \frac{1}{r^2 sin^2\psi}\frac{\partial^2}{\partial\phi^2} \tag{23}$$

In case of dependence on r only, this reduce to

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) \tag{24}$$

Now, we shall use the following non dimensional variables:

$$\acute{r} = C_1\eta r, \acute{u} = C_1\eta u, \acute{t} = C_1^2\eta t, \acute{\gamma_o} = C_1^2\eta\gamma_{o,}\ \acute{\alpha_o} = C_1^2\eta\alpha_o, \acute{\alpha_1} = C_1^2\eta\alpha_1,$$
$$\acute{\sigma_{\iota J}} = \frac{\sigma_{ij}}{\mu_e}, \acute{\theta} = \frac{\theta}{T_o}, \acute{h} = \frac{h}{H_o}, \acute{E} = \frac{E}{\mu_o H_o C_1}, \acute{J} = \frac{J}{\eta H_o C_1} \tag{25}$$

Equations 14–19, 21 and 22 take the following form (dropping the primes for convenience).

$$J = \frac{\partial e}{\partial r} + A\frac{\partial\theta}{\partial r} \tag{26}$$

$$h = -\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) - A\frac{\partial\theta}{\partial r} \tag{27}$$

$$E = \frac{\partial u}{\partial t} + A\frac{\partial\theta}{\partial r} \tag{28}$$

$$\sigma_{rr} = \frac{2\mu_e}{\lambda_e+2\mu_e}\left(1+\alpha_1\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial r} + \frac{\lambda_e}{\lambda_e+2\mu_e}\left(1+\alpha_o\frac{\partial}{\partial t}\right)e\,\frac{\gamma_e\theta_o}{\lambda_e+2\mu_e}\left(1+\gamma_o\frac{\partial}{\partial t}\right)\theta \tag{29}$$

$$\sigma_{\phi\phi} = \sigma_{\psi\psi} = \frac{2\mu_e}{\lambda_e+2\mu_e}\left(1+\alpha_1\frac{\partial}{\partial t}\right)\frac{u}{r} + \frac{\lambda_e}{\lambda_e+2\mu_e}\left(1+\alpha_o\frac{\partial}{\partial t}\right)e\text{-}\frac{\gamma_e\theta_o}{\lambda_e+2\mu_e}\left(1+\gamma_o\frac{\partial}{\partial t}\right)\theta \tag{30}$$

$$\sigma_{r\phi} = \sigma_{r\psi} = \sigma_{\phi\psi} = 0 \tag{31}$$

$$\frac{\partial^2 u}{\partial t^2} = \left(1 + A + \mu_o H_o^2 + \frac{\lambda_e\alpha_o + 2\mu_e\alpha_1}{\rho C_1^2}\frac{\partial}{\partial t}\right)\frac{\partial e}{\partial r} - \frac{\gamma_e\theta_o}{\rho C_1^2}\left(1 + A + \gamma_o\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial r} \tag{32}$$

$$\nabla^2\theta + \frac{K^* C_1^2\eta}{K}\nabla^2\dot{\theta} = \eta C_1^2\ddot{\theta} + \frac{\gamma_e C_1^2}{K}\left(1+\gamma_o\frac{\partial}{\partial t}\right)\ddot{e} \tag{33}$$

Where, $\eta = \frac{\rho C_E}{K}, C_1^2 = \frac{\lambda_e+2\mu_e}{\rho}, A = \frac{k_o}{\mu_o H_o}$.

Equation 32 can be written in the form:

$$\ddot{e} = \left(1 + A + \mu_o H_o^2 + \frac{\lambda_e\alpha_o + 2\mu_e\alpha_1}{\rho C_1^2}\frac{\partial}{\partial t}\right)\nabla^2 e - \frac{\gamma_e\theta_o}{\rho C_1^2}\left(1 + A + \gamma_o\frac{\partial}{\partial t}\right)\nabla^2\theta \tag{34}$$

3. Laplace transform domain

Taking the Laplace transform of Eqs 26–31, 33 and 34 by using homogeneous initial conditions, defined and denoted as

$$\bar{f}(s)\int_0^\infty e^{-st}f(t)dt \qquad ,s>0 \tag{35}$$

We obtain

$$\bar{J}=\frac{d\bar{e}}{dr}+A\frac{d\bar{\theta}}{dr} \tag{36}$$

$$\bar{h}=-\left(\frac{d\bar{u}}{dr}+\frac{\bar{u}}{r}\right)-A\frac{d\bar{\theta}}{dr} \tag{37}$$

$$\bar{E}=s\bar{u}+A\frac{d\bar{\theta}}{dr} \tag{38}$$

$$\nabla^2\bar{\theta}=L_1\bar{\theta}+L_1\bar{e} \tag{39}$$

$$\bar{\sigma}_{rr}=a_1\frac{d\bar{u}}{dr}+a_2\bar{e}-a_3\bar{\theta} \tag{40}$$

$$\bar{\sigma}_{\phi\phi}=\bar{\sigma}_{\theta\theta}=a_1\frac{\bar{u}}{r}+a_2\bar{e}-a_3\bar{\theta} \tag{41}$$

$$\nabla^2\bar{e}=M_1\bar{\theta}+M_1\bar{e} \tag{42}$$

where

$L_1=\frac{K\eta C_1^2s^2}{K+A+K^*\eta C_1^2s^2}$ $,l_2=\frac{\gamma_e C_1^2(1+\gamma_o s)s^2}{K+A+K^*\eta C_1^2s^2}$ $,a_1=\frac{2\mu_e}{\lambda_e+2\mu_e}(1+A+\alpha_1 s)$, $a_2=\frac{\lambda_e}{\lambda_e+2\mu_e}(1+A+\alpha_o s)$,

$a_3=\frac{\gamma_e\theta_o}{\lambda_e+2\mu_e}(1+A+\gamma_o s)$ $,M_1=\frac{\gamma_e\theta_o(1+A+\gamma_o s)L_1}{\mu_o H_o^2\rho C_1^2+\rho C_1^2+(\lambda_e\alpha_o+2\mu_e\alpha_1)s}$, $M_1=\frac{\rho C_1^2s^2+\gamma_e\theta_o(1+A+\gamma_o s)L_2}{\mu_o H_o^2\rho C_1^2+\rho C_1^2+(\lambda_e\alpha_o+2\mu_e\alpha_1)s}$.

4. State space formulation

Choosing as state variables the temperature of heat conduction $\bar{\theta}$and the strain components $\bar{e}$ then Eqs 35 and 38 can be written in the matrix form

$$\nabla^2V(r,s)=A(s)\bar{V}(r,s) \tag{43}$$

where $\bar{V}(r,s)=\begin{bmatrix}\bar{\theta}(r,s)\\ \bar{e}(r,s)\end{bmatrix}$ $,A(s)=\begin{bmatrix}L_1 & L_2\\ M_1 & M_2\end{bmatrix}$.

The formal solution of Eq 43 can be written in the form

$$\bar{V}(r,s)=C\frac{e^{-\sqrt{A(s)}r}}{r}+B\frac{e^{-\sqrt{A(s)}r}}{r} \tag{44}$$

For bounded solution with large r, we have canceled the exponential part has positive power. And at $r = R$ the value of C is given by $C = R\bar{V}(R,s)e^{\sqrt{A(s)}R}$, then Eq 44 reduces to

$$\bar{V}(r,s) = \frac{R}{r}\bar{V}(R,s)e^{-\sqrt{A(s)}R}, r \gg R \quad (45)$$

We will use the will-known Cayley–Hamiltonian theorem to find the form of the matrix $exp\left(-\sqrt{A(s)}(r-R)\right)$. The characteristic equation of the matrix $A(s)$ can be written as

$$k^2 - (L_1 + M_2)k + (L_1M_2 - L_2M_1) \quad (46)$$

The roots of this equation namely k_1 and k_2 satisfy the relations

$$k_1 + k_2 = L_1 + M_2 \quad (47)$$

$$k_1k_2 = L_1M_2 - L_2M_1 \quad (48)$$

The Tailor's series expansion for the matrix exponential of $exp\left(-\sqrt{A(s)}(r-R)\right)$ is given by

$$exp\left(-\sqrt{A(s)}(r-R)\right) = \sum_{n=0}^{\infty}\frac{\left[-\sqrt{A(s)}(r-R)\right]^n}{n!} \quad (49)$$

Using Cayley–Hamiltonian theorem, we can express A^2 and higher orders of the matrix A in terms of I and A where I is the unit matrix of second order.

Thus, the infinite series in Eq 49 can be reduced to

$$exp\left(-\sqrt{A(s)}(r-R)\right) = b_o(r,s)I + b_1(r,s)A \quad (50)$$

where b_o and b_1 are some coefficients depending on s and r.

By Cayley–Hamiltonian theorem, the characteristic roots k_1 and k_2 of the matrix A must satisfy Eq 50, thus we have

$$exp\left(-\sqrt{k_1}(r-R)\right) = b_o + b_1k_1 \quad (51)$$

$$exp\left(-\sqrt{k_2}(r-R)\right) = b_o + b_1k_2 \quad (52)$$

Solving the above linear system of equations, we get

$$b_o = \frac{k_1e^{-\sqrt{k_2}(r-R)} - k_2e^{-\sqrt{k_1}(r-R)}}{k_1 - k_2} \quad (53)$$

$$b_1 = \frac{e^{-\sqrt{k_1}(r-R)} - e^{-\sqrt{k_2}(r-R)}}{k_1 - k_2} \quad (54)$$

Hence, we have

$$exp\left(-\sqrt{A(s)}(r-R)\right) = L_{ij}(r,s), \;\; i,j = 1,2 \tag{55}$$

where

$$L_{11} = \frac{e^{-\sqrt{k_2}(r-R)}(k_1-L_1)+e^{-\sqrt{k_1}(r-R)}(L_1-k_2)}{k_1-k_2} \tag{56}$$

$$L_{12} = \frac{L_2 e^{-\sqrt{k_1}(r-R)} - L_1 e^{-\sqrt{k_2}(r-R)}}{k_1-k_2} \tag{57}$$

$$L_{21} = \frac{M_1 e^{-\sqrt{k_1}(r-R)} - M_2 e^{-\sqrt{k_1}(r-R)}}{k_1-k_2} \tag{58}$$

$$L_{22} = \frac{e^{-\sqrt{k_1}(r-R)}(M_2-k_2)+e^{-\sqrt{k_2}(r-R)}(k_1-M_2)}{k_1-k_2} \tag{59}$$

5. Applications

In order to evaluate the unknown parameters $\overline{\theta}_o(r,s)$ and $\overline{e}_o(r,s)$, we shall use the boundary conditions on the internal surface of the shell, $r = R$ which are given by:
(I) Thermal boundary condition at $r = R$, $\theta(R,t) = \theta_o$

Taking the Laplace transform, this is defined as following:

$$\bar{\theta}_o(r,s) = \frac{\theta_o}{s} \tag{60}$$

(II) Mechanical boundary condition

The internal surface $r = R$ has a rigid foundation, which is rigid enough to prevent any strain $e(R,t) = 0$. Taking the Laplace transform, this is defined as following:

$$\bar{e}(R,s) = \bar{e}_o = 0 \tag{61}$$

Using the Eq 60 and 61 into Eq 45 and using Eqs 56–59, we get

$$\bar{\theta}(r,s) = \frac{R\theta_o}{s(k_1-k_2)r}\left[(k_1-L_2)e^{-\sqrt{k_2}(r-R)} + (k_2-L_1)e^{-\sqrt{k_2}(r-R)}\right] \tag{62}$$

$$\bar{e}(r,s) = \frac{RM_1\theta_o}{s(k_1-k_2)r}\left[(k_2-L_1)e^{-\sqrt{k_1}(r-R)} + (k_1-L_2)e^{-\sqrt{k_2}(r-R)}\right] \tag{63}$$

To find the displacement, taking Laplace transform for Eq 32 using Eqs 62 and 63, we get

$$\bar{u}(r,s) = \frac{R\theta_o}{sr^2(k_1-k_2)r}\left(\left(\left(1+r\sqrt{k_1}\right)\left(B_5(L_1-k_2)\right) - B_4M_1\right)e^{-\sqrt{k_1}(r-R)} + \left(\left(1+r\sqrt{k_2}\right)\left(B_4M_1 + B_5(k_1-L_1)\right)\right)e^{-\sqrt{k_2}(r-R)}\right) \tag{64}$$

To find the radial stress, from Eq 40 and Eqs 62–64 we get

$$\bar{\sigma}_{rr} = \frac{R\theta_o}{sr^2(k_1-k_2)r}\Big\{e^{-\sqrt{k_1}(r-R)}\Big(M_1(B_1+B_3)r^2 + 2B_1B_4M_1\big(1+r\sqrt{k_1}\big) - (L_1-k_2)\Big(B_3r^2 + 2B_1B_5\big(1+r\sqrt{k_1}\big)\Big)\Big) + e^{-\sqrt{k_2}(r-R)}\Big(-M_1(B_1+B_2)r^2 - 2B_1B_4M_1\big(1+r\sqrt{k_1}\big) + (L_1-k_2)\Big(B_3r^2 + 2B_1B_5\big(1+r\sqrt{k_2}\big)\Big) + \Big)\Big\} \quad (65)$$

where $B_4 = \frac{1}{s^2} + \frac{\lambda_e\alpha_o + 2\mu_e\alpha_1}{\rho C_1^2 s^2}$, $B_5 = \frac{\gamma_e\theta_o(1+\gamma_o s)}{\rho C_1^2 s^2}$.

6. Numerical inversion of Laplace transforms

In order to invert the Laplace transforms in the above equations we shall use a numerical technique based on Fourier expansions of functions. Let $\bar{g}(s)$ be the Laplace transform of a given function $g(t)$. The inversion formula of Laplace transforms states that

$$g(t) = \frac{1}{2\pi i}\int_{d-i\infty}^{d+i\infty} e^{st}\bar{g}(s)ds \quad (66)$$

where d is an arbitrary positive constant greater than all the real parts of the singularities of $\bar{g}(s)$. Taking $s= d + iy$, we get

$$g(t) = \frac{e^{dt}}{2\pi}\int_{-\infty}^{\infty} e^{iky}\bar{g}(d+iy)dy \quad (67)$$

This integral can be approximated by

$$g(t) = \frac{e^{dt}}{2\pi}\sum_{k=-\infty}^{\infty} e^{ikt\Delta y}\bar{g}(d+ik\Delta y)\Delta y \quad (68)$$

Taking $\Delta y = \frac{\pi}{t_1}$ we obtain:

$$g(t) = \frac{e^{dt}}{t_1}\left(\frac{1}{2}\bar{g}(d) + Re\left(\sum_{k=1}^{\infty} e^{ikt\pi/t_1}\,\bar{g}(d+ik\pi/t_1)\right)\right) \quad (69)$$

For numerical purposes this is approximated by the function

$$g_N(t) = \frac{e^{dt}}{t_1}\left(\frac{1}{2}\bar{g}(d) + Re\left(\sum_{k=1}^{N} e^{ikt\pi/t_1}\,\bar{g}(d+ik\pi/t_1)\right)\right) \quad (70)$$

where N is a sufficiently large integer chosen such that

$$\frac{e^{dt}}{t_1}Re\left(e^{iN\pi t/t_1}\bar{g}(d+iN\pi/t_1)\right) < \eta \quad (71)$$

where η is a reselected small positive number that corresponds to the degree of accuracy to be achieved Eq 69 is the numerical inversion formula valid for $0 \le t \le t_1$. In particular, we choose$t = t_1$, getting

$$g_N(t) = \frac{e^{dt}}{t_1}\left(\frac{1}{2}\bar{g}(d) + Re\left(\sum_{k=1}^{N}(-1)^k\,\bar{g}(d + ik\pi/t_1)\right)\right) \quad (72)$$

7. Numerical results and discussions

The copper material was chosen for purposes of numerical evaluations and constants of the problem were taken as following (35) in SI units: $K = 386\,N/Ks$, $\alpha_t = 17.8(10)^{-5}\,K^{-1}$, $C_E = 383.1\,m^2/K$, $T_o = 293K$, $\rho = 8954\,kg/m^2$, $\mu_e = 3.86(10)^{10}\,N/m^2$, $\lambda_e = 7{,}76(10)^{10}\,N/m^2$, $\alpha_1 = 3.25(10)^{-2}$, $\alpha_o = 3.25(10)^{-2}$, $R = 1$, $\theta_o = 1$.

In order to study the effect of time t and study the comparison between two models on temperature, radial stress, shear stress, displacement and strain, we now present our results in the form of graphs (Figures 2–9).

Green and Naghdi [7–9] proposed three new thermoelastic theories based on entropy equality rather than the usual entropy inequality. The constitutive assumptions for the heat flux vector are different in each theory. Thus, they obtained three theories they called thermoelasticity of type I, thermoelasticity of type II and thermoelasticity of type III. When the type I theory is linearized we obtain the classical system of thermo-elasticity. The type II theory (is a limiting case of type III) does not admit energy dissipation.

Figure 2 is plotted to show the variation of temperature θ against r for wide range of r $(1 \leq r \leq 3)$ at small time $(t = 0.07)$ for two theories (G-N III) and (G-N II). It is observed from this figure the magnitude of the temperature is greater for (G-N III) model than (G-N II). It can be noted that the speed of propagation of temperature is finite and coincide with the physical behavior of viscoelastic material. Also, we can see from this figure that the boundary condition (60) is satisfied.

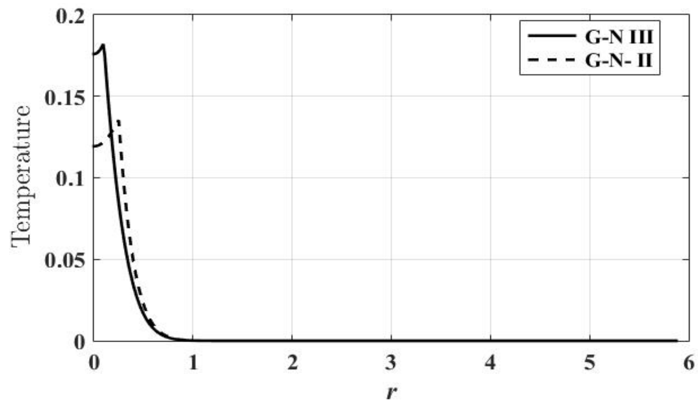

Figure 2. Variation of temperature θ with distance r for two theoris.

Figure 3 shows variation of temperature θ for the coefficient of Ohm and Fourier laws. It is noticed that the modified Fourier and Ohm laws influence is significant, the temperature in the modified model records value higher than these in the old model.

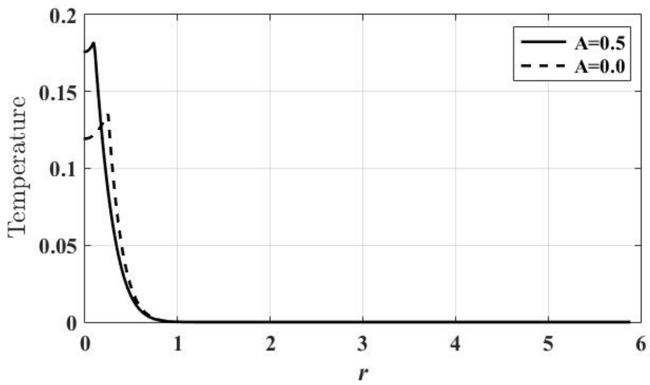

Figure 3. Variation of temperature θ with distance r for the coefficient of Ohm and Fourier laws.

Figure 4 is plotted to show the variation of the radial stress σ_{rr} against r for wide range of r $(1 \leq r \leq 10)$, at small time $(t = 0.07)$, for two theories (G-N III) and (G-N II). It is observed from this figure the magnitude of the radial stress is greater for (G-N III) model than (G-N II). It can be noted that the speed of propagation of stress is finite and coincide with the physical behavior of viscoelastic material.

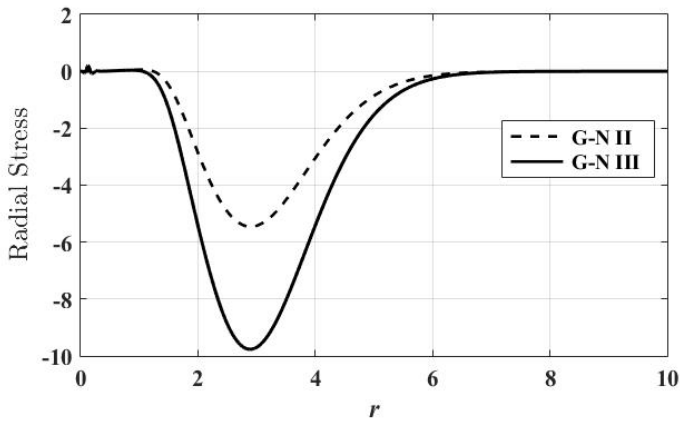

Figure 4. Variation of radial stress σ_{rr} with distance r for two theories.

Figure 5 shows variation of radial stress for the coefficient of Ohm and Fourier laws. It is noticed that the modified Fourier's and Ohm's model effects on the radial stress by increasing their values.

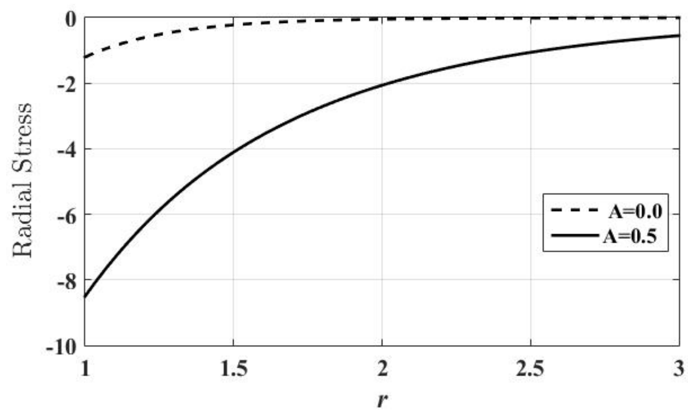

Figure 5. Variation of radial stress σ_{rr} with distance r for the coefficient of Ohm and Fourier laws.

Figure 6 is plotted to show the variation of strain e against r for wide range of $(1 \le r \le 3)$, at small time $(t = 0.07)$, for two theories (G-N III) and (G-N II). It is observed from this figure the magnitude of the strain is greater for (G-N III) model than (G-N II). It can be noted that the speed of propagation of strain is finite and coincide with the physical behavior of viscoelastic material. Also, we can see from this figure that the boundary condition (61) is satisfied.

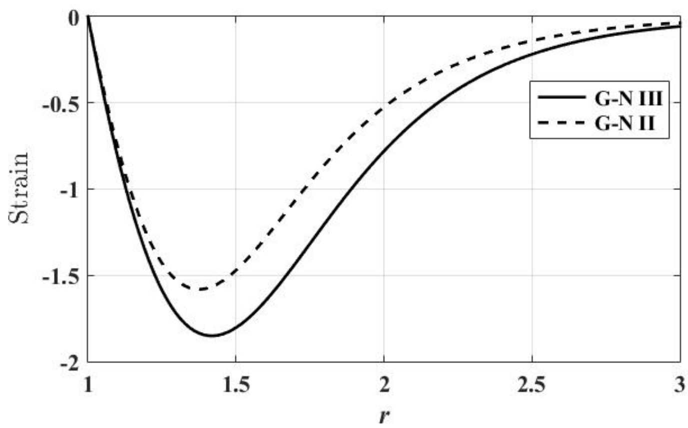

Figure 6. Variation of strain e with distance r for two theories.

Figure 7 shows variation of strain e for the coefficient of Ohm and Fourier laws. It is noticed that the modified Fourier and Ohm laws influence is significant, the strain in the modified model records value higher than these in the old model.

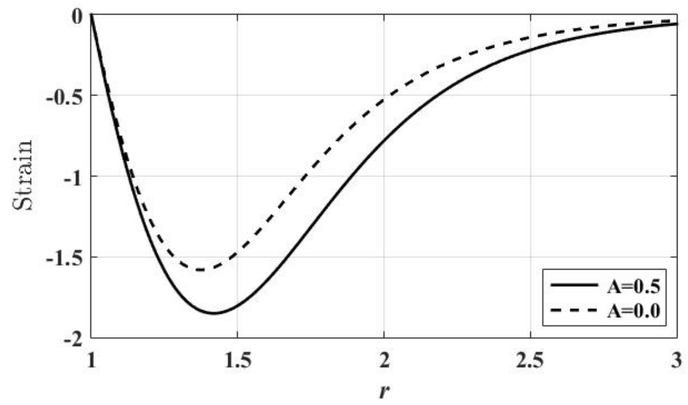

Figure 7. Variation of strain e with distance r for the coefficient of Ohm and Fourier laws.

Figure 8 is plotted to show the variation of displacement u against r for wide range of $(1 \leq r \leq 3)$, at small time$(t = 0.07)$, for two theories (G-N III) and (G-N II). It is observed from this figure the magnitude of the displacement is greater for (G-N III) model than (G-N II). It can be noted that the speed of propagation of displacement is finite and coincide with the physical behavior of viscoelastic material.

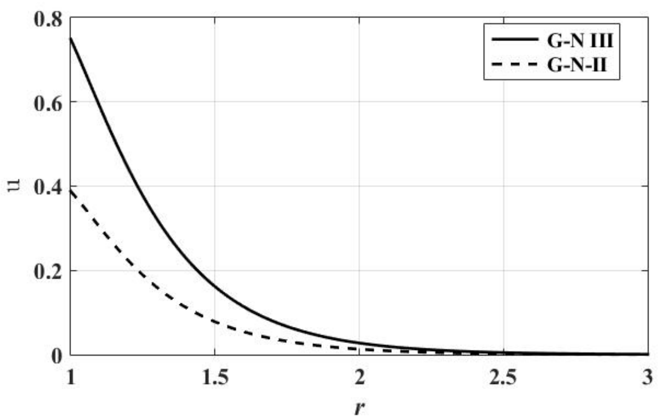

Figure 8. Variation of displacement u with distance r for two theories.

Figure 9 shows the variation of displacement u against r for wide range of$r$ $(1 \leq r \leq 3)$, for different values of time $(t = 0.0, t = 0.5, t = 0.1)$. And we have noticed that, the time t has significant effects on displacement$u$. The increasing of the value of t causes increasing of the value of displacement u, and displacement u vanishes more rapidly.

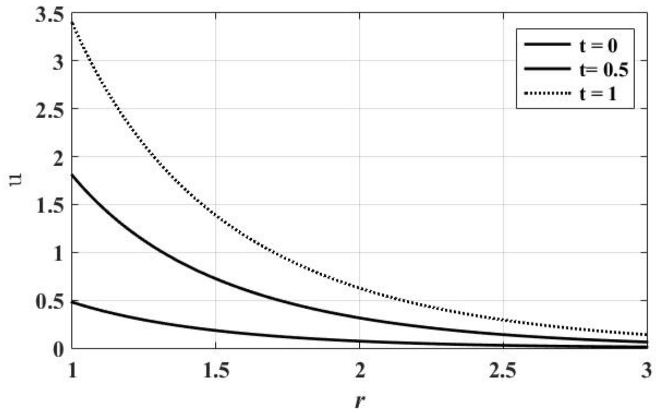

Figure 9. Variation of displacement u with distance r for different value of time t.

8. Conclusion

Green and Naghdi [7–9] developed a generalized theory of thermoelasticity which involves thermal displacement gradient as one of the constitutive variables in contrast to the classical coupled thermoelasticity which includes temperature gradient as one of the constitutive variables. An important feature of this theory is that it does not accommodate dissipation of thermal energy. On this theory the characterization of material response to a thermal phenomenon is based on three types of constitutive response functions. The nature of those three types of constitutive response functions is such that when the respective theories are linearized, type I is same as classical heat conduction equation (based on Fourier's law), whereas type II, the internal rate of production of entropy is taken to be identically zero, implying no dissipation of thermal energy. This model is known as the theory of thermoelasticity without energy dissipation. Type III involves the previous two models as special cases, and admits dissipation of energy in general, in this model, introducing the temperature gradient and thermal displacement gradient as the constitutive variables.

From the above discussion, one can reason that the new model of generalized magneto-thermo-viscoelasticity predicts new qualities for the temperature, displacement, stresses and strain. The impact of the temperature gradient at adequately low temperature may cause new sorts of magneto-thermo-viscoelastic wave with explicit stability properties. The expansion in the estimations of temperature might be clarified as the lost heat producing from the development of electric flow; this heat might be the fundamental motivation behind why the deformation of the medium will in general be ordinary and the magnetic field records esteems more prominent than the qualities in the old model. As indicated by this work, numerous specialists in the field of generalized thermo elasticity have applied Green–Naghdi hypothesis (III-II) for thermo elastic problem and not many of them can effectively applied for magneto thermo Viscoelastic issue. Right now conclude that the greatness of the every single physical amount is more noteworthy for (G-N III) model than (G-N II). It very well may be noticed that the speed of spread of every single physical amount is limited and match with the physical conduct of Viscoelastic material.

Acknowledgements

The authors wish to acknowledge the approval and the support of this research study by the Project NO. SCI-2018-3-9-F-7795 from the Deanship of Scientific Research in Northern Border University, Arar, Saudi Arabia.

Conflict of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Reference

1. Biot MA (1956) Thermo elasticity and irreversible thermo-dynamics. *J Appl Phys* 27: 240–253.
2. Lord HW, Shulman Y (1976) A generalized dynamical theory of thermo elasticity. *J Mech Phys Solid* 15: 299–309.
3. Müller MM, Kaiser E, Bauer P, et al. (1976) Lipid composition of the rat kidney. *Nephron* 17: 41–50.
4. Green AE, Laws N (1972) On the entropy production inequality. *Arch Ration Mech An* 45: 47–53.
5. Green AE, Lindsay KA (1972) Thermo elasticity. *J Elasticity* 2: 1–7.
6. Shuhubi E (1957) Thermo elastic solid, In: Eringen AC, *Continuum Physics*, New York: Academic Press.
7. Green AE, Naghdi PM (1991) A re-examination of the basic postulate of thermo-mechanics. *P Roy Soc A-Math Phy* 432: 171–194.
8. Green AE, Naghdi PM (1993) Thermoelasticity without energy dissipation. *J Elasticity* 31: 189–208.
9. Green AE, Naghdi PM (1992) An unbounded heat wave in an elastic solid. *J Therm Stresses* 15: 253–264.
10. Illyushin AA, Pobedria BE (1970) Fundamentals of the mathematical theory of thermal viscoelasticity.
11. Biot MA (1954) Theory of stress-strain relations in anisotropic viscoelasticity and relaxation phenomena. *J Appl Phys* 25: 1385–1391.
12. Biot MA (1955) Variational principles in irreversible thermodynamics with application to viscoelasticity. *Phys Rev* 97: 1463–1469.
13. Morland LW, Lee EH (1960) Stress analysis for linear viscoelastic materials with temperature variation. *J Rheol* 4: 233–263.
14. Tanner RI (1988) *Engineering Rheology*, Oxford: Oxford University Press.
15. Drozdov AD (1996) A constitutive model in thermoviscoelasticity. *Mech Res Commun* 23: 543–548.
16. Bland DR (1960) *The Theory of Linear Viscoelasticity*, Oxford: Pergamon Press.

17. Lion A (1997) On the large deformation behavior of reinforced rubber at different temperatures. *J Mech Phys Solids* 45: 1805–1834.
18. Liao Z, Hossain M, Yao X, et al. (2020) Thermo-viscoelastic experimental characterization and numerical modelling of VHB polymer. *Int J Nonlin Mech* 118: 103263.
19. Niyonzima I, Jiao Y, Fish J (2019) Modeling and simulation of nonlinear electro-thermo-mechanical continua with application to shape memory polymeric medical devices. *Comput Method Appl M* 350: 511–534.
20. Mehnert M, Hossain M, Steinmann P (2017) Towards a thermo-magneto-mechanical coupling framework for magneto-rheological elastomers. *Int J Solids Struct* 128: 117–132.
21. Mehnert M, Hossain M, Steinmann P (2016) On nonlinear thermo-electro-elasticity. *P Roy Soc A-Math Phys* 472: 20160170.
22. Youssef HM, El-Bary AA, Elsibai KA (2014) Vibration of gold nano beam in context of two-temperature generalized thermoelasticity subjected to laser pulse. *Lat Am J Solids Stru* 11: 2460–2482.
23. Ezzat MA, El-Bary AA (2014) Two-temperature theory of magneto-thermo-viscoelasticity with fractional derivative and integral orders heat transfer. *J Electromagnet Wave* 28: 1985–2004.
24. Ezzat MA, El-Bary AA (2015) State space approach to two-dimensional magneto-thermoelasticity with fractional order heat transfer in a medium of perfect conductivity. *Int J Appl Electrom* 49: 607–625.
25. Ismail MAH, Khamis AK, El-Bary AA, et al. (2017) Effect of rotation of generalized thermoelastic layer subjected to harmonic heat: state-space approach. *Microsyst Technol* 23: 3381–3388.
26. Khamis AK, Ismail AH, Youssef HM, et al. (2017) Thermal shock problem of two-temperature generalized thermoelasticity without energy dissipation with rotation. *Microsyst Technol* 23: 4831–4839.
27. Youssef HM, Elsibai KA, El-Bary AA (2017) Effect of the speed, the rotation and the magnetic field on the Q-factor of an axially clamped gold micro-beam. *Meccanica* 52: 1685–1694.
28. Ezzat MA, El-Karamany AS, El-Bary AA (2017) Thermoelectric viscoelastic materials with memory-dependent derivative. *Smart Struct Sys* 19: 539–551.
29. El-Karamany AS, Ezzat MA, El-Bary AA (2018) Thermodiffusion with two time delays and Kernel functions. *Math Mech Solids* 23: 195–208.
30. Ezzat MA, El-Bary AA (2018) Thermoelectric spherical shell with fractional order heat transfer. *Microsyst Technol* 24: 891–899.
31. El-Bary AA, Atef H (2016) On effect of viscous fractional parameter on infinite thermo Viscoelastic medium with a spherical cavity. Journal of computational and theoretical. *Nanoscience* 13: 1–5.
32. El-Bary AA, Atef M (2016) Modified approach for stress strain equation in the linear Kelvin–Voigt solid based on fractional order. *J Comput Theor Nanos* 13: 1027–1036.
33. Amin MM, El-Bary A, Atef H (2018) Effect of viscous fractional parameter on generalized magneto thermo-viscoelastic thin slim strip exposed to moving heat source. *Mater Focus* 7: 814–823.

34. Amin MM, El-Bary AA, Atef HM (2018) Modification of Kelvin–Voigt model in fractional order for thermoviscoelastic isotropic material. *Mater Focus* 7: 824–832.
35. Sharma K, Kumar P (2013) Propagation of plane waves and fundamental solution in thermoviscoelastic medium with voids. *J Therm Stresses* 36: 94–111.