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Research article

Interaction factors for two elliptical embedded cracks with a wide range of aspect ratios

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Abstract: The value of stress intensity factor may be increased through the interaction of multiple cracks that are in close proximity to one another. We investigated the interaction factors of two equal elliptical cracks with a wide range of aspect ratios. Finite element analysis for a linear elastic solid was used to obtain the interaction factor for embedded cracks in an infinite model subjected to remote tension loading. Relationships between interaction factors and dimensionless distances between the cracks were discussed. The results demonstrated that the interaction factors depend on the crack aspect ratio, whose effect is related to the dimensionless distance. Thus, it is suggested that interaction factors can be reasonably characterized using different dimensionless distances depending on the aspect ratio. Finally, we provide a simple empirical formula for obtaining the interaction factors for embedded cracks.

Keywords: stress intensity factor; embedded crack; multiple crack interaction; interaction factor; finite element analysis; fracture mechanics; fitness-for-service

1. Introduction

The stress intensity factor is an important parameter for evaluating the growth of cracks or predicting failures caused by cracks in flawed components such as pressure vessels and pipes. The stress intensity factor is proportional to remote stress and the square root of crack size. The geometry around the crack also influences the value of the stress intensity factor. For example, cracks in close proximity to one another increase the stress intensity factor of each crack by interacting with each other. The magnitude of the interaction (hereafter the interaction factor) is defined by an increased rate of the stress intensity factor [1]. The interaction factors have been studied intensively because of both their scientific and engineering importance. For example, Isida et al. [2] applied the body force method to closely located elliptical cracks subjected to tension. Their numerical calculations showed that interaction factors depend on the shapes and relative locations of the cracks. Kachanov and Laures investigated the interactions for coplanar penny-shaped cracks using the superposition principle of elasticity theory, and demonstrated that the size of a neighboring crack determines the interaction factor [3]. Xiao et al. also applied the superposition technique to analyze stress and proposed a closes-form solution of stress intensity factors along the crack front of two elliptical cracks [4]. Their solutions clearly indicated that the interaction factor is a function of crack length and depth, as well as the distance between the cracks.

The stress intensity factor of multiple cracks can, in fact, be treated as that of a single crack due to fitness-for-service codes allowing for flaw combination to be used. These combination rules are generally described in a simplified form for practicality in the field of engineering [5,6]. For example, according to the rules provided by ASME B&PV Code Section XI [5], the distance between the cracks is defined by the distance between the bounding boxes, which is the smallest square containing each crack, with sides parallel to the coordinate axes. If multiple cracks are detected during an in-service inspection and the distances are equal to, or less than, the depth of the larger crack, these cracks shall be treated as a single large crack. As a result, the bounding box of this large crack embraces the multiple original cracks. The applicability of this rule has been thoroughly investigated based on extensive research on interaction factors at co-vertices (endpoints of the minor axis) of elliptic cracks, where the largest stress intensity factor for a single crack is observed [7]. However, the distribution of the stress intensity factor along the crack front changes by the multiple crack interaction. Even vertices (endpoints of the major axis), where the stress intensity factor for a single crack is smallest, may be more significant on fracture behavior of the material in some crack alignment. Precise evaluation of the stress intensity factor at vertices and co-vertices of elliptic cracks are important for engineering application. Thus, it is necessary to study the relation between the crack alignment and the interaction features at these points in a systematic manner.

In this study, we investigate the interaction factors at vertices and co-vertices of two equal elliptical cracks with a wide range of aspect ratios. The stress intensity factors and interaction factors for cracks in an infinite solid subjected to uniform remote tension are obtained by finite element analysis (FEA). The relationship between the interaction factors and geometric parameters of the cracks are then discussed. Finally, an empirical formula for evaluating the interaction factors at both vertices and co-vertices in a uniform manner is proposed.

2. Analysis Model

2.1. FEA Model

Figure 1(a) shows the basic geometry of an analysis model in the shape of a solid cube with 900 mm sides. A uniform remote tension σ (= 10 MPa) was applied perpendicularly to the crack surface. In terms of material property, the modulus of elasticity was 206 GPa and the Poisson’s ratio was 0.3. The two equal elliptical cracks were located on the same plane at the cube center. As shown in Figure 1(b), the geometric features of the cracks were defined by three parameters: crack depth a (1.875 mm $\leq a \leq$ 30 mm), half-crack length c (1.875 mm $\leq c \leq$ 30 mm), and distance between the cracks S (1.875 mm $\leq S \leq$ 15 mm). In all cases, either the depth a or the length c was 7.5 mm. Since the solid model is much larger than the crack size, the model can be thought of as an infinite body. Figure 1(c) shows the case where $a/c > 1.0$, referred to as a crack with a large aspect ratio [8]. Although crack depth and length are interchangeable in an infinite body, all pairs of cracks in this study were aligned in the direction of crack depth for simplicity. In Figures 1(b) and 1(c), ϕ is defined as the crack front angle and point A (a co-vertex for $a/c \leq 1.0$ and a vertex for $a/c > 1.0$) represents the closest point to a neighboring crack.

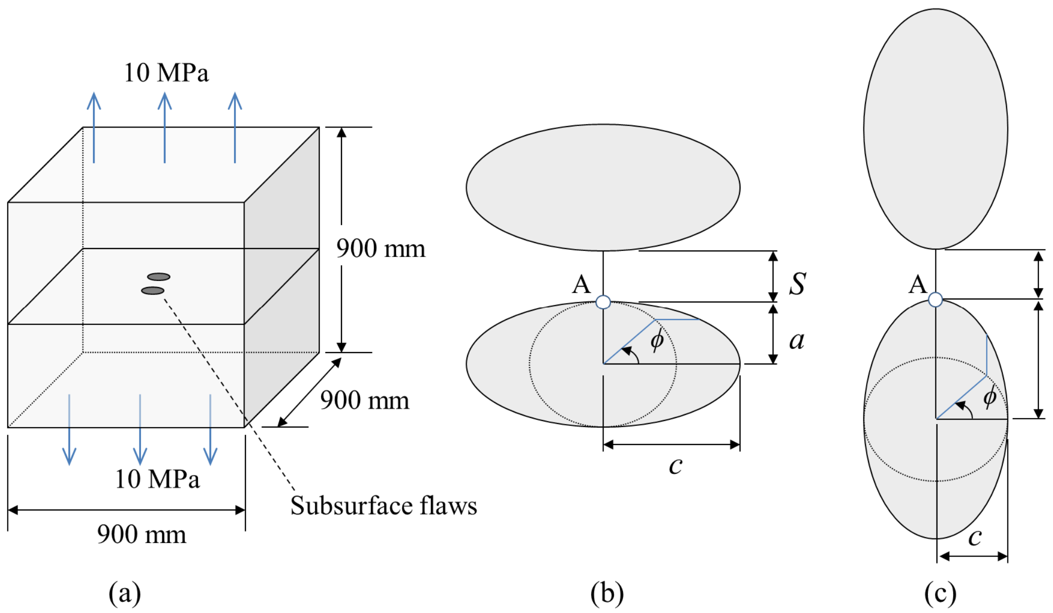

Figure 1. Two equal elliptical cracks in a solid model. (a) Schematic of a solid cube model subjected to uniform remote tension. (b) Interaction at the co-vertex of two elliptical cracks with a small aspect ratio ($a/c \leq 1.0$). (c) Interaction at the vertex of two elliptical cracks with a large aspect ratio ($a/c > 1.0$).

The stress intensity factors along the crack edge were obtained by using J integral calculations implemented through the ABAQUS Standard 6.13 [9]. The finite element meshes were generated by an in-house mesh generator, CRACK-FEM [10], which modeled the crack edge with a tube filled with concentric 20-node hexahedral meshes and the remaining part of the domain with 10-node tetrahedral meshes, as shown in Figures 2(a) and 2(b). The hexahedral mesh region was connected to the surrounding tetrahedral mesh by using a tied contact. Each face of the 20-node elements connected to the crack edge was collapsed, and mid-side nodes adjacent to the face were moved to the quarter points so as to model the singular stress distribution [11]. The mesh generator created around 6,000 hexahedral elements and more than 40,000 tetrahedral elements. A mesh convergence study has been performed to ensure that the discretization error of the stress intensity factor is 0.25% at most.

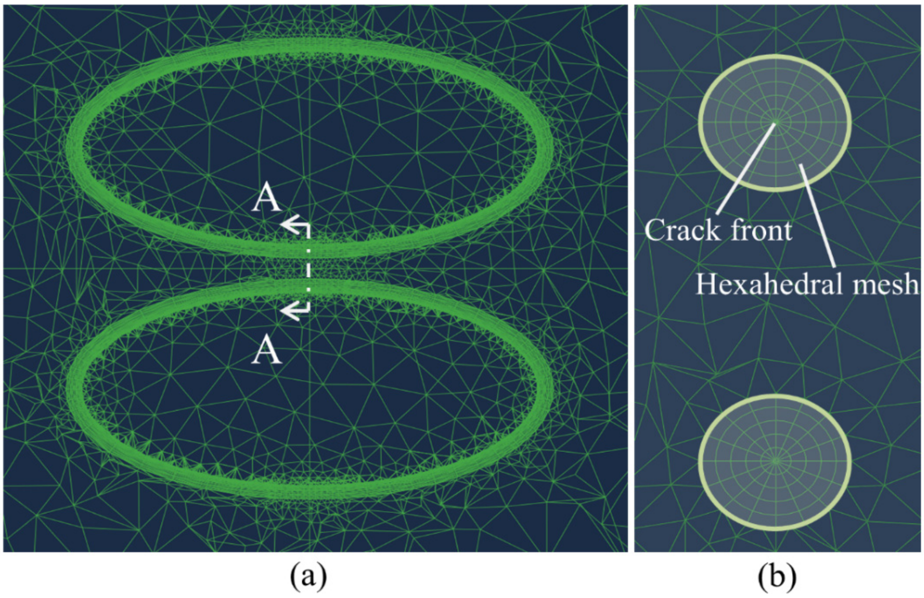

Figure 2. Finite mesh element. (a) Mesh for the crack region. (b) Cross-sectional view (section A–A).

2.2. Stress Intensity Factors and Interaction Factors

The interaction factor is the ratio of stress intensity factors for multiple cracks to those for an isolated crack, and is given as follows:

$$\gamma = \frac{K_{\mathrm{I}}}{K_{\mathrm{I0}}} \tag{1}$$

where K_{I} and K_{I0} are the FEA solutions of the stress intensity factors for multiple and isolated cracks, respectively. The analytical solutions of K_{I0} are also available. For example, the stress intensity factors at point A are given by

$$K_{\mathrm{I0,A}} = \frac{\sigma}{E(k)} \sqrt{\pi a} \qquad (a \leq c) \tag{2}$$

$$K_{\mathrm{I0,A}} = \frac{\sigma}{E(k)} \sqrt{\frac{c}{a}} \sqrt{\pi c} \qquad (a > c) \tag{3}$$

where $E(k)$ is the complete elliptical integral of the second kind with argument k, given by

$$E(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 \phi)^{1/2} d\phi \tag{4}$$

$$k = \sqrt{1 - \left(\frac{a}{c}\right)^2} \qquad (a \leq c) \tag{5}$$

$$k = \sqrt{1 - \left(\frac{c}{a}\right)^2} \qquad (a > c) \tag{6}$$

where ϕ is the crack front angle defined in Figures 1(a) and 1(b). The crack aspect ratios, crack sizes, and values of $K_{\mathrm{I0,A}}$ obtained from Equations (2) and (3) are listed in Table 1. In the table, cracks whose depth or length is smaller or larger than 7.5 mm are called small or large cracks, respectively. Note that FEA systematically underestimated $K_{\mathrm{I0,A}}$ values by 0.5% to 0.7%. A previous work by Doi et al. reported that the error contained in this method for calculating the stress intensity factor for a circular crack in infinite body can be 0.9% [10]. Considering the discretization error of the stress intensity factor described above (< 0.25%), the possible errors in the interaction factors obtained by FEA is probably around 1%.

Table 1. Crack aspect ratios and stress intensity factors at point A ($K_{\mathrm{I0,A}}$) as obtained by Equations (2), (3), and FEA.

Aspect ratio a/c	Crack size	Crack depth a [mm]	Crack length c [mm]	$\sqrt{ac}$ [mm^2]	$K_{\mathrm{I0,A}}$ (Eq. (2)–(3)) [MPa·m$^{0.5}$]	$K_{\mathrm{I0,A}}$ (FEA) [MPa·m$^{0.5}$]
0.25	Small	1.875	7.500	3.750	0.716	0.712
	Large	7.500	30.00	15.00	1.432	1.425
0.5	Small	3.750	7.500	5.303	0.896	0.890
	Large	7.500	15.00	10.61	1.268	1.259
1.0	-	7.500	7.500	7.500	0.978	0.971
2.0	Small	7.500	3.750	5.303	0.634	0.630
	Large	15.00	7.500	10.61	0.896	0.890
4.0	Small	7.500	1.875	3.750	0.358	0.356
	Large	30.00	7.500	15.00	0.716	0.712

3. Results and Discussion

3.1. Interaction at Co-vertices of Cracks with a/c ≤ 1.0

Consider two equal elliptical cracks with $a/c \leq 1.0$ lined up in parallel along the depth direction, as shown in Figure 1(b). Figure 3 shows a typical example of the stress intensity factor solutions (blue dots) and the interaction factors (red dots) for each node along the crack edge in the case where $a/c = 0.25$ and $S/a = 1.0$. For an isolated elliptical single crack, the stress intensity factor reaches a peak at $\phi = 0.5\pi$ and 1.5π. However, the stress intensity factor for an interacted crack reaches the maximum only at $\phi = 0.5\pi$ (co-vertex point A) because of the interaction effect. Close attention should, therefore, be paid to the closest point because fracture initiation may occur here. The interaction factor at the closest point is, therefore, examined in more detail.

The plots in Figures 4(a), (b), and (c) compare the interaction factors for cracks with $a/c = 0.25$ (blue squares), 0.5 (green triangles), and 1.0 (black circles) at the co-vertex ($\phi = 0.5\pi$). Open and closed symbols indicate that the crack size is small and large, respectively (Table 1).

Figure 4(a) shows the interaction factors obtained from FEA as a function of a dimensionless distance S/a. The numerical results for $a/c = 1.0$ by Nisitani and Murakami [1] are also plotted as cross marks here. The discrepancy between the interaction factors obtained by FEA and the reported values is less than 0.015 in each case. The interaction factors clearly increase with decreasing S/a, while the rate of increase depends on the aspect ratio a/c. For example, the interaction factors range from 1.03 to 1.08 when $S/a = 1.0$, which is the combination criterion provided by ASME Code Sec. XI [5]. Due to these differences in the interaction factors, careful consideration of the effect of a/c is required in order to evaluate the interaction using criteria based on S/a. Additionally, small cracks with $a/c = 0.25$ (blue open squares) and 0.5 (green open triangles) show larger interaction factors than those with $a/c = 1.0$, while the values of $K_{I0,A}$ for the former two cracks are lower than those for the later (Table 1). This result indicates that K_I itself is not directly associated with the magnitude of crack interaction.

Figure 4(b) plots the interaction factors at the co-vertex against the dimensionless distance S/c. Since all data points in Figure 4(b) are the same as those given in Figure 4(a), the interaction factors for cracks with $a/c = 1.0$ are equivalent. Regarding the pairs of cracks with $a/c < 1.0$, the interaction factors in Figure 4(b) are shifted to the smaller values of the horizontal axis because $c > a$ (or $S/c < S/a$). As a result, a/c has an opposite effect to that observed in Figure 4(a). Considering the inverse nature of a/c in Figures 4(a) and (b), using a combination of dimensionless distances S/a and S/c may result in the effects of a/c being eliminated. The ratio of distance to the geometrical mean of the depth and length $S/(ac)^{0.5}$ is used here in order to offset the effect of a/c. Figure 4(c) shows the relationship between dimensionless distance $S/(ac)^{0.5}$ and interaction factors at the co-vertex. The interaction factors increase with a decrease in the dimensionless distance, which is similar to the cases presented in Figures 4(a) and (b). However, the effect of aspect ratio is less significant. For example, in the case where $S/(ac)^{0.5} = 1.0$, differences in the interaction factors between each pair of cracks are less than 0.01. $S/(ac)^{0.5}$ may, therefore, be a better parameter for characterizing the interaction factors between embedded cracks with $a/c \leq 1.0$.

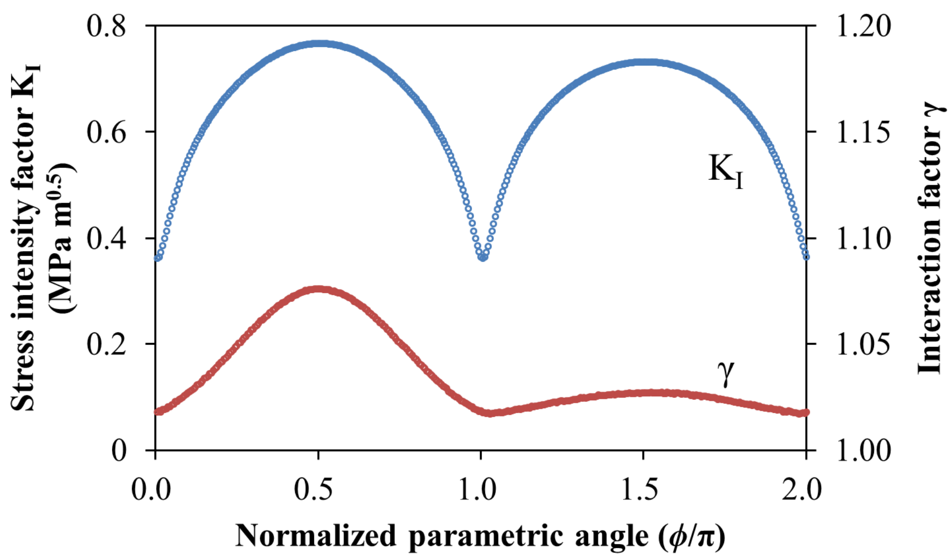

Figure 3. Stress intensity factor K_I and interaction factor γ for interacted elliptical cracks with $S = 1.875$ mm, $a = 1.875$ mm, $c = 7.5$ mm, and $a/c = 0.25$.

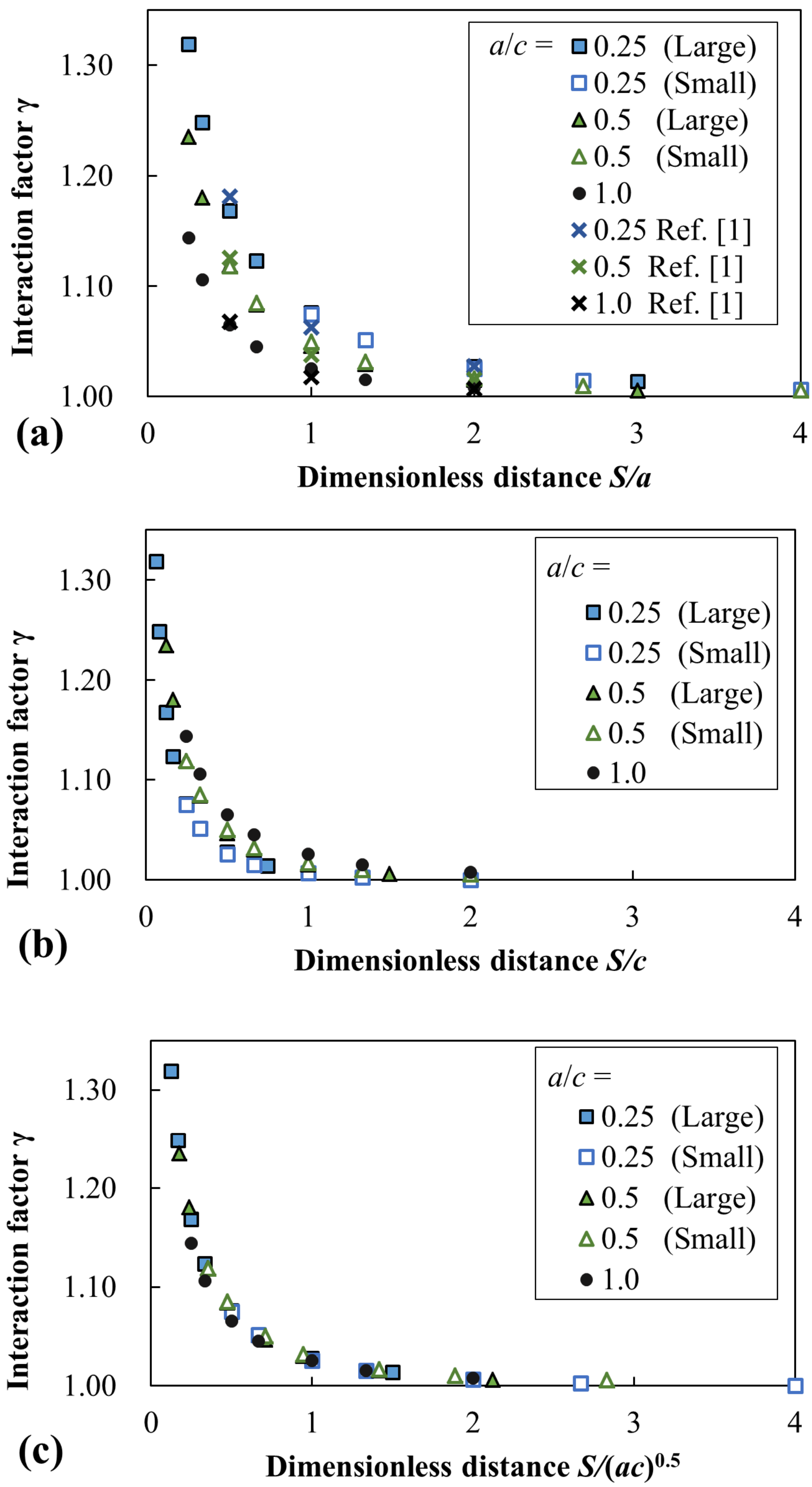

Figure 4. Relationship between interaction factor γ at the co-vertex for two elliptical cracks and dimensionless distances (a) S/a, (b) S/c, and (c) $S/(ac)^{0.5}$. Closed and open symbols indicate that the crack sizes are large and small, respectively. Cross marks indicate the results by Nisitani and Murakami [1].

3.2. Interaction at Vertices of Cracks with a/c > 1.0

The interaction factors for two equal elliptical cracks with *a*/*c* > 1.0 are considered here. Figure 5 illustrates the typical stress intensity factors (blue dots) and interaction factors (red dots) as a function of crack front angle ϕ in the case where *a*/*c* = 4.0 and *S*/*c* = 1.0. There is a peak for the interaction factor at the closest point $\phi = 0.5\pi$ (vertex point A), while the stress intensity factors for the interacted crack reach the local minimum here. The overall interaction factor is smaller than is the case in Figure 3.

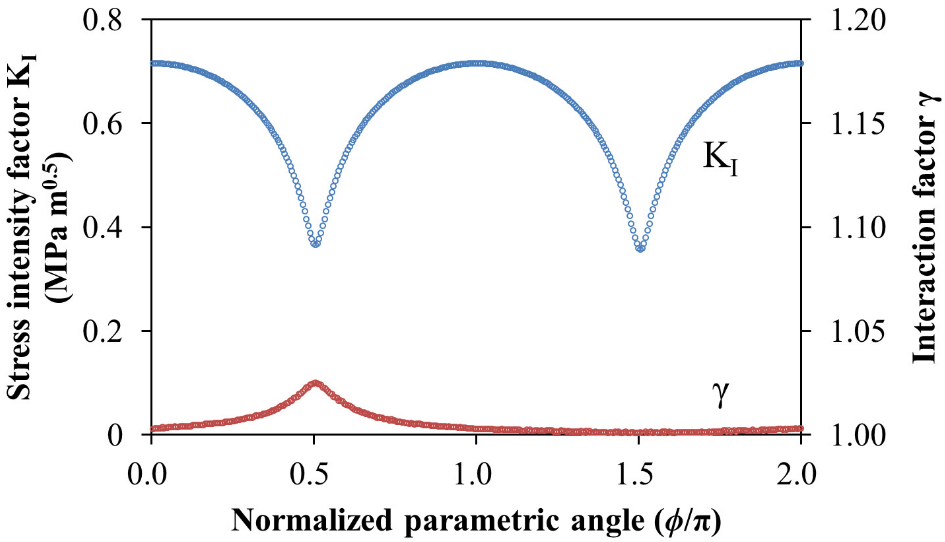

Figure 5. Stress intensity factor K_I and interaction factor γ at the vertex for interacted elliptical cracks with *S* = 1.875 mm, *a* = 7.5 mm, *c* = 1.875 mm, and *a*/*c* = 4.0.

Figures 6(a), 6(b), and 6(c) show the relationship between the dimensionless distance and interaction factors for cracks with *a*/*c* = 1.0 (black circles), 2.0 (yellow triangles), and 4.0 (red squares) at the vertex ($\phi = 0.5\pi$). Open and closed symbols indicate that the sizes of the cracks are small and large, respectively.

Figure 6(a) compares the interaction factors at point A ($\phi = 0.5\pi$) against *S*/*a*. The interaction factors clearly depend on both *a*/*c* and *S*/*a*. The interaction factors for cracks with a larger aspect ratio become smaller and the cracks with *a*/*c* = 4.0 show little interaction when *S*/*a* > 0.5. This effect of aspect ratio is similar to that reported in the previous work based on the superposition principle of the elasticity theory [4].

Figure 6(b) shows the relationship between the interaction factors and *S*/*c*. In contrast to the aspect ratio dependence of stress intensity factors that can be seen in Figure 6(a), the aspect ratios have little effect on the interaction factors. For example, the difference in the interaction factors among the cases where *a*/*c* =1.0, 2.0, and 4.0 with *S*/*c* = 1.0 are less than 0.01.

The interaction factors at the vertex are plotted against $S/(ac)^{0.5}$, as shown in Figure 6(c). In the case where $S/(ac)^{0.5} = 1.0$, the interaction factors range from 1.01 to 1.03 with *a*/*c* decreasing from 4.0 to 1.0. This indicates that the effects of aspect ratio are smaller than that in the case where *S*/*a* is used (Figure 6(a)), though it is still larger than when *S*/*c* is used (Figure 6(b)). *S*/*c*, therefore, appears to be the dominant parameter in the interaction factors for cracks with large aspect ratios.

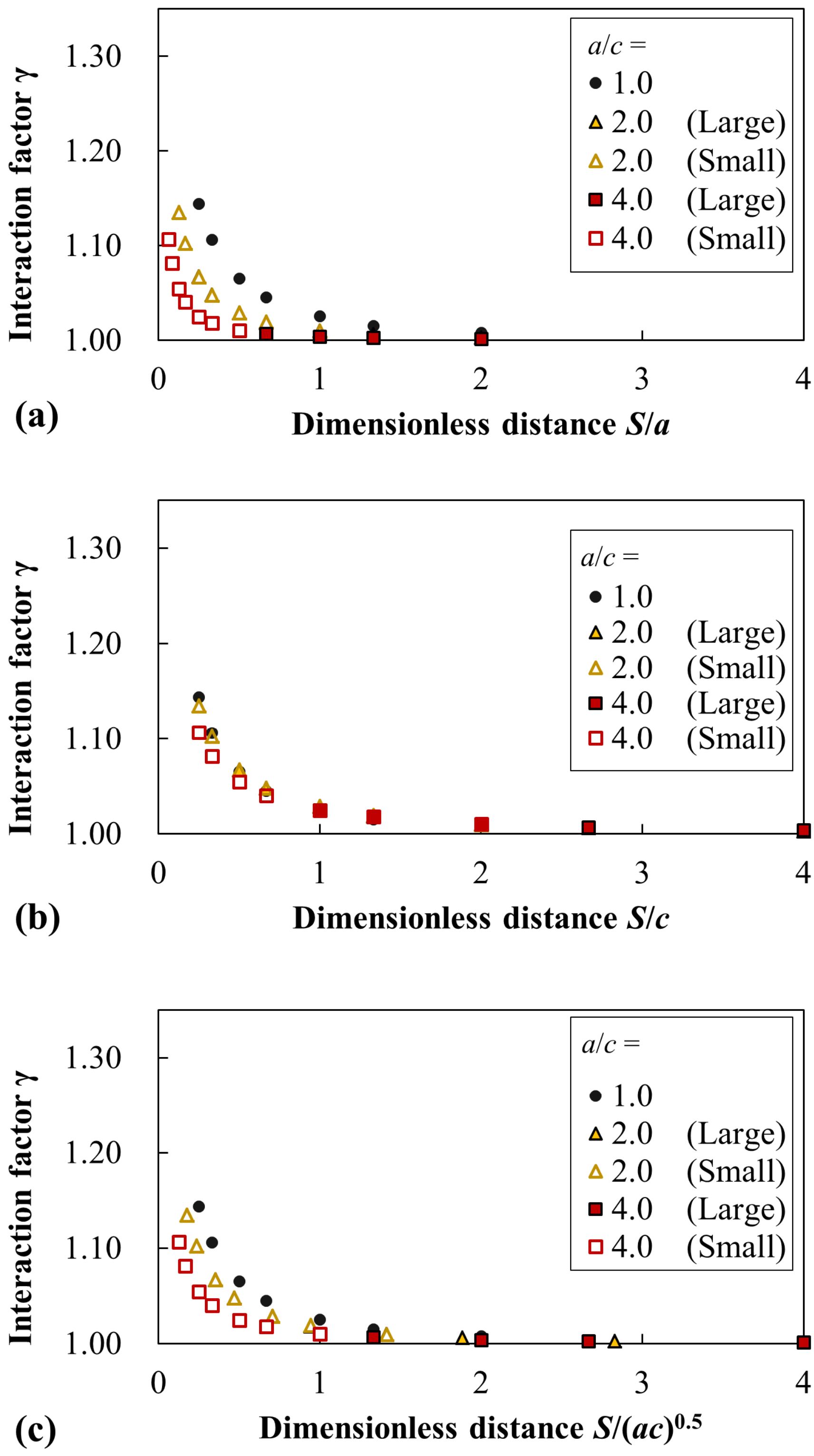

Figure 6. Relationship between interaction factor γ for two elliptical cracks and dimensionless distances (a) S/a, (b) S/c, and (c) $S/(ac)^{0.5}$. Closed and open symbols indicate that the crack sizes are large and small, respectively.

3.3. Empirical Formula for the Interaction Factors

In Section 3.1, there was a result which indicated that the interaction factors at the co-vertex for embedded cracks with small aspect ratios ($0.25 < a/c < 1.0$) are characterized by $S/(ac)^{0.5}$. The results of Section 3.2, however, indicate that S/c is a better parameter for characterizing the interaction factors at the vertex for embedded cracks with large aspect ratios ($1.0 < a/c < 4.0$). These differences may be caused by the different interaction features illustrated in Figures 3 and 5. It appears reasonable, therefore, to consider that cracks with small and large aspect ratios can be characterized by different dimensionless distances, namely $S/(ac)^{0.5}$ and S/c, respectively.

Since the values of the interaction factors in Figure 4(c) are almost the same as for those in Figure 6(b), a unified empirical relationship for the interaction factors and dimensionless distances can be derived:

$$\gamma = 0.990 + 0.040/D \qquad (0.33 \leq D \leq 4.0) \tag{7}$$

$$\gamma = 1.000 \qquad (4.0 < D\,) \tag{8}$$

where D is the dimensionless distance given by

$$D = S/(ac)^{0.5} \qquad (0.25 \leq a/c \leq 1.0) \tag{9}$$

$$D = S/c \qquad (1.0 < a/c \leq 4.0) \tag{10}$$

The range $D < 0.33$ is beyond the scope of Equation (7) because the data points obtained in this region by this study are scarce. The interaction factors are computed using a fit of Equation (7), as shown in Figure 7.

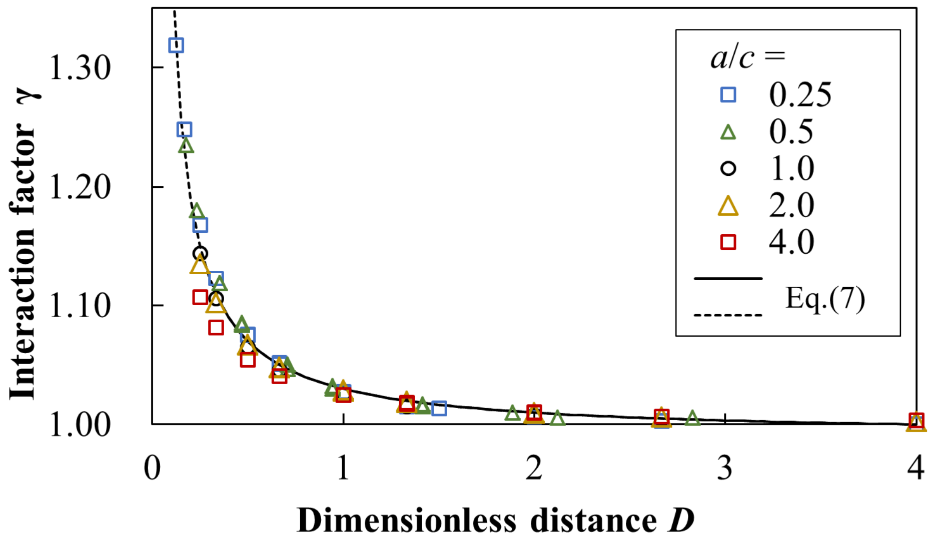

Figure 7. Relationship between interaction factor γ for interacted elliptical cracks and dimensionless distance D. $D = S/(ac)^{0.5}$ ($0.25 \leq a/c \leq 1.0$) and S/c ($1.0 < a/c \leq 4.0$).

This figure compares the interaction factors obtained by FEA (open and closed symbols) and the solutions of Equation (7) (broken line for $D < 0.33$ and solid line for $0.33 \leq D$). This equation is sufficiently accurate for most engineering problems. When $0.33 \leq D$, the maximum absolute difference is 0.016 for $0.25 \leq a/c \leq 1.0$ and 0.029 for $1.0 < a/c \leq 4.0$. Even taking into account FEA errors in the interaction factor described above (around 1%), the maximum absolute difference is sufficiently small for engineering applications. We can thus conclude that Equation (7) provides the best way to estimate predictions of the interaction factors at vertices and co-vertices for two elliptical cracks with a wide range of aspect ratios.

4. Conclusion

We investigated variations in the interaction factors that depended upon the geometrical features of two equal embedded cracks, and analyzed the effect of distance between the cracks as well as their depth and length. The following concluding remarks can be drawn from our results:

1) The dimensionless distance $S/(ac)^{0.5}$ is a good parameter for characterizing the interaction factors at co-vertices. In these cases, the interaction factors are nearly independent of the aspect ratios.
2) The dimensionless distance S/c is also a good parameter for characterizing the interaction factors at vertices. The interaction factors only depend weakly on the aspect ratio in these cases.
3) An empirical formula that can reasonably characterize the interaction factors for elliptical embedded cracks with a wide range of aspect ratios is proposed.

Conflict of Interest

The authors declare that there are no conflicts of interest related to this study.

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