AIMS Energy

http://www.aimspress.com/journal/energy

AIMS Energy, 4(2): 379–396
DOI: 10.3934/energy.2016.2.379
Received: 30 December 2015
Accepted: 06 March 2016
Published: 16 March 2016

Research article

Game-theoretic control of PHEV charging with power flow analysis

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Abstract: Due to an ever-increasing market penetration of plug-in hybrid electric vehicles (PHEVs), the charging demand is expected to become a main determinant of the load in future distribution systems. In this paper, we investigate the problem of controlling in-home charging of PHEVs to accomplish peak load shifting while maximizing the revenue of the distribution service provider (DSP) and PHEV owners. A leader-follower game model is proposed to characterize the preference and revenue expectation of PHEV owners and DSP, respectively. The follower (PHEV owner) decides when to start charging based on the pricing schedule provided by the leader (DSP). The DSP can incentivize the charging of PHEV owners to avoid system peak load. The costs associated with power distribution, line loss, and voltage regulation are incorporated in the game model via power flow analysis. Based on a linear approximation of the power flow equations, the solution of sub-game perfect Nash equilibrium (SPNE) is obtained. A case study is performed based on the IEEE 13-bus test feeder and realistic PHEV charging statistics, and the results demonstrate that our proposed PHEV charging control scheme can significantly improve the power quality in distribution systems by reducing the peak load and voltage fluctuations.

Keywords: game theory; in-home charging; Nash equilibrium; plug-in hybrid electric vehicle; power flow analysis

1. Introduction

Following the increasing ownership of private plug-in hybrid electric vehicles (PHEVs), in-home charging is expected to become one of the common activities of electricity customers in the near future. Since most PHEV owners are used to charge their PHEVs as soon as they arrive home, the peak load associated with charging often appears around the rush hours. In the meantime, the residential load increases due to cooking and other in-home entertainment activities. As a result, the intrinsic peak load situation is aggravated [1].

In order to shift the peak load of a power system, there are two main approaches. One is to schedule the electricity demand by signing a contract between utility company and customers [2]. This approach can be easily implemented, however, its flexibility and options are limited to be popularized in the future. Another approach is based on smart pricing. Since the base load and charging demand are time-varying over a day, the distribution service provider (DSP) can adjust the electricity rate in different periods such that PHEV owners are willing to shift their charging time to off-peak hours for a lower price. Various pricing strategy can be applied, such as critical-peak pricing, real-time pricing, and time-of-use (TOU) pricing [3].

A vital part of smart pricing is to estimate the corresponding charging strategy of PHEV owners. The game-theoretic approach is ideal to address this issue as it can reveal the relationship between the pricing strategy of DSP and the charging strategy of PHEV owners. At the Nash equilibrium, all players in the game have no tendency to alter their decisions. In this way, the DSP can analyze the group performance of all PHEV owners. In literature, game models are proposed to coordinate the charging demand of PHEVs and the revenue of the DSP while keeping the stability of the power grid. In [4], a game theoretic approach is proposed to optimize the schedule of electric vehicle charging in a parking lot with a highly dynamic electricity price during the day time. Beaude *et al.* propose a static non-cooperative game formulation of distributed charging in electric vehicle networks, where several electric vehicles are connected to a common residential distribution transformer and each player aims at choosing an optimal starting time of charging [5]. A hierarchical game approach is applied in [6] with a real-time charging price in the fast charging station. For the in-home charging, Yoon *et al.* propose a Stackelberg game based on demand response [7]. The relevance between the weight factor of each PHEV owner and the optimal price is analyzed, which indicates that the Nash equilibrium can differ when considering the individual differences among followers in the game model. In [8], the authors propose a firm-union bargaining game approach for PHEV charging access control, to determine the starting time of PHEV charging. A central governing procedure to realize the demand-side management in the game model is developed in [9], which is able to schedule load flexibly over multiple time periods. A stochastic optimization model for optimal bidding strategies of electric vehicle (EV) aggregators is studied in [10] and [11] considering EV fleet characteristics, hourly loads and influence of wind-driven generators. In a similar case, a decentralized control method is applied in [12] to coordinate the charging load and the renewable sources. The price competition among EV charging stations is studied in [13]. In [14], the interaction among charging stations is modelled by considering transportation conditions. In [15], the randomness of EV's availability is considered in a day-ahead energy market to shape a general profile of the charging demand in distribution system. A new approach to maintain the system stability through game model is developed in [16], where both aggregators and EVs consider of the quality of energy requirements. More detailed analysis about quality of energy requirements can be developed by modifying the utility function in the game formulation, which involves the impact of line loss and voltage regulation on PHEV charging scheduling. Some recent research works in power engineering propose linear approximation algorithms for power flow analysis [17–19], which may provide some effective and efficient means to assess the impact of power flows in distribution systems. Yet, how to incorporate power flow analysis into the game model of PHEV charging control is still an open issue.

In this paper, we construct a leader-follower game model for PHEV charging control based on the Stackelberg game framework. Each PHEV owner plays the role of follower, who chooses the start time of charging based on the pricing schedule provided by the DSP. When the PHEV owners arrive home, they continue to make decisions between starting PHEV charging and waiting, until midnight when they have to sleep. Once they start charging the PHEVs, their costs are fixed which are not influenced by the alteration of the charging price. For the DSP, he/she can optimize the pricing schedule according to the arrival probability of PHEVs and the base load of residents. The general utility of the DSP is related to the energy cost and the payoff from PHEV owners. In order to characterize the impact of power flows in distribution systems, both line loss and voltage regulation are included in the cost of the DSP in our proposed game model. Specifically, the line loss and voltage regulation can be obtained via traditional power flow analysis, which is nonlinear in nature. As a result, the Nash equilibrium of the game model is obtained through multivariate nonlinear programming with a high computational complexity. To facilitate practical applications, a linear approximation of power flow equations is applied in this paper. For the distribution system under consideration, the methods proposed in [20] and [21] are utilized to analyze line loss and voltage regulation, respectively, based on which the solution of the sub-game perfect Nash Equilibrium is derived. Extensive simulations based on the IEEE 13-bus test feeder and realistic PHEV charging statistics are performed to demonstrate the effectiveness of the proposed PHEV charging control scheme.

The rest of this paper is organized as follows. The system model is introduced in Section 2. The PHEV charging control problem is formulated as a non-cooperative dynamic game in Section 3. The solution of the sub-game perfect Nash Equilibrium is presented in Section 4. In Section 5, a case study is provided to verify our theoretical analysis. The concluding remarks are drawn in Section 6 with future work.

2. System Model

In the electricity market, the retailers can adjust the customers' electricity demand through various pricing schedules [23]. Stability and stochastic analysis of the responding demand can be performed to help reduce the cost of the DSP while protecting the power grid [24]. On the other hand, in energy transactions, the electricity customers are often price-driven [25]. So in our market model, the DSP earns revenue from the fee of charging service, while its cost includes the cost of generation, line loss and voltage regulation. On the other hand, the PHEV owners can benefit from the low electricity price at certain time by adjusting its starting time of charging according to the pricing schedule.

Consider a distribution feeder with N buses. When there are k_n PHEVs charging in the residential houses on bus n, the total electric power consumption of PHEV charging is given by

$$P_c = \sum_{n=1}^{N} k_n P_{per} \tag{1}$$

where P_{per} is the charging power of each PHEV. With a considerable penetration of PHEVs, the line loss on the feeder cannot be ignored. In other words, to distribute a certain amount of load, the DSP has to supply a larger amount of electric power due to the line loss. For the PHEV owners, it is difficulty to estimate the influence of line loss on their charging price. However, for the DSP, the line loss is directly included in the total cost of electricity delivery. In other words, only the DSP investigates the cost associated with the line loss. To maximize the revenue, the DSP can adjust the electricity price according to the line cost. Since the PHEV owners make charging decisions according to the electricity price, they are indirectly influenced by the cost associated with line loss. Through power flow analysis, the line loss caused by PHEV charging can be obtained by the base load and charging load on each bus as well as the admittance matrix of the distribution system. Furthermore, when there is a large number of PHEVs charging at the same time, the voltage drop on some terminal nodes can be severe, which may result in an inferior power quality in the distribution system. Therefore, the DSP needs to take operations on the voltage regulator once it is necessary to increase the voltages of terminal nodes. However, there is a cost associated with the voltage regulation. Here we apply an approach of reactive power compensation as voltage regulation [21], where the cost of voltage regulation can be converted into power compensation. Thus, the actual power consumption of PHEV charging can be calculated as

$$E = P_c + P_l + Q_r \tag{2}$$

where P_c is the total charging load, P_l is the extra line loss caused by PHEV charging in comparison with the case without PHEV charging, and Q_r is the reactive power compensation for voltage regulation determined by base load and charging load of the distribution system. Note that both P_l and Q_r are nonlinear functions which can be obtained through power flow analysis. Based on (2), the total cost of the DSP is given by

$$C = aE \tag{3}$$

where $a > 0$ is the cost coefficient. Note that in practice, the relationship between electric power consumption and cost can be more complicated if we also consider other factors in power distribution, such as operation, maintenance, billing/collection/administration, depreciation, and interest [22]. Yet, an extension to more detailed cost models for the DSP (e.g., quadratic model and translog model [22]) is left for our future work.

In terms of the charging process, we consider a dynamic process over a whole night, which consists of a sequence $\{1, 2, \ldots, T\}$ of time slots. The duration of each time slot is Δt, during which the charging process can be regarded as a static process. In time slot t_i, the DSP publishes a TOU price p_i. Each PHEV owner makes a decision on starting PHEV charging or waiting, based on the TOU price and prior knowledge of historical prices as well as general electricity usage. Besides, the decision makers in time slot t_i can be categorized into two types: one is the newly arriving PHEV owners, the number of whom is constant on each bus in time slot t_i; the other consists of PHEV owners that have made the decisions of waiting before, the number of whom is determined by the strategies of PHEV owners. Mathematically, the relationship between the two parties is given by

$$M_i^n = M_{i,arrival}^n + M_{i-1,wait}^n = M_{i,charge}^n + M_{i,wait}^n \tag{4}$$

where M_i^n is the number of decision makers in time slot t_i on bus n, while $M_{i,arrival}^n$, $M_{i,wait}^n$ and $M_{i,charge}^n$ are the numbers of newly arriving, waiting and charging PHEVs, respectively. Besides, given the charging duration d of PHEVs, we can express the total number of charging PHEVs on bus n in time slot t_i ($k_{n,i}$, which corresponds to k_n in (1)) as follows:

$$k_{n,i} = \sum_{t=i-d+1}^{i} M_{t,charge}^n. \tag{5}$$

Furthermore, the PHEV owners need to concern about the delay discomfort as the cost of waiting. In other words, a long waiting time can only be offset by a considerable revenue of charging. Otherwise, the PHEV owners will continue to wait. In case that they do not charge over night, the owners can refuel PHEVs with gas for their daily commute demand.

The base load is another significant factor for the DSP. Even if there is no PHEV charging, the base load of daily electricity usage of residents may lead to peak load and voltage instability. The load of PHEV charging will aggravate the peak load. Taking the PHEV charging into consideration, the per-slot load in (2) can be expressed as

$$E_i = P_{c,i} + P_{l,i} + Q_{r,i} \tag{6}$$

where E_i, $P_{c,i}$, $P_{l,i}$, and $Q_{r,i}$ are the actual total load, charging load, line loss, and reactive power compensation for voltage regulation in time slot t_i, respectively. Thus, we can express the cost function of the DSP in (3) over all time slots as

$$C = \sum_{i=1}^{T} C_i = a \sum_{i=1}^{T} E_i.$$

3. Formulation of the Non-cooperative Dynamic Game

Generally speaking, the pricing process between the retailer and customers can be modelled by a leader-follower game, where the leader is the retailer and the follower is the group of all customers. Accordingly, the revenue of the follower is the welfare of all potential customers. In other words, each customer always makes a decision to maximize the group interest instead of an individual one, which is not a proper way to estimate the individual charging behaviour. Therefore, we apply a new model with an individual revenue function in order to obtain the total electric power consumption closer to the reality. We also take some factors of power flow into consideration. In this way, the individual characteristic of charging behaviour is distinguished by detailed factors like location, starting time, and individual revenue of charging.

3.1. Game formulation

The charging process can be divided into two steps. At first, the DSP publishes the TOU price in advance. Then, the PHEV owners determine when to start charging. The available information of PHEV owners for decision-making includes the degree of patience to wait, speculative price limit and real-time price; In order to determine a proper pricing schedule, the DSP should have the knowledge about the number of arriving PHEVs in each time slot on each bus of distribution system, the average charging duration, the charging strategies of PHEV owners and the cost of power distribution including the cost of generation, line loss and voltage regulation. Therefore, the charging process can be modelled as a game with incomplete information, since the pricing strategy and utility of the DSP are unknown to the PHEV owners, while the decision criterions of the PHEV owners are unknown to the DSP. To determine the pricing schedule, the DSP needs to analyze the charging strategies of PHEV owners and estimate the total additional electric power consumption due to the charging process. The best pricing strategy for the DSP is to minimize the cost with a maximized revenue. The strategic form of the game is defined by $\Gamma = \{\boldsymbol{M}_i \cup \boldsymbol{D}, \{x_{n,k}\}, \{U_{n,k}\}, \boldsymbol{p}, R(\boldsymbol{p})\}$, the components of which are listed as follows:

- $\boldsymbol{M}_i$ is the set of PHEV owners who play the role of decision makers in time slot t_i on each bus according to the price maker $\boldsymbol{D}$;

- $x_{n,k}$ is the strategy of PHEV owner k on bus n, where $x_{n,k} = \mathcal{C}$ denots starting PHEV charging and $x_{n,k} = \mathcal{W}$ represents waiting;
- $U_{n,k}$ is the utility function of PHEV owner k on bus n, which is characterized by the speculative price limit, historical price, and real time price;
- $\boldsymbol{p} = \{p_1, p_2, \ldots, p_T\}$ is the vector of TOU pricing schedule provided by the DSP;
- $R(\boldsymbol{p})$ is the utility function of the DSP, which depends on the price vector $\boldsymbol{p}$.

3.2. Utility function of PHEV owner

We denote the PHEV owner in each residential house as $k \in \{1, 2, \ldots, K\}$. The utility function of PHEV owner k is denoted by $U_k(x_k, \boldsymbol{p}, p_{\text{lim}}, w, i)$, where $x_k \in \{\mathcal{C}, \mathcal{W}\}$ is the decision of starting PHEV charging or waiting, $\boldsymbol{p}$ is the vector of the real-time price, p_{lim} is the speculative price limit, and time slot t_i is the decision time of the PHEV owner. To model of the charging behaviour in response to the real-time price, two assumptions are made as follows.

1. The speculative price limit increases with the increasing waiting time according a discounting process $p_{\text{lim}}(t) = w^t p_{\text{lim}}(0)$;
2. The speculative price limit is subject to an exponential distribution.

Here, $w \in (0, 1)$ is the delay discomfort coefficient of the PHEV owner. The physical meaning is that the speculative price limit reduces by a factor of w after waiting for each time slot with duration Δt. The details of the two assumptions are discussed in Subsections 3.2.2 and 3.4, respectively.

3.2.1. Utility of charging

According to the consumer choice behaviour model in [26], the individual speculative price limit p_{lim} can be expressed by the real-time price p_i adding a deviation ε. Here ε is a random variable influenced by unobservable characteristics. Due to different past purchasing experience, purchase demand and waiting time, the value of p_{lim} varies. And the real-time price is also time-varying. As a result, ε, the deviation of speculative price limit and real-time price can indicate the price gap between expectation and reality. If the speculative price limit is greater than or equal to the real-time price, the PHEV owner tends to start charging. Otherwise, a decision of waiting is likely to be made by the PHEV owner. Mathematically, we have

$$U_k(\mathcal{C}, \boldsymbol{p}, p_{\text{lim}}, w, i) = p_{\text{lim}} - p_i \tag{7}$$

where p_i is the price in time slot t_i, obtained from vector $\boldsymbol{p}$.

3.2.2. Utility of waiting

In the decision making process of PHEV owner, the utility of waiting in time slot t_i can be expressed with the expected utility of charging later. However, due to the inconvenience of waiting, the speculative price limit after waiting is lower. According to [27], the satisfaction rating usually decreases along with time, and the tendency of decreasing gradually slows down. Therefore, in our model, we approximate the speculative price limit after waiting for t time slots as $\omega^t p_{\text{lim}}$. Accordingly, the utility of waiting for one time slot is given by

$$U_k(\mathcal{C}, \boldsymbol{p}, p_{\text{lim}}, w, i+1) = w p_{\text{lim}} - p_{i+1}. \tag{8}$$

Similarly, the utility of charging in t time slots later is given by

$$U_k(\mathcal{C}, \boldsymbol{p}, p_{\mathrm{lim}}, w, i+t) = w^t p_{\mathrm{lim}} - p_{i+t}. \tag{9}$$

Based on (9), the utility of waiting can be expressed by

$$\begin{aligned} U_k(\mathcal{W}, \boldsymbol{p}, p_{\mathrm{lim}}, w, i) &= \frac{1}{T-i} \sum_{t=i+1}^{T} U_k(\mathcal{C}, \boldsymbol{p}, p_{\mathrm{lim}}, w, t) \\ &= \frac{1}{T-i} \left(W_i p_{\mathrm{lim}} - \sum_{t=i+1}^{T} p_t \right) \end{aligned} \tag{10}$$

where W_i is a constant determined by the discount factor w, given by

$$W_i = \frac{w^{T-i} - 1}{w - 1}. \tag{11}$$

From (7) and (10), the utility function of PHEV owners can be obtained.

3.3. Utility function of DSP

In general, the utility of DSP depends on the revenue from charging and the relative cost. The latest models in [28–30] define the utility of DSP as the revenue minus the cost. In these models, the revenue can be calculated based on electricity consumption multiplied by the price. The cost needs to be estimated and can involve factors such as generation cost, line loss, as well as voltage regulation. Since a time-varying system is considered in this paper, these factors are also time-varying. Mathematically, the utility of DSP in each time slot can be expressed by the revenue minus the cost given by

$$\begin{aligned} U_i &= p_i P_{c,i} - a E_i \\ &= (p_i - a) P_{c,i} - a P_{l,i} - a Q_{r,i}. \end{aligned} \tag{12}$$

Thus, the optimal pricing strategy $\boldsymbol{p}^*$ of the DSP can be obtained by solving the following nonlinear programming problem:

$$\max_{\boldsymbol{p}} \quad \sum_{i=1}^{T} U_i \tag{13}$$

$$\text{s.t.} \quad p_i \in (p_{\mathrm{lim}}, \infty). \tag{14}$$

In (13), it is obvious that the utility of the DSP depends on not only the pricing strategy and the cost of electricity consumption, but also the demand response in terms of PHEV charging scheduling as well as the cost on the power grid. Furthermore, the utility function of the DSP is a time varying function, which means that directly increasing the charging price in the peak hour is not a proper solution. The reason is that, on one hand, the charging load before the peak hour will be maintained. As a result, it will be too late to reduce the charging load on the peak hour. On the other hand, in essence, the control of the charging load is to redistribute the number of charging PHEVs in each time slot. Due to the continuity of time, to make the off-peak price cheap is another proper strategy to incentivize the PHEV owners' tendency of off-peak charging. In this way, the DSP can realize the goal of peak load shifting with revenue maximization.

3.4. Charging power

In time slot i, given the TOU price vector $\boldsymbol{p}$, whether to charge the PHEV is determined by the speculative price limit $p_{\lim}$. Based on (7) and (10), the condition of charging is given by

$$U_k(x_k = \mathcal{C}) > U_k(x_k = \mathcal{W}). \tag{15}$$

Equivalently, we have

$$p_{\lim} - p_i > \frac{1}{T-i}\left(W_i p_{\lim} - \sum_{t=i+1}^{T} p_t\right). \tag{16}$$

Then, the value of $p_{\lim}$ is given by

$$p_{\lim} > \frac{(T-i)\,p_i - \sum\limits_{t=i+1}^{T} p_t}{T-i-W_i}. \tag{17}$$

Given the discount factor w, the right hand side of (17) is a function of the TOU price vector. Here, we define a variable $B(i)$ as

$$B(i) \triangleq \frac{(T-i)\,p_i - \sum\limits_{t=i+1}^{T} p_t}{T-i-W_i}. \tag{18}$$

Then, we can regard $B(i)$ as a threshold function of the speculative price limit, so that the only determinant factor is $p_{\lim}$. In other words, when $p_{\lim}$ is larger than $B(i)$, it is more beneficial for the owner to start PHEV charging right away. Otherwise, waiting will be a proper choice. In this way, we can estimate an individual decision by a unified function, which indicates the group performance of PHEV owners. Based on the statistics obtained by investigating historical data, the distribution of speculative price limit can be obtained. In general, the number of customers under certain speculative price limit decreases with the increase of the value of the speculative price limit. According to [31], exponentially distributed reservation prices are widely used. Here, the reservation price means an expected acceptable price for customers, which is similar to the definition of speculative price limit in our model. However, conventional exponential distribution starts from zero, while the electricity price can hardly be zero due to the cost associated with the power flow analysis. So we set an additional lower bound ($p_{\min}$) for the speculative price limit. Accordingly, the speculative price limit can be modelled as $p_{\lim} - p_{\min} \sim \exp(\lambda)$, where λ is a constant obtained from historical data. In this way, we can estimate the expected number of charging PHEVs, given by

$$\overline{M}^{n}_{i,charge} = M^n_i \int_{B(i)-p_{\min}}^{\infty} \lambda e^{-\lambda x} dx = M^n_i e^{-\lambda\left[B(i)-p_{\min}\right]}. \tag{19}$$

Based on the expected number of charging PHEVs, the expected charging power can be calculated based on (1) and (5).

3.5. Line loss

Consider every bus on the feeder as a PQ bus. For the DSP, the average load in time slot i on each bus, i.e., $\bar{s}_{i,1}, \bar{s}_{i,2}, \ldots, \bar{s}_{i,N}$, are available. Adding the electric power consumption of PHEV charging, the actual load on each bus is given by

$$\hat{s}_{i,n} = \bar{s}_{i,n} + P_{per} \sum_{t=i-d+1}^{i} \overline{M}^{n}_{t,charge} \tag{20}$$

where d is the average charging duration of each PHEV. Generally speaking, for a distribution feeder, the line loss can be calculated by the voltage drop and the impedance between each two connected buses, given by

$$P_l = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\left(v_i - v_j\right)^2}{\left|Z_{i,j}\right|} \tag{21}$$

where v_i is the voltage on bus i, and $Z_{i,j}$ is the impedance between bus i and j. There are many approaches of power flow analysis to calculate the voltage on each bus with the load data. Here we apply an approach of linear approximation [20] to simplify the computation. Mathematically, we have

$$\boldsymbol{v}_i = V_0 e^{j\theta_0} \left(\mathbf{1} + \frac{\boldsymbol{Z}\hat{\boldsymbol{s}}_i}{V_0^2}\right) \tag{22}$$

where $\boldsymbol{v}_i$ is the voltage vector, $V_0 e^{j\theta_0}$ is the voltage on the slack bus, $\boldsymbol{Z}$ is the impedance matrix, and $\hat{\boldsymbol{s}}_i$ is the total load vector. Based on (20), (21), and (22), we can calculate the total line loss as a linear function of the load. Specifically, the line loss resulted from PHEV charging can be calculated as

$$P_{l,i} = P_l\left(\hat{\boldsymbol{s}}_i\right) - P_l\left(\bar{\boldsymbol{s}}_i\right). \tag{23}$$

3.6. Voltage regulation

Another electricity distribution cost is resulted from voltage regulation. There are various approaches to deal with the voltage drop. In this paper, we consider that the voltage is regulated by reactive power compensation. The corresponding cost of voltage regulation depends on the compensated reactive power. For example, if the voltage regulator is installed on bus m, the load on bus m is $\bar{s}_m = 0 - jQ_m$. Then the compensated reactive power can be obtained via nonlinear programming. The solution is the minimum value of Q_m that satisfies $v_n > v_{\min}, \forall n$, where $v_{\min}$ is the minimum voltage based on power quality requirement and v_n can obtained from (22).

4. Solution of Sub-game Perfect Nash Equilibrium

Recall from the previous section, the utility of the DSP (13) is a complex nonlinear function. As a result, the existence of the solution cannot be proved directly. Besides, the solution of the sub-game perfect Nash equilibrium (SPNE) is also hard to be obtained. Therefore, in this section, we will simplify (13) and propose a practical approach to solve the problem through iterations of linear functions.

Given the pricing vector $\boldsymbol{p}$, a constant sequence of $B(i)$ can be obtained according to (18). Then we can calculate the charging rate in each time slot, given by

$$\mu_{i,n} = e^{-\lambda\left[B(i) - p_{\min}\right]} = \frac{\overline{M}^{n}_{i,charge}}{M^n_i}. \tag{24}$$

Based on (4) and together with the charging rate μ_i^n and the arrival number $M_{i,arrival}^n$, the number of PHEVs that start charging in time slot i can be rewritten recursively, given by

$$\overline{M}_{i,charge}^{n} = \begin{cases} \mu_{1,n} M_{1,arrival}^{n} & i = 1 \\ \mu_{i,n} \sum_{t=1}^{i} \left[M_{t,arrival}^{n} \prod_{j=t}^{i-1} (1 - \mu_{i,n}) \right] & i > 1. \end{cases} \tag{25}$$

Since given the price vector $\boldsymbol{p}$, the values of $\mu_{i,n}$ and $M_{i,arrival}^n$ are all available, thus the number of PHEVs that start to charge in time slot i can also be obtained as a constant sequence. According to (20), the total charging load on bus n in time slot i is given by

$$s_{c,i,n} = P_{per} \sum_{t=i-d+1}^{i} \overline{M}_{t,charge}^{n}. \tag{26}$$

Up to now, we have converted the price into a sequence of uncoupled variables $s_{c,i,n}$, which only depends on the pricing schedule provided by the DSP. In this way, (13) can be rewritten as a function of $s_{c,i,n}$, given by

$$U(\boldsymbol{s}) = \sum_{i=1}^{T} \left[(p_i - a) P_{c,i}(\boldsymbol{s}) - a P_{l,i}(\boldsymbol{s}) - a Q_{r,i}(\boldsymbol{s}) \right] \tag{27}$$

where $\boldsymbol{s} \triangleq \left[s_{c,1,1}, \ldots, s_{c,T,N} \right]$ is the charging load vector. If SPNE exists, the solution should be either one of the stationary points or on the boundary of the domain of definition. The partial derivative function of (27) is given separately in the following. For the charging consumption $P_{c,i}$, it is easy to calculate the partial derivative function, given by

$$\frac{\partial P_{c,i}}{\partial s_{c,i,n}} = p_i - a. \tag{28}$$

According to (21), the partial derivative function of $P_{l,i}$ is given by

$$\frac{\partial P_{l,i}}{\partial s_{c,i,n}} = \sum_{m=1}^{N} \sum_{n=1}^{N} \left[\frac{2(v_m - v_n)}{\left| Z_{m,n} \right|} \frac{\partial (v_m - v_n)}{\partial s_{c,i,n}} \right], \tag{29}$$

where v_n has been given in (22). Thus, we have

$$v_{i,n} = V_0 e^{j\theta_0} \left[1 + \frac{1}{V_0^2} \sum_{m=1}^{N} Z_{m,n} \left(\bar{s}_{i,m} + s_{c,i,m} \right) \right]. \tag{30}$$

The partial derivative of $v_{i,n}$ is given by

$$\frac{\partial v_{i,n}}{\partial s_{c,i,n}} = \frac{e^{j\theta_0}}{V_0} Z_{n,n}. \tag{31}$$

Therefore, the partial derivative function of P_l is a homogeneous linear function of the vector $\boldsymbol{s}$.

Assume bus f experiences the most significant voltage drop on the feeder. Also, we consider a typical placement of the voltage regulator, i.e., at the beginning of the feeder (on bus 2). Then, we have $\bar{s}_{i,2} = 0$ and $s_{c,i,2} = -Q_{r,i}$. Together with (30), the compensated reactive power is given by

$$Q_{r,i} = \frac{1}{Z_{2,f}}\left[\sum_{m=1,m\neq 2}^{N} Z_{m,f}\left(\bar{s}_{i,m} + s_{c,i,m}\right) - \left(\frac{v_{i,f}}{V_0 e^{j\theta_0}} - 1\right)V_0^2\right] \tag{32}$$
$$v_{i,f} = v_{\min}.$$

The partial derivative function of $Q_{r,i}$ is a constant, given by

$$\frac{\partial Q_r}{s_{c,i,n}} = \frac{\Delta Z_{n,f}}{Z_{2,f}}. \tag{33}$$

Combining (28), (31), and (32), the condition of the stationary points is given by a set of linear equations as follows

$$\begin{bmatrix} p_1 - \alpha_1 \\ \vdots \\ p_T - \alpha_{T\times N} \end{bmatrix} - a \begin{bmatrix} \delta_{1,1} & \cdots & \delta_{1,T\times N} \\ \vdots & \ddots & \vdots \\ \delta_{T\times N,1} & \cdots & \delta_{T\times N,T\times N} \end{bmatrix} \begin{bmatrix} s_{c,1,1} \\ \vdots \\ s_{c,T,N} \end{bmatrix} = 0 \tag{34}$$

where α and δ are all constants determined by the power grid and electricity cost. In this way, given an initial value of $\boldsymbol{p}$, the charging load vector $\boldsymbol{s}$ can be obtained through (24)-(26). Then a new value of $\boldsymbol{p}$ can be obtained from (34). Based on the iterations, the final result will converge to an optimal pricing schedule. Even though some variables may exceed the bounds, we can replace them by the boundary values and remove the corresponding equations in (26). Since both $\boldsymbol{p}$ and $\boldsymbol{s}$ are continuous on the bounded domain of definition, we can always find a solution from (26), i.e., the solution of SPNE always exists.

5. Case Study

Our case study is based on the IEEE 13-Bus distribution test feeder. Each bus is assumed to be a service transformer in a residential area, as shown in Figure 1. Both the daily base load and PHEV charging load are delivered through the same feeder. The voltage regulator is installed close to the slack bus. Besides, the rated power and voltage on the slack bus are 4.16 MVA and 4.16 kV, respectively. The hourly base load is obtained from the data set of Electric Reliability Council of Texas (ERCOT) [32], which manages the electric power for 24 million Texas customers - representing about 90 percent of the state's electric load. For the in-home charging station, we use the parameters of CT4000 Level 2 Commercial Charging Stations [33] in our simulations. The charging power is 7.2 kW and the charging duration is approximately 3 hours. The TOU price provided by the DSP varies every 30 minutes.

Four cases are considered in our simulations. In case 1, the PHEV owners start charging as soon as they arrive home; In case 2, the optimal TOU pricing is calculated based on our model with the speculative price limit discount factor $w = 0.98$ and the distribution parameter of speculative price limit $\lambda = 1$; In case 3, we reduce the value of w to 0.96 to evaluate the impact of delay discomfort as the cost of waiting; In case 4, we halve the value of λ to evaluate the impact of speculative price limit.

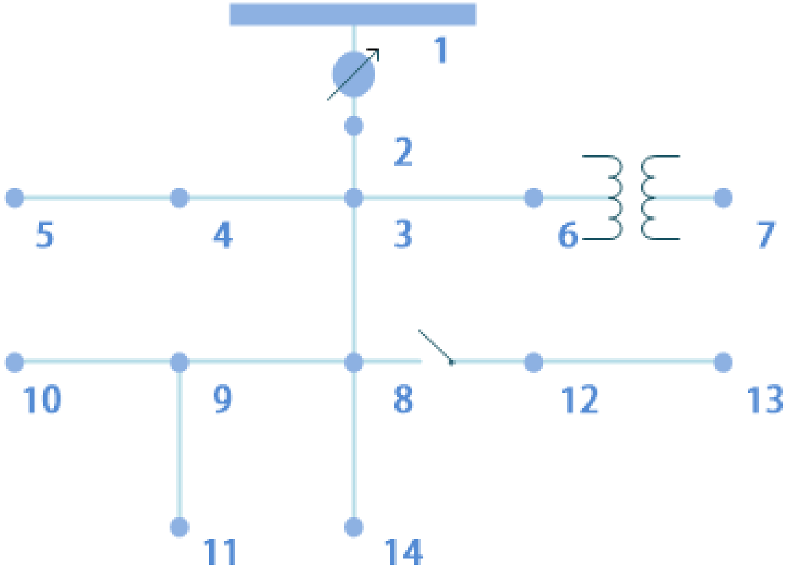

Figure 1. IEEE 13-bus distribution test feeder.

In the following, we will observe the influence of PHEV charging control on the load, charging rate μ, line loss, and voltage regulator consumption. Besides, to simplify the simulations, we define a per unit price with respect to the base price a, given by

$$\eta_i = \frac{p_i}{a} \quad (p.u.) . \tag{35}$$

Since a is a constant, according to (35) and η_i, we can ignore the impact of a on the results.

In Figure 2, we can find that the TOU price delays the peak load of charging effectively. The standard deviation of the load over the night has been reduced by 27.1%. In case 1, the peak load comes at around 7:00 pm and in case 2, the dominant peak load is mainly at 9:00 pm. Figure 3 shows the charging rate in each case. In case 1, the charging demand of every PHEV can be satisfied while in case 2 not. Due to the high base load and arriving home late, some of the PHEV owners will keep waiting until the midnight. If they cannot find a proper time to start charging, they can refuel their PHEVs with gas for the driving demand in the next day. Besides, it should be noticed that in our model, the peak load shifting is postponed. The off-peak load period between early arrival peak of PHEVs and the peak of base load is much shorter compared with the charging duration, such that the charge rate is always low then.

Figure 4 and Figure 5 demonstrate the line loss and voltage regulation in each case. The delivery efficiency has been improved. Specifically, for the voltage regulation, the total consumption is decreased by 14.9%. And the details of the voltage range after regulation is shown in Table 1. More importantly, we can observe that the fluctuation of the voltage regulation has been remitted. Accordingly, the number of time to operate the voltage regulator is reduced, which can significantly reduce the wear and tear of voltage regulator.

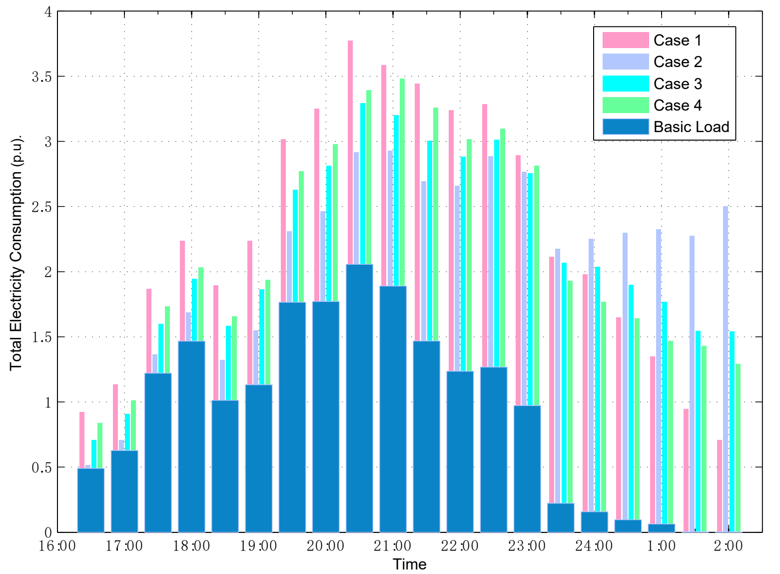

Figure 2. The total electricity consumption in each case.

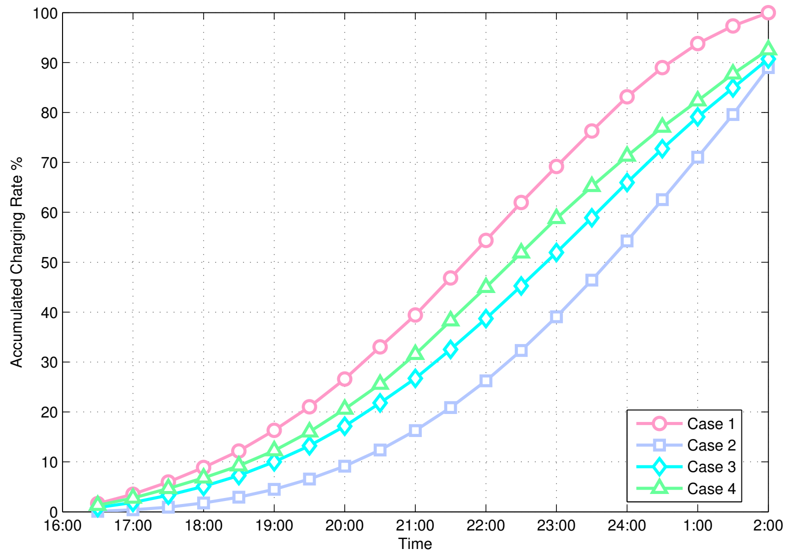

Figure 3. The accumulated charging rate in each case.

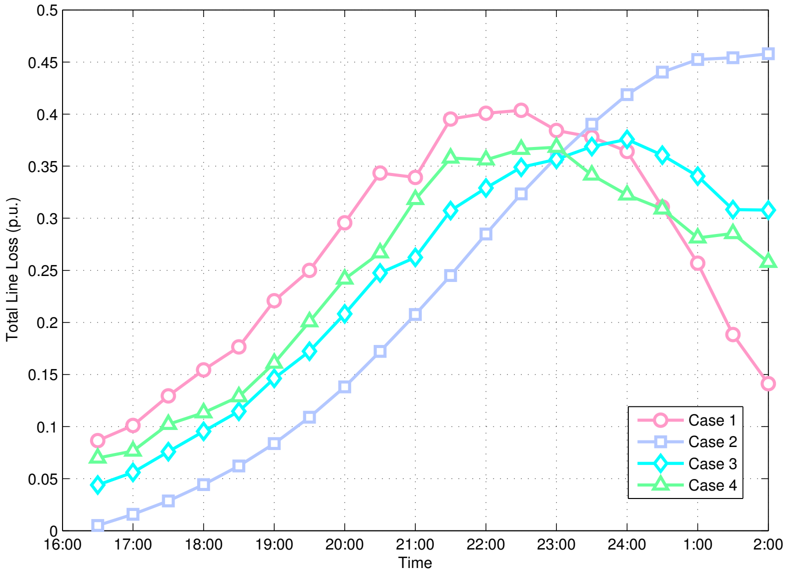

Figure 4. The total line loss in each case.

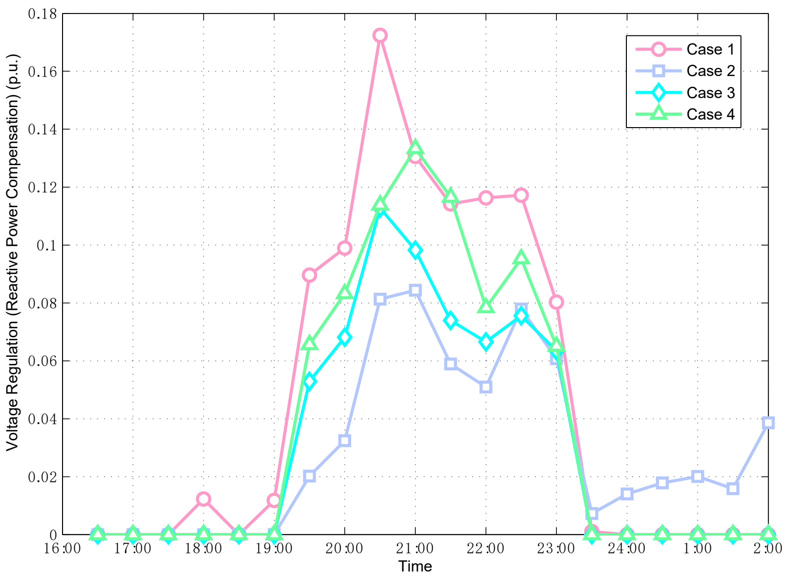

Figure 5. The voltage regulation (reactive power compensation) in each case.

Table 1. Maximum and minimum voltages of each bus.

Bus number	Maximum voltage / p.u.	Minimum voltage / p.u.
3	0.9925+j0.0772	0.9962 + j0.0386
4	0.9929+j0.0810	0.9964 + j0.0405
5	0.9932+j0.0841	0.9965 + j0.0420
6	0.9935+j0.0821	0.9967 + j0.0410
7	0.9939+j0.0837	0.9969 + j0.0418
8	0.9880+j0.1316	0.9940 + j0.0658
9	0.9880+j0.1346	0.9944 + j0.0680
10	0.9914+j0.1401	0.9957 + j0.0700
11	0.9887+j0.1344	0.9944 + j0.0671
12	0.9899+j0.1360	0.9949 + j0.0679
13	0.9892+j0.1355	0.9946 + j0.0677
14	0.9881+j0.1329	0.9940 + j0.0664

For the decision factor of PHEV owners, we can find an obvious relevance to the peak load shifting result. The value of w close to 0 means the intolerance of waiting. In other words, as long as the PHEV owners are patient enough, they can always find a proper time to start charging without aggravating the peak load. Moreover, the value of λ denotes the PHEV owner' sensitivity to the price fluctuation. The more the value of λ is close to 1, the more the TOU price can influence the charging preference.

Since the solution to our model is iterative, the traditional methods of computational complexity analysis can hardly be applied here. Therefore, we use the number of iterations to indicate the computational cost of our solution. As shown in Figure 6, the per unit price η_i converges within about 10 iterations. And in each iteration, we mainly solve the linear equations (34). According to the result, the computational cost is acceptable.

6. Conclusion

In this paper, we propose a leader-follower game model between PHEV owners and the DSP. The utility of each PHEV owner is formulated by considering the distribution of speculative price limit. For the utility of the DSP, we take two factors of power flow analysis into consideration, i.e., line loss and voltage regulation. To obtain the solution of the optimal price, we apply an approximated linear function in power flow analysis and propose the corresponding optimal algorithm. Simulation results demonstrate that our proposed model can improve the power quality of the distribution system, in terms of lower system peak load and less voltage fluctuations. In our future work, we will take the randomness of PHEV owners into consideration, including the randomness in arrival time and tolerance of price variation, as well as the stochastic base load.

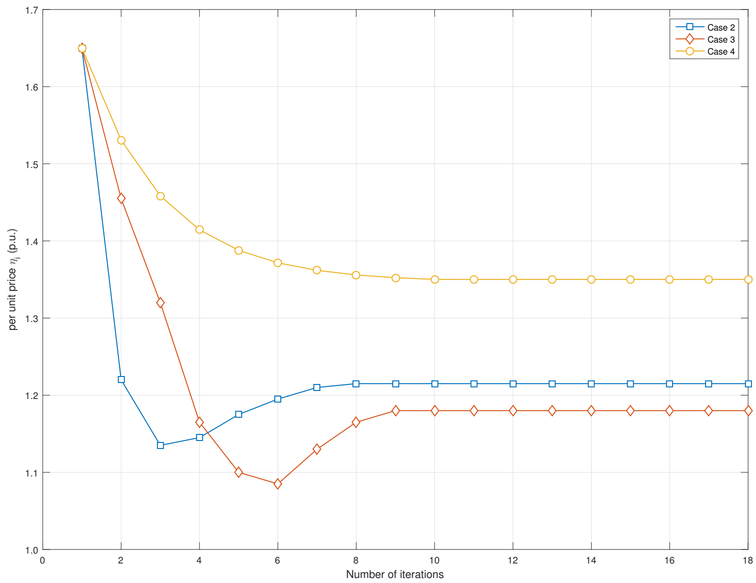

Figure 6. Convergence of the per unit price.

Acknowledgments

This work is supported by Alberta Innovates Technology Futures (AITF) postdoctoral fellowship and a research grant from the Natural Science and Engineering Research Council (NSERC) of Canada.

Conflict of Interest

All authors declare no conflict of interest in this article.

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