



*Research article*

## **Forecasting hourly WTI oil front monthly price volatility densities**

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## **Supplementary**

### **Appendix**

The appendix focuses on daily data for the period January 2023 to April 2023. Figure A.1 plots the three (2+1) volatility factors. Table A reports a static forecast with associated forecast fit measures. Figure A.2 reports the fit, actual, and standard errors. Table A.2 reports the measures for the three machine learning techniques: Lasso, Ridge, and Decision Forest. We have not optimised the hyperparameter values for optimal performance. Figure A.3 reports the three machine learning techniques together with the static forecast, including two standard errors. Figures A.3 and A.4 report a neural network for  $V_{1t}$  and  $V_{2t}$ , respectively.

**Table A.1.** Characteristics volatility-factors WTI oil daily prices.

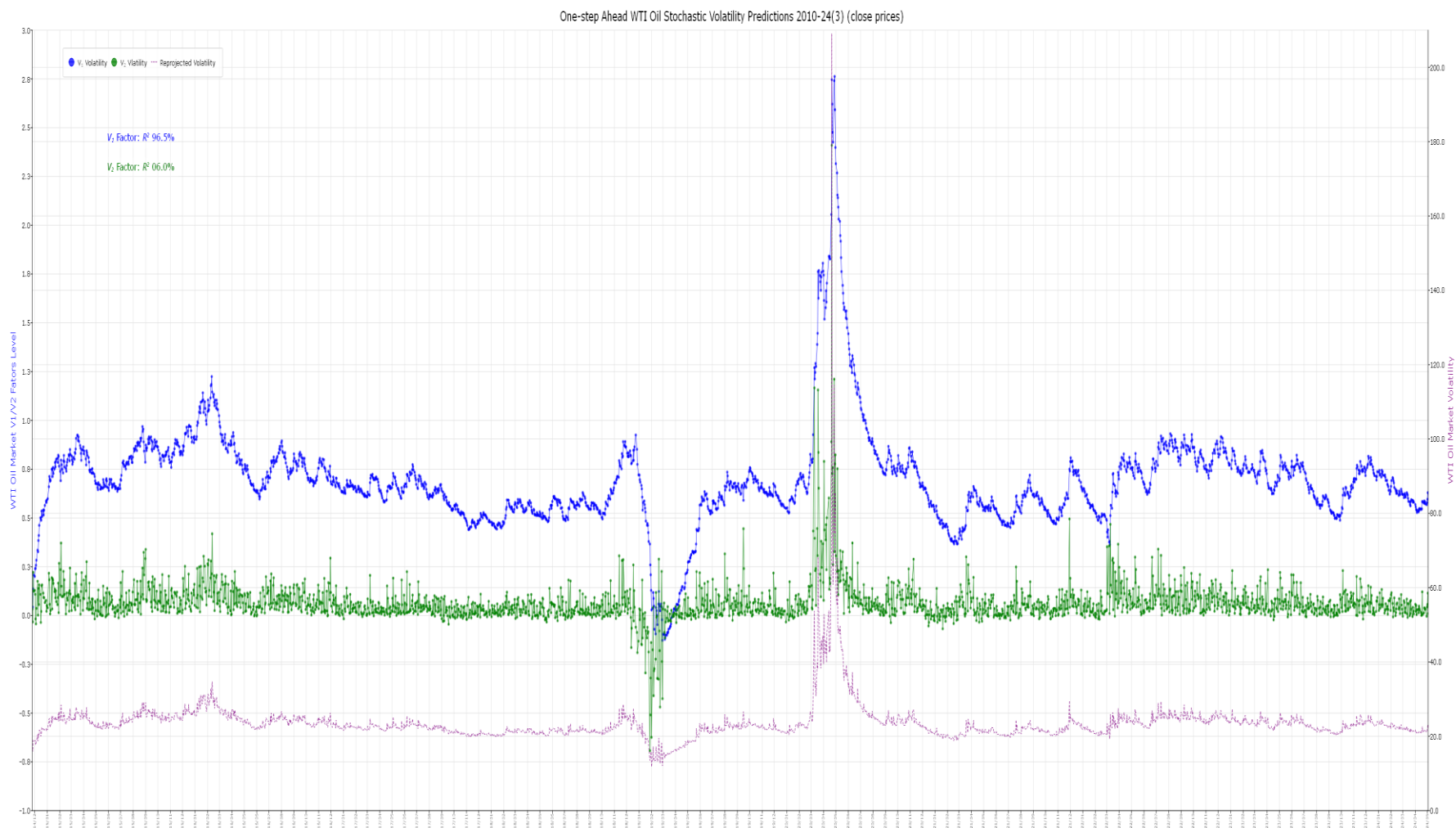
Category	Mean (all)/ Mode	Median Std.dev.	Maximum/ Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Cramer- Mises	Serial dep Q(12)	VaR (1%; 2.5%)
Factor $V_{1t}$	0.69165	0.66716 0.26304	2.7626 -0.1275	14.6064 2.32520	-0.08342 0.07476	2.9556 {0.2281}	4.9654 {0.0000}	25099.4 {0.0000}	0.0031 0.1469
	BDS-Z- statistic ( $e$ = 1) m=2	m=3	m=4	m=5	Phillips- Perron	Augmented DF-test		Breusch- Godfrey	CVaR (1%; 2.5%)
Category	122.9288 {0.0000}	143.0326 {0.0000}	169.4305 {0.0000}	207.3663 {0.0000}	-4.2775 {0.0034}	-4.5796 {0.0011}		9022.9 {0.0000}	-0.0690 0.0148
	Mean (all)/ Mode	Median Std.dev.	Maximum/ Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Cramer- von- Mises	Serial dependence Q(12)	VaR (1%; 2.5%)
Factor $V_{2t}$	0.06585	0.04679 0.11034	2.4103 -0.6927	105.5532 6.21107	0.06215 0.20374	17.1314 {0.0002}	30.839 {0.0000}	132.798 {0.0000}	-0.1153 -0.0265
	BDS-Z- statistic ( $e$ = 1) m=2	m=3	m=4	m=5	Phillips- Perron	Augmented DF-test		Breusch- Godfrey	CVaR (1%; 2.5%)
Category	11.4337 {0.0000}	12.8488 {0.0000}	13.9678 {0.0000}	15.1503 {0.0000}	-50.214 {0.0000}	-13.9766 {0.0000}		10.8923 {0.0000}	-0.2912 -0.1470
	Mean (all)/ Mode	Median Std.dev.	Maximum/ Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Cramer- von- Mises	Serial dependence Q(12)	VaR (1%; 2.5%)
Volatility exp( $V_{1t}+V_{2t}$ ) (yearly)	23.61411	22.79810 6.54268	209.1165 11.7859	298.8589 13.10155	-0.02123 0.11677	5.5495 {0.0624}	19.797 {0.0000}	9418.18 {0.0000}	15.223 17.192
	BDS-Z- statistic ( $e$ = 1) m=2	m=3	m=4	m=5	Phillips- Perron	Augmented DF-test		Breusch- Godfrey	CVaR (1%; 2.5%)
Category	67.1534 {0.0000}	72.5761 {0.0000}	77.3409 {0.0000}	83.5409 {0.0000}	-41.978 {0.0000}	-5.6931 {0.0000}		258.117 {0.0000}	12.837 14.825
	Mean (all)/ Mode	Median Std.dev.	Maximum/ Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Cramer- von- Mises	Serial dependence Q(12)	VaR (1%; 2.5%)

**Table A.2.** WTI oil daily volatility ordinary OLS ( $qr$ ) step ahead fit measures.

Daily Estimated Stochastic Volatility Forecast Fit Measures (Eviews)					
Contracts	Error Measures	Factor 1 $V_{1t}$	Factor 2 $V_{2t}$	Reprojected Volatility	
Ftse100 spot index (UK)	Root Mean Square Error (RMSE)	0.01436	0.03857	0.16740	
	Mean absolute Error (MAE)	0.01184	0.03050	0.13308	
	Mean absolute percent error (MAPE)	1.7662	1551.470	7.21080	
	Theil inequality coefficient (U1)	0.01070	0.34703	0.04462	
	Bias proportion		0.0007	0.0415	0.0378
	Variance Proportion		0.0001	0.1692	0.1225
	Covariance Proportion		0.9992	0.7893	0.8398
	Theil U2 Coefficient	0.93584	2.31389	1.79626	
	Symmetric MAPE	1.7548	73.206	7.00502	

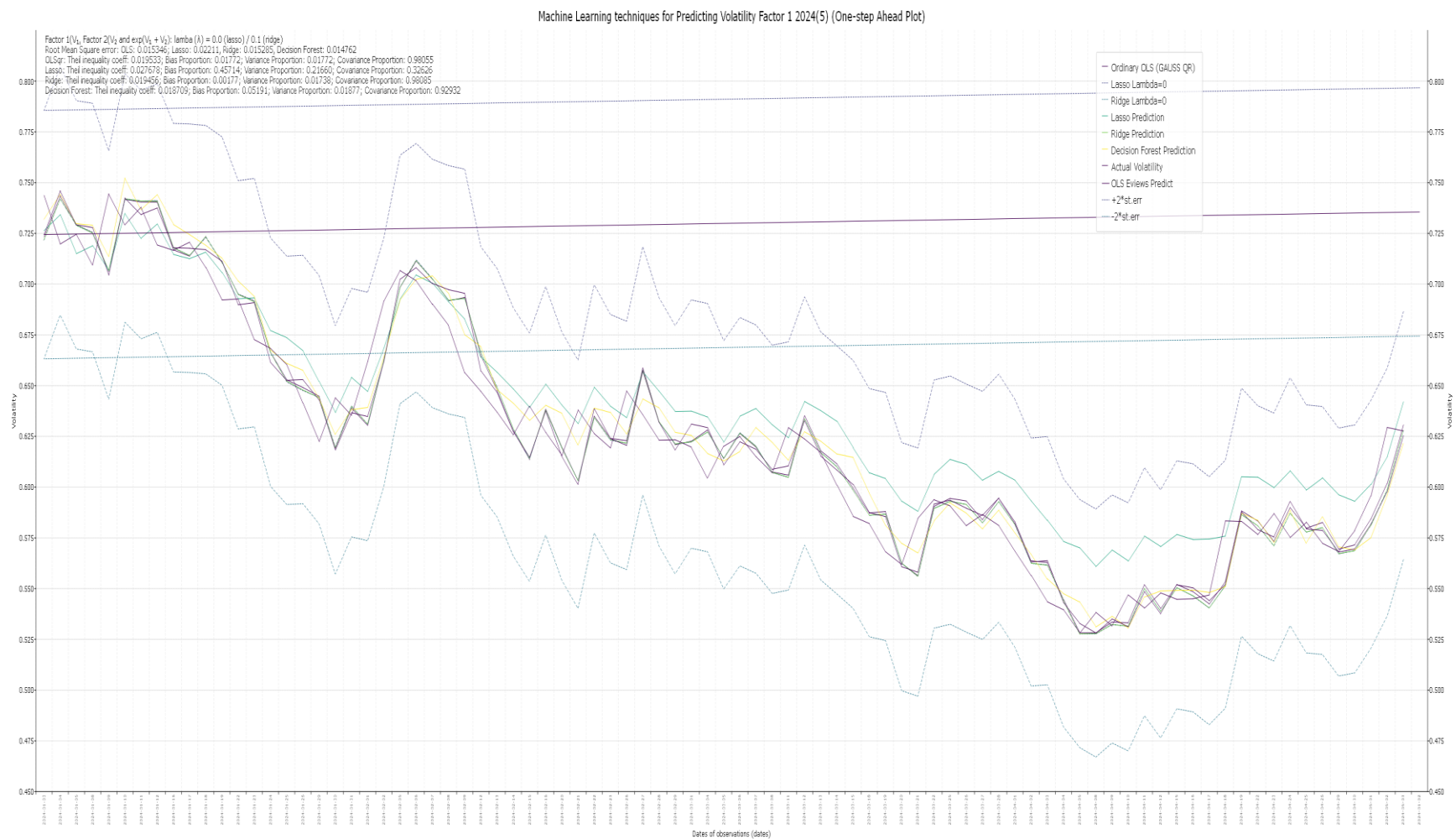
**Table A.3.** Projection fit for machine learning regression models.

<b>Daily Estimated Stochastic Volatility Forecast Fit Measures</b>				
Category		Factor 1 $V_{1t}$	Factor 2 $V_{2t}$	Reprojected Volatility ( $e^{(V_{1t}+V_{2t})}$ )
<b>Lasso Regression</b>				
<i>(l = 0.0)</i>				
<i>Ridge Regression</i>	RMSE	0.015346	0.036834	1.09599
<i>(l = 0.0)</i>	MSE	0.011879	0.029463	0.87104
	MAPE	1.878833	455.3575	3.89490
	Theil inequality coefficient	0.019533	6.40554	0.00111
	1 (U1)			3
	Bias Proportion	0.00173	0.04434	0.00003
	Variance Proportion	0.01772	0.17177	0.19182
	Covariance Proportion	0.98055	0.78389	0.80815
	Theil U2 Coefficient	0.62386	0.48754	3.07155
	Symmetric MAPE	0.02240	0.74716	0.04623
<i>Lasso Regression</i>				
<i>(l = 0.05)</i>				
	RMSE	0.022211	0.039863	0.64344
	MSE	0.019648	0.034452	0.50198
	MAPE	3.267560	541.2264	2.23897
	Theil inequality coefficient	0.027678	5.68713	0.00406
	1 (U1):			8
	Bias Proportion	0.45714	0.29925	0.00407
	Variance Proportion	0.21660	0.63805	0.01897
	Covariance Proportion	0.32626	0.06271	0.97696
	Theil U <sub>2</sub> Coefficient	1.46367	1.11529	1.06054
	Symmetric MAPE	0.03662	0.75878	0.02662
<b>Ridge Regression</b>				
<i>(l = 0.1)</i>				
	RMSE	0.015285	0.036715	1.08430
	MSE	0.011832	0.029404	0.86094
	MAPE	1.871757	455.7302	1.66536
	Theil inequality coefficient	0.019456	6.38953	0.00110
	1 (U1):			1
	Bias Proportion	0.00177	0.04459	0.00003
	Variance Proportion	0.01738	0.17531	0.18838
	Covariance Proportion	0.98085	0.78010	0.81159
	Theil U2 Coefficient	0.61936	0.47915	3.00591
	Symmetric MAPE	0.02231	0.74569	0.04570
<b>Decision Forest</b>				
	RMSE	0.014762	0.033259	0.47812
	MSE	0.012508	0.027112	0.37235
	MAPE	1.991256	337.5840	1.66536
	Theil inequality coefficient	0.018709	6.59757	0.00048
	1 (U1):			5
	Bias Proportion	0.05191	0.00510	0.01146
	Variance Proportion	0.01877	0.43315	0.00497
	Covariance Proportion	0.92932	0.56175	0.98356
	Theil U2 Coefficient	0.57565	0.61513	0.59142
	Symmetric MAPE	0.02353	0.72987	0.01974

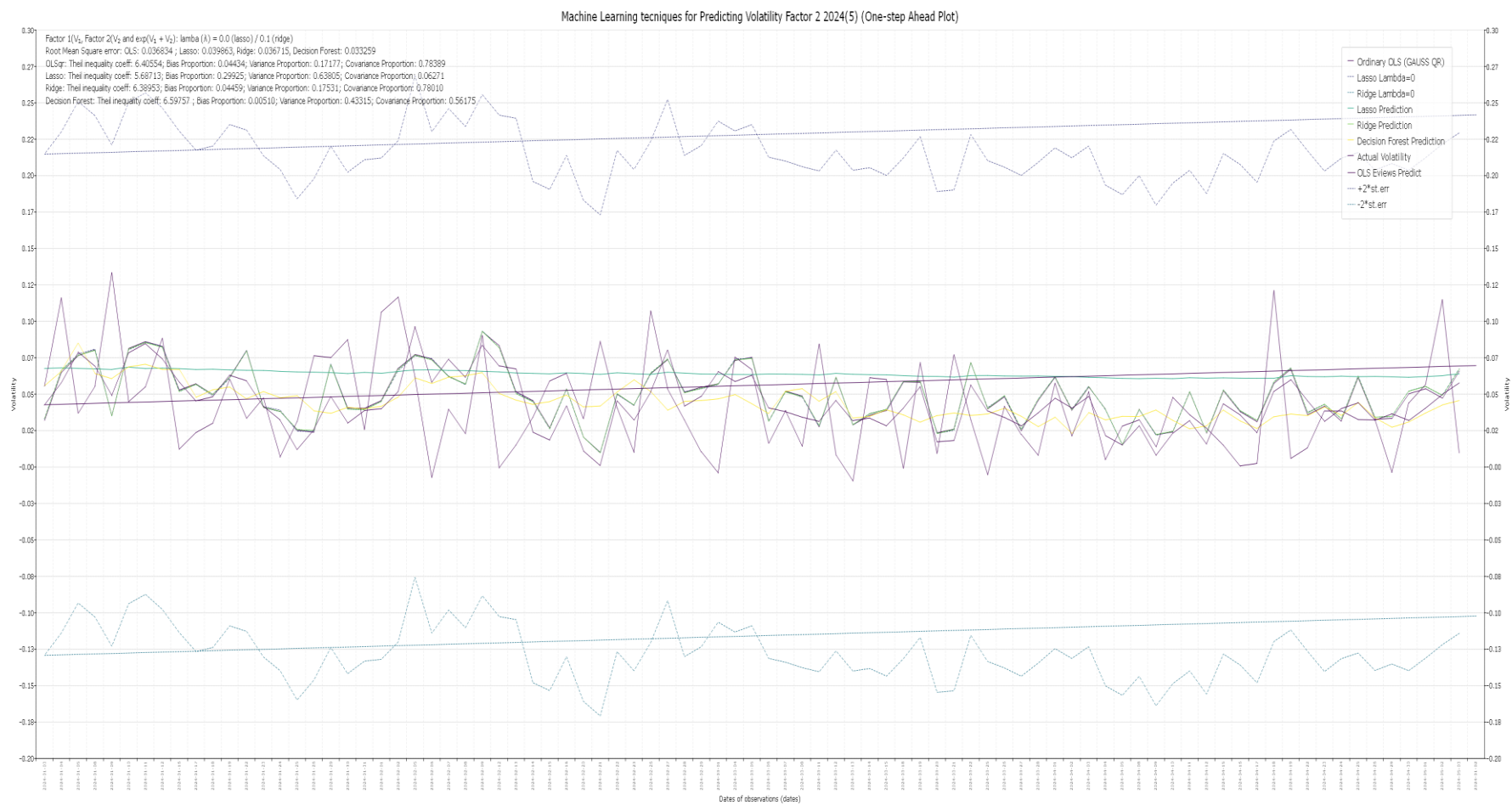


See Figure 8 above for interpretation of volatility time series.

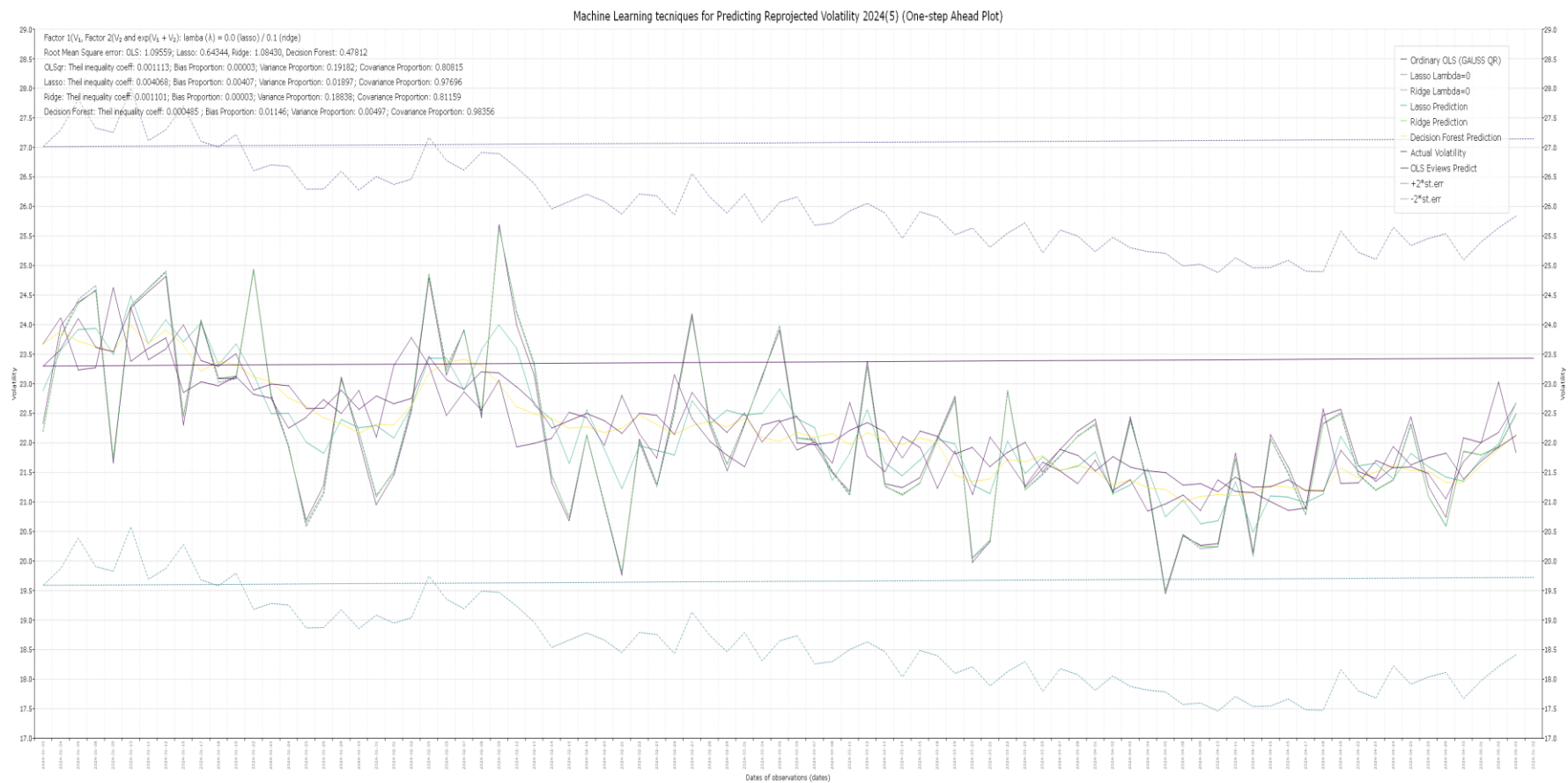
**Figure A.1.** Daily volatility paths for two factors ( $V_1$  and  $V_2$ ) and re-projected  $\exp(V_1+V_2)$  volatility (2014–2024(3)).



See Figure 10 above for interpretation of volatility time series.

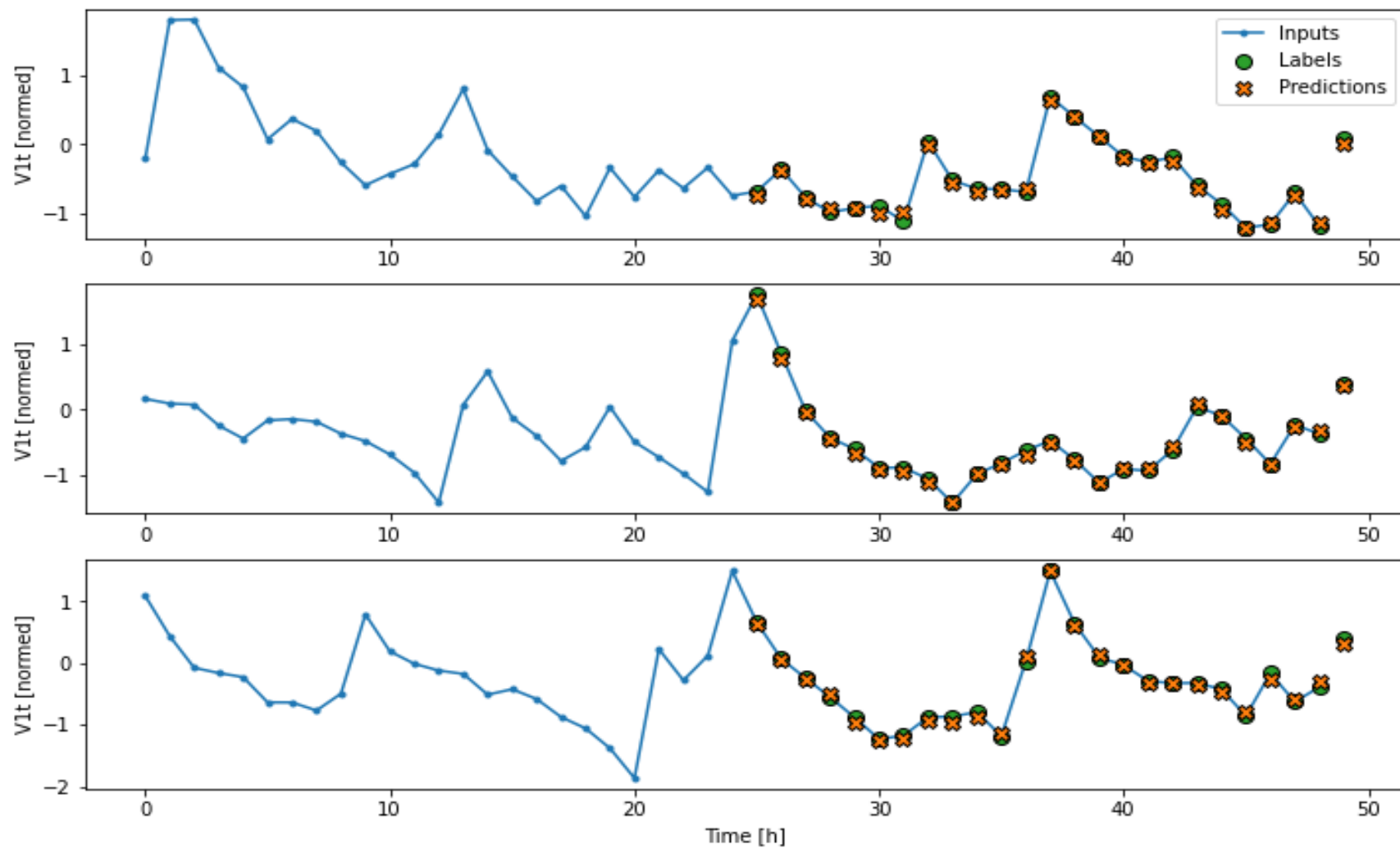


See Figure 10 above for interpretation of volatility time series.



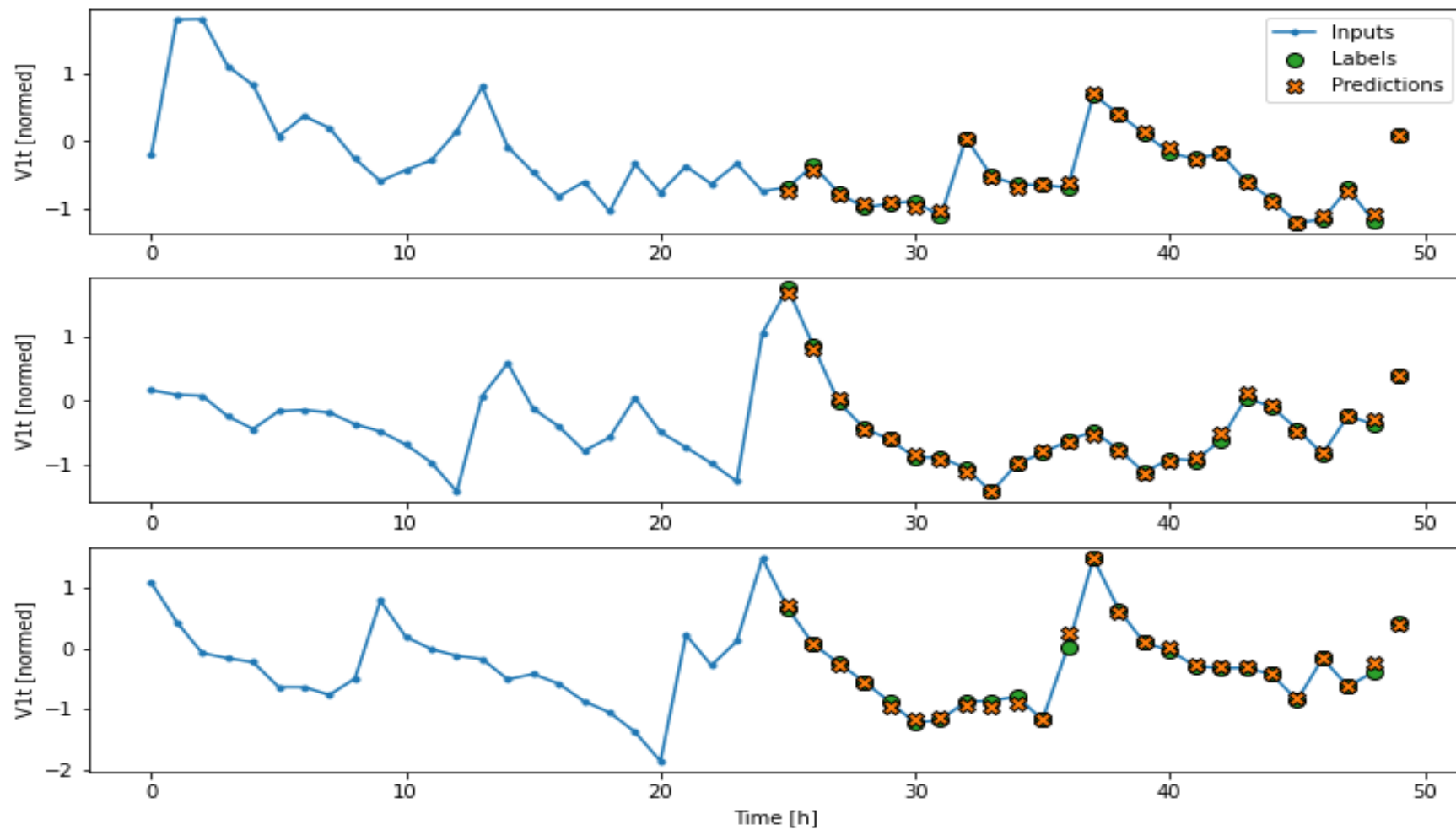
See Figure 10 above for interpretation of volatility time series.

**Figure A.2.** Daily machine learning prediction model fits.

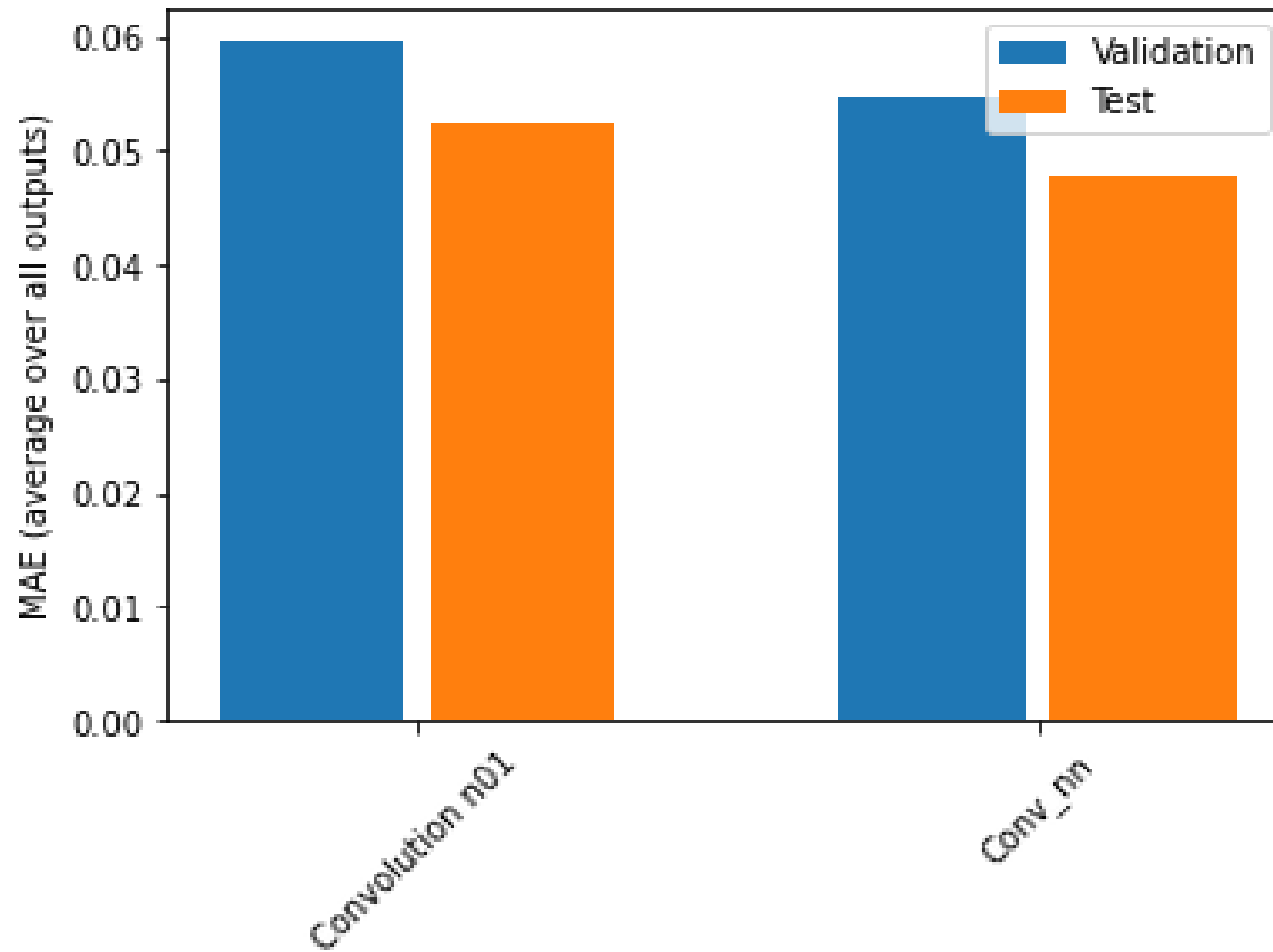


See Figure 11 above for interpretation of volatility time series



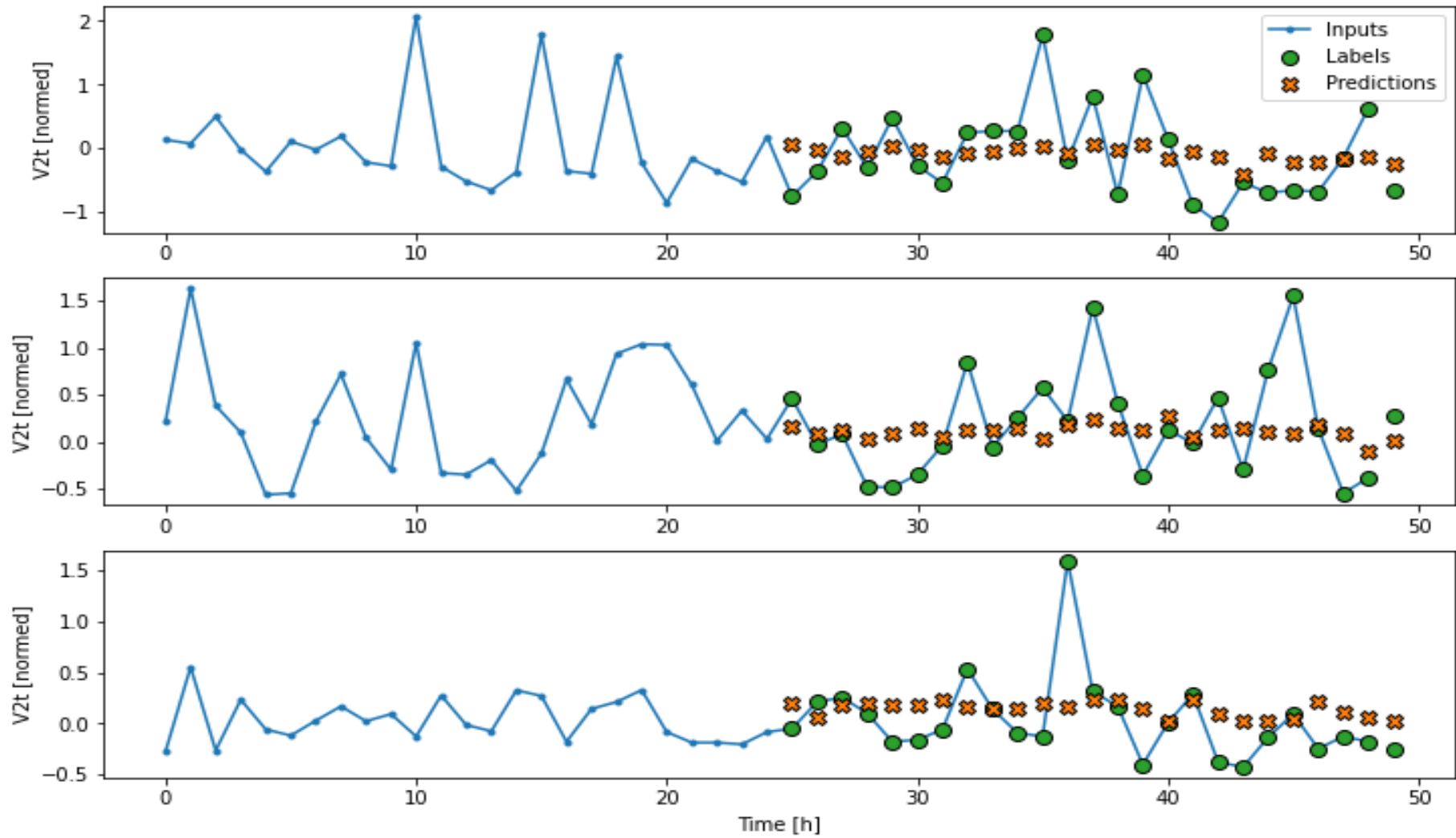


See Figure 11 above for interpretation of volatility time series

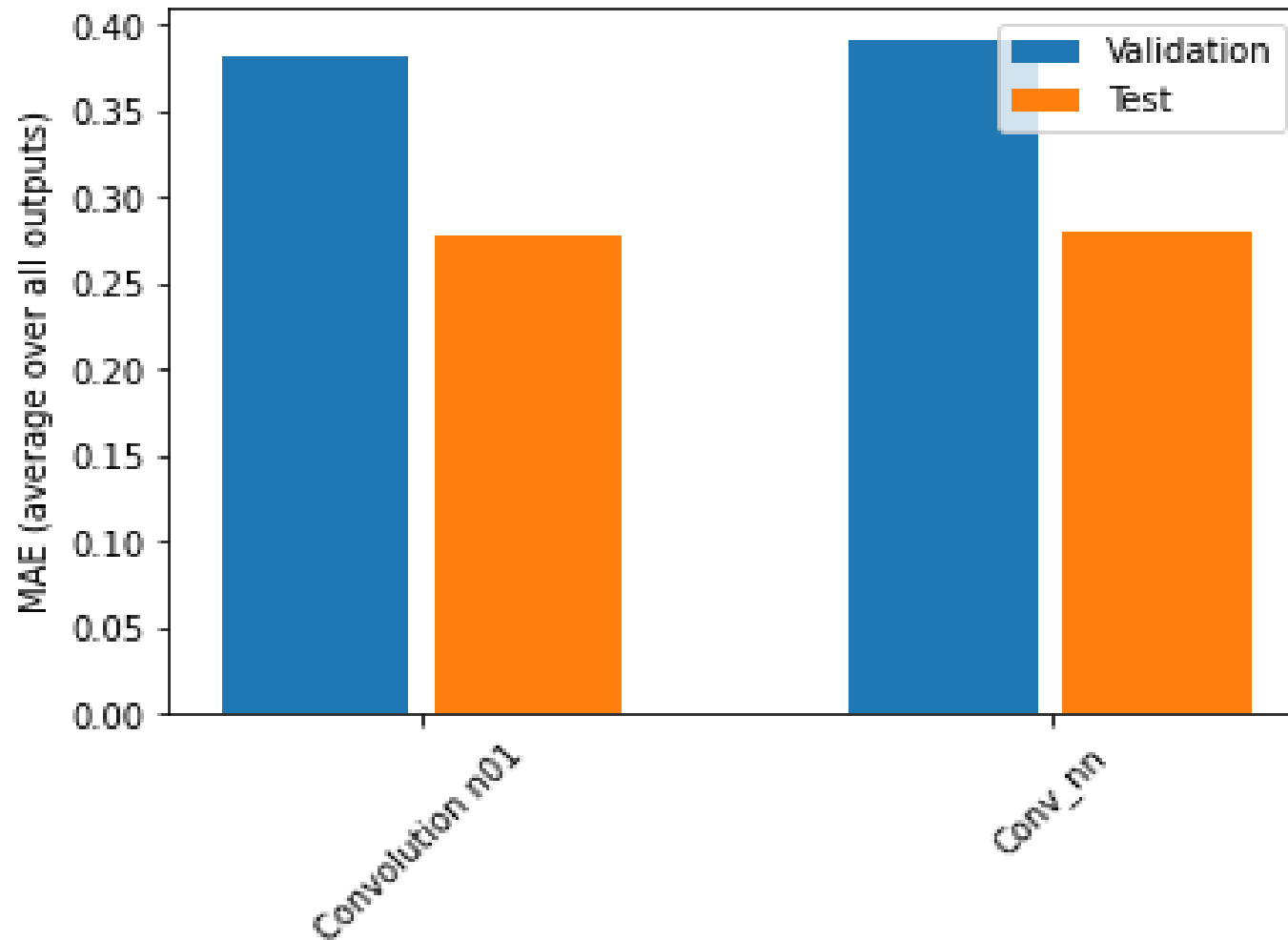


See Figure 11 above for interpretation of volatility time series.

**Figure A.3.** Daily neural network CNN/RNN/LSTM results and performance for WTI oil  $V_{It}$ .



See Figure 12 above for interpretation of volatility time series



See Figure 12 above for interpretation of volatility time series.

**Figure A.4.** Daily neural network CNN/RNN/LSTM results and performance for WTI oil  $V_{2t}$ .



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