



Research article

Efficacy of monetary policy in a currency union? Evidence from Southern Africa's Common Monetary Area

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Supplementary

Appendix A: Temporal Disaggregation

One of the challenges that researchers have to address is the lack of macroeconomic data (such as GDP, Inflation rates) at desired frequencies (quarterly, monthly). Several temporal disaggregation methods have been developed in recent years to address this problem. Temporal disaggregation is a process of estimating a high-frequency time series data using low-frequency data (Sax and Steiner, 2013). These methods can generally be classified into two categories; a) models based on an indicator series, e.g. Chow-Lin (1971) and Litterman (1983), and b) models developed without an indicator, e.g. Denton (1971). These techniques are particularly useful in this analysis because in estimating an SVAR model, all variables must have the same frequency.

A.1 The Chow-Lin Approach

The temporal disaggregation procedure adopted in this paper was developed by Chow and Lin (1971). It is commonly referred to as the best linear unbiased estimator (BLUE) because it uses a regression approach that relates the unknown frequency series to a set of known high-frequency series.

Suppose, without loss of generality, that we have annual values of n years of a given time series y_a , the goal is to disaggregate y_a into a quarterly series y_q with $4n$ observations. The Chow-Lin approach to this problem is based on x_q , some observed quarterly indicator related to y_a . The relationship between the disaggregated series and the indicator is,

$$\hat{Y}_q = X_q\beta + \varepsilon_q \quad (1)$$

where \hat{Y}_q is a $(4n \times 1)$ vector of the estimated quarterly series, X_q is the vector $(n \times 1)$ of observed quarterly series, β is the vector of unknown parameters and is estimated using the Generalised Least

Square (GLS) method, ε is vector of stochastic disturbances with mean, $E(\varepsilon) = 0$ and covariance $E(\varepsilon\varepsilon') = \sigma^2 I = V_q$, σ^2 is a constant. The Chow-Lin can be adopted to our case in the following three steps:

Step 1: Finding an Aggregation Matrix

Since \hat{Y}_q is a high frequency matrix of the unobserved series, the Chow-Lin approach transforms equation (1) into a low frequency matrix of the observed series Y_a . This is achieved by pre-multiplying equation (1) by the aggregation matrix $C = c' \otimes I_n$ such that $Y_a = C\hat{Y}_q$, where $c' = [1, 1, 1, 1]$ and \otimes denotes the kronecker product. The result of the aggregated model is,

$$Y_a = X_a\beta + \varepsilon_a \quad (2)$$

where $X_a = CX_q$, $\varepsilon_a = C\varepsilon_q$ is the vector of aggregated disturbances with mean $E(\varepsilon_a) = CE(\varepsilon_q) = 0$ and covariance $E(\varepsilon_a\varepsilon_a') = \sigma^2 C I C' = V_a$. β describes the parameters that characterize the relationship between Y_a and X_a .

Step 2: Finding the Chow-Lin disaggregation equation

The next step is to establish the equation to disaggregate annual data to quarterly estimates. The optimal coefficient is determined by applying the GLS estimation method to the quarterly regression, thus

$$\hat{\beta}_{GLS} = [X_a'(CV_qC')^{-1}X_a]^{-1} X_a'(CV_qC')^{-1}Y_a \quad (3)$$

In order to find the Chow-Lin equation that disaggregates annual data to quarterly data, we follow from equation (2) where $\varepsilon_a = C\varepsilon_q$. We can re-write ε_q as the subject of the formula and expand the function further as shown below,

$$\begin{aligned} \varepsilon_q &= V_q C' (C V_q C')^{-1} \varepsilon_a \\ \varepsilon_q &= V_q C' (C V_q C')^{-1} (Y_a - X_a \hat{\beta}_{GLS}) \end{aligned} \quad (4)$$

Equations (3) and (4) can be substituted into (1) to give the Chow-Lin equation that disaggregates annual data to quarterly estimates as shown below,

$$\hat{Y}_q = X_q \hat{\beta}_{GLS} + V_q C' (C V_q C')^{-1} (Y_a - X_a \hat{\beta}_{GLS}) \quad (5)$$

Step 3: Estimating the Covariance matrix under Chow-Lin Assumptions

A major drawback of the Chow-Lin approach is that the covariance matrix V_q is unknown. Chow-Lin (1971) proposed two assumptions under which V_q could be better estimated, which are

- i. the disturbances are not serially correlated, each with variance σ^2 , then $V_q = \sigma^2 I$
- ii. the quarterly disturbances ε_q , follow a simple autoregressive structure of first order, AR(1) as,

$$\varepsilon_t = \rho \varepsilon_{t-1} + \mu_t \quad |\rho| < 1 \quad \forall t \quad (6)$$

where μ_t is the white noise process; $\mu \sim i.i.d(0, \sigma_\mu^2)$, $E(\mu_t) = 0$ and $E(\mu_t^2) = \sigma^2$. Based on these assumptions, the variance-covariance matrix V_q takes the form,

$$V_q = \frac{\sigma^2}{1 - \rho^2} \begin{bmatrix} 1 & \rho & \rho^2 & \dots & \rho^{4n-1} \\ \rho & 1 & \rho & \dots & \rho^{4n-2} \\ \rho^2 & \rho & 1 & \dots & \rho^{4n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{4n-1} & \rho^{4n-2} & \dots & \dots & 1 \end{bmatrix} \quad (7)$$

To estimate the autoregressive parameter ρ , Chow-Lin (1971) suggested a polynomial that needs to be solved*. If a sufficient length of quarterly data is available, then one may estimate ρ from the OLS residuals of equation (1).

In this study, where the objective is to generate monthly GDP estimates from quarterly aggregates, if the monthly residuals follow an autoregressive parameter, then the first order auto-correlation of the quarterly residuals forms a polynomial expression in the autoregressive coefficient of the monthly residuals (Karan, 2013). Therefore, a process similar to the GLS can be constructed to obtain results implied by equations (4) and (6).

A.2 Estimating Monthly GDP

The data used in this estimation procedure is from the period from February 2000 to December 2018. The exports of goods and services were identified as a suitable indicator for economic growth. The results of the monthly economic output series estimated using Chow-Lin (1971) are shown in figure 1. The results show that the estimated monthly GDP exhibits similar movements to the quarterly data for all countries. The results also show larger volatility for South Africa starting from the period around 2008. These patterns could be explained by changes observed in figure 4.1 during the same period. Since South Africa and Namibia are the largest exporters in the CMA, the effects of the external shocks will be greater compared to Lesotho and Eswatini.

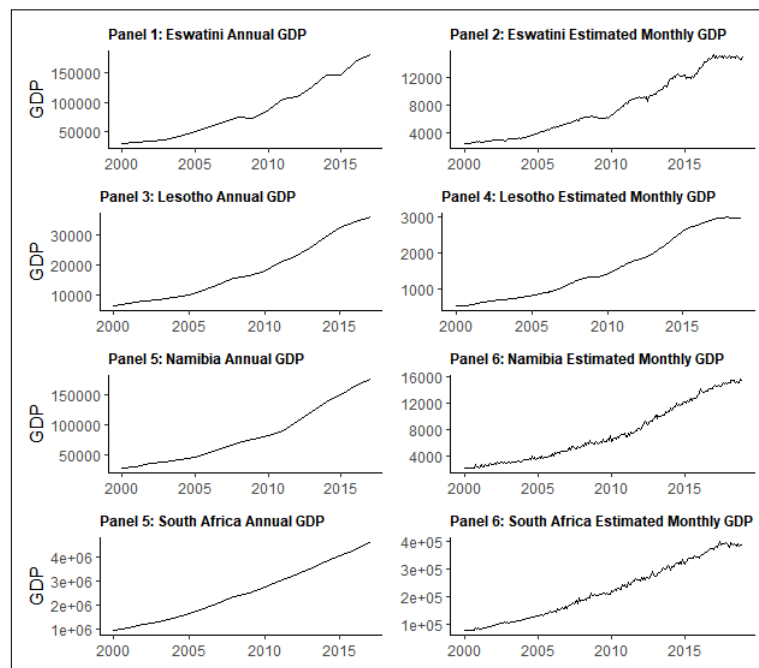


Figure 1. Estimated monthly GDP, 2000M2 - 2018M12

*The following polynomial needs to be solved, $\hat{\rho}_a = \frac{\rho+1)(\rho^2+1)^2}{2(\rho^2+\rho+2)}$ where $\hat{\rho}_a$ is the estimated first-order autocorrelation coefficient from the OLS residuals of the annual-data regression (4.2)

Appendix B: Stationarity test (Augmented Dickey Fuller) results

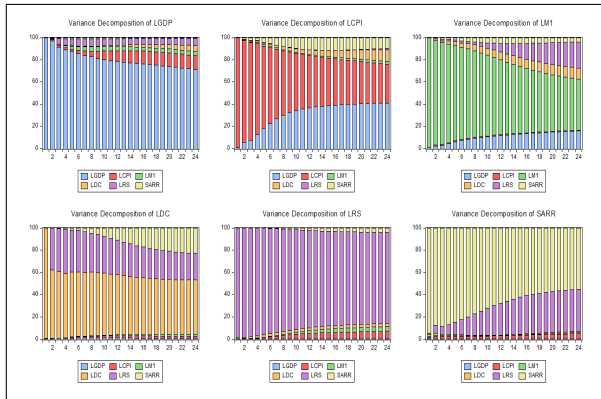
Table 1. ADF test results by country, 2000M2-2018M12

	Country	Levels		First Differences		Decision
		Constant	Trend	Constant	Trend	
LM2	ESW	-0.94	-2.18	-11.50***	-11.51***	I(1)
	LES	-1.72	-2.31	-9.44***	-9.48***	I(1)
	NAM	-1.84	-3.97***	-11.55***	-11.53***	I(1)
	RSA	-1.66	-0.079	-9.31***	-9.47***	I(1)
LCPI	ESW	-0.20	-3.22*	-10.42***	-10.34***	I(1)
	LES	-1.96	-1.33	-6.94***	-7.21***	I(1)
	NAM	-3.37**	-4.37***	-5.55***	-5.99***	I(1)
	RSA	-0.32	-1.92	-5.91***	-5.89***	I(1)
LDC	ESW	-2.94**	-2.95	-15.38***	-15.35***	I(1)
	LES	-1.95	-3.36*	-11.38***	-11.38***	I(1)
	NAM	-4.93***	-1.43	-7.47***	-8.28***	I(1)
	RSA	-2.78*	-0.74	-6.96***	-7.36***	I(1)
LRS	ESW	-1.26	-1.04	-4.59***	-4.72***	I(1)
	LES	-1.85	-1.75	-8.06***	-8.04***	I(1)
	NAM	-2.39	-2.41	-12.02***	-12.12***	I(1)
	RSA	-2.44	-3.45**	-10.41***	-10.45***	I(1)
LGDP	ESW	-1.64	-3.44**	-11.91***	12.05***	I(1)
	LES	-2.37	1.84	-7.19***	-7.69***	I(1)
	NAM	-1.21	-2.75	-13.51***	-13.57***	I(1)
	RSA	-3.61***	-1.27	-8.68***	-9.47***	I(1)
SARR	RSA	-1.99	-2.31	-4.39***	4.40***	I(1)

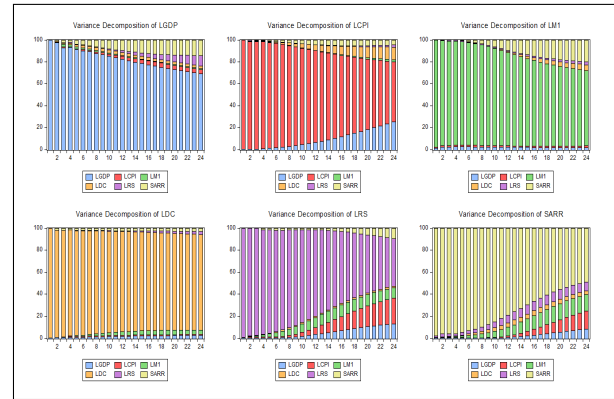
Note: The Augmented Dickey-Fuller (ADF) test is performed with the null hypothesis that the series has a unit root. The results present the value of the test-statistic, where (*) $\rightarrow p < 0.1$, (**) $\rightarrow p < 0.05$, and (***) $\rightarrow p < 0.01$.

Appendix C: Forecast Error Variance Decomposition

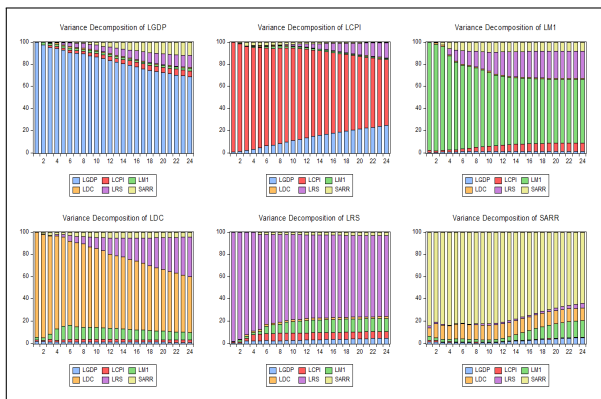
(a) Eswatini



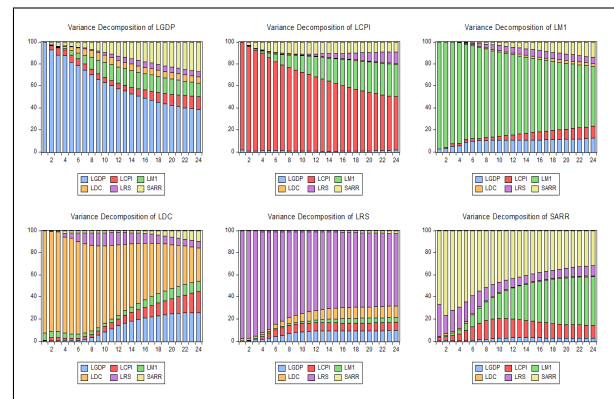
(b) Lesotho



(c) Eswatini

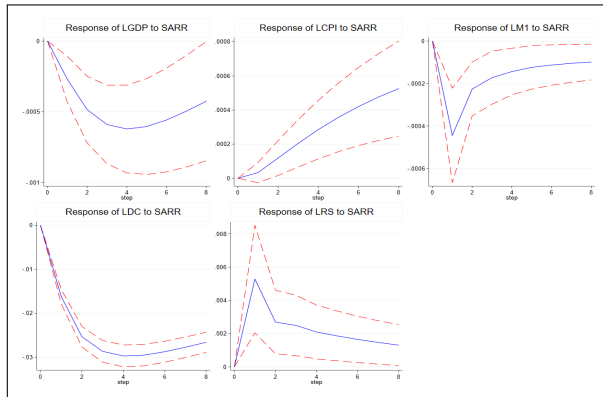


(d) Lesotho

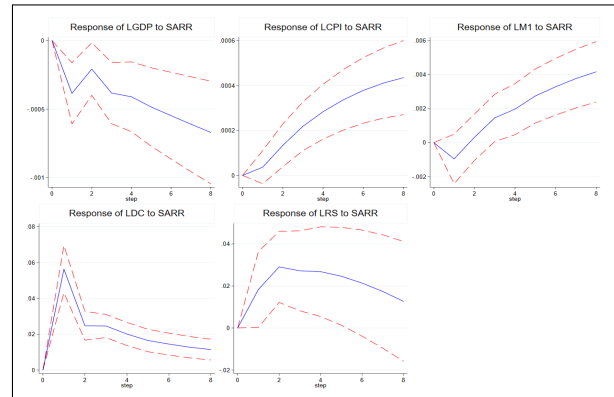


Appendix D: Check for Model Robustness - Impulse Response Functions to Repo Rate Shock

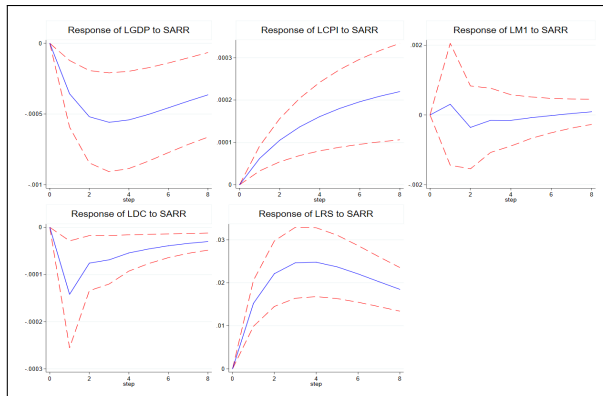
(a) Eswatini



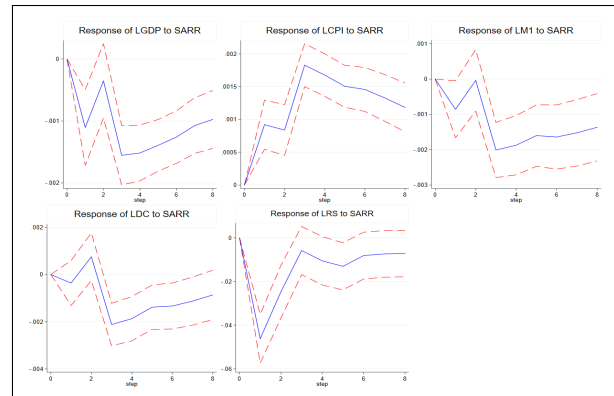
(b) Lesotho



(c) Namibia

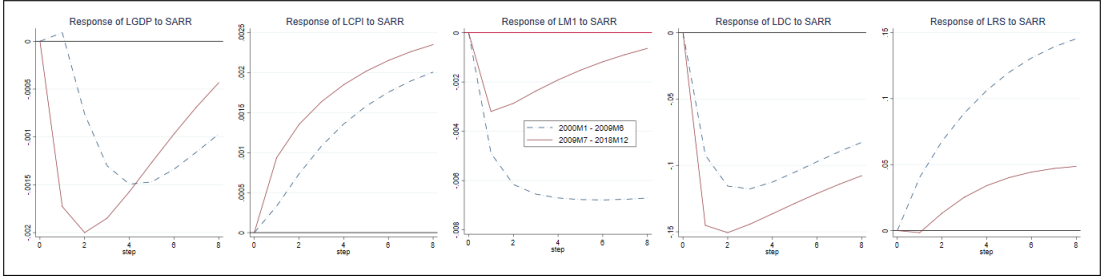


(d) South Africa

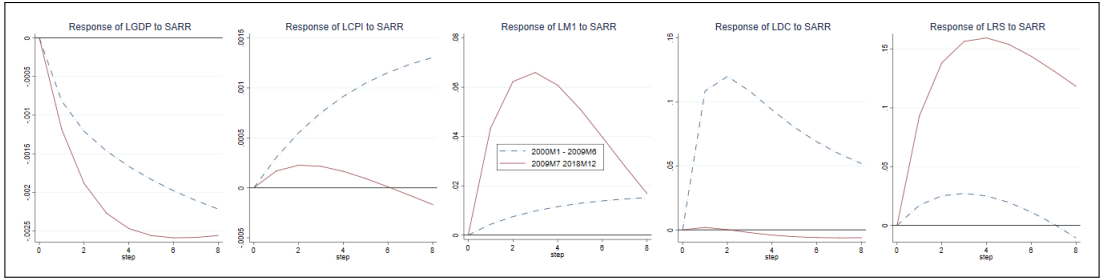


Appendix E: Check for Model Robustness - Break Point Tests

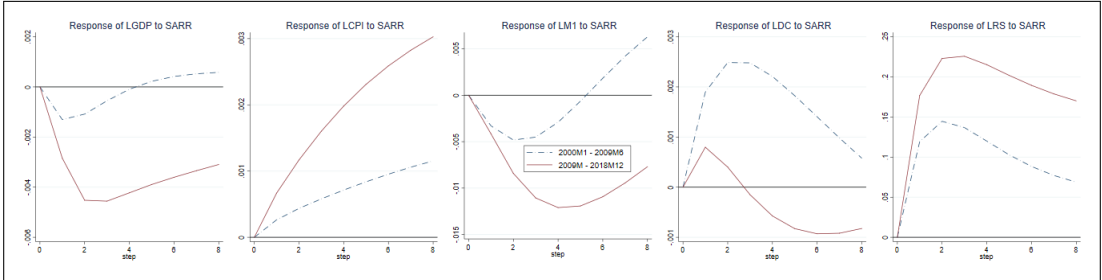
(a) Eswatini



(b) Lesotho



(c) Namibia



(b) South Africa

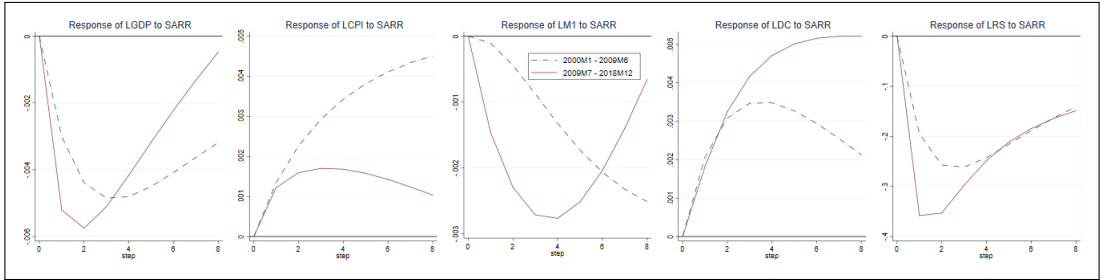


Table 2. Joint residual heteroskedasticity and normality tests

Country	Normality		Heteroskedasticity	
	Jarque-Bera	Prob.	Chi-square	Prob.
Eswatini	22.3542	0.2835	17.2822	0.3676
Lesotho	16.7221	0.5773	14.5703	0.5563
Namibia	12.3066	0.6849	16.9862	0.3835
South Africa	18.6076	0.4899	22.2764	0.1346

Table 3. Lagrange-Multiplier test for serial autocorrelation

	Chi-square	Lags	Significance level
Eswatini	$\chi^2(49) = 57.6425$	1	0.1860
Lesotho	$\chi^2(49) = 64.2429$	1	0.1467
Namibia	$\chi^2(49) = 56.0724$	1	0.2268
South Africa	$\chi^2(49) = 55.0792$	1	0.2556



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