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Research article

Valuing tradeability in exponential Lévy models

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Supplementary

A. Appendices

A.1. Appendix A: Dynamics of $(Y_t)_{t\geq 0}$ under $\mathbb{Q}^{(1)}$

In this appendix, we derive, for any finite time horizon $T > 0$, the dynamics of the Lévy process $(Y_t)_{t\in[0,T]}$ under the particular measure transformation (20). To this end, we denote by $(X_t^c)_{t\geq 0}$ and $(X_t^d)_{t\geq 0}$ — and $(Y_t^c)_{t\geq 0}$, $(Y_t^d)_{t\geq 0}$ — the continuous and discontinuous parts of $(X_t)_{t\geq 0}$ — and $(Y_t)_{t\geq 0}$ respectively —, i.e. we set

$$X_t^c := b_X t + \sigma_X W_t^X, \qquad X_t^d := X_t - X_t^c, \qquad t \geq 0 \tag{A.1}$$

— and analogously $Y_t^c := b_Y t + \sigma_Y W_t^Y$, $Y_t^d := Y_t - Y_t^c$, $t \geq 0$. Then, from the independence of the diffusion and jump parts, we note that

$$\left.\frac{d\mathbb{Q}^{(1)}}{d\mathbb{Q}}\right|_{\mathcal{F}_t} = \frac{e^{1\cdot X_t}}{\mathbb{E}^{\mathbb{Q}}\left[e^{1\cdot X_t}\right]} = \frac{e^{1\cdot X_t^c} e^{1\cdot X_t^d}}{\mathbb{E}^{\mathbb{Q}}\left[e^{1\cdot X_t^c}\right]\mathbb{E}^{\mathbb{Q}}\left[e^{1\cdot X_t^d}\right]} =: \left.\frac{d\mathbb{Q}^{(1),c}}{d\mathbb{Q}}\right|_{\mathcal{F}_t} \left.\frac{d\mathbb{Q}^{(1),d}}{d\mathbb{Q}}\right|_{\mathcal{F}_t} \tag{A.2}$$

Combining this fact with Girsanov's theorem for multidimensional correlated Brownian motion and the properties of $(X_t)_{t\geq 0}$ and $(Y_t)_{t\geq 0}$, we obtain, for $m \in \mathbb{N}$, $(\theta_0, \ldots, \theta_{m-1}) \in \mathbb{R}^m$ and $0 \leq t_0 < t_1 < \ldots < t_m \leq T$, that

$$\mathbb{E}^{\mathbb{Q}^{(1)}}\left[\exp\left\{i\sum_{j=0}^{m-1}\theta_j(Y_{t_{j+1}} - Y_{t_j})\right\}\right]$$

$$
\begin{aligned}
&= \mathbb{E}^{\mathbb{Q}}\Big[\frac{d\mathbb{Q}^{(1)}}{d\mathbb{Q}}\Big|_{\mathcal{F}_T} \exp\Big\{ i \sum_{j=0}^{m-1} \theta_j (Y_{t_{j+1}} - Y_{t_j}) \Big\}\Big] \\
&= \mathbb{E}^{\mathbb{Q}}\Big[\frac{d\mathbb{Q}^{(1),c}}{d\mathbb{Q}}\Big|_{\mathcal{F}_T} \exp\Big\{ i \sum_{j=0}^{m-1} \theta_j (Y^c_{t_{j+1}} - Y^c_{t_j}) \Big\}\Big] \mathbb{E}^{\mathbb{Q}}\Big[\exp\Big\{ i \sum_{j=0}^{m-1} \theta_j (Y^d_{t_{j+1}} - Y^d_{t_j}) \Big\}\Big] \\
&= \exp\Big\{ i \sum_{j=0}^{m-1} \theta_j \rho\sigma_X\sigma_Y (t_{j+1} - t_j) \Big\} \mathbb{E}^{\mathbb{Q}^{(1),c}}\Big[\exp\Big\{ i \sum_{j=0}^{m-1} \theta_j (\tilde{Y}^c_{t_{j+1}} - \tilde{Y}^c_{t_j}) \Big\}\Big] \mathbb{E}^{\mathbb{Q}}\Big[\exp\Big\{ i \sum_{j=0}^{m-1} \theta_j (Y^d_{t_{j+1}} - Y^d_{t_j}) \Big\}\Big] \\
&= \prod_{j=0}^{m-1} \mathbb{E}^{\mathbb{Q}^{(1)}}\Big[e^{i\theta_j \left[(\tilde{Y}^c_{t_{j+1}} - \tilde{Y}^c_{t_j}) + \rho\sigma_X\sigma_Y (t_{j+1} - t_j) \right]} \Big] \prod_{j=0}^{m-1} \mathbb{E}^{\mathbb{Q}^{(1)}}\Big[e^{i\theta_j (Y^d_{t_{j+1}} - Y^d_{t_j})} \Big] \\
&= \prod_{j=0}^{m-1} \mathbb{E}^{\mathbb{Q}^{(1)}}\Big[e^{i\theta_j (Y_{t_{j+1}} - Y_{t_j})} \Big]
\end{aligned}
\tag{A.3}
$$

where

$$
\tilde{Y}^c_t := Y^c_t - \rho\sigma_X\sigma_Y t = b_Y t + \sigma_Y \tilde{W}^Y_t, \qquad \tilde{W}^Y_t := W^Y_t - \rho\sigma_X t \tag{A.4}
$$

and we have used the fact that $(\tilde{W}^Y_t)_{t\in[0,T]}$ is, under $\mathbb{Q}^{(1),c}$, a Brownian motion — in fact Girsanov's theorem tells us that the processes $(\tilde{W}^X_t)_{t\in[0,T]}$, $\tilde{W}^X_t := W^X_t - \sigma_X t$, and $(\tilde{W}^Y_t)_{t\in[0,T]}$ are correlated Brownian motions under $\mathbb{Q}^{(1)}$. This shows that $(Y_t)_{t\in[0,T]}$ has independent increments under $\mathbb{Q}^{(1)}$.

Showing that $(Y_t)_{t\in[0,T]}$ has stationary increments under $\mathbb{Q}^{(1)}$ is easily done and follows from the identity

$$
\begin{aligned}
\mathbb{E}^{\mathbb{Q}^{(1)}}\Big[\exp\Big\{ i \sum_{j=0}^{m-1} \theta_j (Y_{t_{j+1}} - Y_{t_j}) \Big\}\Big] &= \mathbb{E}^{\mathbb{Q}}\Big[\frac{d\mathbb{Q}^{(1)}}{d\mathbb{Q}}\Big|_{\mathcal{F}_T} \exp\Big\{ i \sum_{j=0}^{m-1} \theta_j (Y_{t_{j+1}} - Y_{t_j}) \Big\}\Big] \\
&= \mathbb{E}^{\mathbb{Q}}\Big[\frac{d\mathbb{Q}^{(1)}}{d\mathbb{Q}}\Big|_{\mathcal{F}_T} \exp\Big\{ i \sum_{j=0}^{m-1} \theta_j Y_{(t_{j+1} - t_j)} \Big\}\Big] = \mathbb{E}^{\mathbb{Q}^{(1)}}\Big[\exp\Big\{ i \sum_{j=0}^{m-1} \theta_j Y_{(t_{j+1} - t_j)} \Big\}\Big]
\end{aligned}
\tag{A.5}
$$

Finally, it is clear that equivalent measure transformations do not alter both the starting value and the path continuity of processes. Hence, $(Y_t)_{t\in[0,T]}$ is also under $\mathbb{Q}^{(1)}$ càdlàg and satisfies $Y_0 = 0$. This shows that $(Y_t)_{t\in[0,T]}$ is under $\mathbb{Q}^{(1)}$ again a Lévy process.

Deriving the characteristic exponent of $(Y_t)_{t\in[0,T]}$ under $\mathbb{Q}^{(1)}$ is now easily done using the equation

$$
\mathbb{E}^{\mathbb{Q}^{(1)}}\Big[e^{i\theta Y_t} \Big] = e^{i\theta\rho\sigma_X\sigma_Y t}\, \mathbb{E}^{\mathbb{Q}^{(1),c}}\Big[e^{i\theta \tilde{Y}^c_t} \Big] \mathbb{E}^{\mathbb{Q}}\Big[e^{i\theta Y^d_t} \Big] \tag{A.6}
$$

which can be derived as in (A.3). This gives that the Lévy exponent of $(Y_t)_{t\in[0,T]}$ under $\mathbb{Q}^{(1)}$, $\Psi^{(1)}_Y(\cdot)$, is given by

$$
\Psi^{(1)}_Y(\theta) = -i(b_Y + \rho\sigma_X\sigma_Y)\theta + \frac{1}{2}\sigma_Y^2\theta^2 + \int_{\mathbb{R}} (1 - e^{i\theta y} + i\theta y \mathbb{1}_{\{|y|\le 1\}}) \Pi_Y(dy) \tag{A.7}
$$

i.e. $(Y_t)_{t\in[0,T]}$ is under $\mathbb{Q}^{(1)}$ an $\mathbf{F}$-Lévy process with triplet $(b_Y + \rho\sigma_X\sigma_Y, \sigma_Y^2, \Pi_Y)$

A.2. Appendix B: Proofs - deterministic illiquidity horizon

Proof of Proposition 1. Due to the discussion preceding Proposition 1, we only need to show that $\mathfrak{C}^{\star}_{\mathbf{E}}(\cdot)$ has enough regularity, i.e. in particular that

i) $x \mapsto \mathfrak{C}^{\star}_{\mathbf{E}}(\mathcal{T}, x)$ is, for any $\mathcal{T} \in (0, T_D)$, twice continuously differentiable,
ii) $t \mapsto e^{-\tilde{r}t}\mathfrak{C}^{\star}_{\mathbf{E}}(\mathcal{T}, x)$ is, for any $x \in (0, \infty)$, continuously differentiable,
iii) $(\mathcal{T}, x) \mapsto \mathfrak{C}^{\star}_{\mathbf{E}}(\mathcal{T}, x)$ is continuous on $[0, T_D] \times [0, \infty)$.

We start by briefly outlining the proof of i). Since this part does not involve any martingale arguments, we refer the reader for details to Cont and Voltchkova (2005) and Voltchkova (2005). To see i), one first notices that the European-type option $\mathfrak{C}^{\star}_{\mathbf{E}}(\cdot)$ can be re-expressed in terms of the function

$$u(\mathcal{T}, \xi) = \mathbb{E}^{\mathbb{Q}^{(1)}}\left[e^{-\tilde{r}\mathcal{T}}(e^{\xi + Y_{\mathcal{T}}} - 1)^{+}\right] \tag{A.8}$$

as

$$\mathfrak{C}^{\star}_{\mathbf{E}}(\mathcal{T}, x) = \mathbb{E}^{\mathbb{Q}^{(1)}}[e^{-\tilde{r}\mathcal{T}}(xe^{Y_{\mathcal{T}}} - 1)^{+}] = \mathbb{E}^{\mathbb{Q}^{(1)}}\left[e^{-\tilde{r}\mathcal{T}}\left(e^{\log(x) + Y_{\mathcal{T}}} - 1\right)^{+}\right] = u\left(\mathcal{T}, \log(x)\right) \tag{A.9}$$

Therefore, in order to show the smoothness of $x \mapsto \mathfrak{C}^{\star}_{\mathbf{E}}(\mathcal{T}, x)$ it is enough to prove the smoothness of $u(\cdot)$ in the log-moneyness coordinate. To this end, two facts can be combined. First, as noted in Cont and Voltchkova (2005) and Voltchkova (2005), Condition (41) ensures that Y_t has, for any $t \in [0, T_D]$, a smooth, at least C^2, ($\mathbb{Q}^{(1)}$-)density with derivatives vanishing at infinity. We denote this density in the following by $q_t(\cdot)$. Secondly, setting $\tilde{q}_t(y) := q_t(-y)$, we can rewrite $u(\cdot)$ as a convolution of the form

$$u(\mathcal{T}, \xi) = e^{-r\mathcal{T}} \int_{\mathbb{R}} \left(e^{\xi + y} - 1\right)^{+} q_{\mathcal{T}}(y)\, dy = e^{-r\mathcal{T}} \int_{\mathbb{R}} (e^{z} - 1)^{+}\, \tilde{q}_{\mathcal{T}}(\xi - z)\, dz \tag{A.10}$$

Therefore, the decay of $q_{\mathcal{T}}(\cdot)$ and in particular of its derivatives (Cont and Voltchkova (2005), Voltchkova (2005)) allows one to use the dominated convergence theorem to differentiate under the integral sign and to obtain that $x \mapsto \mathfrak{C}^{\star}_{\mathbf{E}}(\mathcal{T}, x)$ is twice continuously differentiable.

We now prove ii) using Fourier methods. This approach was similarly used in Cont and Voltchkova (2005) and relies on a seminal article of Carr and Madan (Carr and Madan (1999)). Recall, for an integrable function $f(\cdot)$, the definition of the Fourier transform, $\mathcal{F}$, and Fourier inverse, $\mathcal{F}^{-1}$,

$$\mathcal{F}f(\xi) := \int_{\mathbb{R}} f(y)e^{iy\xi}\, dy, \qquad \mathcal{F}^{-1}f(y) := \frac{1}{2\pi}\int_{\mathbb{R}} f(\xi)e^{-i\xi y}\, d\xi \tag{A.11}$$

and that both operators can be extended to isometries on the space of square-integrable functions. As noted in i), Condition (41) ensures that Y_t has, for any $t \in [0, T_D]$, a smooth, C^2, ($\mathbb{Q}^{(1)}$-)density which we will denote again by $q_t(\cdot)$. Therefore, the characteristic function of $Y_{\mathcal{T}}$ at θ, $\chi_{\mathcal{T}}(\theta)$, can be expressed as

$$e^{-\mathcal{T}\Psi^{(1)}_{Y}(\theta)} = \chi_{\mathcal{T}}(\theta) = \int_{\mathbb{R}} e^{i\theta y} q_{\mathcal{T}}(y)\, dy \tag{A.12}$$

We now consider, for $k \in \mathbb{R}$, the modified call price defined by

$$\mathfrak{c}_{\mathcal{T}}(k) := e^{k} \int_{k}^{\infty} e^{-\tilde{r}\mathcal{T}}(e^{y} - e^{k})q_{\mathcal{T}}(y)\, dy \tag{A.13}$$

and easily see that with $k := \log\left(\frac{K}{x}\right)$, $x \in (0,\infty)$ and $K \in (0,\infty)$, it satisfies that

$$x \cdot \mathfrak{c}_{\mathcal{T}}(k) = x \cdot e^k \, \mathbb{E}^{\mathbb{Q}^{(1)}}\left[e^{-\tilde{r}\mathcal{T}}(e^{Y_{\mathcal{T}}} - e^k)^+\right] = e^k \, \mathbb{E}_x^{\mathbb{Q}^{(1)}}\left[e^{-\tilde{r}\mathcal{T}}(E_{\mathcal{T}} - K)^+\right] \tag{A.14}$$

Additionally, we set $\mathfrak{c}^*_{\mathcal{T}}(k) := e^{-\tilde{r}t}\mathfrak{c}_{\mathcal{T}}(k)$. Arguing as in Carr and Madan (1999) one sees that Condition (29) implies both the integrability and square-integrability of the discounted modified call price $k \mapsto \mathfrak{c}^*_{\mathcal{T}}(k)$. Furthermore one readily derives, using (A.13), that

$$\mathcal{F}\mathfrak{c}^*_{\mathcal{T}}(v) = \int_{\mathbb{R}} \mathfrak{c}^*_{\mathcal{T}}(k)e^{ikv}\,dk = \frac{e^{-\tilde{r}T_D}\chi_{\mathcal{T}}(v-2i)}{(iv+1)(iv+2)} \tag{A.15}$$

Notice that this expression is clearly differentiable with respect to t and that one obtains

$$\partial_t \mathcal{F}\mathfrak{c}^*_{\mathcal{T}}(v) = \frac{e^{-\tilde{r}T_D}\chi_{\mathcal{T}}(v-2i)\,\Psi^{(1)}_Y(v-2i)}{(iv+1)(iv+2)} \tag{A.16}$$

From the Lévy-Khintchine formula/representation, one additionally sees that $\Psi^{(1)}_Y(v-2i) = O(|v|^2)$ (as $|v| \to \infty$) — hence at ∞ the denominator compensates $\Psi^{(1)}_Y(v-2i)$. Combining these arguments with the fact that, under (41),

$$|\chi_{\mathcal{T}}(z)| \leq C(\mathcal{T})\exp(-c(\mathcal{T})|z|^{\gamma}) \quad \text{for some} \quad \gamma > 0^1 \quad \text{and “constants”} \quad C(\mathcal{T}), c(\mathcal{T}) > 0 \tag{A.17}$$

and, in particular, that $\mathcal{T} \mapsto C(\mathcal{T})$, $\mathcal{T} \mapsto c(\mathcal{T})$ can be chosen to be continuous (by the continuity of $\mathcal{T} \mapsto \chi_{\mathcal{T}}(z)$), tells us that (A.16) is in any case dominated (locally in $\mathcal{T}$) by an integrable function that does not have any $\mathcal{T}$-dependency.[2] Finally, this allows us to use the dominated convergence theorem in order to conclude that

$$\partial_t \mathfrak{c}^*_{\mathcal{T}}(k) = \partial_t \mathcal{F}^{-1}\mathcal{F}\mathfrak{c}^*_{\mathcal{T}}(k) = \mathcal{F}^{-1}\partial_t\mathcal{F}\mathfrak{c}^*_{\mathcal{T}}(k) \tag{A.18}$$

which shows, in particular by means of Relation (A.14) with $K = 1$, that $t \mapsto e^{-\tilde{r}t}\mathfrak{C}^{\star}_{\mathbf{E}}(\mathcal{T}, x)$ is for any $x \in (0,\infty)$ differentiable. The continuity of the derivative is easily seen from (A.18) and (A.16) and the dominated convergence theorem, by noting that $t \mapsto \chi_{\mathcal{T}}(v-2i)$ is continuous (recall that $\mathcal{T} = T_D - t$).

Finally, iii) is a direct consequence of Relation (A.14) and the continuity of $(\mathcal{T}, k) \mapsto \mathfrak{c}_{\mathcal{T}}(k)$, which follows again from (A.15) by means of Fourier inversion and the dominated convergence theorem. This finalizes the proof.

Proof of Lemma 1. The first part of *a*), i.e. the non-decreasing property follows directly from the path properties of exponential Lévy models. As this is easily proved, we focus on showing the convexity of the American-type option. To start, let $\mathcal{T} \in [0, T_D]$ be arbitrary but fixed. We define, for any initial value $x \in [0,\infty)$ and any stopping time $\tau \in \mathfrak{T}_{[0,\mathcal{T}]}$, the two value functions $V(\cdot)$ and $V^*(\cdot)$ by

$$V(\tau, x) := \mathbb{E}_x^{\mathbb{Q}^{(1)}}\left[e^{-\tilde{r}\tau}(E_\tau - 1)^+\right] \tag{A.19}$$

and

$$V^*(x) := \sup_{\tau \in \mathfrak{T}_{[0,\mathcal{T}]}} V(\tau, x) \tag{A.20}$$

[1] $\gamma = 2$ if $\sigma \neq 0$ and $\gamma = \alpha$ if $\sigma = 0$ and the second condition is satisfied. This was already noted in Voltchkova (2005) (Sato (1999)).
[2] It suffices to take, for a given (compact) $\mathcal{T}$-neighborhood U, $C^\star := \max_{t\in U} C(t)$ and $c^\star := \min_{t\in U} c(t)$ in (A.17).

and note that $V^*(x) = \mathfrak{C}^\star_{\mathbf{A}}(\mathcal{T}, x)$. Given two initial values x_1 and x_2 and an arbitrary $\lambda \in [0,1]$, we set $\tilde{x} := \lambda x_1 + (1-\lambda)x_2$ and fix some $\epsilon > 0$. By definition of $V^*(\cdot)$, we can find a stopping time τ_ϵ satisfying $V^*(\tilde{x}) \le V(\tau_\epsilon, \tilde{x}) + \epsilon$. Furthermore, from the (strong) Markov property of $(E_t)_{t\in[0,T_D]}$ and the properties of the pay-off function, we have that

$$V(\tau_\epsilon, \tilde{x}) \le \lambda V(\tau_\epsilon, x_1) + (1-\lambda)V(\tau_\epsilon, x_2) \tag{A.21}$$

which implies that

$$V^*(\tilde{x}) \le V(\tau_\epsilon, \tilde{x}) + \epsilon \le \lambda V(\tau_\epsilon, x_1) + (1-\lambda)V(\tau_\epsilon, x_2) + \epsilon \le \lambda V^*(x_1) + (1-\lambda)V^*(x_2) + \epsilon \tag{A.22}$$

Since ϵ was arbitrary, this gives the convexity of the American-type option.

Property b) follows directly by noting that, for $0 \le \mathcal{T}_1 \le \mathcal{T}_2 \le T_D$, any stopping time $\tau \in \mathfrak{T}_{[0,\mathcal{T}_1]}$ also satisfies $\tau \in \mathfrak{T}_{[0,\mathcal{T}_2]}$. Therefore, we are left with Part c). To prove this last part, we use the (strong) Markov property of $(E_t)_{t\in[0,T_D]}$ as well as the property that, for $x, y \in [0,\infty)$, $\left|(x-1)^+ - (y-1)^+\right| \le |x-y|$ holds. We then obtain, for a fixed $\mathcal{T} \in [0, T_D]$, that

$$\begin{aligned}
&\left| \sup_{\tau\in\mathfrak{T}_{[0,\mathcal{T}]}} \mathbb{E}_x^{\mathbb{Q}^{(1)}}\left[e^{-\tilde{r}\tau}(E_\tau - 1)^+\right] - \sup_{\tau\in\mathfrak{T}_{[0,\mathcal{T}]}} \mathbb{E}_y^{\mathbb{Q}^{(1)}}\left[e^{-\tilde{r}\tau}(E_\tau - 1)^+\right]\right| \\
&\qquad \le \sup_{\tau\in\mathfrak{T}_{[0,\mathcal{T}]}} \left|\mathbb{E}_x^{\mathbb{Q}^{(1)}}\left[e^{-\tilde{r}\tau}(E_\tau - 1)^+\right] - \mathbb{E}_y^{\mathbb{Q}^{(1)}}\left[e^{-\tilde{r}\tau}(E_\tau - 1)^+\right]\right| \\
&\qquad \le \left|x-y\right| \cdot \sup_{\tau\in\mathfrak{T}_{[0,\mathcal{T}]}} \mathbb{E}^{\mathbb{Q}^{(1)}}\left[e^{-(\tilde{r}-\Phi_Y^{(1)}(1))\tau} e^{Y_\tau - \tau\Phi_Y^{(1)}(1)}\right]
\end{aligned} \tag{A.23}$$

Since the process $\left(e^{Y_t - t\Phi_Y^{(1)}(1)}\right)_{t\in[0,T_D]}$ is known to be a ($\mathbb{Q}^{(1)}$-)martingale, we can take

$$C := \begin{cases} 1, & \text{if } \tilde{r} \ge \Phi_Y^{(1)}(1), \\ e^{-(\tilde{r}-\Phi_Y^{(1)}(1))\mathcal{T}}, & \text{otherwise,} \end{cases} \tag{A.24}$$

and obtain from (A.23) that

$$|\mathfrak{C}^\star_{\mathbf{A}}(\mathcal{T}, x) - \mathfrak{C}^\star_{\mathbf{A}}(\mathcal{T}, y)| \le C|x-y| \tag{A.25}$$

Proof of the smooth-fit property in Proposition 3. This part provides a proof of Equation (60), i.e. we show that, for all $\mathcal{T} \in (0, T_D]$, we have

$$\partial_x \mathfrak{L}^\star(\mathcal{T}, \mathfrak{b}_s(\mathcal{T})) = 1 - \partial_x \mathfrak{C}^\star_{\mathbf{E}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T})) \tag{A.26}$$

For this equation to hold, it is sufficient to have that, for any $\mathcal{T} \in (0, T_D]$, the function $x \mapsto \mathfrak{C}^\star_{\mathbf{A}}(\mathcal{T}, x)$ is in $\mathfrak{b}_s(\mathcal{T})$ differentiable with $\partial_x \mathfrak{C}^\star_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T})) = 1$. We show that this is true.

First, we recall that for a Lévy process $(Z_t)_{t\ge0}$ on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ a fixed level $z \in \mathbb{R}$ is said to be regular for (z, ∞), if we have that

$$\mathbb{P}_z(\tau_z^+ = 0) = 1 \tag{A.27}$$

where τ_z^+ is given by

$$\tau_z^+ := \inf\{t \ge 0 : \; Z_t \in (z, \infty)\} \tag{A.28}$$

and we set as usual $\inf \emptyset = \infty$. As noted for instance in Kyprianou (2006), Theorem 6.5, any Lévy process of infinite variation has the particularity that the point 0 is regular for the interval $(0, \infty)$. Since we have assumed that $\sigma_Y \neq 0$, the ($\mathbb{Q}^{(1)}$-)Lévy process $(Y_t)_{t\geq 0}$ has clearly infinite variation (Sato (1999), Applebaum (2009)). Therefore, it suffices to show that the regularity of 0 for $(0, \infty)$ and $(Y_t)_{t\geq 0}$ implies the smooth-fit property of $\mathfrak{C}^{\star}_{\mathbf{A}}(\cdot)$. We show it by adapting the proof of Theorem 4.1. in Lamberton and Mikou (2011):

Let us fix $\mathcal{T} \in (0, T_D]$. We start by noting that

$$\lim_{h\downarrow 0} \frac{\mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T}) + h) - \mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T}))}{h} = 1 \tag{A.29}$$

This directly follows since any $x \geq \mathfrak{b}_s(\mathcal{T})$ satisfies that $\mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, x) = x - 1$. Therefore, we only need to show that

$$\lim_{h\uparrow 0} \frac{\mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T}) + h) - \mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T}))}{h} = 1 \tag{A.30}$$

First, we obtain from $\mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T})) = (\mathfrak{b}_s(\mathcal{T}) - 1)^+$ and $\mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, x) \geq (x-1)^+$ that, for any $h < 0$,

$$\frac{\mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T}) + h) - \mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T}))}{h} \geq \frac{(\mathfrak{b}_s(\mathcal{T}) + h - 1)^+ - (\mathfrak{b}_s(\mathcal{T}) - 1)^+}{h} \tag{A.31}$$

This gives that

$$\liminf_{h\uparrow 0} \frac{\mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T}) + h) - \mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T}))}{h} \geq 1 \tag{A.32}$$

To show that

$$\limsup_{h\uparrow 0} \frac{\mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T}) + h) - \mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T}))}{h} \leq 1 \tag{A.33}$$

we consider, for $h < 0$, the optimal stopping problem related to $\mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T}) + h)$: First, we define the stopping time

$$\begin{aligned} \tau_h &:= \inf\{t \in [0, \mathcal{T}) : \ (\mathfrak{b}_s(\mathcal{T}) + h)\, e^{Y_t} \geq \mathfrak{b}_s(\mathcal{T})\} \\ &= \inf\left\{t \in [0, \mathcal{T}) : \ Y_t \geq \log\left(\frac{\mathfrak{b}_s(\mathcal{T})}{\mathfrak{b}_s(\mathcal{T}) + h}\right)\right\} \end{aligned} \tag{A.34}$$

and note from the regularity of 0 for the set $(0, \infty)$ that $\tau_h \to 0$ a.s. when $h \uparrow 0$. This can be seen by the following argument: On the almost sure set $\{\tau_0^+ = 0\}$, we can find for any $t_0 \in (0, \mathcal{T})$ a point $u \in [0, t_0]$ such that $Y_u > 0$. Then, taking $h < 0$ small enough (i.e. near enough to zero) gives that $Y_u > \log\left(\frac{\mathfrak{b}_s(\mathcal{T})}{\mathfrak{b}_s(\mathcal{T})+h}\right)$. Consequently, $\lim_{h\uparrow 0} \tau_h \leq t_0$ a.s. and from the arbitrariness of $t_0 \in (0, \mathcal{T})$ this already gives that $\lim_{h\uparrow 0} \tau_h = 0$.
Next, noting that

$$\mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T})) \geq \mathbb{E}^{\mathbb{Q}^{(1)}}_{\mathfrak{b}_s(\mathcal{T})}\left[e^{-\tilde{r}\tau_h}\left(E_{\tau_h} - 1\right)^+\right] \tag{A.35}$$

and combining this inequality with the optimality of the stopping time τ_h for the starting value $\mathfrak{b}_s(\mathcal{T}) + h$ gives, for $h < 0$, that

$$\frac{\mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T}) + h) - \mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T}))}{h} = \frac{\mathbb{E}^{\mathbb{Q}^{(1)}}_{\mathfrak{b}_s(\mathcal{T})+h}\left[e^{-\tilde{r}\tau_h}\left(E_{\tau_h} - 1\right)^+\right] - \mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T}, \mathfrak{b}_s(\mathcal{T}))}{h}$$

$$\leq \mathbb{E}^{\mathbb{Q}^{(1)}}\left[e^{-\tilde{r}\tau_h}\frac{\left((\mathfrak{b}_s(\mathcal{T})+h)e^{Y_{\tau_h}}-1\right)^+-\left(\mathfrak{b}_s(\mathcal{T})e^{Y_{\tau_h}}-1\right)^+}{h}\right] \tag{A.36}$$

Since $x \mapsto (x-1)^+$ is continuously differentiable in a neighbourhood of $\mathfrak{b}_s(\mathcal{T})$, we have that

$$\lim_{h\uparrow 0}\frac{\left((\mathfrak{b}_s(\mathcal{T})+h)e^{Y_{\tau_h}}-1\right)^+-\left(\mathfrak{b}_s(\mathcal{T})e^{Y_{\tau_h}}-1\right)^+}{h}=1 \tag{A.37}$$

Finally, using Lemma 1.*c*) allows us to apply the dominated convergence theorem, to obtain that

$$\limsup_{h\uparrow 0}\frac{\mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T},\mathfrak{b}_s(\mathcal{T})+h)-\mathfrak{C}^{\star}_{\mathbf{A}}(\mathcal{T},\mathfrak{b}_s(\mathcal{T}))}{h}\leq 1 \tag{A.38}$$

which gives the result.

A.3. Appendix C: Proofs - stochastic illiquidity horizon

Proof of Proposition 4. First, we note that the continuity of $x \mapsto \mathfrak{C}^{R,\star}_{\mathbf{E}}(x)$ on $[0,\infty)$ follows from the dominated convergence theorem, by combining Condition (73) with Representations (71) and (74). Additionally, the continuity of $x \mapsto \partial_x\mathfrak{C}^{R,\star}_{\mathbf{E}}(x)$ on $(0,\infty)$ follows analogously using (71), the continuity of $x \mapsto \mathfrak{C}^{\star}_{\mathbf{E}}(t_R,x)$ for all $t_R>0$, and the inequality

$$\left|\mathfrak{C}^{\star}_{\mathbf{E}}(t_R,x)-\mathfrak{C}^{\star}_{\mathbf{E}}(t_R,y)\right|\leq e^{-(\tilde{r}-\Phi^{(1)}_Y(1))t_R}|x-y|, \qquad \forall x,y\in(0,\infty) \tag{A.39}$$

Therefore, we are left with the proof of Equations (78), (79). Here, we start by re-considering the $\tilde{r}$-killed version of $(E_t)_{t\geq 0}$, $(\bar{E}_t)_{t\geq 0}$, i.e. the process whose transition probabilities are given by

$$\mathbb{Q}^{(1)}_x(\bar{E}_t\in A)=\mathbb{E}^{\mathbb{Q}^{(1)}}_x\left[e^{-\tilde{r}t}\,\mathbb{1}_A(E_t)\right] \tag{A.40}$$

and identify, without loss of generality, its cemetery state with $\partial \equiv 0$. We then re-express $\mathfrak{C}^{R,\star}_{\mathbf{E}}(\cdot)$ as solution to an optimal stopping problem: We view the stochastic illiquidity horizon T_R as jump time of a corresponding Poisson process[3] $(N_t)_{t\geq 0}$ with intensity $\vartheta>0$ and consider, for any $z=(n,x)\in\mathbb{N}_0\times[0,\infty)$, the (strong) Markov process $(Z_t)_{t\geq 0}$ defined by means of $Z_t := (n+N_t,\bar{E}_t)$, $\bar{E}_0=x$, on the state domain $\mathcal{D}:=\mathbb{N}_0\times[0,\infty)$. Then, $\mathfrak{C}^{R,\star}_{\mathbf{E}}(\cdot)$ can be equivalently written as

$$\mathfrak{C}^{R,\star}_{\mathbf{E}}(x)=\widetilde{V}_E((0,x)) \tag{A.41}$$

where, for $z=(n,x)\in\mathcal{D}$, the value function $\widetilde{V}_E(\cdot)$ is defined, under the measure $\mathbb{Q}^{(1),Z}_z$ having initial distribution $Z_0=z$, by

$$\widetilde{V}_E(z):=\mathbb{E}^{\mathbb{Q}^{(1),Z}}_z\left[G(Z_{\tau_{\mathcal{S}}})\right], \qquad G(z):=(x-1)^+ \tag{A.42}$$

and $\tau_{\mathcal{S}}:=\inf\{t\geq 0: Z_t\in\mathcal{S}\}$, $\mathcal{S}:=(\mathbb{N}\times(0,\infty))\cup(\mathbb{N}_0\times\{0\})$, is a stopping time that is $\mathbb{Q}^{(1),Z}_z$-almost surely finite for any $z=(n,x)$.[4] Furthermore, the stopping domain $\mathcal{S}$ forms (under an appropriate

[3] Our assumptions on T_R clearly imply that the Poisson process is independent of $(\bar{E}_t)_{t\geq 0}$.

[4] The finiteness of this stopping time directly follows from the properties (e.g. finiteness of the first moment) of the exponential distribution of any intensity $\vartheta>0$.

product-metric) a closed set in $\mathcal{D}$.[5] Therefore, standard arguments based on the strong Markov property of $(Z_t)_{t\geq 0}$ (Peskir and Shiryaev (2006)) imply that $\widetilde{V}_E(\cdot)$ solves the following problem

$$\mathcal{A}_Z \widetilde{V}_E(z) = 0, \qquad \text{on } \mathcal{D} \setminus \mathcal{S} \tag{A.43}$$

$$\widetilde{V}_E(z) = G(z), \qquad \text{on } \mathcal{S} \tag{A.44}$$

where $\mathcal{A}_Z$ denotes the infinitesimal generator of the process $(Z_t)_{t\geq 0}$. To complete the proof, it therefore suffices to note that (for any suitable function $V : \mathcal{D} \to \mathbb{R}$) the infinitesimal generator $\mathcal{A}_Z$ can be re-expressed as

$$\begin{aligned}\mathcal{A}_Z V((n,x)) &= \mathcal{A}_N^n V((n,x)) + \mathcal{A}_{\bar{E}}^x V((n,x)) \\ &= \vartheta\left(V((n+1,x)) - V((n,x))\right) + \mathcal{A}_E^x V((n,x)) - \tilde{r}V((n,x))\end{aligned} \tag{A.45}$$

where $\mathcal{A}_N$ denotes the infinitesimal generator of the Poisson process $(N_t)_{t\geq 0}$ and the notation $\mathcal{A}_N^n$, $\mathcal{A}_{\bar{E}}^x$, and $\mathcal{A}_E^x$ is used to indicate that the generators are applied to n and x respectively. Indeed, recovering $\mathfrak{C}_{\mathbf{E}}^{R,\star}(\cdot)$ via (A.41) while noting Relation (A.45) and the fact that for any $x \in [0,\infty)$ we have

$$\widetilde{V}_E((1,x)) = G((1,x)) = (x-1)^+ \tag{A.46}$$

finally gives the claim.

Proof of Proposition 5. To start, we note that, under $\tilde{r} \leq \Phi_Y^{(1)}(1)$, the American-type switching option $\mathfrak{C}_{\mathbf{A}}^{R,\star}(\cdot)$ reduces to its European counterpart $\mathfrak{C}_{\mathbf{E}}^{R,\star}(\cdot)$. As earlier, this is a direct consequence of the fact that the process $(e^{-\tilde{r}t}E_t)_{t\geq 0}$ then becomes a ($\mathbb{Q}^{(1)}$-)submartingale. In this case, the result directly follows via Proposition 4, with $\mathfrak{b}_s^R = \infty$.

For $\tilde{r} > \Phi_Y^{(1)}(1)$, we first note that Theorem 1 in Mordecki (2002) implies the existence of a finite optimal stopping boundary $\mathfrak{b}_s^R > 0$. Indeed, this follows by combining Lemma 2 with the fact that, with

$$\mathfrak{C}_{\mathbf{A}}^{\infty,\star}(x) := \sup_{\tau \in \mathfrak{T}_{[0,\infty)}} \mathfrak{C}^\star(\tau, x) \tag{A.47}$$

$$\{x \in [0,\infty) : \mathfrak{C}_{\mathbf{A}}^{\infty,\star}(x) = (x-1)^+\} \subseteq \{x \in [0,\infty) : \mathfrak{C}_{\mathbf{A}}^{R,\star}(x) = (x-1)^+\} \tag{A.48}$$

and by arguing as in Section 3.2.3. Therefore, by viewing the stochastic illiquidity horizon T_R as jump time of a corresponding Poisson process $(N_t)_{t\geq 0}$ with intensity $\vartheta > 0$, we can re-express our optimal stopping problem in the following form: We consider, for any $z = (n,x) \in \mathbb{N}_0 \times [0,\infty)$, the (strong) Markov process $(Z_t)_{t\geq 0}$ defined by means of $Z_t := (n + N_t, \bar{E}_t)$, $\bar{E}_0 = x$, on the state domain $\mathcal{D} := \mathbb{N}_0 \times [0,\infty)$ and identify again its cemetery state with $\partial \equiv 0$. Then, we note that

$$\mathfrak{C}_{\mathbf{A}}^{R,\star}(x) = \widetilde{V}_A((0,x)) \tag{A.49}$$

where, for $z = (x,n) \in \mathcal{D}$, the value function $\widetilde{V}_A(\cdot)$ is defined, under the measure $\mathbb{Q}_z^{(1),Z}$ having initial distribution $Z_0 = z$, by

$$\widetilde{V}_A(z) := \mathbb{E}_z^{\mathbb{Q}^{(1),Z}}\left[G(Z_{\tau_S})\right], \qquad G(z) := (x-1)^+ \tag{A.50}$$

[5]We note that several choices of a product-metric on $\mathcal{D}$ give the closedness of the set $\mathcal{S}$. In particular, one may choose on $\mathbb{N}_0$ the following metric

$$d_{\mathbb{N}_0}(m,n) := \begin{cases} 1 + |2^{-m} - 2^{-n}|, & m \neq n, \\ 0, & m = n, \end{cases}$$

and consider the product-metric on $\mathcal{D}$ obtained by combining $d_{\mathbb{N}_0}(\cdot,\cdot)$ on $\mathbb{N}_0$ with the Euclidean metric on $[0,\infty)$.

and $\tau_{\mathcal{S}} := \inf\{t \geq 0 : Z_t \in \mathcal{S}\}$, $\mathcal{S} := (\mathbb{N} \times (0,\infty)) \cup (\mathbb{N}_0 \times \{0\}) \cup (\{0\} \times [\mathfrak{b}_s^R, \infty))$ is a stopping time that is $\mathbb{Q}_z^{(1),Z}$-almost surely finite for any $z = (n, x)$. Furthermore, the stopping domain $\mathcal{S}$ forms (under an appropriate product-metric) a closed set in $\mathcal{D}$.[6] Therefore, standard arguments based on the strong Markov property of $(Z_t)_{t\geq 0}$ (Peskir and Shiryaev (2006)) imply that $\widetilde{V}_A(\cdot)$ solves the following problem

$$\mathcal{A}_Z \widetilde{V}_A(z) = 0, \qquad \text{on } \mathcal{D} \setminus \mathcal{S} \tag{A.51}$$

$$\widetilde{V}_A(z) = G(z), \qquad \text{on } \mathcal{S} \tag{A.52}$$

where $\mathcal{A}_Z$ denotes the infinitesimal generator of the process $(Z_t)_{t\geq 0}$. To complete the proof, we therefore argue as in the proof of Proposition 4, i.e. we recover $\mathfrak{C}_{\mathbf{A}}^{R,\star}(\cdot)$ via (A.49) and combine Relation (A.45) with the fact that for any $x \in [0,\infty)$ we have

$$\widetilde{V}_A((1,x)) = G((1,x)) = (x-1)^+ \tag{A.53}$$

Since Equation (83) is naturally satisfied, this leads to the required problem. The continuity of the function $x \mapsto \mathfrak{C}_{\mathbf{A}}^{R,\star}(\cdot)$ directly follows from its convexity (Lemma 2). Therefore, the proof is complete.

A.4. Appendix D

In this appendix, we briefly derive a semi-analytical solution to the free-boundary problem of Proposition 6, when the dynamics of $(S_t)_{t\geq 0}$ and $(E_t)_{t\geq 0}$ are given by (91) and (8), (92) and assuming non-positive jumps, i.e. $\varphi \leq 0$. This is used to obtain numerical results in Section 5.4.

To start, we first note that, under the given dynamics and with $\tilde{b} := b + \rho\sigma_X\sigma$, the free-boundary problem reads:

1. If $\tilde{r} \leq \tilde{b}$, the (absolute) tradeability premium $\mathfrak{L}^{R,\star}(\cdot)$ satisfies

$$\mathfrak{L}^{R,\star}(x) = 0, \qquad \forall x \in [0,\infty) \tag{A.54}$$

2. If $\tilde{r} > \tilde{b}$, the pair $(\mathfrak{L}^{R,\star}(\cdot), \mathfrak{b}_s^R)$ solves the following free-boundary problem:

$$\frac{1}{2}\sigma^2 x^2 \partial_x^2 \mathfrak{L}^{R,\star}(x) + (\tilde{b} - \lambda(e^{\varphi}-1))x\partial_x \mathfrak{L}^{R,\star}(x) + \lambda\left(\mathfrak{L}^{R,\star}(xe^{\varphi}) - \mathfrak{L}^{R,\star}(x)\right) - (\tilde{r}+\vartheta)\mathfrak{L}^{R,\star}(x) = 0 \tag{A.55}$$

on $x \in (0, \mathfrak{b}_s^R)$ and subject to the boundary conditions

$$\mathfrak{L}^{R,\star}(\mathfrak{b}_s^R) = \mathfrak{b}_s^R - 1 - \mathfrak{C}_{\mathbf{E}}^{R,\star}(\mathfrak{b}_s^R) \tag{A.56}$$

$$\partial_x \mathfrak{L}^{R,\star}(\mathfrak{b}_s^R) = 1 - \partial_x \mathfrak{C}_{\mathbf{E}}^{R,\star}(\mathfrak{b}_s^R) \tag{A.57}$$

$$\mathfrak{L}^{R,\star}(0) = 0 \tag{A.58}$$

Therefore, it is sufficient to focus on the non-trivial case, i.e. we assume from now on that $\tilde{r} > \tilde{b}$. Here, we decompose the full domain $[0,\infty)$ into two intervals, $I_1 := [0, \mathfrak{b}_s^R)$ and $I_2 := [\mathfrak{b}_s^R, \infty)$, derive solutions $V_1(\cdot)$ and $V_2(\cdot)$ on these respective domains and combine them to recover $\mathfrak{L}^{R,\star}(\cdot)$ via

$$\mathfrak{L}^{R,\star}(x) = \begin{cases} V_1(x), & x \in I_1, \\ V_2(x), & x \in I_2. \end{cases} \tag{A.59}$$

[6] As earlier, this property can be obtained under the product-metric considered in Footnote 5.

We now turn to the derivation of these solutions. First, it is clear that, on I_2, $V_2(x) = x - 1 - \mathfrak{C}^{R,\star}_{\mathbf{E}}(x)$ must hold. Hence, we only need to derive an expression for $V_1(\cdot)$. Here, we start by noting that $\Phi^{(1)}_Y(\theta)$, the Laplace exponent of $(Y_t)_{t\geq 0}$ under $\mathbb{Q}^{(1)}$, is well-defined for all $\theta \in \mathbb{R}$. Furthermore, it can be easily seen that $\theta \mapsto \Phi^{(1)}_Y(\theta)$ is convex and satisfies $\Phi^{(1)}_Y(0) = 0$ and $\lim\limits_{|\theta|\to\infty} \Phi^{(1)}_Y(\theta) = \infty$. Consequently, the equation $\Phi^{(1)}_Y(\theta) = y$ has, for any $y > 0$, two solutions, a positive and a negative root. In the sequel, we denote by $\left(\Phi^{(1)}_Y\right)^{-1,+}(y)$ its positive root and by $\left(\Phi^{(1)}_Y\right)^{-1,-}(y)$ its negative root. Using this notation, one easily shows that, under $\varphi \leq 0$, the general solution of the homogeneous equation (A.55) on I_1 takes the form

$$V_1(x) = c_1^+ x^{\gamma_+} + c_1^- x^{\gamma_-} \tag{A.60}$$

where $\gamma_+ = \left(\Phi^{(1)}_Y\right)^{-1,+}(\tilde{r} + \vartheta)$, $\gamma_- = \left(\Phi^{(1)}_Y\right)^{-1,-}(\tilde{r} + \vartheta)$ and c_1^+, c_1^- are constants to be determined. Therefore, to conclude, we only need to derive c_1^+, c_1^- and $\mathfrak{b}^R_s$ and make use of Conditions (A.56)-(A.58). First, we note that (A.58) implies that $c_1^- \equiv 0$. Additionally, Conditions (A.56) and (A.57) give the following equations:

$$c_1^+ (\mathfrak{b}^R_s)^{\gamma_+} = \mathfrak{b}^R_s - 1 - \mathfrak{C}^{R,\star}_{\mathbf{E}}(\mathfrak{b}^R_s) \tag{A.61}$$

$$\gamma_+ c_1^+ (\mathfrak{b}^R_s)^{\gamma_+ - 1} = 1 - \partial_x \mathfrak{C}^{R,\star}_{\mathbf{E}}(\mathfrak{b}^R_s) \tag{A.62}$$

The latter system can now be solved to obtain c_1^+ and $\mathfrak{b}^R_s$. First, rewritting (A.62) gives that

$$c_1^+ = \frac{(\mathfrak{b}^R_s)^{1-\gamma_+}}{\gamma_+}\left(1 - \partial_x \mathfrak{C}^{R,\star}_{\mathbf{E}}(\mathfrak{b}^R_s)\right) \tag{A.63}$$

Then, inserting this result in (A.61) leads to the following non-linear equation in $\mathfrak{b}^R_s$:

$$\mathfrak{b}^R_s = 1 + \mathfrak{C}^{R,\star}_{\mathbf{E}}(\mathfrak{b}^R_s) + \frac{\mathfrak{b}^R_s}{\gamma_+}\left(1 - \partial_x \mathfrak{C}^{R,\star}_{\mathbf{E}}(\mathfrak{b}^R_s)\right) \tag{A.64}$$

Therefore, solving the latter equation for $\mathfrak{b}^R_s$ allows us to subsequently derive c_1^+. This finally allows us to recover the tradeability premium $\mathfrak{L}^{R,\star}(\cdot)$ via (A.59).

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