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Research article

Intra-specific diversity and adaptation modify regime shifts dynamics under environmental change

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Supplementary

Supplementary 1. Prey density equilibria using Cardano's method

Equation (2.1) described up to three complex or real non vanishing equilibria. They were computed using Cardano's method, and we only focused on the positive real solutions to find population density equilibria. During simulations, such density equilibria were used as the initial condition for the prey population.

$$\alpha(s)\cdot a(s)\cdot N^3 - r(s)\cdot a(s)\cdot N^2 + (B\cdot a(s)+\alpha(s))\cdot N - r(s) = 0 \tag{A.1}$$

To find the required equilibria, we thus have to solve the polynomial equation:

$$AX^3 + CX^2 + DX + E = 0 \tag{A.2}$$

where

$$\begin{cases} A = \alpha(s)\cdot a(s) \\ C = -r(s)\cdot a(s) \\ D = B\cdot a(s)+\alpha(s) \\ E = -r(s) \end{cases} \qquad \begin{cases} p = \frac{-C^2}{3A^2} + \frac{C}{A} \\ q = \frac{C}{27A}\cdot\frac{2B^2}{A^2} - 9\frac{C}{A} + \frac{D}{A} \end{cases} \tag{A.3}$$

Following Cardano's method, these equilibria were computed such as in (A.3). Then, a Δ was defined as $\Delta = -4p^3 + 27q^2$. Up to three equilibria, such as in the bistability case (see Figure 1), existed depending on the sign of Δ.

$$\text{If } \Delta < 0 : \quad \begin{cases} U = \sqrt[3]{\frac{-q + \sqrt{\frac{-\Delta}{27}}}{2}} \\ V = \sqrt[3]{\frac{-q - \sqrt{\frac{-\Delta}{27}}}{2}} \end{cases} \qquad \left\{ z_0 = U + V \right. \tag{A.4}$$

$$\text{If } \Delta > 0 : \quad \begin{cases} U = \sqrt[3]{\frac{-q + i\sqrt{\frac{\Delta}{27}}}{2}} \\ j = \exp^{i\frac{2\pi}{3}} \end{cases} \qquad \begin{cases} z_0 = U + \bar{U} \\ z_1 = jU + \bar{jU} \\ z_2 = j^2U + \bar{j^2U} \end{cases} \tag{A.5}$$

$$\text{If } \Delta = 0 : \quad \begin{cases} z_0 = \frac{3q}{p} \\ z_1 = \frac{-3q}{2p} \end{cases} \tag{A.6}$$

The equilibrium z found in (A.4)–(A.6), were used to compute the population equilibria $N = z - \frac{C}{3A}$.

Supplementary 2. Initial conditions of PDE simulations and protocol

The initial condition of PDE experiments was defined using an initial total density N_0, and a mean population trait s_0. These initial values were estimated from the eco-evolutionary equilibrium of the adaptive dynamics model (2.10). From this equilibrium, the initial trait distribution was estimated as a Gaussian, centered around s_0. The initial standard deviation σ was set to 0.02.

$$\begin{cases} f(s) = \frac{\alpha}{\sqrt{2\pi\sigma}} \cdot e^{-\frac{(s-s_0)^2}{2\sigma}} \\ N_0 = \int_s f(s)ds \end{cases} \tag{A.7}$$

System (A.7) thus gave the condition for a Gaussian distribution of prey density around the mean trait s_0. To estimate this initial distribution, the parameter α (A.7) was computed by considering discrete intervals on the trait distribution, all of width ds, that is the width of the boxes in the PDE model.

$$\alpha = \left(\frac{ds}{N_0} \cdot \sum_s \frac{e^{-\frac{(s-s_0)^2}{2\sigma}}}{\sqrt{2\pi\sigma}}\right)^{-1} \tag{A.8}$$

Once the parameter α was estimated as in (A.8), $f(s)$ (A.7) estimated the prey density for each box, using the corresponding median trait value.

To reach an equilibrium distribution before starting the shift experiments, every run waited for the trait distribution to stabilize until the time $t = 80$ (Figure 2). The results of this stabilizing simulation were then discarded. After this time, a perturbation as predation B change was launched until a maximum time t_{max}.

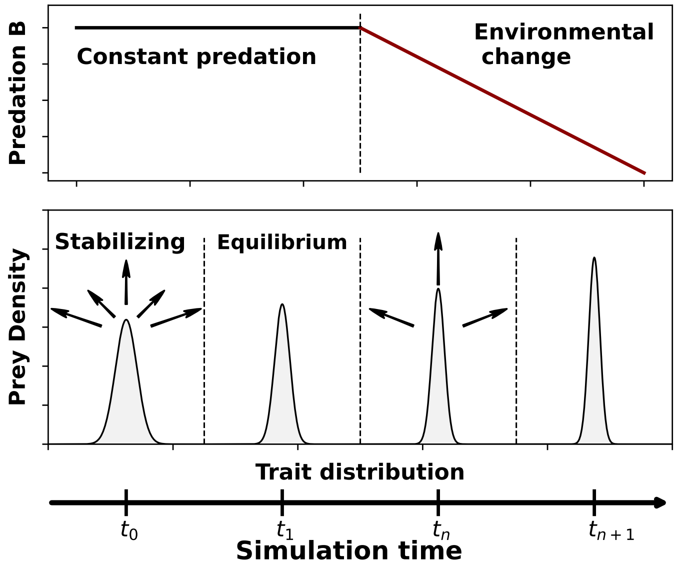

Figure A1. Drawing of predation change experiments for PDE simulations. Simulations started from the estimated distribution at t_0. The prey distribution was first simulated for $t = 80$ to reach an equilibrium. Predation change was then launched after $t = 80$.

During experiments, the whole trait distribution was recorded. The total population density and mean population trait were then computed and used to detect regime shifts in the prey population. Shifts were detected using a numerical approximation of the second partial derivative of $\frac{dN}{dt}$ (2.10) on N. Thus, a change of sign estimated the middle of S-shaped shifts in population density (see Figure 1).

Supplementary 3. Adimentionalized ecological model

Here, Eq (2.1) could be adimentionalized to find bifurcation points in the system, and thus the tipping points for the prey population. To adimentionalize the model, we wrote $x = h \cdot N$, the dimensionless prey density, and $t = q \cdot \tau$, the dimensionless time. We also wrote $q = \frac{1}{r}$, the dimensionless growth rate, and $h = \frac{1}{\sqrt{a}}$, the dimensionless predator attack rate.

$$\frac{dx}{d\tau} = \frac{q}{h}(r - \alpha N)N - \frac{q}{h}\frac{aBN^2}{1 + aN^2} \tag{A.9}$$

$$\frac{dx}{d\tau} = (1 - A_t x)x - \frac{B_t x^2}{1 + x^2} \tag{A.10}$$

We then wrote $A_t = \frac{\alpha}{r\sqrt{a}}$, and $B_t = \frac{B\sqrt{a}}{r}$. Thus, the adimentionalized model (A.10) could be used to compute tipping points.

$$\begin{cases} f(x) = g(x) \\ f'(x) = g'(x) \\ f(x) = 1 - A_t x \text{ and } g(x) = \frac{B_t x}{1+x^2} \end{cases} \tag{A.11}$$

Hence, critical predation thresholds B_t, defining the tipping points, were defined by the system (A.11) as a function of prey density x, considered to be the equilibrium. Thus, thresholds B, not adimentionalized, were computed from B_t.

Supplementary 4. Effect of $\bar{n}$ on the tipping points

The environmental variability term, from the PDE model, was added to (2.1) as an additional mortality $-\sqrt{\frac{\bar{n}}{N}}$. Then, the resulting model (A.12) was adimentionalized in a similar fashion as in Supplementary 3, using $x = \frac{N}{\sqrt{a}}$, and $\tau = t \cdot r$.

$$\frac{dN}{dt} = N(r - \alpha N - \sqrt{\frac{\bar{n}}{N}}) - \frac{aBN^2}{1 + aN^2} \tag{A.12}$$

$$\frac{dx}{d\tau} = x(1 - A_t x - \sqrt{\frac{\bar{n}}{x}}P) - \frac{B_t x^2}{1 + x^2} \tag{A.13}$$

In (A.13), we wrote $A_t = \frac{\alpha}{r\sqrt{a}}$, $B_t = \frac{B\sqrt{a}}{r}$, and $P = a^{\frac{1}{4}}$.

$$\begin{cases} 1 - A_t x - \frac{\sqrt{\bar{n}}}{\sqrt{x}}P = \frac{B_t x}{1+x^2} \\ -A_t + \frac{1}{2}\frac{\sqrt{\bar{n}}}{\sqrt{x^3}}P = \frac{B_t(1x^2)}{(1+x^2)^2} \end{cases} \tag{A.14}$$

Solving system (A.14) gave $B_t = \frac{\frac{1}{x} - \frac{3}{2}\frac{\sqrt{\bar{n}}}{\sqrt{x^3}}}{x}(1 + x^2)$, the predation threshold as a function of x, and $\bar{n}$. The partial differential of B_t on $\bar{n}$ was then computed to discuss the effects of an increase in environmental variability, such as in (A.15).

$$\frac{\partial B_t}{\partial \bar{n}} = \frac{-3}{2}\frac{1}{2\sqrt{x^5}\sqrt{\bar{n}}}(1 + x^2) \tag{A.15}$$

As $\frac{\partial B_t}{\partial \bar{n}} < 0$, the environmental variability $\bar{n}$ had a negative influence on critical predation thresholds B_t (see Figure 6). Thus, increasing environmental variability seemed to displace regime shift thresholds to lower predation values, thus favoring shifts to low prey densities at lower B values.