



Supplementary material

Supplementary Material for “How do classroom-turnover times depend on lecture-hall size?”

Joseph Benson¹, Mariya Bessonov², Korana Burke³, Simone Cassani⁴, Maria-Veronica Ciocanel⁵, Daniel B. Cooney⁶ and Alexandria Volkening^{7,*}

¹ Mathematics, Statistics, and Computer Science, Macalester College, Saint Paul, MN 55105, USA

² Department of Mathematics, NYC College of Technology, Brooklyn, NY 11201

³ Department of Mathematics, University of California Davis, Davis, CA 95616

⁴ Department of Mathematics, University at Buffalo, Buffalo, NY 14260

⁵ Department of Mathematics and Department of Biology, Duke University, Durham, NC 27708

⁶ Department of Mathematics and Center for Mathematical Biology, University of Pennsylvania, Philadelphia, PA 19104

⁷ Department of Mathematics, Purdue University, West Lafayette, IN 47907

* **Correspondence:** Email: avolkening@purdue.edu; Tel: +17654961052.

S1. Overview

To support the reproducibility of our results, this Supplementary Material includes:

- information about the lecture-hall domains in which we simulate student movement (Section S2.1);
- details about the initial conditions and student destinations in our model (Section S2.2);
- details about how we implement boundary conditions in our model (Section S2.3);
- information about the numerical implementation of our model (Section S2.4);
- a summary of the figure-specific values of our model parameters and details on how we compute heat-map images (Section S2.5); and
- additional simulation figures, including timelines of student density for different separation times between classes and an example agent trajectory for an outlier in our simulations (Section S3).

The code that we developed to simulate our model is available at [1].

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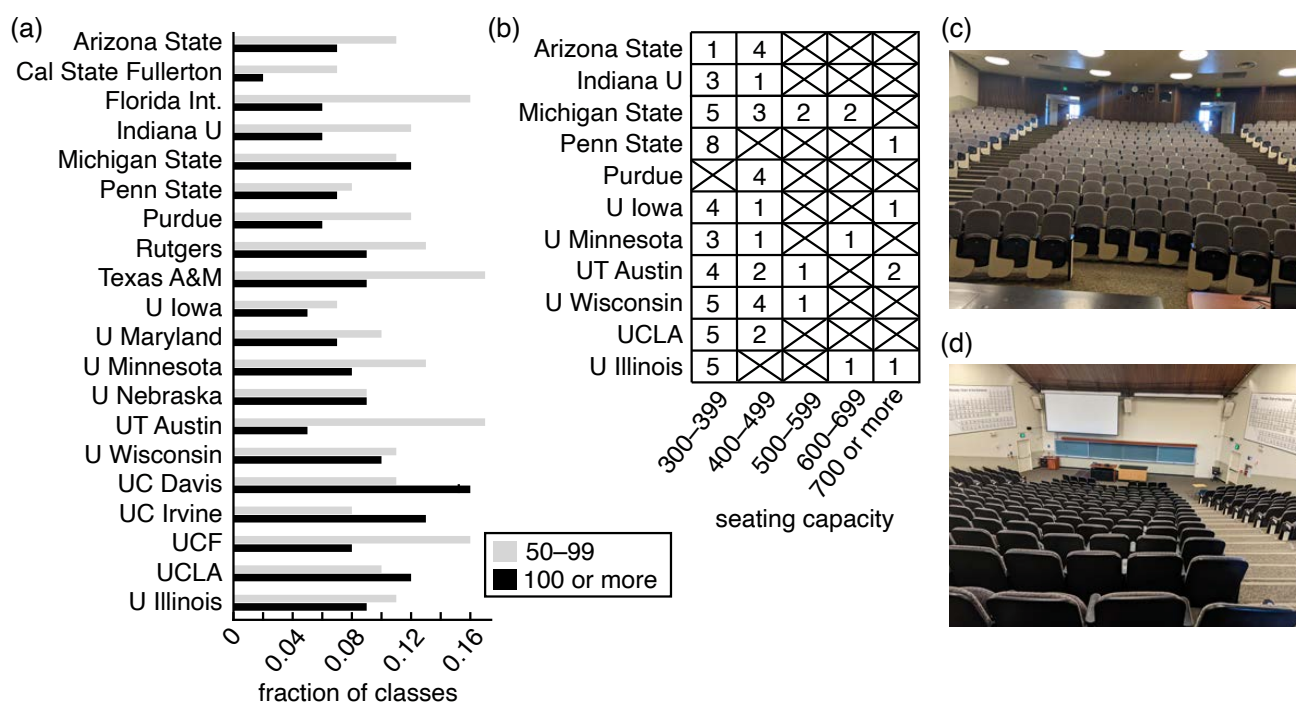
S2. Additional modeling and implementation details

We provide additional details on our lecture-hall domains, initial conditions and student destinations, boundary conditions, and numerical implementation, as well as information on how to reproduce our figures, in Sections S2.1–S2.5, respectively.

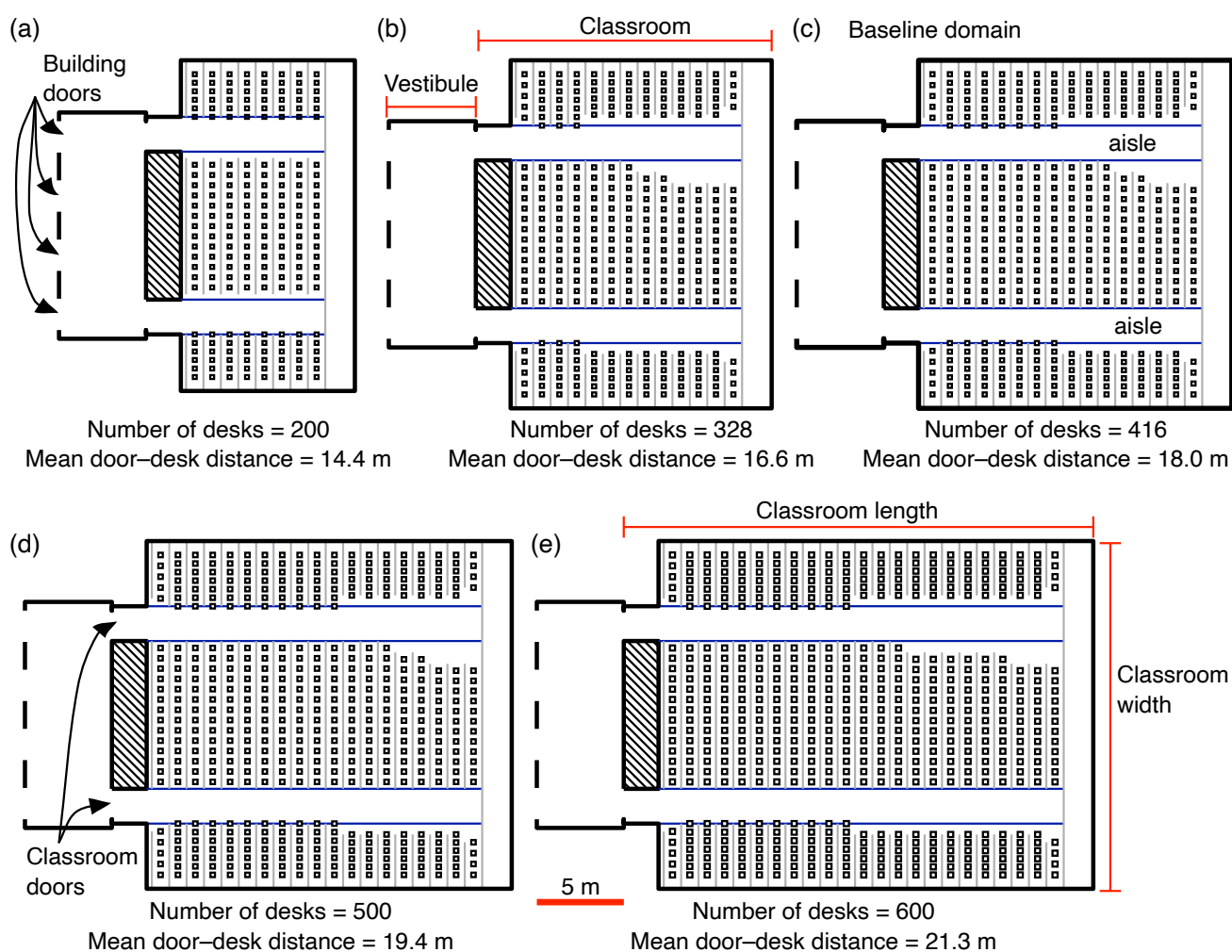
S2.1. Lecture-hall domains

As we show in Supplementary Figure 1, lecture halls that can accommodate 300 or more students are not uncommon in large colleges and universities in the United States. Motivated by the broad range of classroom sizes in Supplementary Figure 1(b), we thus construct five domains, pictured in Supplementary Figure 2. Our baseline domain is modeled after our measurements of Rock Hall [2] at the University of California, Davis (see Supplementary Figures 1(c)–(d)) and has 416 desks. In this domain, which is meant to be a simplified approximation of Rock Hall, the average distance from each desk to its nearest-neighboring desk is 0.543 m, measured between desk centers, and we do not directly account for the presence of stairs in Supplementary Figures 1(c)–(d). We also consider two smaller classrooms that hold 200 and 328 desks, respectively, and two larger lecture halls with 500 and 600 desks, respectively. Our 328-desk room models an academic space with a capacity that is in between two other large classrooms at the University of California, Davis [3] with capacity for 369 and 285 people, respectively. Motivated by our measurements of Rock Hall, we assume building doors are 1.8 m wide, internal classroom doors are 1.75 m wide, and classroom aisles are 2.0 m wide.

Because our goal is to investigate the role of lecture-hall size, rather than room geometry, in classroom-turnover times, we account for different room sizes by adjusting the length (and to a much lesser extent, the width) of the classroom only. Our 600-person domain has a classroom length of 27 m (not including the vestibule) and width of 20 m; similarly, our 500-person room is 23 m long and 20 m wide, our 416-person room is 20 m long and 20 m wide, our 328-person room is 17 m long and 20 m wide, and our 200-person room is 12 m long and 19 m wide. We use the same vestibule geometry (namely 5 m long and 13 m wide) and door sizes for all of our domains; see Supplementary Figure 2. See our code on GitLab [1] for the coordinates of desk centers, doors, and aisles in our domains.



Supplementary Figure 1. Overview of classroom sizes at example universities and colleges in the United States. (a) Across the twenty institutions that we show (namely examples of public, Big Ten Academic Alliance and large institutions with data available here [4]), on average about 8% of classes have 100 or more students and about 12% have 50–99 students. Importantly, these percentages are in terms of classes, rather than students; this means that the percentage of students who find themselves in a large class is much higher. (Our estimates are unweighted averages and based on data from *CollegeData.com* [4].) (b) A 2009 report associated with The Ohio State University highlighted the number of large lecture halls by size at different universities, and we show a sample of this data [5]. (c)–(d) As an example academic space, Rock Hall [2] at the University of California, Davis has a capacity for 416 people.



Supplementary Figure 2. Overview of our lecture-hall domains. All of these domains are based on adapting our rough estimate of Rock Hall at the University of California, Davis. The same scale bar applies to panels (a)–(e). We indicate aisle boundaries in blue, desk-row boundaries in gray, and desks as squares; see Figure 1(a) in the main manuscript for details. (a) Our smallest lecture hall contains 200 desks, with an average distance of 0.5405 m between each desk center and its nearest-neighbor desk center. (b) Our next classroom has 328 desks, and the average distance between nearest desk centers is 0.5419 m. (c) Our baseline domain, approximated from our measurements of Rock Hall, contains 416 desks with an average separation of 0.543 m between desks. (d) Our second largest domain has 500 desks separated by an average distance of 0.5428 m, and (e) our largest lecture hall contains 600 desks with an average separation of 0.5405 m between nearest-neighbor desks. We refer to the horizontal length of the classroom space in our domains as the “classroom length” and the vertical height as the “classroom width”. The vestibule is the same size in all of our domains, and we include four building doors (between the vestibule and outside), two internal classroom doors (between the vestibule and classroom), and two aisles in all cases.

S2.2. Assigning student destinations and initial conditions

As we overview in Section II B of the main manuscript, we assign each student i a desk with coordinates $\mathbf{D}_i^{\text{desk}}$ and a building door with coordinates $\mathbf{D}_i^{\text{bldg}}$; we also assign them two intermediate destinations, a classroom door $\mathbf{D}_i^{\text{class}}$ and an aisle destination $\mathbf{D}_i^{\text{aisle}}$; see Eq (2.1) in the main manuscript. We follow the process below:

$$\mathbf{D}_i^{\text{desk}} = \mathbf{d}_j \text{ with } j \sim \text{Unif}^{\text{discrete}}(1, N^{\text{desks}}) \text{ (student } i\text{'s desk is chosen uniformly at random),} \quad (\text{S1})$$

$$\mathbf{D}_i^{\text{bldg}} = \mathbf{b}_k + (0.5, \beta_i) \text{ with } k \sim \text{Unif}^{\text{discrete}}(1, 4) \text{ (} i \text{ chooses their door } k \text{ uniformly at random),} \quad (\text{S2})$$

$$\mathbf{D}_i^{\text{aisle}} = \begin{cases} (D_{i,1}^{\text{desk}}, a_{\text{up}}) & \text{if } |D_{i,2}^{\text{desk}} - a_{\text{up}}| < |D_{i,2}^{\text{desk}} - a_{\text{low}}| \text{ (} i \text{ uses the aisle closer to their desk)} \\ (D_{i,1}^{\text{desk}}, a_{\text{low}}) & \text{otherwise (} i \text{ uses the lower aisle if it is closer to their desk)} \end{cases}, \quad (\text{S3})$$

$$\mathbf{D}_i^{\text{class}} = \begin{cases} \mathbf{c}_{\text{up}} + \mathbf{w}_i & \text{if } |D_{i,2}^{\text{desk}} - a_{\text{up}}| < |D_{i,2}^{\text{desk}} - a_{\text{low}}| \text{ (} i \text{ uses the door associated with } \mathbf{D}_i^{\text{desk}}) \\ \mathbf{c}_{\text{low}} + \mathbf{w}_i & \text{otherwise (} i \text{ chooses the classroom door associated with } \mathbf{D}_i^{\text{desk}}) \end{cases}, \quad (\text{S4})$$

where \mathbf{d}_j marks the (x, y) -coordinates of the center of the j th desk; $\text{Unif}^{\text{discrete}}(1, n)$ is the discrete uniform distribution from 1 to n ; N^{desks} is the number of desks in the lecture hall; \mathbf{b}_k is the midpoint of the k th door into the building, and there are $n = 4$ building doors in each of our domains; $\mathbf{D}_i^{\text{desk}} = (D_{i,1}^{\text{desk}}, D_{i,2}^{\text{desk}})$ are the coordinates of student i 's desk; a_{up} (respectively, a_{low}) is the y -coordinate of the center of the upper (respectively, lower) aisle (see Figure 1(a) in the main manuscript); \mathbf{c}_{up} (respectively, \mathbf{c}_{low}) marks the midpoint of the classroom door that leads into the upper (respectively, lower) aisle of the classroom; $\mathbf{w}_i = (w_{i,1}, w_{i,2})$ with $w_{i,k} \sim \text{Unif}(0, \mu)$ for $k \in \{1, 2\}$; and $\beta_i \sim \text{Unif}(-0.5, 0.5)$ m. We use $\mu = 0.01$ m for students in the entering class, and $\mu = 0$ m for exiting students.

For each student i in the entering class, we include a small amount of noise in $\mathbf{D}_i^{\text{class}}$ in Eq (S4) to help prevent rare occasions where we observed pairs of agents getting stuck along the vestibule–classroom wall early on in our model-development process. In this rare setting, particularly in crowded conditions, some agents were pushed against the vestibule–classroom wall and moving toward one another in order to reach their classroom destinations; by adding some noise to $\mathbf{D}_i^{\text{class}}$, we help prevent these agents from getting stuck at a head-to-head standstill in their attempt to move in directly opposing directions. Additionally, we add 0.5 m to the x -coordinate of \mathbf{b}_k when computing $\mathbf{D}_i^{\text{bldg}}$ in Eq (S3) to ensure new entering students are inside the building boundary. While this choice is motivated by entering students, we add 0.5 m to the x -coordinate of $\mathbf{D}_k^{\text{bldg}}$ for both exiting and entering students so that students in both classes have the same distance to travel in our bidirectional simulations.

Each student i in the exiting class begins active at their desk, so $\mathbf{X}_i(0) = \mathbf{D}_i^{\text{desk}}$; see Eq (S1). In our simulations with only an entering class, we assume that 2% of the entering class starts in the vestibule. This means that we include $N^{\text{early}} = 0.02N^{\text{enter}}$ early-arriving students in the entering class when $N^{\text{exit}} = 0$; these early-arriving students have a starting point in the vestibule, modeling students waiting outside of the classroom. Specifically, if student i is an early-arriving student, then they are in \mathcal{A}_0 , and we set $\mathbf{X}_i(0)$ to be a position selected uniformly at random from a grid of points in the vestibule (with a step size of 1 m, specified to ensure that students are not randomly initialized at locations that are unrealistically close). With the exception of these $N^{\text{early}} \geq 0$ early-arriving students, all of the other students in the entering class start at a building door when they become active; in particular, if student i becomes active at time t_i^{active} , then $\mathbf{X}(t_i^{\text{active}}) = \mathbf{D}_i^{\text{bldg}}$. In Eq (S2), if $N^{\text{enter}} - N^{\text{early}}$ (respectively, N^{exit}) is not divisible by four (the number of building doors), we use a distribution that is approximately uniform

to assign each student a building door k from the set of all doors.

S2.3. Boundary conditions

For each student $i \in \mathcal{A}_t$, after calculating $\mathbf{V}_i(t + \Delta t)$ according to Eq (2.5) in the main manuscript, we adjust $\mathbf{V}_i(t + \Delta t)$ as necessary to account for no-flux boundary conditions along the building walls, row boundaries, and—depending on student i 's row status $R_i(t)$ in Eq (2.2) in the main manuscript—aisle boundaries (see Figures 1 and 2 in the main manuscript). First, we note that our building outline can be represented by a set of line segments, and all of the walls that we consider are straight and parallel to either the x - or y -axis. For each student $i \in \mathcal{A}_t$, we find the point $\mathbf{b}_i^{\text{bldg}}$ on the building-boundary line segments that is closest to their current position $\mathbf{X}_i(t)$ and compute the unit vector $\mathbf{e}_i^{\text{bldg}} = (\mathbf{b}_i^{\text{bldg}} - \mathbf{X}_i(t)) / \|\mathbf{b}_i^{\text{bldg}} - \mathbf{X}_i(t)\|$. Second, since the boundaries of all of the aisles and rows are also line segments that are parallel to the x - and y -axes, respectively, we follow a similar process for the desk rows and aisles (Figure 1(a) in the main manuscript). In particular, we find the point \mathbf{b}_i^{r} on the desk-row boundaries that is closest to $\mathbf{X}_i(t)$ and compute the unit vector $\mathbf{e}_i^{\text{r}} = (\mathbf{b}_i^{\text{r}} - \mathbf{X}_i(t)) / \|\mathbf{b}_i^{\text{r}} - \mathbf{X}_i(t)\|$. If $R_i(t) = 0$, we also find the point \mathbf{b}_i^{a} on the aisle boundaries that is closest to $\mathbf{X}_i(t)$ and calculate $\mathbf{e}_i^{\text{a}} = (\mathbf{b}_i^{\text{a}} - \mathbf{X}_i(t)) / \|\mathbf{b}_i^{\text{a}} - \mathbf{X}_i(t)\|$.

Third, we compute student i 's updated velocity, \mathbf{V}_i^{**} , which accounts for our no-flux boundary conditions, in a three-step process, as below:

$$\mathbf{v}_i = \mathbf{V}_i(t + \Delta t) - \underbrace{\mathbf{f}^{\text{bnd}}(\mathbf{X}_i(t), \mathbf{V}_i(t + \Delta t), \mathbf{e}_i^{\text{bldg}}, \mathbf{b}_i^{\text{bldg}})}_{\text{accounts for the building walls}} \quad (\text{S5})$$

$$\mathbf{v}_i^* = \begin{cases} \mathbf{v}_i - \underbrace{\mathbf{f}^{\text{bnd}}(\mathbf{X}_i(t), \mathbf{v}_i, \mathbf{e}_i^{\text{a}}, \mathbf{b}_i^{\text{a}})}_{\text{accounts for the aisles}} & \text{if } R_i(t) = 0 \\ \mathbf{v}_i & \text{otherwise} \end{cases} \quad (\text{S6})$$

$$\mathbf{v}_i^{**} = \mathbf{v}_i^* - \underbrace{\mathbf{f}^{\text{bnd}}(\mathbf{X}_i(t), \mathbf{v}_i^*, \mathbf{e}_i^{\text{r}}, \mathbf{b}_i^{\text{r}})}_{\text{accounts for the desk rows}} \quad (\text{S7})$$

where $\mathbf{f}^{\text{bnd}}(\mathbf{x}, \mathbf{v}, \mathbf{e}, \mathbf{b}) = f^{\text{bnd}}(\mathbf{x}, \mathbf{b}) \text{proj}_{\mathbf{e}}(\mathbf{v})$ when both $\|\mathbf{x} - \mathbf{b}\| \leq 2b^{\text{bnd}}$ and $\text{proj}_{\mathbf{e}}(\mathbf{v}) \geq 0$, and $\mathbf{f}^{\text{bnd}} = 0$ otherwise. Here the factor $f^{\text{bnd}}(\mathbf{x}, \mathbf{b})$ serves as a smoothed heaviside function, so that students who are near a boundary and moving toward it experience a gradual decrease in their speed. This factor is given by:

$$f^{\text{bnd}}(\mathbf{x}, \mathbf{b}) = \begin{cases} \frac{1}{2} + \frac{1}{2} \tanh\left(10(b^{\text{bnd}} - \|\mathbf{b} - \mathbf{x}\|)\right) & \text{if } \mathbf{b} \text{ is on the bldg. boundary and not a door frame} \\ \frac{1}{2} + \frac{1}{2} \tanh\left(10(b^{\text{tight}} - \|\mathbf{b} - \mathbf{x}\|)\right) & \text{if } \mathbf{b} \text{ is on an aisle, row, or door-frame boundary.} \end{cases} \quad (\text{S8})$$

We choose $b^{\text{tight}} < b^{\text{bnd}}$ to account for student movement in the tighter spaces of door frames, aisles, or rows. Finally, with a slight abuse of notation, we set $\mathbf{V}_i(t + \Delta t) = \mathbf{v}_i^{**}$.

S2.4. Numerical implementation

We simulate our model using MATLAB, and our code is available on GitLab [1]. As we summarize in Figure 2 in the main manuscript, at each time step $\Delta t = 0.01$ s, we synchronously calculate the

updated velocity for each student, adjust these velocities to account for the appropriate no-flux boundary conditions, and update each student's position. We then synchronously update the row status R_i and current destination \mathbf{D}_i of each student i ; check whether any exiting students have left the building and, if so, remove them from the active set; and add any new arriving students to the simulation. We use the “inpolygon” function in MATLAB to determine whether students are inside the classroom and whether exiting students have left the building in Eqs (2.3)–(2.4) in the main manuscript. We implement our stochastic differential equation for student velocity in Eq (2.5) in the main manuscript using the Euler–Maruyama method [6]; and we implement our differential equation for student position in Eq (2.6) in the main manuscript using a forward Euler scheme. We simulate from time $t = 0$ to $t = t_{\max}$ with a time step of $\Delta t = 0.01$ s. As a result of our approach to implementing no-flux boundary conditions through forces, we note that it is important to choose Δt sufficiently small; too large of a Δt value makes agents more likely to step through walls. We selected a time step that prevents this based on our observations of model simulations; if one adapted our model to include stronger forces, we expect a smaller time step would be needed to reliably implement no-flux boundary conditions using our approach.

S2.5. Instructions for reproducing our figures

Our model parameters are in Table 2 in the main manuscript. Because we investigate different classroom sizes and separation times, we use simulation-specific values of N^{enter} , N^{exit} , N^{early} , t_{\max} , and t_{sep} . To support reproducibility of our work, we include these parameters, as well as details on our heat-map images, by figure below:

- **Heat-map Images:** The heat maps in Figures 5(d)–(h) and 7(g)–(i) in the main manuscript and Supplementary Figure 4 are based on 100 simulations for each parameter condition, and here we describe our process for creating each of these figures. At the simulation time point of interest, we discretize our domain from $x = 0$ m to $x = 25$ m with $\Delta x = 1$ m and from $y = 0$ m to $y = 20$ m with $\Delta y = 0.5$ m. For each of our 100 simulations, we find the number of students in each grid square at the given time and compute the mean velocity across the students in that grid square. To produce heat maps of mean student speed, we take the mean of these values across 100 simulations, omitting the velocity in cases where there are no students in a given grid square. Each grid square contains at most one desk. Because our grid step and desk positions do not line up perfectly, there are grid squares in our classroom in which no desks are located (e.g., see the narrow horizontal strips in the center desk region that appear as black in Figure 5(h) in the main manuscript).
- **Figure 4:** $N^{\text{enter}} = 400$ students, $N^{\text{exit}} = 0$ students, $N^{\text{early}} = 0.02 \times 400 = 8$ students, $t_{\max} = 300$ s, and $t_{\text{sep}} = 0$ s;
- **Figure 5:** $N^{\text{enter}} = 400$ students, $N^{\text{exit}} = 0$ students, $N^{\text{early}} = 0.02 \times 400 = 8$ students, $t_{\max} = 280$ s, and $t_{\text{sep}} = 0$ s;
- **Figure 6:** $N^{\text{enter}} = N^{\text{early}} = 0$ students, $N^{\text{exit}} = 400$ students, $t_{\max} = 280$ s, and $t_{\text{sep}} = 0$ s;
- **Figures 7 and 8(h) and Supplementary Figures 3 and 4(i)–(p):** $N^{\text{enter}} = N^{\text{exit}} = 400$ students, $N^{\text{early}} = 0$ students, $t_{\max} = 450$ s, and $t_{\text{sep}} = 90$ s;
- **Figures 8(a)–(b) and 8(e):** $N^{\text{enter}} = N^{\text{exit}}$ is the class size in the figure legend, $N^{\text{early}} = 0$ students, $t_{\max} = 450$ s, and $t_{\text{sep}} = 90$ s.

- **Figure 8(c):** Conditions are the same as in Figures 8(d)–(f), as described above and below, for our bidirectional simulations. Under “No social”, we set $B^{\text{col}} = B^{\text{rep}} = 0 \text{ m/s}^2$; include $N^{\text{exit}} = 0$ students; provide N^{enter} as the class size in the legend; include $N^{\text{early}} = 0.02N^{\text{enter}}$ students; set $t_{\text{max}} = 400 \text{ s}$ for the 200-person class, $t_{\text{max}} = 100 \text{ s}$ for the 300-person class with $\alpha = 10.67$ students/s, $t_{\text{max}} = 600 \text{ s}$ for the 400- and 500-person classes (in some cases we simulate for much longer than necessary), and $t_{\text{max}} = 50 \text{ s}$ with $\alpha = 200.67$ students/s for the 600-person class; and use $t_{\text{sep}} = 0 \text{ s}$. Under “Enter only”, we include $N^{\text{exit}} = 0$ students; provide N^{enter} as the class size in the legend; include $N^{\text{early}} = 0.02N^{\text{enter}}$ students; set $t_{\text{max}} = 300 \text{ s}$ for the 200- and 300-person classes, 280 s for the 400-person class, 550 s for the 500-person class, and 320 s for 600-person class; and use $t_{\text{sep}} = 0 \text{ s}$.
- **Figure 8(d):** $N^{\text{enter}} = N^{\text{exit}}$ is the class size in the figure legend; $N^{\text{early}} = 0$ students; $t_{\text{max}} = 450 \text{ s}$ for the 600- and 400-person classes and 500 s for the 500-, 300-, and 200-person classes; and $t_{\text{sep}} = 120$ seconds.
- **Figure 8(f):** $N^{\text{enter}} = N^{\text{exit}}$ is the class size in the figure legend; $N^{\text{early}} = 0$ students; $t_{\text{max}} = 450 \text{ s}$ for the 600- and 500-person classes, 410 s for the 400-person class, 500 s for the 300-person class, and 600 s for the 200-person class; and $t_{\text{sep}} = 60 \text{ s}$.
- **Figure 8(g) and Supplementary Figures 4(q)–(x):** $N^{\text{enter}} = N^{\text{exit}} = 400$ students, $N^{\text{early}} = 0$ students, $t_{\text{max}} = 450 \text{ s}$, and $t_{\text{sep}} = 120 \text{ s}$.
- **Figure 8(i) and Supplementary Figures 4(a)–(h):** $N^{\text{enter}} = N^{\text{exit}} = 400$ students, $N^{\text{early}} = 0$ students, $t_{\text{max}} = 410 \text{ s}$, and $t_{\text{sep}} = 60 \text{ s}$.

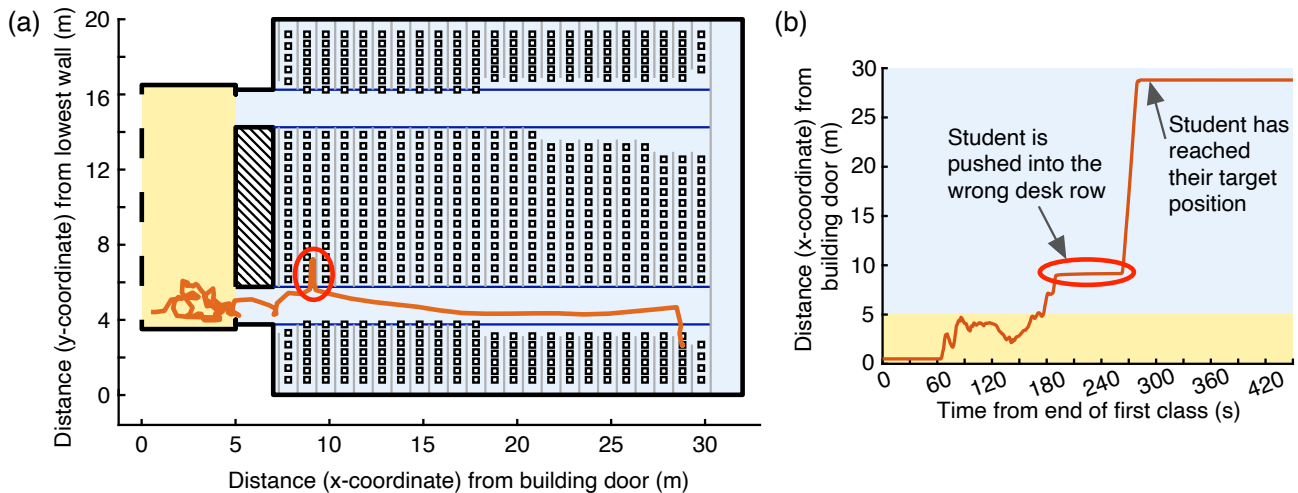
S3. Additional simulation examples

In Supplementary Figure 3, we highlight an example of a simulation outlier. In particular, we provide the trajectory of an example agent who takes longer than $Q_3 + 1.5(Q_3 - Q_1)$ to reach their desk, for the case of a 600-person room with a separation time of 60 s . Here Q_3 is the 75th percentile of $\{t_i^{\text{final}}\}_i$ in the entering class and Q_1 is the 25th percentile of $\{t_i^{\text{final}}\}_i$ in the entering class, as determined using the “prtile” function in MATLAB. The student shown spends about 120 s in the vestibule, attempting to enter the classroom; note that we omit plotting the other students that are moving around this outlier agent. Once in the classroom, they are pushed into the wrong desk row by other agents. At this point, near $t = 180 \text{ s}$ in Supplementary Figure 3(b), the student’s row status is still zero because they are not in their desired row; thus, they still experience repulsive forces from the aisle walls, in addition to building walls, desk rows, and other students. As a result, this agent spends about a minute in the wrong row, before attractive forces toward their desired aisle destination overwhelm the repulsive forces that the agent experiences from the aisle walls and other students.

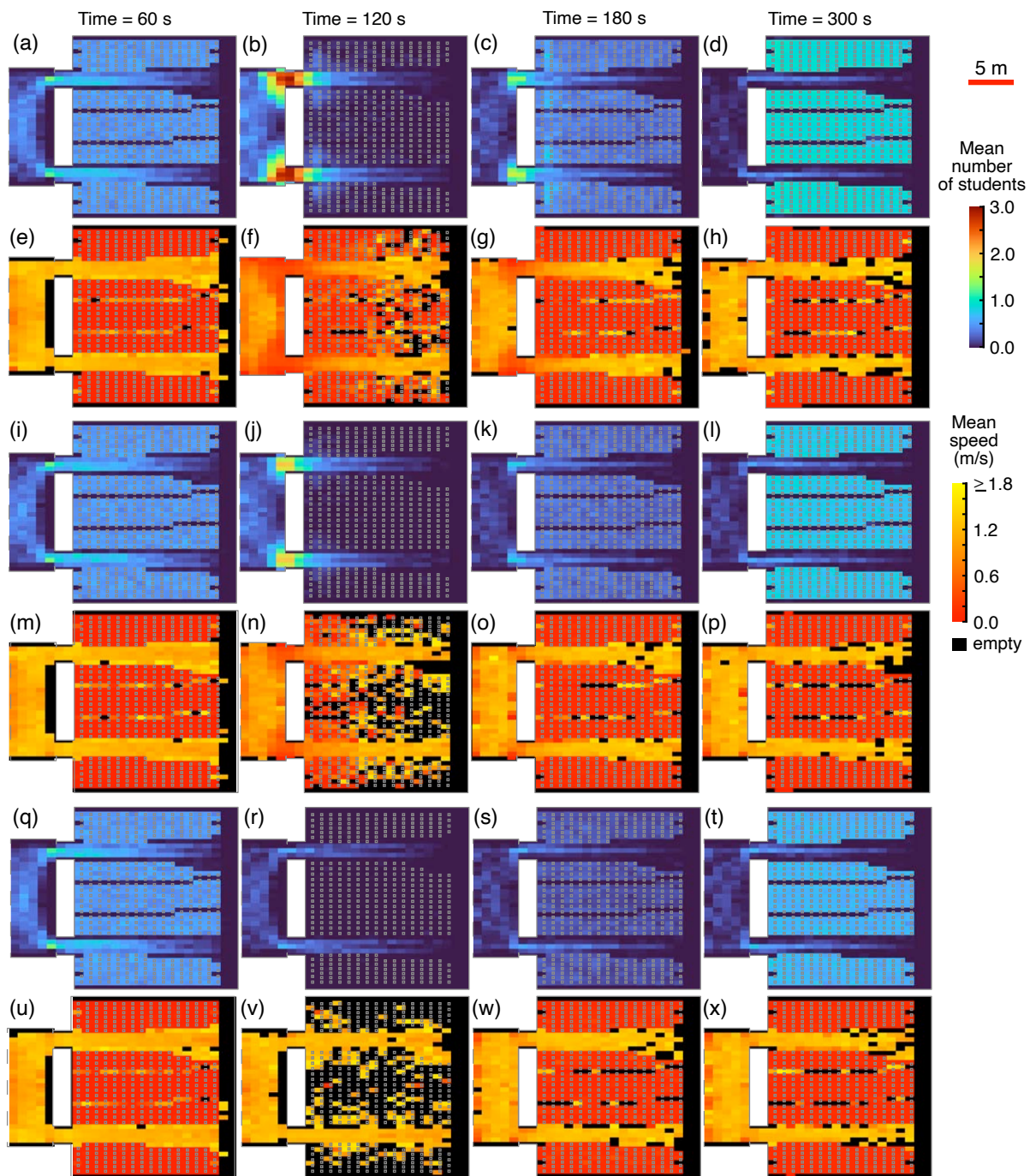
The outlier highlighted in Supplementary Figure 3 has a travel time that falls outside of the range covered by the gray bars in Figure 8(c) in the main manuscript. Because our model is a simplification that does not include many real-life dynamics that may prolong classroom turnover in large lecture halls (e.g., pauses due to students talking to one another or rare events like individuals from the prior class forgetting something in the classroom and needing to come back to look for it), we allow these rare outliers in our simulations; importantly, they do not affect the interpretation of our results.

In Supplementary Figure 4, we show the mean mass (number of students in grid squares with area 1.5 m^2) and speed of students across 100 simulations of 400 entering and 400 exiting students

in our Rock Hall, under different separation times between when the first class ends and when entering students begin arriving. This figure expands on Figures 8(g)–(i) in the main manuscript, providing more timepoints. Crowding occurs near the classroom doors, and the congestion decreases as the separation time grows from 60 s in Supplementary Figures 4(a)–(d) to 90 s in Supplementary Figures 4(i)–(l) and 120 s in Supplementary Figures 4(q)–(t). Crowding is accompanied by low speeds near classroom doors, and speeds in the vestibule grow as the separation time increases (e.g., compare Supplementary Figure 4(f) with a separation time of 60 s to Supplementary Figure 4(v) with a separation time of 120 s).



Supplementary Figure 3. Example of an entering student with an outlier travel time, for the case of 600 entering and 600 exiting students in our 600-person room with a separation time of 60 s. We define “outlier” as an agent with a travel time above $Q_3 + 1.5(Q_3 - Q_1)$ or below $Q_1 - 1.5(Q_3 - Q_1)$, where we compute the 25th percentile Q_1 and 75th percentile Q_3 across our 100 simulations; also see Figure 8(c) in the main manuscript. (a) We show the trajectory of an example outlier agent, and we omit plotting the other agents in the simulation to make the trajectory visible. The example student attempts to enter the classroom, but is pushed back in the vestibule by the exiting class. Once the agent finally enters the classroom, they are pushed through our aisle-wall boundaries into the wrong row by other students. (b) To highlight the time that this student spends in different regions of the classroom, we also show the agent’s x -coordinate in time. When the student is pushed into the wrong desk row, they are “stuck” for about a minute, until the attractive forces that they feel toward their aisle destination overcome the repulsive forces that they feel from the aisle-wall boundaries and other students. This dynamic is rare in our simulations and may result from high student density; it leads to a travel time that falls outside of the gray bars in Figure 8(c) in the main manuscript, representing an outlier.



Supplementary Figure 4. Mean student mass and speed across 100 simulations of 400 entering and 400 exiting students in our Rock Hall, under different separation times. This figure expands on Figures 8(g)–(i) in the main manuscript. Mean mass (number of students in grid squares with area 1.5 m^2) for a class-separation time of (a)–(d) 60 s, (i)–(k) 90 s, and (q)–(l) 120 s across time. Similarly, (e)–(h), (m)–(n), and (u)–(x) show mean student speeds in grid squares at different times for a separation time of 60, 90 and 120 s, respectively. We use the method described in Section S2.5 to compute the mean number of students in each grid square and the mean speed of students in these grid squares. Scale bar of 5 m applies to all images, and times at the top of the figure apply to the six images in each column.

To facilitate further exploration into Figure 8 in the main manuscript, Supplementary Tables 1 and 2 provide summary statistics for each of our sets of 100 simulations for various class sizes and simulation conditions. The scenarios that we consider are pedestrians entering or exiting the classroom in the absence of social forces; a single entering class without an exiting class; a single exiting class without an entering class; and cases of bidirectional movement with two consecutive classes under separation times of 60, 90 or 120 s between when the first class ends and when students from the next class begin arriving at the building doors. We present the mean, median, 75th percentile, and 90th percentile of the travel times in Supplementary Tables 1 and 2 for entering and exiting students, respectively, showing how these quantities vary for class sizes ranging from 200 to 600 students.

Supplementary Table 1. Summary statistics for entrance times across 100 simulations for various scenarios and class sizes. We present the (a) mean, (b) median, (c) 75th percentile, and (d) 90th percentile travel times for all of the entering students across our 100 simulations; also see Figure 8 in the main manuscript and Supplementary Table 2. “No Social” refers to setting $B^{\text{rep}} = B^{\text{col}} = 0$ in Eq (2.12) in the main manuscript, so that students experience no forces from each other; this means that, from each student’s perspective, they are the only student in the classroom. “Enter Only” refers to an entering class in the absence of an exiting class, and the cases of bidirectional movement with separation times of 60 , 90 and 120 s are labelled “60 s Delay”, “90 s Delay”, and “120 s Delay”, respectively.

Classroom Scenario	Class Size				
	200	300	400	500	600
No Social	13.73	15.66	16.76	17.84	19.33
Enter Only	14.61	16.62	18.03	19.51	21.28
120 s Delay	14.65	16.73	18.25	19.80	22.35
90 s Delay	15.52	18.74	22.075	27.09	34.84
60 s Delay	17.54	24.65	33.09	43.65	55.83

(a) Mean travel time for entering students in seconds

Classroom Scenario	Class Size				
	200	300	400	500	600
No Social	13.45	15.29	16.36	17.41	18.85
Enter Only	14.24	16.24	17.5	18.81	20.45
120 s Delay	14.28	16.33	17.75	19.09	21.00
90 s Delay	14.87	17.45	19.35	21.89	25.92
60 s Delay	15.79	19.39	23.43	29.13	36.57

(b) Median travel time for entering students in seconds

Classroom Scenario	Class Size				
	200	300	400	500	600
No Social	16.10	18.59	20.00	21.44	23.60
Enter Only	17.05	19.52	21.21	22.91	25.21
120 s Delay	17.08	19.68	21.47	23.24	25.96
90 s Delay	18.01	21.38	24.37	28.31	34.91
60 s Delay	19.74	26.67	36.96	48.89	62.06

(c) 75th percentile entrance time in seconds

Classroom Scenario	Class Size				
	200	300	400	500	600
No Social	18.63	21.65	23.45	25.29	27.9
Enter Only	19.72	22.63	24.75	26.745	29.60
120 s Delay	19.81	22.8	25.005	27.17	30.555
90 s Delay	21.38	26.12	31.73	39.85	54.53
60 s Delay	25.87	42.01	62.235	84.67	113.70

(d) 90th percentile entrance time in seconds

Supplementary Table 2. Summary statistics for exit times across 100 simulations for various scenarios and class sizes. We present the (a) mean, (b) median, (c) 75th percentile, and (d) 90th percentile travel times for all of the exiting students across our 100 simulations; also see Figure 8 in the main manuscript and Supplementary Table 1. “No Social” refers to setting $B^{\text{rep}} = B^{\text{col}} = 0$ in Eq (2.12) in the main manuscript, so that students experience no forces from each other; this is equivalent to a single student exiting the classroom. The case of an exiting class in the absence of an entering class is labelled “Exit Only”. The cases of bidirectional movement with separation times of 60, 90 and 120 s between when the first class ends and when students from the next class begin arriving are labelled “60 s Delay”, “90 s Delay”, and “120 s Delay”, respectively.

Classroom Scenario	Class Size				
	200	300	400	500	600
No Social	73.54	75.43	76.54	77.73	79.18
Exit Only	74.91	77.35	78.54	80.11	82.02
120 s Delay	75.21	77.14	78.90	80.76	82.74
90 s Delay	76.25	78.54	80.64	83.99	87.43
60 s Delay	77.33	82.17	86.89	92.21	97.23

(a) Mean travel time for exiting students in seconds

Classroom Scenario	Class Size				
	200	300	400	500	600
No Social	73.685	75.34	76.42	77.76	79.2
Exit Only	75.08	77.99	78.97	80.78	82.795
120 s Delay	75.575	77.62	79.37	81.07	82.83
90 s Delay	75.925	77.28	78.63	80.925	82.45
60 s Delay	76.09	77.62	79.37	81.07	87.26

(b) Median travel time for exiting students in seconds

Classroom Scenario	Class Size				
	200	300	400	500	600
No Social	95.155	96.94	97.8	99.18	100.97
Exit Only	96.305	98.98	100.48	102.15	104.32
120 s Delay	96.795	98.57	100.4	102.45	104.4
90 s Delay	96.955	99.21	101.24	104.02	106.55
60 s Delay	100.77	106.61	110.125	114.05	117.03

(c) 75th percentile exit time in seconds

Classroom Scenario	Class Size				
	200	300	400	500	600
No Social	112.325	114.385	115.515	117.01	118.46
Exit Only	113.65	116.00	117.185	118.89	120.855
120 s Delay	113.475	115.59	117.33	118.94	120.98
90 s Delay	116.66	120.99	123.59	127.225	131.42
60 s Delay	119.90	129.705	139.205	150.50	165.225

(d) 90th percentile exit time in seconds

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