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Research article

Interaction between spatial perception and temporal perception enables preservation of cause-effect relationship: Visual psychophysics and neuronal dynamics

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Supplementary

S1. Derivation of Eq (24) of main text

$$\begin{bmatrix} -\frac{B_3}{P_3} & B_3 \\ \frac{1}{B_3} - \frac{B_3}{{P_3}^2} & \frac{B_3}{P_3} \end{bmatrix} = \begin{bmatrix} -\frac{B_2}{P_2} & B_2 \\ \frac{1}{B_2} - \frac{B_2}{{P_2}^2} & \frac{B_2}{P_2} \end{bmatrix} * \begin{bmatrix} -\frac{B_1}{P_1} & B_1 \\ \frac{1}{B_1} - \frac{B_1}{{P_1}^2} & \frac{B_1}{P_1} \end{bmatrix}$$

After matrix multiplication, we obtained the following four equations:

$$-\frac{B_3}{P_3} = \frac{B_1 B_2}{P_1 P_2} + \frac{B_2}{B_1} - \frac{B_1 B_2}{P_1^2} \tag{S1}$$

$$B_3 = -\frac{B_1 B_2}{P_2} + \frac{B_1 B_2}{P_1} \tag{S2}$$

$$\frac{1}{B_3} - \frac{B_3}{P_3^2} = -\frac{B_1}{P_1 B_2} + \frac{B_1 B_2}{P_1 P_2^2} + \frac{B_2}{B_1 P_2} - \frac{B_1 B_2}{P_1^2 P_2} \tag{S3}$$

$$\frac{B_3}{P_3} = \frac{B_1}{B_2} - \frac{B_1 B_2}{P_2^2} + \frac{B_1 B_2}{P_1 P_2} \tag{S4}$$

Addition of Eqs (S1) and (S4) gives us:

$$B_1 B_2 \left(\frac{1}{P_1 P_2} + \frac{1}{B_1^2} - \frac{1}{P_1^2}\right) - B_1 B_2 \left(\frac{1}{B_2^2} - \frac{1}{P_2^2} + \frac{1}{P_1 P_2}\right) = 0$$

i.e.,

$$B_1 B_2 \left(\frac{1}{P_1 P_2} + \frac{1}{B_1^2} - \frac{1}{P_1^2} - \frac{1}{B_2^2} + \frac{1}{P_2^2} - \frac{1}{P_1 P_2}\right) = 0$$

or,

$$\frac{1}{B_1^2} - \frac{1}{P_1^2} = \frac{1}{B_2^2} - \frac{1}{P_2^2} \tag{S5}$$

$$\text{Hence: } \left(\frac{1}{B_1^2} - \frac{1}{P_1^2}\right)^2 = \left(\frac{1}{B_2^2} - \frac{1}{P_2^2}\right)^2 \tag{S6}$$

From Eq (S3), we have:

$$B_3\left(\frac{1}{B_3^2} - \frac{1}{P_3^2}\right) = -\frac{B_1 B_2}{P_1}\left(\frac{1}{B_2^2} - \frac{1}{P_2^2}\right) + \frac{B_1 B_2}{P_2}\left(\frac{1}{B_1^2} - \frac{1}{P_1^2}\right) \tag{S6a}$$

Likewise from Eq (S5), we get:

$$B_3\left(\frac{1}{B_3^2} - \frac{1}{P_3^2}\right) = \left(\frac{1}{B_1^2} - \frac{1}{P_1^2}\right)\left(-\frac{B_1 B_2}{P_1} + \frac{B_1 B_2}{P_2}\right) \tag{S6b}$$

Taking the square of Eq (S6b) on both sides, we arrive at:

$${B_3}^2\left(\frac{1}{B_3^2} - \frac{1}{P_3^2}\right)^2 = \left(\frac{1}{B_1^2} - \frac{1}{P_1^2}\right)^2\left(-\frac{B_1 B_2}{P_1} + \frac{B_1 B_2}{P_2}\right)^2 \tag{S6c}$$

Now putting the value of the B_3 from Eq (S2) into Eq (S6c), we get:

$$\left(\frac{1}{B_3^2} - \frac{1}{P_3^2}\right)^2 = \left(\frac{1}{B_1^2} - \frac{1}{P_1^2}\right)^2$$

S2. Centrality analysis

We constructed the hemisphere-wise networks for visual-spatial and time perception, in which the corresponding brain regions involved in the tractography analysis were nodes. Then we calculated the centrality parameter for each node which signifies the importance of that node in the information flow or connectivity in the network. Using the DSI studio procedure (http://dsi-studio.labsolver.org), we computed each node's eigenvector centrality and PageRank centrality. Results of the centrality analysis are shown in Tables S1 and S2, which highlight that area V5 has the highest centrality in the networks and is the most significant node in the network. Our observations show consonance in our findings from the 3-tesla scanner and 7-tesla scanner.

Table S1. Eigenvector centrality and PageRank centrality of the different brain regions as nodes of the network for Subject-1 regarding time perception and spatial perception.

Subject 1 (MRI field: 3 tesla)				
	Brain regions active during "Spatial Perception"			
	Eigenvector Centrality (Weighted)		PageRank Centrality (Weighted)	
Brain Regions as nodes	Left Hemisphere	Right Hemisphere	Left Hemisphere	Right Hemisphere
MT / V5	0.5590	0.6577	0.4088	0.3834
Superior Parietal Lobule	0.4103	0.0156	0.1535	0.0366
Posterior Parietal Cortex	0.3505	0.3811	0.2154	0.1897
Intraparietal Sulcus	0.4700	0.6493	0.0688	0.3538
	Brain regions active during "Time Perception"			
	Eigenvector Centrality (Weighted)		PageRank Centrality (Weighted)	
Brain Regions as nodes	Left Hemisphere	Right Hemisphere	Left Hemisphere	Right Hemisphere
Prefrontal Cortex	0.1347	0.0752	0.0610	0.0540
Premotor Cortex	0.0933	0.0000	0.0456	0.0268
Inferior Parietal Cortex	0.0413	0.2979	0.0343	0.1576
Putamen (Basal Ganglia)	0.4145	0.0000	0.1680	0.0268
MT / V5	0.6949	0.6985	0.4088	0.3661

Table S2. Eigenvector centrality and PageRank centrality of the different brain regions as nodes of the network for Subject-2 regarding time perception and spatial perception.

Subject 1 (7T)				
	Space Perception			
	Eigenvector Centrality (Weighted)		Pagerank Centrality (Weighted)	
	Left Hemisphere	Right Hemisphere	Left Hemisphere	Right Hemisphere
MT / V5	0.5182	0.6152	0.2683	0.3172
Superior Parietal Lobule	0.2248	0.1850	0.1199	0.1050
Posterior Parietal Cortex	0.5145	0.6119	0.2448	0.3133
Intraparietal Sulcus	0.5181	0.3319	0.2471	0.1594

Continued on next page

	Time Perception			
	Eigenvector Centrality (Weighted)		Pagerank Centrality (Weighted)	
	Left Hemisphere	Right Hemisphere	Left Hemisphere	Right Hemisphere
Prefrontal Cortex	0.0014	0.0000	0.0211	0.0316
Premotor Cortex	0.0000	0.0147	0.0205	0.0370
Inferior Parietal Cortex	0.0037	0.0000	0.0323	0.0316
Putamen (Basal Ganglia)	0.5629	0.5508	0.2558	0.2520
MT / V5	0.6051	0.6190	0.3292	0.2957

S3. Derivation of Eq (35) of main text

$$|S|^2 = |X|^2 + |T|^2 \tag{S6d}$$

The object is moving in one spatial dimension. Suppose that the coordinates of the starting and stopping points are (x_1, t_1) and (x_2, t_2) in the retinotopic space, respectively. In the perceptual space, corresponding coordinates are (x_1', t_1') and (x_2', t_2'), respectively. We used Pythagoras's theorem to calculate $\vec{S}$ (vector addition of temporal vector and spatial vector).

Let us put a constraint that $|S|^2$ is equal in the retinotopic space and perceptual space and take that the: $\vec{X} = \alpha\ \vec{x}$ and $\vec{T} = \beta\ \vec{t}$

In retinotopic space, we have from Eq (S6d):

$$|X|^2 = \alpha^2(x_2 - x_1)^2 \text{ and } |T|^2 = \beta^2(t_2 - t_1)^2$$

Thus
$$|S|^2 = \alpha^2(x_2 - x_1)^2 + \beta^2(t_2 - t_1)^2 \tag{S7}$$

Likewise in perceptual space, we have from Eq(S6d):

$$|X|^2 = \alpha^2(x_2^* - x_1^*)^2 \text{ and } |T|^2 = \beta^2(t_2^* - t_1^*)^2$$

Hence
$$|S|^2 = \alpha^2(x_2^* - x_1^*)^2 + \beta^2(t_2^* - t_1^*)^2 \tag{S8}$$

Rewriting the transformation equation (Eq (28) in the main text) as below:

$$x^* = \frac{x - Pt}{\sqrt{1 - \left(\frac{P}{k}\right)^2}} \quad \text{and} \quad t^* = \frac{t - \frac{Px}{k^2}}{\sqrt{1 - \left(\frac{P}{k}\right)^2}} \tag{S9}$$

Putting values of the x* and t* from Eq (S9) into Eq (S8):

$$|S|^2 = \alpha^2 \frac{(x_2 - Pt_2 - x_1 + Pt_1)^2}{1 - \left(\frac{P}{k}\right)^2} + \beta^2 \frac{\left(t_2 - \frac{Px_2}{k^2} - t_1 + \frac{Px_1}{k^2}\right)^2}{1 - \left(\frac{P}{k}\right)^2} \tag{S9a}$$

i.e., $|S|^2 = \frac{\alpha^2}{1-\left(\frac{P}{k}\right)^2}\left((x_2-x_1)^2 + P^2(t_1-t_2) + 2P(x_2-x_1)(t_1-t_2)\right) + \frac{\beta^2}{1-\left(\frac{P}{k}\right)^2}\left((t_2-t_1)^2 + \frac{P^2}{k^4}(x_1-x_2)^2 + \frac{2P}{k^2}(x_2-x_1)(t_1-t_2)\right)$ (S9b)

or $$|S|^2 = \left(\frac{\alpha^2}{1-\left(\frac{P}{k}\right)^2} + \frac{P^2\beta^2/k^4}{1-\left(\frac{P}{k}\right)^2}\right)(x_2-x_1)^2 + \left(\frac{P^2\alpha^2}{1-\left(\frac{P}{k}\right)^2} + \frac{\beta^2}{1-\left(\frac{P}{k}\right)^2}\right)(t_2-t_1)^2 + \left(\frac{2P\alpha^2}{1-\left(\frac{P}{k}\right)^2} + \frac{2P\beta^2/k^2}{1-\left(\frac{P}{k}\right)^2}\right)(x_2-x_1)(t_1-t_2) \quad \text{(S9c)}$$

Comparing the terms of $\alpha^2$and β^2 in Eq (S9c) with Eq(S7), we get following three equtions:

$$\frac{\alpha^2}{1-\left(\frac{P}{k}\right)^2} + \frac{P^2\beta^2/k^4}{1-\left(\frac{P}{k}\right)^2} = \alpha^2 \quad \text{(S10)}$$

$$\frac{P^2\alpha^2}{1-\left(\frac{P}{k}\right)^2} + \frac{\beta^2}{1-\left(\frac{P}{k}\right)^2} = \beta^2 \quad \text{(S11)}$$

$$\frac{2P\alpha^2}{1-\left(\frac{P}{k}\right)^2} + \frac{2P\beta^2/k^2}{1-\left(\frac{P}{k}\right)^2} = 0 \quad \text{(S12)}$$

Simplifying Eqs (S10) or (S11), or (S12) yields the same results, i.e.,

$$\alpha^2 = -\frac{\beta^2}{k^2} \quad \text{(S13)}$$

Putting value of β^2 from Eq (S13) into Eq (S8), we obtain the following:

$$|S|^2 = \alpha^2(x_2^* - x_1^*)^2 - \alpha^2 k^2 (t_2^* - t_1^*)^2 \quad \text{(S14)}$$

Now let us take: $x_2^* - x_1^* = x$ and $t_2^* - t_1^* = t$, thereby Eq (S14) becomes:

$$|S|^2 = \alpha^2 x^2 - \alpha^2 k^2 t^2 \quad \text{(S15)}$$

where α is a scalar. For α =1, we note the equality of retinotopic space (Eq (S7)) and perceptual space (Eq (S8)). Putting α = 1 in Eq (S15), we arrive at:

$$|S|^2 = x^2 - k^2 t^2$$

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