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Research article

Convolutional neural network with group theory and random selection particle swarm optimizer for enhancing cancer image classification

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Supplementary

Definition (group). Given a set of elements G and a binary multiplication operation $\otimes$, then the group G is defined if:

•Closure: $\forall g,h \in G,\ g \otimes h \in G$

•Associativity: $\forall g,h,j \in G,\ (g \otimes h) \otimes j = g \otimes (h \otimes j)$

•Identity: $\forall g \in G,\ \exists e \in G,\ g \otimes e = g$

•Inverses: $\forall g \in G,\ \exists g^{-1} \in G,\ g \otimes g^{-1} = e$

Definition (abelian group). An abelian group embodies a commutative binary operation:

• Commutativity: $\forall g,h \in G,\ g \otimes h = h \otimes g$

Definition (permutation). A permutation p of a given set X is a function that arranges its members into an ordered sequence. So it is a bijective mapping of $f: X \to X$ from X to itself, $p = \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ p(x_1) & p(x_2) & \cdots & p(x_n) \end{pmatrix}$.

Definition (permutation group). A permutation group G is a group with the elements of some permutations of a given set X.

Definition (symmetric group). A symmetric group S_n is a group with the elements of all permutations of a given set X, where n is the number of letters in X and S_n has the cardinality of $n!$.
Definition (cycle). A cycle is a permutation of some elements in the given set X or its subset S that maps those elements to each other in a cyclic form, while keeping others fixed. A cyclic form is called a i - cycle if there are i elements in the set $(a_1 \quad a_2 \quad \cdots \quad a_i)$, and it maps a_1 to a_2, a_2 to a_3, …, a_{i-1} to a_i and a_i back to a_1.
Definition (group action). A group action is the transformation from one element to another of a group on a set. Given a group G and a set X, let $X = \{x,\ y,\ z, \dots\}$, the group action of G on X, is a bijective mapping of $f\colon X \to X$ so that $\forall x \in X,\ f(x) = gx = y \in X$ and there exists $f^{-1},\ f^{-1}(y) = x$.
Definition (orbit). An orbit is the subset of a given set X composed of the elements that can be reached by particular group actions of a given group G. For $x \in X,\ Oribt(g, x) = \{gx | g \in G\}$.
Definition (orbital plane). An orbital plane is the partition of a given set X where different partition results have disjoint elements but share the same collections of element positions of cycles in order.
Definition (conjugation). For $f, g, h \in G$, define f and h are conjugate by g if $f = ghg^{-1}$, and conjugation can be symmetric and transitive.
Definition (conjugacy class). The conjugacy class is a set that contains all conjugate elements of the generator element. For $f, g \in G$, the conjugacy class of element f is $CC(f) = \{gfg^{-1} | g \in G\}$. If G is abelian, then $CC(f) = \{gfg^{-1} | g \in G\} = \{gg^{-1}f | g \in G\} = \{f | g \in G\}$, the only conjugate element is f itself in $CC(f)$.