Convolutional neural network with group theory and random selection particle swarm optimizer for enhancing cancer image classification

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Supplementary

Definition (group). Given a set of elements G and a binary multiplication operation $\otimes$, then the group G is defined if:

• Closure: $\forall g, h \in G, g \otimes h \in G$
• Associativity: $\forall g, h, j \in G, (g \otimes h) \otimes j = g \otimes (h \otimes j)$
• Identity: $\forall g \in G, \exists e \in G, g \otimes e = g$
• Inverses: $\forall g \in G, \exists g^{-1} \in G, g \otimes g^{-1} = e$

Definition (abelian group). An abelian group embodies a commutative binary operation:

• Commutativity: $\forall g, h \in G, g \otimes h = h \otimes g$

Definition (permutation). A permutation $p$ of a given set $X$ is a function that arranges its members into an ordered sequence. So it is a bijective mapping of \( f: X \rightarrow X \) from $X$ to itself, $p = (x_1, x_2, \ldots, x_n)$. $p(x_1), p(x_2), \ldots, p(x_n)$.

Definition (permutation group). A permutation group $G$ is a group with the elements of some permutations of a given set $X$. 
Definition (symmetric group). A symmetric group $S_n$ is a group with the elements of all permutations of a given set $X$, where $n$ is the number of letters in $X$ and $S_n$ has the cardinality of $n!$.

Definition (cycle). A cycle is a permutation of some elements in the given set $X$ or its subset $S$ that maps those elements to each other in a cyclic form, while keeping others fixed. A cyclic form is called a $i$ - cycle if there are $i$ elements in the set $(a_1 \ a_2 \ \ldots \ a_i)$, and it maps $a_1$ to $a_2$, $a_2$ to $a_3$, ..., $a_{i-1}$ to $a_i$ and $a_i$ back to $a_1$.

Definition (group action). A group action is the transformation from one element to another of a group on a set. Given a group $G$ and a set $X$, let $X = \{x, y, z, \ldots\}$, the group action of $G$ on $X$, is a bijective mapping of $f:X \rightarrow X$ so that $\forall x \in X$, $f(x) = gx = y \in X$ and there exists $f^{-1}$, $f^{-1}(y) = x$.

Definition (orbit). An orbit is the subset of a given set $X$ composed of the elements that can be reached by particular group actions of a given group $G$. For $x \in X$, $\text{Orbit}(g,x) = \{gx|g \in G\}$.

Definition (orbital plane). An orbital plane is the partition of a given set $X$ where different partition results have disjoint elements but share the same collections of element positions of cycles in order.

Definition (conjugation). For $f, g, h \in G$, define $f$ and $h$ are conjugate by $g$ if $f = ghg^{-1}$, and conjugation can be symmetric and transitive.

Definition (conjugacy class). The conjugacy class is a set that contains all conjugate elements of the generator element. For $f, g \in G$, the conjugacy class of element $f$ is $\text{CC}(f) = \{gf \ g^{-1} | g \in G\}$. If $G$ is abelian, then $\text{CC}(f) = \{gf \ g^{-1} | g \in G\} = \{gg^{-1}f | g \in G\} = \{f | g \in G\}$, the only conjugate element is $f$ itself in $\text{CC}(f)$. 