



Research article

High-Frequency Trading with Machine Learning Algorithms and Limit Order Book Data

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A. Section in Appendix

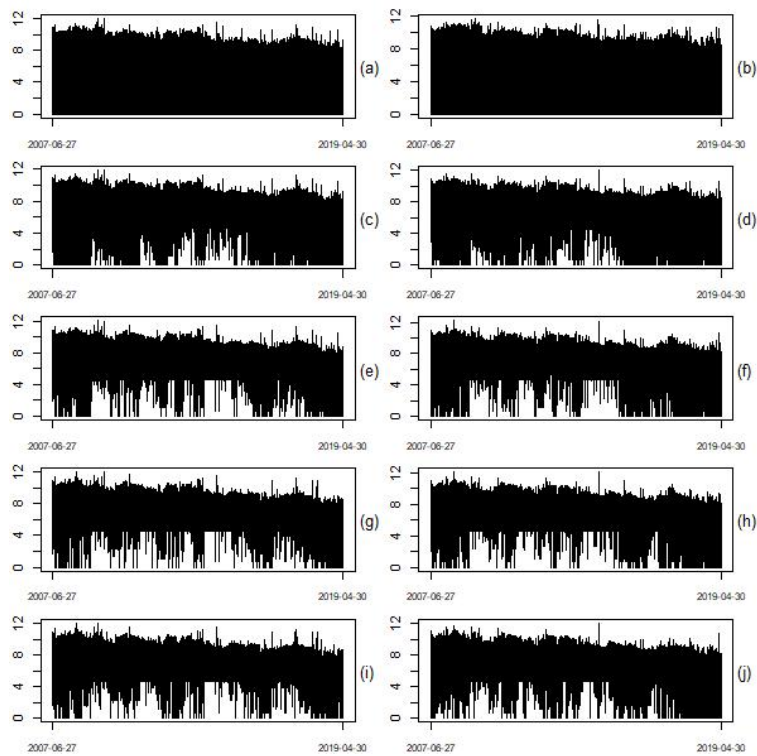


Figure A1. Logarithm of volume variables over the whole observation period: $\log(V_{j,t}^{ask})$ $j = 1, \dots, 5$ corresponding to $a), c), e), g), i)$, respectively, and $\log(V_{j,t}^{bid})$ $j = 1, \dots, 5$ corresponding to $b), d), f), h), j)$, respectively.

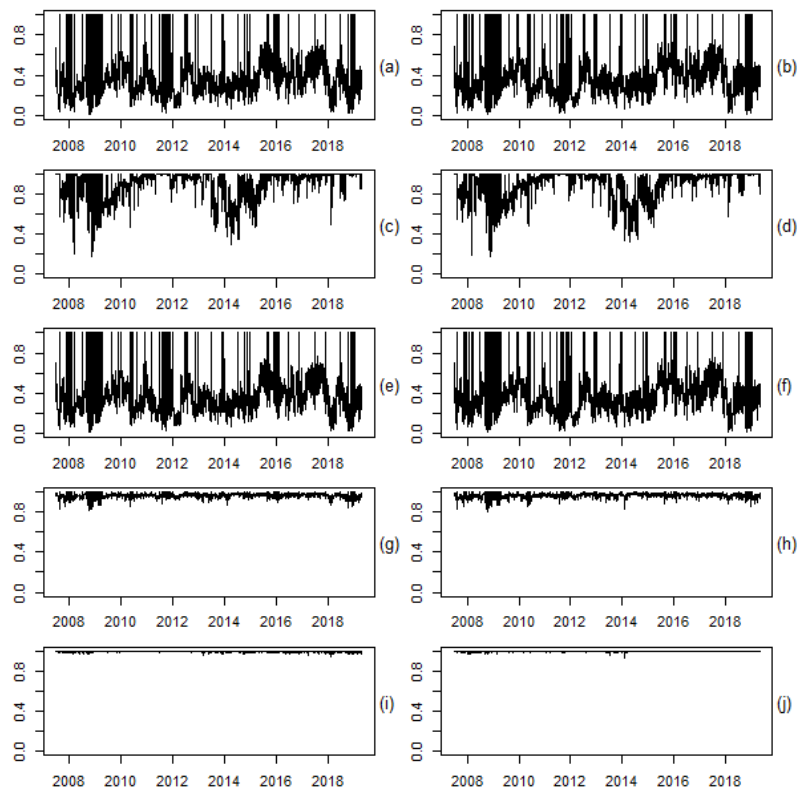


Figure A2. Daily percentage of missing values for *BOn* (a), *SOn* (b), *pdBOn* (c), *pdSON* (d), *tdBOn* (e), *tdSON* (f), *evBOn* (g), *evSON* (h), *ehBOn* (i), *ehSON* (j)

Table A1. Collection of lagged and doubly lagged feature vectors considered for the optimization procedure depending on which 1-dimensional feature X_i $i \in J = \{1, \dots, 18\}$ corresponds to the highest cumulative return, i.e most profitable strategy.

Lags	i	Feature Vectors
1	$i = 1$	$II : (X_{1,\bar{t}}, X_{j,\bar{t}}) \quad j \in J \setminus \{1\}$ $III : (X_{1,\bar{t}}, X_{15,\bar{t}}, X_{j,\bar{t}}) \quad j \in J \setminus \{1, 15\}$ $IV : (X_{1,\bar{t}}, X_{15,\bar{t}}, X_{18,\bar{t}}, X_{j,\bar{t}}) \quad j \in J \setminus \{1, 15, 18\}$
2	$i = 1$	$II^* : (X_{1,\bar{t}}, X_{1,\bar{t}-1}, X_{j,\bar{t}-1}, X_{j,\bar{t}}) \quad j \in J \setminus \{1\}$ $III^* : (X_{1,\bar{t}}, X_{1,\bar{t}-1}, X_{15,\bar{t}}, X_{15,\bar{t}-1}, X_{j,\bar{t}-1}, X_{j,\bar{t}})$ $j \in J \setminus \{1, 18\}$ $IV^* : (X_{1,\bar{t}}, X_{1,\bar{t}-1}, X_{15,\bar{t}}, X_{15,\bar{t}-1}, X_{18,\bar{t}}, X_{18,\bar{t}-1}, X_{j,\bar{t}-1}, X_{j,\bar{t}})$ $j \in J \setminus \{1, 15, 18\}$
1	$i = 15$	$II : (X_{15,\bar{t}}, X_{j,\bar{t}}) \quad j \in J \setminus \{1\}$ $III : (X_{1,\bar{t}}, X_{15,\bar{t}}, X_{j,\bar{t}}) \quad j \in J \setminus \{1, 15\}$ $IV : (X_{1,\bar{t}}, X_{15,\bar{t}}, X_{18,\bar{t}}, X_{j,\bar{t}}) \quad j \in J \setminus \{1, 15, 18\}$
2	$i = 15$	$II^* : (X_{15,\bar{t}}, X_{15,\bar{t}-1}, X_{j,\bar{t}-1}, X_{j,\bar{t}}) \quad j \in J \setminus \{1\}$ $III^* : (X_{1,\bar{t}}, X_{1,\bar{t}-1}, X_{15,\bar{t}}, X_{15,\bar{t}-1}, X_{j,\bar{t}-1}, X_{j,\bar{t}}) \quad j \in J \setminus \{1, 18\}$ $j \in J \setminus \{1, 18\}$ $IV^* : (X_{1,\bar{t}}, X_{1,\bar{t}-1}, X_{15,\bar{t}}, X_{15,\bar{t}-1}, X_{18,\bar{t}}, X_{18,\bar{t}-1}, X_{j,\bar{t}-1}, X_{j,\bar{t}})$ $j \in J \setminus \{1, 15, 18\}$
1	$i \in \{16, 17, 18\}$	$II : (X_{18,\bar{t}}, X_{j,\bar{t}}) \quad j \in J \setminus \{18\}$ $III : (X_{18,\bar{t}}, X_{1,\bar{t}}, X_{j,\bar{t}}) \quad j \in J \setminus \{1, 18\}$ $IV : (X_{18,\bar{t}}, X_{1,\bar{t}}, X_{15,\bar{t}}, X_{j,\bar{t}}) \quad j \in J \setminus \{1, 15, 18\}$
2	$i \in \{16, 17, 18\}$	$II^* : (X_{18,\bar{t}}, X_{18,\bar{t}-1}, X_{j,\bar{t}-1}, X_{j,\bar{t}}) \quad j \in J \setminus \{18\}$ $III^* : (X_{18,\bar{t}}, X_{18,\bar{t}-1}, X_{1,\bar{t}}, X_{1,\bar{t}-1}, X_{j,\bar{t}-1}, X_{j,\bar{t}})$ $j \in J \setminus \{1, 18\}$ $IV^* : (X_{18,\bar{t}}, X_{18,\bar{t}-1}, X_{1,\bar{t}}, X_{1,\bar{t}-1}, X_{15,\bar{t}}, X_{15,\bar{t}-1}, X_{j,\bar{t}-1}, X_{j,\bar{t}})$ $j \in J \setminus \{1, 15, 18\}$
1	$i \notin \{1, 15, 16, 17, 18\}$	$II : (X_{i,\bar{t}}, X_{j,\bar{t}}) \quad j \in J \setminus \{i\}$ $III : (X_{i,\bar{t}}, X_{1,\bar{t}}, X_{j,\bar{t}}) \quad j \in J \setminus \{i, 1\}$ $IV : (X_{i,\bar{t}}, X_{1,\bar{t}}, X_{15,\bar{t}}, X_{j,\bar{t}}) \quad j \in J \setminus \{i, 1, 15\}$ $V : (X_{i,\bar{t}}, X_{1,\bar{t}}, X_{15,\bar{t}}, X_{18,\bar{t}}, X_{j,\bar{t}}) \quad j \in J \setminus \{i, 1, 15, 18\}$
2	$i \notin \{1, 15, 16, 17, 18\}$	$II^* : (X_{i,\bar{t}}, X_{i,\bar{t}-1}, X_{j,\bar{t}-1}, X_{j,\bar{t}}) \quad j \in J \setminus \{i\}$ $III^* : (X_{i,\bar{t}}, X_{i,\bar{t}-1}, X_{1,\bar{t}}, X_{1,\bar{t}-1}, X_{j,\bar{t}-1}, X_{j,\bar{t}}) \quad j \in J \setminus \{i, 1\}$ $IV^* : (X_{i,\bar{t}}, X_{i,\bar{t}-1}, X_{1,\bar{t}}, X_{1,\bar{t}-1}, X_{15,\bar{t}}, X_{15,\bar{t}-1}, X_{j,\bar{t}-1}, X_{j,\bar{t}})$ $j \in J \setminus \{i, 1, 15\}$ $V^* : (X_{i,\bar{t}}, X_{i,\bar{t}-1}, X_{1,\bar{t}}, X_{1,\bar{t}-1}, X_{15,\bar{t}}, X_{15,\bar{t}-1}, X_{18,\bar{t}}, X_{18,\bar{t}-1}, X_{j,\bar{t}-1}, X_{j,\bar{t}}) \quad j \in J \setminus \{i, 1, 15, 18\}$

Table A2. Predictive Performance p_i of SVM for each regarded evaluation set E_i $i \in \{1, \dots, 11\}$ with tuning values $\gamma \in \{0.1, 0.5, 1, 2\}$ and C , where $\bar{p} = \sum_1^{11} p_i$ corresponds to the average predictive performance.

γ	C	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	\bar{p}
0.1	0.1	50.46	50.87	50.95	51.73	51.79	51.21	51.1	50.94	51	52.44	51.14	46.97
0.1	0.5	50.62	50.91	51.11	52.03	52.29	51.67	51.24	50.77	51.08	52.02	50.98	47.06
0.1	0.7	50.55	50.92	51.13	52.08	52.31	51.71	51.28	50.76	51.07	52.04	50.88	47.06
0.1	1	50.69	50.98	51.14	51.99	52.32	51.64	51.3	50.66	51.03	52.01	51.06	47.07
0.1	2	50.64	50.85	51.21	51.99	52.38	51.74	51.48	50.66	51.01	51.99	51.13	47.09
0.1	3	50.88	50.89	51.4	52.18	52.41	51.77	51.44	50.62	51.01	51.86	51.08	47.13
0.1	5	50.78	50.82	51.44	52.17	52.51	51.72	51.49	50.32	51.03	51.76	51.15	47.1
0.1	10	50.78	50.78	51.37	52.12	52.43	51.82	51.34	50.46	50.99	51.51	51.18	47.07
0.1	30	50.79	50.78	51.48	52.21	52.34	51.69	51.18	50.44	51.23	51.47	50.92	47.05
0.1	50	50.78	50.58	51.4	52.12	52.43	51.73	51.16	50.34	51.32	51.52	50.89	47.02
0.1	100	50.76	50.59	51.39	52.2	52.48	51.72	51.09	50.16	51.36	51.39	50.98	47.01
0.5	0.1	50.59	50.87	51.56	51.96	52.81	51.83	51.23	50.42	51.23	51.6	51.12	47.1
0.5	0.5	50.74	50.76	51.51	51.77	52.67	51.96	51.14	50.38	51.42	51.54	51.07	47.08
0.5	0.7	50.74	50.76	51.59	51.83	52.66	52.02	51.13	50.27	51.27	51.35	51.19	47.07
0.5	1	50.77	50.72	51.69	51.68	52.76	52.11	51.12	50.26	51.3	51.25	51.06	47.06
0.5	2	50.66	50.73	51.69	51.59	52.69	52.16	51.16	50.17	51.33	51.29	50.85	47.03
0.5	3	50.5	50.74	51.68	51.58	52.62	52.15	51.09	50.09	51.44	51.03	50.98	46.99
0.5	5	50.54	50.47	51.65	51.62	52.71	52.12	51.08	50.19	51.27	50.9	50.85	46.95
0.5	10	50.35	50.43	51.54	51.58	52.63	52.08	50.9	50.11	51.24	50.92	51	46.9
0.5	30	49.95	49.9	51.65	51.08	52.63	51.96	50.98	50.23	50.94	51.22	51.03	46.8
0.5	50	49.93	50.02	51.62	50.87	52.55	51.95	50.73	50.23	50.85	50.93	51.02	46.72
0.5	100	49.8	49.98	51.79	50.86	52.48	51.98	50.78	49.99	50.85	51.12	51.02	46.72
1	0.1	50.14	50.7	51.46	51.91	52.67	52.11	51.15	50.71	51.31	51.52	51.01	47.06
1	0.5	50.47	50.68	51.77	51.82	52.59	52.1	50.96	50.07	51.2	51.05	50.83	46.96
1	0.7	50.61	50.43	51.71	51.76	52.56	52.03	50.84	50.14	51.07	51.26	50.86	46.94
1	1	50.43	50.42	51.77	51.54	52.51	52.03	50.86	50.05	51.07	51.15	50.77	46.88
1	2	50.5	50.13	51.7	51.59	52.34	52.1	50.83	50.16	50.78	50.91	50.86	46.82
1	3	50.4	50.07	51.77	51.3	52.46	51.97	50.79	50.26	50.89	50.97	50.89	46.82
1	5	50.21	50.12	51.67	51.15	52.38	51.99	50.76	50.26	51.01	50.94	50.98	46.79
1	10	50	50.16	51.69	51.13	52.32	51.99	50.77	50.18	50.83	50.69	50.98	46.73
1	30	49.83	50.01	51.5	50.69	52.09	52.02	50.45	49.78	50.49	50.47	50.9	46.52
1	50	49.83	49.9	51.4	50.71	51.97	51.9	50.4	49.53	50.44	50.77	50.9	46.48
1	100	49.85	50.18	51.34	50.74	52.06	51.76	50.62	49.41	50.44	50.82	50.71	46.49
2	0.1	50.17	50.7	51.62	51.78	52.57	52.05	50.91	50.1	51.07	51.3	50.75	46.92
2	0.5	50.48	49.83	51.6	51.43	52.16	52.12	50.79	50	50.73	50.7	51	46.74
2	0.7	50.16	49.91	51.44	51.36	52.1	51.95	50.71	49.98	50.58	50.64	50.95	46.65
2	1	50.11	50.06	51.59	51.08	52.14	51.57	50.64	49.95	50.67	50.62	51.05	46.62
2	2	49.72	50.14	51.58	51.08	51.98	51.46	50.61	49.92	50.52	50.82	50.98	46.57
2	3	49.61	50.1	51.48	50.91	52.04	51.55	50.44	49.72	50.38	50.81	50.94	46.5
2	5	49.7	50.19	51.53	50.78	51.86	51.64	50.4	49.37	50.3	50.94	50.75	46.45
2	10	49.62	50.52	51.53	50.49	51.77	51.52	50.48	49.48	50.23	50.77	50.17	46.38
2	30	49.64	50.55	51.04	50.34	51.91	51.54	50.35	49.44	50.06	50.76	50.28	46.33
2	50	49.74	50.68	50.91	50.38	51.91	51.53	50.39	49.58	50.14	50.74	50.59	46.38
2	100	49.59	50.85	50.86	50.41	52.05	51.83	50.37	49.71	50.12	50.54	50.72	46.42

Table 3. Predictive Performance p_i of SVM for each regarded evaluation set E_i $i \in \{1, \dots, 11\}$ with tuning values $\gamma \in \{3, 5, 8, 10\}$ and C , where $\bar{p} = \sum_1^{11} p_i$ corresponds to the average predictive performance.

γ	C	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	\bar{p}
3	0.1	50.17	50.69	51.67	51.68	52.22	51.88	50.91	49.97	50.95	51.36	50.89	46.87
3	0.5	49.94	50.02	51.55	51.23	52.15	51.72	50.74	49.71	50.39	50.8	50.96	46.6
3	0.7	49.85	50.2	51.55	51.24	52.19	51.61	50.54	49.67	50.39	50.75	50.94	46.58
3	1	49.66	50.11	51.46	50.88	52.15	51.59	50.49	49.82	50.36	50.94	50.73	46.52
3	2	49.72	50.46	51.6	50.94	52.14	51.63	50.38	49.29	50.02	50.87	50.57	46.47
3	3	49.53	50.39	51.35	50.53	52.04	51.51	50.37	49.3	50	50.71	50.49	46.35
3	5	49.88	50.53	51.01	50.58	52.09	51.5	50.44	49.49	50.11	50.55	50.65	46.4
3	10	49.76	50.54	50.86	50.5	52.15	51.72	50.22	49.72	50.03	50.6	50.71	46.4
3	30	50.1	50.84	50.67	50.75	51.64	51.05	50.07	49.58	50.19	50.05	50.63	46.3
3	50	49.8	50.76	50.55	50.48	51.56	51.11	50.22	49.82	50.16	50.08	50.56	46.26
3	100	49.5	50.48	50.67	50.29	51.42	50.71	50.3	50.07	50.1	50.47	50.41	46.2
5	0.1	50.02	50.24	51.42	51.21	52.23	52	50.73	49.93	50.85	51.19	50.81	46.72
5	0.5	49.67	50.54	51.38	50.77	52.2	51.77	50.35	49.43	50.03	50.85	50.66	46.47
5	0.7	49.8	50.54	51.3	50.75	52.17	51.57	50.17	49.54	50.2	50.69	50.79	46.46
5	1	49.65	50.49	51.14	51.02	52.27	51.53	50.14	49.62	50.19	50.59	50.9	46.46
5	2	49.87	50.72	50.81	50.63	51.57	51.46	50.08	49.46	50.14	50.27	50.68	46.31
5	3	49.78	50.71	50.83	50.6	51.49	51.1	49.97	49.53	50.21	50.08	50.42	46.23
5	5	49.95	50.72	50.6	50.48	51.1	50.94	50.09	49.82	50.04	49.93	50.34	46.17
5	10	50.11	50.44	50.18	50.25	51.09	50.71	50.07	50.04	50.05	50.12	50.39	46.12
5	30	49.95	50.35	50.48	50.18	51.23	50.6	49.9	50.07	50.02	50.37	50.04	46.1
5	50	49.94	50.29	50.31	50.48	51.26	50.66	49.82	50.03	49.94	50.34	49.89	46.08
5	100	49.95	50.24	50.25	50.31	51.11	50.85	49.59	50.03	49.97	50.5	49.89	46.06
8	0.1	50.05	50.28	51.37	51.08	52.15	51.73	50.49	50.35	50.7	51.26	50.56	46.67
8	0.5	49.47	50.7	50.9	50.86	51.75	51.43	50.12	49.95	50.15	50.49	50.38	46.35
8	0.7	49.77	50.59	50.81	50.73	51.42	51.3	50.08	49.94	50.14	50.28	50.31	46.28
8	1	49.77	50.64	50.7	50.77	51.16	51.1	50.28	49.85	50.11	50.34	50.53	46.27
8	2	49.59	50.57	50.4	50.38	51.32	50.65	50.1	50.09	49.86	50.25	50.38	46.13
8	3	49.59	50.34	50.37	50.35	51.4	50.64	49.97	50.11	50.03	50.02	50.15	46.08
8	5	49.94	50.35	50.46	50.43	51.44	50.79	49.78	50.14	49.83	50.32	50.03	46.13
8	10	50.11	50.1	50.33	50.48	51.16	50.75	49.59	50.19	49.95	49.94	50.05	46.05
8	30	50.09	49.98	50.57	50.46	51.15	50.46	49.62	50.24	49.91	49.65	50.28	46.03
8	50	50.11	50.03	50.65	50.33	51.12	50.51	49.64	50.14	49.67	49.55	50.41	46.01
8	100	50.25	50.41	50.12	50.06	51.17	50.49	49.74	50.24	49.66	49.86	50.09	46.01
10	0.1	50.07	50.39	51.15	50.97	51.99	51.58	50.58	50.31	50.51	51.39	50.57	46.63
10	0.5	49.77	50.61	50.79	50.75	51.42	51.3	50.32	50.08	50.13	50.62	50.23	46.34
10	0.7	49.7	50.68	50.58	50.78	51.33	51.06	50.19	50.01	49.91	50.45	50.44	46.26
10	1	49.66	50.66	50.35	50.56	51.22	50.86	50.09	49.98	49.82	50.36	50.26	46.15
10	2	49.54	50.52	50.38	50.43	51.47	50.78	50.03	50.1	49.76	50.14	50.28	46.12
10	3	49.73	50.33	50.52	50.65	51.35	50.83	49.63	50.22	49.98	50.16	50.08	46.12
10	5	49.8	50.29	50.71	50.67	51.26	50.88	49.56	50.35	49.9	49.9	50.23	46.13
10	10	49.98	49.98	50.48	50.44	51.15	50.46	49.64	50.32	49.92	49.64	50.31	46.03
10	30	50.06	50.37	50.07	50.09	51.11	50.48	49.69	49.98	49.72	49.7	50.13	45.95
10	50	50.38	50.6	50.07	49.96	51.11	50.54	49.61	50.24	49.83	49.97	50.36	46.06
10	100	50.16	50.45	50.13	49.95	50.75	50.69	49.79	50.2	49.99	49.87	50.04	46

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