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Research article

A new hybrid form of the skew-t distribution: estimation methods comparison via Monte Carlo simulation and GARCH model application

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Supplementary

Appendix A

The commonly used conditional innovation distributions in GARCH-type volatility models include

Normal Distribution

For the normal distributed innovations, the density function is given by

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \tag{A.1}$$

Student-t Distribution

For the student-t distributed innovations, the density function is given by

$$f(z; v) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)}\,\Gamma\left(\frac{v}{2}\right)} \frac{1}{\left[1+\frac{z^2}{v-2}\right]^{\left(\frac{v+1}{2}\right)}} \tag{A.2}$$

where v is the degrees of freedom, $2 < v \leq \infty$ and $\Gamma(.)$ is the gamma function.

Generalized error Distribution

For the generalized error distributed innovations, the density function is given by

$$f(z; v) = \frac{v}{\kappa_v 2^{1+v^{-1}} \Gamma(\kappa^{-1})} e^{-\frac{1}{2}\left|\frac{z}{\kappa_v}\right|^v} \tag{A.3}$$

where v is the degrees of freedom, $0 < v < \infty$, $\kappa_v = \sqrt{\left(\frac{2^{\frac{-2}{v}} \Gamma(v^{-1})}{\Gamma(3v^{-1})}\right)}$ and $\Gamma(.)$ denote the gamma function.

Skew Normal Distribution

For the skew normal distributed innovations, the density function is given by

$$f(z) = \frac{1}{\kappa\pi} e^{\frac{-(z-\xi)^2}{2\kappa^2}} \int_{-\infty}^{\alpha\frac{z-\xi}{\kappa}} e^{\frac{-t^2}{2}} \partial t, \tag{A.4}$$

where $\alpha$is the skew parameter and (ξ, κ) are the location and scale parameters, respectively.

Skew Student-t Distribution

For the skew Student-t distributed innovations, the density function is given by

$$f(z; v) = \frac{\Gamma\left(\frac{v+1}{2}\right)\left(\frac{2}{\xi+\frac{1}{\xi}}\right)}{\sqrt{\pi(v-2)}\Gamma\left(\frac{v}{2}\right)} \frac{s}{\left[1+\frac{(sz+m)^2 \xi^{-2I_t}}{v-2}\right]^{\left(\frac{1+v}{2}\right)}} \tag{A.5}$$

where v is the degree of freedom, $\Gamma(.)$ denote the gamma function, ξ is the asymmetry parameter, and $s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2}$, $m = \frac{\Gamma\left(\frac{v+1}{2}\right)\sqrt{v-2}}{\sqrt{\pi}\Gamma\left(\frac{v}{2}\right)}\left(\xi - \frac{1}{\xi}\right)$, $I_t = \begin{cases} 1 & if \quad z_t \geq -\frac{m}{s} \\ -1 & if \quad z_t < -\frac{m}{s} \end{cases}$.

Skew Generalized Error Distribution

For the generalized error distributed innovations, the density function is given by

$$f(z|v, \eta) = C\, exp\left(-\frac{1}{[1-sign(z-\kappa)\eta]^v \varphi^v} |z-\kappa|^v\right) \tag{A.6}$$

where v is the degrees of freedom, η is the skew parameter $(-1 < \eta < 1)$,$C = v[2\varphi\Gamma(v^{-1})]^{-1}$, $\kappa = 2\eta A S(\eta)^{-1}$,

$$A = \Gamma\left(\frac{2}{v}\right)\Gamma\left(\frac{1}{v}\right)^{-\frac{1}{2}}\Gamma\left(\frac{3}{v}\right)^{-\frac{1}{2}}, \varphi = \Gamma\left(\frac{1}{v}\right)^{\frac{1}{2}}\Gamma\left(\frac{3}{v}\right)^{-\frac{1}{2}} S(\eta)^{-1}, \text{ and } S(\eta) = \sqrt{1 + 3\eta^2 - 4A^2\eta^2}.$$

Generalized Hyperbolic Distribution

For the generalized hyperbolic distributed innovations, the density function is given by

$$f(z;\lambda,\alpha,\beta,\delta,\mu)=\frac{\{\alpha^2-\beta^2\}^{\frac{\lambda}{2}}}{\sqrt{2\pi}\alpha^{\lambda-\frac{1}{2}}\delta^{\lambda}\Psi_{\lambda}\left\langle\delta\sqrt{\alpha^2-\beta^2}\right\rangle}(\delta^2+\langle z-\mu\rangle^2)^{\frac{\left(\lambda-\frac{1}{2}\right)}{2}}$$

$$\times\Psi_{\lambda-\frac{1}{2}}\left\{\alpha\sqrt{\delta^2+\langle z-\mu\rangle^2}\right\}exp(\beta\{z-\mu\}) \quad \text{(A.7)}$$

where δ is scale parameter, μ is location parameter, β is the asymmetry parameter, λ,α are real parameters, Ψ_{λ} is the modified Bessel function of third order.

Johnson Reparametrized (SU) Distribution

For the Johnson reparametrized (SU) distributed innovations, the density function is given by

$$f(z;\eta,\tau,\upsilon,\vartheta)=\frac{\vartheta}{\eta\sqrt{1+\left(\frac{z-\tau}{\eta}\right)^2}}\phi\left[\upsilon+\vartheta\,sinh^{-1}\left(\frac{z-\tau}{\eta}\right)\right] \quad \text{(A.8)}$$

where, ϕ is the density function of $N(0,1)$, τ,η are location and scale parameters, respectively, while υ,ϑ denote the skew and kurtosis parameters, respectively.

Normal Inverse Gaussian Distribution

For the normal inverse gaussian distributed innovations, the density function is given by

$$f(z;\alpha,\beta,\delta,\mu)=\frac{\alpha\delta\,exp\left(\delta\sqrt{\alpha^2-\beta^2}+\beta(z-\mu)\right)K_1\left(\alpha\sqrt{\delta^2+(z-\mu)^2}\right)}{\pi\sqrt{\delta^2+(z-\mu)^2}} \quad \text{(A.9)}$$

where δ is scale parameter, μ is location parameter, β is the asymmetry parameter, α is the shape parameter, K_1 is the modified Bessel function of third order.

Generalized Hyperbolic Skew Student-t Distribution

For the generalized hyperbolic skew student-t distributed innovations, the density function is given by

$$f(z;\alpha,\beta,\delta,\mu)=\frac{2^{\frac{1-\nu}{2}}\delta^{\nu}|\beta|^{\frac{\nu+1}{2}}exp\{\beta(z-\mu)\}\left(\sqrt{\beta^2\{\delta^2+(z-\mu)^2\}}\right)K_{\frac{\nu+1}{2}}}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi}\left\{\sqrt{\delta^2+(z-\mu)^2}\right\}^{\frac{\nu+1}{2}}} \quad \text{(A.10)}$$

where δ is scale parameter, μ is location parameter, β is the asymmetry parameter, $\alpha\rightarrow|\beta|$ is the shape parameter, K_1 is the modified Bessel function of third order.