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Research article

The impact of time-varying risk on stock returns: an experiment of cubic piecewise polynomial function model and the Fourier Flexible Form model

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Supplementary

Appendix

Table A.1. Summary of stocks.

Stock Exchange	Number of Stocks	Percentage to whole Sample	Average Annualized Return	Standard Deviation
NYSE	2198	16.21%	10.59%	10.99%
NASDAQ	7636	56.33%	12.32%	20.13%
AMEX	1105	8.15%	10.98%	16.91%
NYSE & NASDAQ	1031	7.60%	14.12%	13.55%
NYSE & AMEX	556	4.10%	14.10%	13.72%
NASDAQ & AMEX	829	6.11%	11.24%	20.17%
NYSE & NASDAQ & AMEX	202	1.49%	14.61%	16.15%
Total Sample	13557	100%	12.08%	17.06%

Illustration A.1. Piecewise polynomial matrix with 2 knots.

$$S_2 = \begin{bmatrix} 1^0 & 1^1 & 1^2 & 1^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2^0 & 2^1 & 2^2 & 2^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3^0 & 3^1 & 3^2 & 3^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4^0 & 4^1 & 4^2 & 4^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \left(\frac{t}{3}\right)^0 & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots \\ \left(\frac{2t}{3}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ t^0 & t^1 & t^2 & t^3 & \left(\frac{2t}{3}\right)^0 & \left(\frac{2t}{3}\right)^1 & \left(\frac{2t}{3}\right)^2 & \left(\frac{2t}{3}\right)^3 & \left(\frac{t}{3}\right)^0 & \left(\frac{t}{3}\right)^1 & \left(\frac{t}{3}\right)^2 & \left(\frac{t}{3}\right)^3 \end{bmatrix}$$

Illustration A.2. Piecewise polynomial matrix with 3 knots.

$$S_3 = \begin{bmatrix}
1^0 & 1^1 & 1^2 & 1^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2^0 & 2^1 & 2^2 & 2^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3^0 & 3^1 & 3^2 & 3^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{t}{4}\right)^0 & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{t}{2}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{3t}{4}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
t^0 & t^1 & t^2 & t^3 & \left(\frac{3t}{4}\right)^0 & \left(\frac{3t}{4}\right)^1 & \left(\frac{3t}{4}\right)^2 & \left(\frac{3t}{4}\right)^3 & \left(\frac{t}{2}\right)^0 & \left(\frac{t}{2}\right)^1 & \left(\frac{t}{2}\right)^2 & \left(\frac{t}{2}\right)^3 & \left(\frac{t}{4}\right)^0 & \left(\frac{t}{4}\right)^1 & \left(\frac{t}{4}\right)^2 & \left(\frac{t}{4}\right)^3
\end{bmatrix}$$

Illustration A.3. Piecewise polynomial matrix with 4 knots

$$
S_4 = \begin{bmatrix}
1^0 & 1^1 & 1^2 & 1^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2^0 & 2^1 & 2^2 & 2^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3^0 & 3^1 & 3^2 & 3^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{t}{5}\right)^0 & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{2t}{5}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{3t}{5}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{4t}{5}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
t^0 & t^1 & t^2 & t^3 & \left(\frac{4t}{5}\right)^0 & \left(\frac{4t}{5}\right)^1 & \left(\frac{4t}{5}\right)^2 & \left(\frac{4t}{5}\right)^3 & \left(\frac{3t}{5}\right)^0 & \left(\frac{3t}{5}\right)^1 & \left(\frac{3t}{5}\right)^2 & \left(\frac{3t}{5}\right)^3 & \left(\frac{2t}{5}\right)^0 & \left(\frac{2t}{5}\right)^1 & \left(\frac{2t}{5}\right)^2 & \left(\frac{2t}{5}\right)^3 & \left(\frac{t}{5}\right)^0 & \left(\frac{t}{5}\right)^1 & \left(\frac{t}{5}\right)^2 & \left(\frac{t}{5}\right)^3
\end{bmatrix}
$$

Illustration A.4. Piecewise polynomial matrix with 5 knots.

$$S_5 = \begin{bmatrix}
1^0 & 1^1 & 1^2 & 1^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2^0 & 2^1 & 2^2 & 2^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3^0 & 3^1 & 3^2 & 3^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{t}{6}\right)^0 & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{t}{3}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{t}{2}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{2t}{3}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{5t}{6}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
t^0 & t^1 & t^2 & t^3 & \left(\frac{5t}{6}\right)^0 & \left(\frac{5t}{6}\right)^1 & \left(\frac{5t}{6}\right)^2 & \left(\frac{5t}{6}\right)^3 & \left(\frac{2t}{3}\right)^0 & \left(\frac{2t}{3}\right)^1 & \left(\frac{2t}{3}\right)^2 & \left(\frac{2t}{3}\right)^3 & \left(\frac{t}{2}\right)^0 & \left(\frac{t}{2}\right)^1 & \left(\frac{t}{2}\right)^2 & \left(\frac{t}{2}\right)^3 & \left(\frac{t}{3}\right)^0 & \left(\frac{t}{3}\right)^1 & \left(\frac{t}{3}\right)^2 & \left(\frac{t}{3}\right)^3 & \left(\frac{t}{6}\right)^0 & \left(\frac{t}{6}\right)^1 & \left(\frac{t}{6}\right)^2 & \left(\frac{t}{6}\right)^3
\end{bmatrix}$$