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Research article

MatCalib: a Matlab software package for Bayesian modeling of radiocarbon ages subject to temporal order constraints

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1. Derivation of the integrated likelihood function

Define $a_i = \frac{N_i}{2}$ and $b_{i,j} = \frac{[r_{i,j} - \boldsymbol{\mu}(\theta_{i,j})]^2}{2\omega_{i,j}^2}$. According to Bayes' theorem, the posterior probability density function of hyperparameter δ can be written as:

$$p(\delta|\mathbf{r}_i, \boldsymbol{\sigma}_i, \boldsymbol{\theta}_i) = \frac{L(\mathbf{r}_i|\boldsymbol{\theta}_i, \boldsymbol{\sigma}_i, \delta)\, f(\delta)}{L(\mathbf{r}_i|\boldsymbol{\theta}_i, \boldsymbol{\sigma}_i)}, \quad \text{(A1)}$$

where $L(\mathbf{r}_i|\boldsymbol{\theta}_i, \boldsymbol{\sigma}_i)$ is the integrated likelihood function. As $p(\delta|\mathbf{r}_i, \boldsymbol{\sigma}_i, \boldsymbol{\theta}_i)$ is a proper probability density function, it satisfies $\int_0^\infty p(\delta|\mathbf{r}_i, \boldsymbol{\sigma}_i, \boldsymbol{\theta}_i) d\delta = 1$. Thus, integrating Eq. A1 on both sides and arranging items yields:

$$\begin{aligned} L(\mathbf{r}_i|\boldsymbol{\theta}_i, \boldsymbol{\sigma}_i) &= \int_0^{+\infty} L(\mathbf{r}_i|\boldsymbol{\theta}_i, \boldsymbol{\sigma}_i, \delta) f(\delta)\, d\delta \\ &= \int_0^{+\infty} \prod_{j=1}^{N_i} \frac{1}{\sqrt{2\pi\delta}\,\omega_{i,j}} exp\left(-\frac{b_{i,j}}{\delta}\right) \frac{1}{\delta}\, d\delta \quad \text{(A2)} \\ &\propto \int_0^{+\infty} \left(\frac{1}{\delta}\right)^{a_i+1} \exp\left(-\frac{\sum_{j=1}^{N_i} b_{i,j}}{\delta}\right) d\delta \end{aligned}$$

$$= \Gamma(a_i)\left(\sum_{j=1}^{N_i} b_{i,j}\right)^{-a_i} \int_0^{+\infty} \frac{\left(\sum_{j=1}^{N_i} b_{i,j}\right)^{a_i}}{\Gamma(a_i)} \left(\frac{1}{\delta}\right)^{a_i+1} \exp\left(-\frac{\sum_{j=1}^{N_i} b_{i,j}}{\delta}\right) d\delta$$

$$= \Gamma(a_i)\left(\sum_{j=1}^{N_i} b_{i,j}\right)^{-a_i},$$

where $\int_0^{\infty} \frac{\left(\sum_{j=1}^{N_i} b_{i,j}\right)^{a_i}}{\Gamma(a_i)} \left(\frac{1}{\delta}\right)^{a_i+1} \exp\left(-\frac{\sum_{j=1}^{N_i} b_{i,j}}{\delta}\right) d\delta = 1$ is the integration of the posterior probability density function of hyperparameter δ, which follows the inverse gamma distribution with a_i and $\sum_{j=1}^{N_i} b_{i,j}$ being the shape and scale parameter, respectively, and $\Gamma(a_i)$ denotes the gamma function.

2. Matlab code for implementing Bayesian radiocarbon age modeling

Please see supplementary 2.